Kaunas University of Technology

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Summary

The need for increased engagement of workers in pension fund investment globally have in recent years reached Lithuania with both researchers and policy makers alike paying attention to the issues. This research therefore looks to deploy big data analytic techniques such as data collection and cleaning as well as modelling methods such as BATS, ARIMA and NAÏVE models to classify, rank and forecast pension funds. In order for the decision of what funds could be selected by the prospective subscribers, judgmental forecasting is also employed in the tail end of the decision making process completing the multi criteria decision making process with both qualitative and quantitative criteria.

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Santrauka

Globaliai išaugęs darbuotojų, įsitraukiančių į pensijų fondų investicijas, poreikis pastaraisiais metais pasiekė ir Lietuvą. Akivaizdu, kad šiai sričiai skiriamas vis didesnis tiek tyrėjų, tiek politikų dėmesys. Šiame tyrime išsamiai pristatomi tokie analizės būdai, kaip duomenų rinkimas ir apdorojimas, taip pat tokie modeliavimo metodai, kaip BATS, ARIMA ir NAÏVE, skirti pensijų fondų klasifikavimui, įvertinimui ir prognozavimui. Aktualizuojant tai, kokie fondai būtų tinkami potencialiems pensijų fondų dalyviams, į tyrimą įtraukiami ir kiti prognoziniai, daugiakriteriniai sprendimai, aprėpiantys tiek kokybinius, tiek kiekybinius kriterijus.

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List of abbreviation and terms
Abbreviations:

Doc. – associate professor;

Dr. – doctor .

MCDM – Multi Criteria Decision Making

MCDA – Multi Criteria Decision Analysis

AHP – Analytic Hierarchy Process

MAUT – Multi-Attribute Utility Theory

ISI – Institute for Scientific Information

MCDM – Multi Criteria Decision Analysis

EC – Economic Capital

ANP – Analytic Network Process

CBR – Case Based Reasoning

DEA – Data Envelopment Analysis

SMART – Simple Multi-Attribute Rating Technique

ELECTRE – *Elimination et Choix treduisantë la Realite*

PROMETHEE – Preference Ranking Organization Method for Enrichment Evaluation

SAW – Simple Additive Weighting

EU – European Union

PPP – Public-Private Partnerships

OECD – Organization for Economic Co-operation and Development

DB – Defined Benefits

DC – Defined Contribution

NDC – Notional or nonfinancial Defined Contribution

PAYG – Pay As You Go

ARIMA – Autoregressive Integrated Moving Average

MAPE – Mean Absolute Percentage Error

MSE – Mean Square Error

RMSE – Root Mean Square Error

BATS – Box-cox Arimal-errors Trend Seasonal-components

Terms:

Multi Criteria Decision Making – methods for making decisions when multiple criteria (or objectives) need to be considered together, in order to rank or choose between the alternatives being evaluated.

Pension – A pension is a retirement fund for an employee paid into by the employer, employee, or both, with the employer usually covering the largest percentage of contributions. When the employee retires, she's paid in an annuity calculated by the terms of the pension.

Portfolio – A portfolio is a collection of financial investments like stocks, bonds, commodities, cash, and cash equivalents.

Pillar / Tier – A term adopted by most European countries and used to identify the different methods of funding pension provision.

Forecasting – Forecasting is the art of predicting what will happen in the future by taking into account past and present occurrences.

Quantitative Measures – It can simply be used to define the object's attributes in terms of area, perimeter, intensity, color, shape, and so on to offer a measure of the object's morphology or structure.

Qualitative Measures – measurement of data that can be put into numbers.

Introduction

Consumers of financial services have a critical problem of financial partner stability in the current crisis, and several elements should be considered when selecting this partner (Novickytė & Rabikauskaitė 2017). Because state insurance services are unreliable, people may feel compelled to seek additional services, and it may be necessary for Lithuanians to choose a pension portfolio as a partner in the provision of their pension funds. When selecting a pension portfolio, the customer should consider a variety of aspects, including a solid business reputation, losses-free activities, financial stability, and investment potential. Multi-criteria research methodologies can be used to undertake this partner selection (Voronova 2011).

Multi Criteria Decision Making is concerned with the structure and resolution of multi-criteria decision and planning problems. The major goal of this study is to assist decision-makers in situations where there are numerous options for solving an issue. When there is no one ideal solution for these challenges, it is usually required to leverage the decision maker's desire to differentiate between options (Albayrak & Erensal 2005). The solution to the problem can be construed in a variety of ways. It could be equivalent to selecting the "best" option from a group of options (where "best" can be understood as "the decision maker's most desired option"). Another key aspect of "solving" is to pick a small group of solid options and sort them into distinct preference groups. To uncover all "efficient" or "non-dominated" options, an extreme interpretation is used (Aruldoss, Lakshmi & Venkatesan 2013). The focus of the study is on the decision-making process in Lithuania while selecting a pension portfolio. The study's topic is the use of expert analysis to construct a multi-criteria evaluation system. The investigation is based on the following factual data: Pension portfolios for Lithuania from 2017 until 2021. This study prioritizes the essential elements and sub-factors of selecting a pension portfolio using a multi-criteria analysis technique. The introduced model works for both tier II and III, but the aim of this study is to test it on data collected on tier III only. We investigate the impact of customer preferences in selecting a pension portfolio using the Analytic Hierarchy Process (AHP) method. The author's personal experience and an investigation of literary sources inform the selection of criteria and sub-criteria.

The study proceeds in the following manner: after the literature review on MCDM in Chapter 1, a brief discussion on various MCDM methods and applications is given in the next section, On the basis of studies primarily undertaken in Lithuania, a bibliometric inquiry into pension portfolio assessment is done… Next in Chapter 2, the key pension portfolios providers in Lithuania are identified and the criteria for pension portfolio selection are stated. Then, Section 3 presents the findings of the analysis as well as a case study on the evaluation of Lithuanian pension portfolios. Our study's findings and outcomes are presented in the conclusion.

Research problems

The situation globally as regards pension contribution and collection of benefits in recent times had become rather discouraging as governments struggle to accumulate enough wealth during the lifetime of a worker to enable them pay a pension after retirement. This has led to complete erosion of trust and confidence in the previously upheld principle that if you work hard and long enough, contributed to your pension in safe keep with the government, you would be well taken care of when you are done working and your twilight days come. Grandparents now who were well off in their days and worked many years now live in relative penury and get just a little token from the government as pension. The case in Lithuania is not far-fetched and this has become a dilemma to the government, policy makers, pension fund administrators and researchers alike. The question of "how can workers be encouraged to contribute more towards their own wellbeing in future after retirement" remains primary. Other question would be "how do pension fund managers, including the state pension run pension fund maintain a profitable investment strategy"? Hence ensuring that contributions made by workers don't end up in the funding of state white elephant projects as was the practice in the past. Smart and sustainable investments by shrewd and disciplined managers would see the value of funds rise, the risks associated to the funds mitigated, returns on contributions at the time of maturity satisfactory to the workers and ultimately confidence in the system assured for younger workers and future generations to participate in.

Today's workforce is mainly comprised of young individuals who work within the gig economy, taking up jobs and roles at intermittent intervals while studying, travelling the world or just going through various stages of growth and life crisis. They do not maintain a single employer over their working lifetimes but tend to move easily from one project to another. This creates the situation where it is not easy for the state to collect all contributions towards the pension of this individuals. Also it has fostered the practice of informal employment and payment of salaries or wages in cash between employers and employees. There is also the issue of eroded trust by this young generation to the state pension system as they see their parents and grandparents struggle to live a good life in their old age. This has developed the attitude of not wanting to lose the present value of their earnings to a depreciated future value. This has become a known challenge for the problem and the global push towards private pension fund managers participating in this space has very recently reached Lithuania, with the adoption of three pillars of pension fund contribution and management. There is however not much sensitization of the masses towards this changes and what they entail. How the different pillars vary and how different funds can be combined is not an explored topic in literature, and the criteria both quantitative and qualitative that can ensure that funds are properly evaluated through data, industry expertise and concise knowledge before decisions are made by employees to contribute into them has received minimal attention

Aim and Objectives of the study

This research therefore aims to create a framework for the digitalization of pension fund selection, contribution and portfolio management. Since by default the Lithuanian state pension contribution

(SODRA) is compulsory for all, there is a possibility to contribute to other funds and create a portfolio of funds. The following would be the objective for the research:

1. Carry out a thorough of literature concerning multi criteria decision making and identify the various decision making models
2. Investigate the current pension fund structure in globally and relate to the Lithuanian case, identifying pillars and tiers.
3. Collect useful data on pension funds focusing on measurable and judgmental criteria for decision making
4. Build a quantitative analytic black box containing algorithms for evaluating, ranking, trending and forecasting models with quantitative variable for decision making.
5. Show how the use of qualitative variables in carrying out judgmental forecasting strengthens the case for multi criteria decision making in pension funds selection

1. Literature review

1.1. Analysis of multi criteria decision making.

Decision-making questions in the real world are typically too challenging and ill-structured to be considered by testing a single criterion that will lead to the optimal decision. In actuality, such a one-dimensional approach simplifies the true nature of the situation, and it may lead to irrational conclusions. (Novickyt & Rabikauskait 2017). The simultaneous consideration of all possible variables that are linked to the issue will be a more attractive technique. Multi-criteria Decision Making (MCDM) is an applied topic of organizational study that focuses on the creation and deployment of decision-making tools and methodologies to address specific decision-making difficulties involving many competing needs, goals, or objectives (Zopounidis & Doumpos 2002). Over the past few decades, Multi-Criteria Decision Analysis has seen an enormous amount of use. Its function has increased significantly in various application areas, particularly as new methods are being developed and as old methods are improving. This paper analyzes several popular decision-making methods for multi-criteria (MCDM) and determines their applicability to various circumstances by analyzing their relative benefits and disadvantages (Velasquez & Hester 2013).

1.2. Multi Criteria Decision Making (MCDM) in Business: A Practical Application

As part of organizational analysis, MCDM has evolved to design computational and mathematical methods to enable the subjective assessment of performance by those who make the decisions (Zavadskas, Turskis & Kildienė 2014). The MCDM has been established by a number of investigations (Zavadskas, Turskis, Antucheviciene & Zakarevicius 2012; Zavadskas, Turskis 2010). MCDM techniques and applications have been utilized in prior studies to solve problems in domains such as supply chain management (Rajesh & Ravi 2015), operation research, and soft computing in recent years (Roszkowska & Wachowicz 2015; Angilella & Mazzù 2015; Zhu, Xu, Zhang & Hong 2015; Bouyssou & Marchant 2015; Vasto-Terrientes, Valls, Słowiński & Zielniewicz 2015), manufacturing systems, technology and information management (Oztaysi 2014), material (Zavadskas, Kaklauskas, Trinkunas & Trinkuniene 2004), energy, environment and sustainability (Soltani, Hewage, Reza & Sadiq 2015; Zavadskas, Turskis & Bagočius 2015; Şengül, Eren, Eslamian Shiraz, Gezder & Şengül 2015), tourism management (Akincilar & Dagdeviren 2014), construction and project management (Monghasemi, Nikoo, Fasaee & Adamowski 2015), strategic management (Hosseini, Nasab & Milani 2012), risk management and safety (Ilangkumaran, Karthikeyan, Ramachandran, Boopathiraja, & Kirubakaran 2015), quality management (Certa, Antonella, Lupo, Toni & Passannanti, Gianfranco 2015), and GIS (Latinopoulos & Kechagia 2015).

T. Saaty (1977, 1980, 1994) has developed the Analytic Hierarchy Process (AHP) which is one of the most commonly used and best known MCDM strategies. AHP is a powerful and adaptable decision-making approach that uses weighted scoring to help people set goals and make the best

choices. Despite five major disadvantages, AHP is widely used in scientific study and industrial practice to make multi-criteria judgments. Some researchers (Hajeeh & Al-Othman 2005) regard it as a tool of intuition, while others (Cheng & Li 2001) regard it as a subjective methodology. In the economic-financial sphere, multi-criteria decision-making and rating assessment of economic themes are widely utilized. Based on bibliometric data collected on multiple-criteria decision-making (MCDM) and Multi-Attribute Utility Theory (MAUT) using the Institute for Scientific Information (ISI) database over a five-year period (2002-2006), the research authors (Wallenius, Dyer, Fishburn, Steuer, Zionts & Deb 2008) note that the number of MAUT publications is increasing in line with the growth of management science/operations. According to the authors of a bibliometric study (Wallenius, Dyer, Fishburn, Steuer, Zionts & Deb 2008), management and business difficulties have increased by around 40%, while computer science has increased by 20%, the environment has doubled, and all engineering fields have expanded as well.

An interdisciplinary field of science is the method of study of hierarchies. Lithuanian scientists did research that included an evaluation of the practical application of MCDM. The realistic approach to the use of multi-criteria decision analysis (MCDA (AH)) is evident in the works of Lithuanian researchers, such as the use of multi-criteria analysis for complex evaluation of factors in the marketing climate of new construction companies (Žvirblis & Buraca 2013), the application of the Promethee approach to assess Lithuanian banks' trustworthiness in relation to client enterprises (Gineviius 2011) and the selection of the best real estate investment option (Gineviius 2011). Laurinaviius (2011) discussed the use of the Analytic Hierarchy Process (AHP) and ELECTRE (Elimination Et Choix Treduisant La Realite) to evaluate the weighting ratios of the factor parameters used by DEA techniques to evaluate the effectiveness of investment projects of business subjects claiming investment assistant from Economic Capital (EC). The features of multi-criteria analysis were used to analyze the risks of different investment projects by Shevchenko, Ustinovichius, and Andrukeviius (2008).

This study on multi criteria decision recognizes that many works have been proposed in establishing the best optimal solution for a problem utilizing different approaches in it, and each of the MCDM approaches has its own uniqueness. Many applications employ MCDM to identify system weaknesses, which may then be managed by utilizing the right solution to solve the problem. Our study therefore make contribution in this field with the aim of analyzing and assessing pension portfolio selection in Lithuania using multi-criteria techniques, we offer a proper and advanced pension portfolio assessment model. This research allows for the evaluation and comparison of pension portfolios, as well as the adaptation of multi-criteria methods that combine numerous pension portfolio ratios into a single indication. The results of various multi-criteria decision approaches are integrated into a single indicator and used to evaluate the performance of funds.

1.2.1. Multi criteria Decision Making Methods Used in Pension Portfolio Selection

Pension portfolio selection and management is the process of creating a portfolio of assets (treasury bills, stocks, mutual funds, bonds, and so on) that maximizes a retiree's utility. The

solution to this problem can be broken down into two stages (Doumpos & Zopounidis (1997, 1995)): (1) assessing the various securities to determine which ones best meet the pensioner's preferences, (2) the quantity of capital that will be invested in the first stage of each of the securities chosen. Markowitz's mean-variance approach is used to apply these two phases in traditional portfolio theory (1952). Recently, however, researchers in finance (Zopounidis & Doumpos 2002) and MCDA researchers (Zeleny 2011) have emphasized the multi-dimensional nature of the problem. In this multi-dimensional circumstance, MCDA discrete evaluation methods provide significant assistance in evaluating securities according to the pensioner's policy. The focus of study in this area has been on modeling and depiction of a pensioner's policy, aims, and objectives in a mathematical model. All relevant factors describing the output of the securities are aggregated by the model and their overall assessment is given. Securities with a higher overall evaluation are chosen for portfolio development in the study's final stage. Several research addressing the application of MCDA evaluation methodologies in portfolio selection and management are included in Table 1.

Table 1. Multi criteria Decision Making Methods Used in Pension Portfolio Selection (*Source: Author*)

Studies	**Methods**	**Citation**
Portfolio selection through hierarchies.	Analytic hierarchy process (AHP)	Saaty, Rogers and Pell (1980)
Pension funding and investment: a multiple criteria decision making approach	Goal Programming	Sharif (1985)
An application of a multicriteria approach to portfolio comparisons.	ELECTRE	Martel, N. T. Khoury, and Bergeron (1988)
An integrated multi objective portfolio management system	Single Decision Model (SDM)	Colson and Bruyn (1989)
L' aide `a la d´ecision en gestion de portefeuille	ELECTRE	Szala (1990)
The relationship between riskreturn characteristics of mutual funds and their size.	PROMETHEE	Khoury and Martel (1990)
Comparaison performancetaille des fonds mutuels par une analyse multicrit`ere.	PROMETHEE	Martel, Khoury and M'Zali (1991)
On the use of the MINORA decision aiding system to portfolio selection and management	Utilities Additives (UTA)	Zopounidis (1993)
M´ethode multicrit`ere de s´election de portefeuilles indiciels internationaux	ELECTRE	Khoury, Martel, and Veilleux (1993)
On the use of multi-criteria decision aid methods to portfolio selection	Utilities Additives (UTA)	Zopounidis, Godefroid and Hurson (1995)
Designing a multi criteria decision support system for portfolio selection and management	Utilities Additives (UTA)	Zopounidis, Godefroid and Hurson (1995)
On the use of multi-criteria decision aid methods to portfolio selection	ELECTRE	Hurson and Zopounidis (1995)
Gestion de Portefeuille et Analyse Multicrit`ere	Utilities Additives (UTA)	Hurson and Zopounidis (1996)

Portfolio selection using the idea of reference solution	BIPOLAR	Dominiak (1997)
A multicriteria approach for selecting a portfolio manager	PROMETHEE	Hababou and Martel (1998)
Multi criteria decision making and portfolio management with arbitrage pricing theory	ELECTRE	Hurson and Ricci (1998)
Stock evaluation using a preference disaggregation methodology	Utilities Additives Discriminantes (UTADIS)	Zopounidis, Doumpos and Zanakis (1999)
A decision support system based on multiple criteria for portfolio selection and composition	Utilities Additives Discriminantes (UTADIS)	Zopounidis and Doumpos (2000)
Multi criteria sorting methodology: Application to financial decision problems.	Multi-group Hierarchical Discrimination Method (MHDIS)	Doumpos, Zopounidis and Pardalos (2000)
Multicriteria approaches for portfolio selection	Measuring attractiveness through a categorical-based evaluation technique (MACBETH)	Costa and Soares (2001)
A Multi criteria Decision Making at Portfolio Management	Analytic Hierarchy Process (AHP)	Charouz and Ramík (2010)
Latvian Pension Funds: Multi-Criteria Analysis and Consumer Assessment	Analytic Hierarchy Process (AHP)	Voronova (2011)
Investment funds‘ assessment	SAW	Stankevičienė and Bernatavičienė (2012)
Mutual funds’ performance appraisal using stochastic multi criteria acceptability analysis	SMAA	Babalos, Philippas, Doumpos and Zompounidis (2012)
Performance evaluation of Turkish pension funds by using ELECTRE method	ELECTRE	Uygurtürk (2013)
Financial performance of pension companies operating in Turkey with TOPSIS analysis method.	TOPSIS	İşseveroğlu, and Sezer (2015)
Islamic Pension Funds’ Performance in Turkey	TOPSIS	Icke, B. T., & Akbaba, C. (2015)
The evaluation of the II pillar pension’s funds: An integrated approach using multi-criteria decision methods	Simple Additive Weighing (SAW), SR, GA, TOPSIS, VIKOR and COPRAS	Novickytė and Dičpinigaitienė (2015)
Social security issues: II pillar pension funds‘ performance in Lithuania	SAW, VS, GA, TOPSIS, VIKOR and COPRAS	Novickytė, Rabikauskaitė and Pedroja (2016)
A multi-criteria decision-making approach to performance evaluation of mutual funds	Analytic hierarchy process (AHP) and Data Envelopment Analysis (DEA)	Jakšic, Mimovic and Lekovic (2017)
Measuring the performance of private pension sector by TOPSIS multi criteria decision-making method	TOPSIS	Gurol and Imam (2018)

Research on Decision-Making Decision of Pension Model Based on Hesitant Fuzzy Set	Fuzzy	Zhang (2019)
Comparative performance assessment with entropy weighted ARAS and COPRAS methods of private pension companies	ARAS and COPRAS	Bayrakci and Aksoy (2019)
The Application of Weighted Decision Matrix for the Selection of Non-state Pension Provision Strategy	Weighted Decision Matrix (WDM)	Pukała, Vnukova, Achkasova and Gorokhovatskyi (2020).
A fuzzy multi-criteria decision-making method for purchasing life insurance in India	Fuzzy	Pattnaik, Mohanty, Mohanty, Chatterjee, Jana and Diaz (2021)

In this paper we propose an application of the Analytic Hierarchy Process (AHP) to a partial task in portfolio management is then proposed. Any portfolio manager working for a pension management firm faces the challenge of transforming his assets into cash. It is necessary to choose which pension portfolio on the market is the best to invest in. Of course, there are several specialized areas that make the situation tough, such as law, contracts, and internal needs (criteria). This is a multicriteria decision making (MCDM) problem, and the Analytic Hierarchy Process (AHP) appears to be a good method for tackling it.

1.3. Overview and Forms of MCDM

For at least the last two decades, Multi-Criteria Decision Making (MCDM) has been one of the most rapidly expanding problem areas. Over the previous few decades, decision-making in business has changed, and practitioners are increasingly conscious of this trend. Since the 1960s, many solutions have been proposed and developed in principle to handle this problem in a variety of ways (Wang & Triantaphyllou 2004). There are two basic theoretical streams to consider: Multi-objective decision-making methods that assume continuous solution spaces (and thus continuous mathematics) attempt to find the best compromise solutions and generally presume that the problem may be handled as a mathematical programming model. Continuous mathematics is exceedingly beautiful and powerful, allowing many variations of a fundamental model or approach to be made quickly. This is generally the domain of theoreticians. Unfortunately, in practice, mathematical programming does not manage the majority of MCDM-problems., thus practitioners can only employ these attractive and complex methods to a limited extent. The second stream focuses on discrete choice spaces, i.e., decision spaces with a finite number of options, and incorporates discrete mathematics approaches, which are less mathematically elegant than the first. This branch is referred to as “Multi-Attribute Decision Making.” The broader term MCDM is used in this work. These models don’t strive to identify an optimal solution; instead, they strive to rank either a ranking of relevant actions that is “ideal” in terms of various criteria or they aim to identify the “ideal” actions among the current ones (Triantaphyllou 2000). Despite the fact that this sort of issue is far more common and relevant in reality, there are far fewer solutions available, and

assessing their effectiveness is far more difficult than in the continuous case. As a result, one of the most significant, but also one of the most difficult question to answer is, “Which method is the best for a given problem?”. In this section, some common MCDM methods are analyzed with each method’ uniqueness and abnormalities.

1.3.1. **Multi-Attribute Utility Theory (MAUT)**

MAUT is an expected utility theory that can be used to determine the best approach in a given problem by assigning a utility to each possible result and calculating the best utility (Konidari & Mavrakis 2007). The main advantage of MAUT is that it accounts for uncertainty. It can be given a utility, which is a property that isn’t taken into account in many MCDM approaches. It is comprehensive at every step of the approach and can account for and include the preferences of each outcome. This amount of precision is convenient, but it can lead to many possible drawbacks. At every step of the procedure, an incredible amount of input is necessary to accurately record the preferences of the decision maker, making this method extremely data intensive. This level of input and quantity of data may not be available for every decision-making scenario. Decision-makers' judgments must also be precise, assigning particular weights to each of the outcomes, necessitating firmer assumptions at every level. This can be difficult to apply exactly and is highly subjective. The capacity to take into account uncertainty is a major strength of MAUT, and it is frequently used in common MAUT applications.

1.3.2. **Analytic Hierarchy Process (AHP)**

AHP is a theory of evaluation based on pair-based comparisons that derives priority scales from professional assessments (Saaty 2008). It is one of the most often used MCDM systems, and it has a number of advantages as well as disadvantages. Its ease of use is one of its benefits. The use of pairwise comparisons will make it possible to weight coefficients for decision-makers and to compare alternatives with relative ease. It is modular, and because of its hierarchical nature, it can easily change in size to accommodate decision-making issues. While sufficient data is required to successfully execute pairwise comparisons, it is not nearly as data heavy as MAUT. Interdependence between needs and alternatives has been a problem for the mechanism. It may be subject to changes in judgment and ranking criteria due to the methodology of pairwise comparisons, and it “does not allow [individuals] to assess one instrument in isolation, but in relation with the others, without separating strength and weaknesses” (Konidari & Mavrakis 2007). One of its strongest critiques is that rank reversal is vulnerable to the general form of AHP. While adequate data is required for pairwise comparisons to be effective, it does not require nearly as much data as MAUT. Interdependence between needs and options has been a problem for the mechanism. Due to the approach of pairwise comparisons, it may be prone to changes in judgment and ranking criteria, and it “does not permit [individuals] to assess one method in isolation, but in relation to the others, without differentiating strength and weaknesses” (Konidari & Mavrakis 2007). But to prevent this approach, issues where alternatives are widely added will do well. The general form of AHP (Saaty, 2006) can be considered Analytic Network Process (ANP) and is

more concerned with network structure. It allows for dependency in terms of benefits and requires self-reliance. It can rank element groups or clusters in order of importance. It is superior to AHP at dealing with dependency and “may foster intricate, networked decision-making with different intangible criteria” (Tsai, Chang, & Lin, 2010). Aside from the issues with AHP, one of its major flaws is that “it ignores the varied effects amongst clusters” (Wang 2012).

1.3.3. **Fuzzy Theory**

Fuzzy set theory is a branch of classical set theory that “allows many issues related to the handling of inaccurate and uncertain data to be tackled” (Balmat, Lafont, Maifret & Pessel 2011). It does have many advantages. Inadequate information and the growth of accessible knowledge are taken into consideration by fuzzy logic (Balmat, Lafont, Maifret & Pessel 2011). This enables imprecise input. This allows for a few recommendations to cover topics of great complexity. Fuzzy systems can also be difficult to build because of drawbacks. In certain instances, before being able to be used in the real world, they can require multiple simulations. Engineering, economics, environmental, social, medical, and management are just a few of the fields where fuzzy set theory has been created and utilized. The availability of imprecise input takes advantage of many of these types of issues. These types of applications prefer a process that accepts vagueness and can be checked many times before real-world implementation (Velasquez & Hester 2013).

It accepts input that really isn't precise. This enables several recommendations to cover a wide range of topics. Fuzzy systems can also be difficult to build because of drawbacks. In certain instances, before being able to be used in the real world, they can require multiple simulations. In areas such as medical, economic, social, environmental, engineering and management, the Fuzzy set theory is developed and used. The availability of imprecise input takes advantage of many of these types of issues. These applications prefer a method that tolerate ambiguity and may be tested multiple times before being implemented in the real world (Velasquez & Hester 2013).

1.3.4. **TOPSIS**

In a multidimensional computational space, TOPSIS is a method for determining which alternative is closest to the perfect solution while being the furthest away from the perfect negative solution. (Yan, Luo, Qin, Zhou, Guan, & Zhang 2008). It does have several benefits. It has a fundamental mechanism. It's simple to use and program. The number of steps remains constant regardless of the number of attributes. A drawback is that the association of attributes is not considered by its use of Euclidean Distance. It’s challenging to balance traits and retain consistency in judgment, especially when there are more of them TOPSIS is being used in supply chain and logistics management, architecture, and engineering. and production processes, management of industry and marketing, environmental management, management of human capital and management of water resources (Velasquez & Hester 2013). This is another technique where its ease of use has sustained the success of its use. TOPSIS has verified the answers suggested by other MCDM approaches for many of the uses seen in the literature review. Because of its flexibility and

capability to maintain the same number of steps regardless of the magnitude of the challenge, it can be used as a decision-making tool to compare and contrast different techniques or to stand alone.

1.3.5. **Goal Programming**

Goal Programming is a logical type of programming capable of selecting from an infinite number of alternatives. One of its strengths is that it has the ability to deal with large-scale issues. Depending on the case, the capacity to generate infinite alternatives offers a major advantage over other approaches. Its failure to weight coefficients is a big downside. Many applications require the use of other methods, such as AHP, to accurately weight the coefficients. In combination with other approaches to accommodate proper weighting, several of these applications have been used (Velasquez & Hester 2013). By doing so, while still being able to choose from infinite options, it removes one of its drawbacks. This fits a general trend in which MCDM techniques are most commonly used in applications that avoid most of their drawbacks.

Other MCDM methods that have been applied in various recognized fields are: Data Envelopment Analysis (DEA), Simple Multi-Attribute Rating Technique (SMART), Case Based Reasoning (CBR), ELECTRE (*Elimination et Choix treduisantë la Realite*), PROMETHEE and SAW. The analysis of MCDM and applications shown that it has been applied in various fields including investment projects, they do not, however, attempt to combine portfolio indications into unified whole. (Jasienė and Kočiūnaitė 2007, Lieksnis 2010, etc.). Methods of multi-criteria decision will help systematize indicators (Pendaraki and Zopounidis 2003, Alptekin 2009, etc). It was proposed to use the Analytic Hierarchy Process (AHP) technique to study and appraise pension portfolio selection in Lithuania because of its unique ability to break a challenging MCDM problem into a systematic hierarchy procedure. This paper facilitates the assessment and evaluation of pension portfolios, as well as the adaptation of multi-criteria approaches in which different percentages of pension funds are integrated into a single measure.

1.4. Pension Portfolio Assessment

The word “pension” comes from Latin “payment”. It is related to the financial security and cases provided by law for the residents of the country. People who reach the retirement age receive benefits for their current expenses. That is the function of the social security system because a pension system is the basis of a social security system (Rudyte & Berzinskiene 2012). Pension portfolios are one of the most essential supporting structures for the majority of citizens’ social safety. When income is no longer available (income dries up), the pension fund program theoretically transfers most of the assets amassed during working life to post-retirement (Muralidhar 2001). Hundreds of thousands of participants in a pension portfolio may live in different places, are unfamiliar with one another, and are unable to arrange themselves in order to acquire and reflect an overall accord. As a result, pension portfolio management firms confront issues similar to those outlined above, namely, how to match the needs of pension pool investors

and pension management company shareholders (Novickyt & Rabikauskait 2017). Limited liability companies with one shareholder (parent company) are pension portfolio managing companies in Lithuania. Initially used in Latin America and the post-Soviet environment, such a second pillar pension fund management model is not very common (Strumskis & Balkeviˇcius 2016).

Many risks are borne by pension portfolio members in Lithuania. Participants can pick the sort of pension fund to join (low or high risk), as well as their risk tolerance and expected short and long-term returns on investment. A fundamental understanding of the financial market is essential for such a decision-making process; however, this is not the case in this subject (Strumskis & Balkevicius 2016). An examination into people's understanding of financial investing was conducted in order to analyze participants' expectations, attitudes toward the success of their pension portfolios, and establish what criteria they use to select a pension portfolio. For the investigation of pension portfolios participant approach, a multi-criteria decision process was employed and reviewed by a company specializing in this.

1.5. Pillars, Tiers, and Pension Governance

The example of financialization is instructive in highlighting the theoretical limitations in pension governance research. The variety of critical institutions that are not only thought to characterize governance of pension provision in ideal style regimes, but also conditions and restricts academic thinking on governance logics, may be one explanation for this bias (Briganti 2008). The classic discursive structure of the World Bank's three pillars, which has direct implications for understanding governance, is perhaps the most significant. First pillar pension schemes (e.g. flat, means-tested, earnings-related or universal) are supposedly administered by public bodies. Second pillar schemes earnings-related schemes based on individual or collective labour market arrangements and executed by companies (e.g. in forms of book reserves), separate pension funds or outsourced financial sector actors. Third pillar pensions are about private supplementary pensions paid by individuals and administered by selected private entities (Holzmann & Palmer 2006).

While there is an argument that a lack of a rigorous structure for addressing governance types and practices is a key theoretical void that renders the old three-pillar division obsolete, empirical facts in this thematic field have pushed the pillar model to crisis. The fact that many public pension systems require private administration and implementation poses the most significant challenge in terms of pillar typologies in Europe and the European Union (EU), whose legislation heavily relies on the three-pillar model. From the standpoint of EU law, this has been extremely troublesome, as it leaves the pillar to which a scheme belongs up to interpretation, which has, for example, placed first-pillar schemes outside of EU jurisdiction into its interpretative horizon. Most notably, the regulation 1408/71 is currently interpreted to cover all types of public-private partnerships, completely blurring the distinction between the first and second pillars in terms of different

schemes (Briganti 2008). As a result, there have been demands for further research into the governance of Public-Private Partnerships (PPPs) that deal with pensions (Orenstein 2008).

The emerging practice of viewing pension schemes in terms of pillars and tiers acknowledges the separation of private and public administration within pillars, at least indirectly. There is a lot of variance in these interpretations. One influential handbook categorization is shown in Table 2. The distinction of pillars into layers or tiers has had a lot of analytical importance, particularly in comparative studies, and it's been closely tied to the research question asked. For example, Whiteford and Whitehouse (2006) divided the first pillar into just two levels in a recent study of OECD countries (excluding Malta, Cyprus, Estonia, Romania, and Bulgaria): first tier safety nets to avoid old-age poverty and second tier structures to ensure a sufficient replacement rate for earnings during working life. There were some variations between these tiers. The first tier may include basic flat rate plans that guarantee a minimum pension (often linked to earnings-related pensions) or general social assistance for the poor that is not tied to age (Germany until 2006). The most popular schemes in the second tier were defined benefits (DB) plans. In these programs, the amount an employee earns in retirement is determined by contributions made during their working lives as well as some indicator of individual earnings (Johanson & Sorsa 2010). The defined contribution (DC) plan, in which workers deposit their contributions into individual accounts, was the second most common scheme. The retirement income is derived from the contributions' accrued capital and investment returns. Notional or nonfinancial DC (NDC) is a variant of the pure DC scheme that can be used in a variety of schemes (Clark & Whiteside 2003).

Table 2. Generic description of pension systems via pillars and tiers
Source: Immergut and Anderson (*2007*)

	First pillar	**Second pillar**	**Third pillar**
Third tier	(e.g. Swedish individual funded accounts)	Voluntary occupational pensions (e.g. UK)	Voluntary private pension (life insurance)
Second tier	Earnings-related part of pensions (e.g. French supplementary occupational pension schemes) for employees and/or self employed	Government 25or25lem ai occupational pension (e.g. Danish tax deductible occupational pension schemes)	Government 25or25lem ai private pension (e.g. German Riester-Rente)
First tier	Government 25or25lem ai private pension (e.g. German RiesterRente) Means-tested part (e.g. Swedish guarantee pension)	Mandatory occupational pension (e.g. Swiss second pillar or de facto mandatory Dutch occupational pensions)	Mandatory private pension (e.g. Portuguese Plafonamento)

In Lithuania to be precise, the pension accumulation system includes the three pillar. The first pillar (SODRA) is the state social security pension, the second pillar allow accumulating additional

funds for old age and the third pillar is voluntary contribution to a pension fund in a life insurance scheme. The purpose of which is to enable a person to secure a well-off old age and to independently accumulate his/her own pension.

In Lithuania, private corporate entities, more specifically investment corporations founded by banks or insurance firms, handle second and third pillar pension funds. This indicates that pension funds are contractual in Lithuania. Fund management organizations, like all other businesses, have a primary goal of increasing shareholder value. However, in order to do so, the company must have consumers, which it can only do if it provides value to them as well. It should be ensured through competition between enterprises in a truly free market scenario. Because the market for mutual connections between businesses and their customers is free, both the firm and the client can choose whether or not to engage into an agreement, and there is plenty of room for negotiation — this may apply to third-pillar pension plans (Strumskis & Balkeviius 2016 The situation is significantly different for second-pillar pension plans: once chosen, participation is mandatory. A participant can switch from one fund to another, or from one company's fund to another, but there is no way to completely abandon the system.

Table 3. Pension Funds for Tier II and III (*Source: Author*).

Pension Funds

Tier II

1.SEB pension 1954-1960
2. SEB pension 1961-1967
3. SEB pension 1968-1974
4. SEB pension 1975-1981
5. SEB pension 1982-1988
6. SEB pension 1989-1995
7. SEB pension 1996-2002
8. SEB Asset Preservation Fund
9. INVL pension 1996-2002
10. INVL pension 1989-1995
11. INVL pension 1982-1988
12. INVL pension 1975-1981
13. INVL pension 1968-1974
14. INVL pension 1961-1967
15. INVL pension 1954-1960
16. INVL pension Asset Preservation Fund
17. Pension 1996-2002
18. Pension 1989-1995
19. Pension 1982-1988
20. Pension 1975-1981
21. Pension 1968-1974
22. Pension 1961-1967
23. Pension 1954-1960
24. Asset Preservation Pension Fund
25. Luminor 1996-2002
26. Luminor 1989-1995
27. Luminor 1982-1988
28. Luminor 1975-1981
29. Luminor 1968-1974
30. Luminor 1961-1967
31. Luminor 1954-1960
32.Asset Preservation Pension Fund
33. AVIVA Y3
34.AVIVA Y2
35. AVIVA Y1
36. AVIVA X3
37. AVIVA X2
38. AVIVA X1
39. AVIVA B
40. AVIVA S

Tier III

1. SEB pension 18+
2. SEB pension 50+
3. SEB pension 58+
4. INVL Drąsus
5. INVL Apdairus
6. INVL STABILO III 58+ / INVL Stabilus
7. INVL EXTREMO III 16+
8. INVL MEDIO III 47+
9. Pension Fund 100 (limited termination)
10. Pension Fund 60 (limited termination)
11. Pension Fund 30 (limited termination)
12. Luminor Pension 1 plus
13.Luminor Pension 2 plus
14. Luminor Pension 3 plus
15. Luminor Pension for employee 1 plus
16. Luminor Pension for employee 2 plus

Finally, there is just one state PAYG system, which is experiencing some issues and is not particularly tempting to high-income individuals, even before electing to join in the second pillar. Because participants and shareholders in Lithuanian pension funds do not engage directly, Wilson (2008) proposes that the triangle be "straightened." The pension fund management sector appears to be stymied by two obstacles: shareholders on the one hand, demand profitability, and on the other, participants demand that resources be used to achieve their objectives.

1.6. Section Summary

The unreliability of governmental insurance services encourages Lithuanians to seek out other services, such as a three-pillared pension accumulation structure. The first pillar (SODRA) is the state social security pension, the second pillar allow accumulating additional funds for old age and the third pillar is independently accumulated pension based on an investment life insurance agreement. Fund management organizations have a primary goal of increasing shareholder value. However, in order to do so, the company must have consumers, which it can only do if it provides value to them as well. The pension portfolio managing companies face problems such as how the needs of the investors in the pension pool and the shareholders in the pension managing companies can be matched. This study, though born out of curiosity, sought to address the gap in the literature and take a novel approach to identify how different pillars vary and how different funds can be combined choosing a pension portfolio using a multi-criteria analysis methodology to prioritize the essential elements and sub-factors, and deduce the criteria both quantitative and qualitative that can ensure that funds are properly evaluated through data, industry expertise and concise knowledge before decisions are made by employees.

2. Research Methods

To evaluate pension portfolio, and decide on some of the criteria necessary for selection, literature review was carried out, and the quantitative and qualitative forecasting perspective were selected. The qualitative forecasting is not purely guesswork, it is a well-developed structured approach to obtaining good forecasts without using historical data. There are many methods for calculating a pension fund‘s risk adjusted results. Some common examples are; Sharpe 22 ratio, Treynor‘s scale, and Jensen Alpha. Returns must be risk-adjusted before comparison in order to make fair comparisons (Bodie, Kane & Marcus 2014). If an investor has a single investment portfolio or several, the performance assessment model they use is determined by the number of investment portfolios they have (Bodie et al. 2014). Different risk-adjusted success metrics have their own set of benefits and drawbacks (Bodie et al. 2014). Sharpe-ratio can only be used as a performance assessment metric when an investor only has one portfolio, which is his or her entire investment fund (Bodie et al. 2014). When comparing two mutual funds that invest in the same market, the Sharpe ratio is only useful (Larsson & Gustafsson 2017). The aim of an investment plan is to make a medium-to-long-term investment. It is based on projections of expected returns, costs, and correlations over long periods of time. Although it may be tempting to base such estimates on recent capital market experience, this is usually not a good idea. It will be extremely beneficial to be able to recognize large asset classes that will perform particularly well in the future. Many investors will look for information that will allow them to do so. Prices may change to restore equilibrium relationships as they attempt to act on such facts. Attempts to outperform the market sow the seeds of their own demise. A short-term trading strategy is not an investment strategy. It is, instead, a key component of a strategy that is tailored to a specific investor‘s needs and circumstances.

The quality of judgmental forecasts has improved as a direct result of implementing well-structured and systematic approaches which is extremely beneficial as it allows us to be able to recognize large asset classes that will perform particularly well in the future. Pension funds information may change to restore equilibrium relationships, thus it is important to recognise that judgmental forecasting is subjective and comes with limitations, however, implementing systematic and well-structured approaches can confine these limitations and markedly improve forecast accuracy of these pension funds. Quantitative criteria was chosen because there is enough data available and relevant enough to make a forecast. Quantitative forecasting approaches come in a variety of shapes and sizes, and are frequently developed within certain fields for particular objectives. Each methodology has its own set of characteristics, accuracies, and expenses that must be considered while choosing a tecnique. In most quantitative prediction problems, time series data (obtained at regular intervals across time) or cross-sectional data are used (collected at a single point in time).

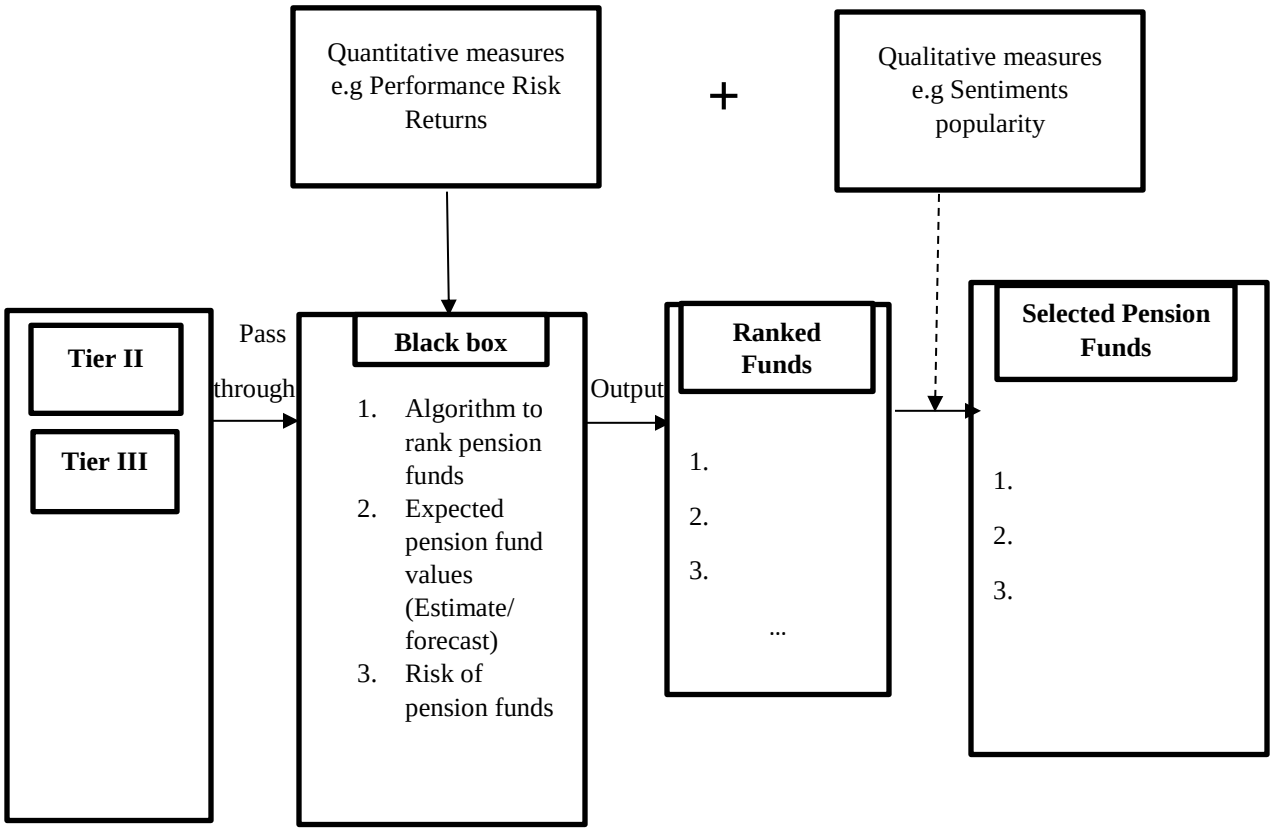

Fig. 1. Research Model (*Source: Author*)

In this paper, the focus is to forecast the future of pension funds in Lithuania, using the time series domain. Quantitative criteria allows us to form judgement by judgemental forecasting, research has shown that the accuracy of judgmental forecasting improves when the forecaster has (i) important domain knowledge, and (ii) more timely, up-to-date information. A judgmental approach can be quick to adjust to such changes, information or events Lawrence, Goodwin, O'Connor and Önkal (2006).

2.1. Time series models

Time series analysis is a set of techniques for deriving useful statistics and other characteristics from time series data (Kang, Sohn, Park, Lee & Yoon 2011, Sobu & Wu 2012; Mori & Takahashi 2012). Parametric and non-parametric time series models are the two most common forms of time series models. The parametric techniques assume that the fundamental stationary stochastic process has a structure that can be characterized by only a few variables (Huang & Lu 2010). All of the time series models used in this study are parametric. For the provided data, single, double, centered, and weighted moving average models were tested with various orders and intervals.

Because time series data can display a wide range of patterns, it‘s often useful to break it down into numerous components, each representing a different underlying pattern category. Because

time series data can display a wide range of patterns, it‘s often useful to break it down into numerous components, each representing a different underlying pattern category. Time series patterns are of three types; trend, seasonality and cycles. When a time series is decomposed into components, the trend and cycle is usually combined into a single trend-cycle component (sometimes called the trend for simplicity). As a result, a time series can be divided into three parts: a seasonal component, a trend-cycle component and a remainder component (containing anything else in the time series). Extracting these components from a time series oftentimes can help to improve forecast accuracy, and also improve understanding of the time series. Time series models used for forecasting in this paper include decomposition models, exponential smoothing models, Naïve model, and ARIMA models.

2.2. Naïve model

The naive technique is a very simple technique that is based on the premise that the last observed value will occur in the following time interval, hence the naive approach is identical to the last seen value. This method simply involves the application of a single time series point. Naïve forecasts are the most cost-effective forecasting model, and provide a benchmark against which more sophisticated models can be compared (Godwin 2014). This method works quite well for economic and financial time series, which often have patterns that are difficult to reliably and accurately predict. If the time series is believed to have seasonality, the seasonal naïve approach may be more appropriate where the forecasts are equal to the value from last season.

2.3.Exponential Smoothing in Time Series Analysis:

Exponential smoothing was proposed in the late 1950s (Brown 1959), and has inspired some of the most effective forecasting techniques. Exponentially smoothed forecasts are weighted averages of previous observations, with the weights decaying exponentially as the observations get older. That is, the larger the related weight, the more recent the observation. This framework produces accurate forecasts fast and for a wide range of time series, which is a significant benefit for industrial applications. The objective behind exponential smoothing is to smooth the original series in the same manner that the moving average smooths it, and then use the smoothed series to forecast future values of the variable of interest. In exponential smoothing, we want the more recent values of the series to have a bigger influence on the forecast of future values than the more distant data. Exponential smoothing is a straightforward and practical method of forecasting that involves constructing a forecast using an exponentially weighted average of previous observations. The current observation is given the maximum weight, with the observation immediately before it receiving less weight, and the observation before that receiving even less weight, and so on, with exponential decay of past data influence (Aczel 2014).

This method predicts the one next period value based on the past and current value. It involves averaging of data such that the nonsystematic components of each individual case or observation cancel out each other. The exponential smoothing method is used to predict the short term predication. Alpha, Gamma, Phi, and Delta are the parameters that estimate the effect of the time series data. Alpha is used when seasonality is not present in data. Gamma is used when a series has a trend in data. Delta is used when seasonality cycles are present in data. A model is applied according to the pattern of the data.

Of all forecasting techniques, this one is the most extensively employed. It is utilized when the data pattern is roughly horizontal and requires little calculation (i.e., there is no neither cyclic variation nor pronounced trend in the historical data).

Assume a time series has been observed to be $y_1, y_2, ..., y_n$. Formally, the simple exponential smoothing equation takes the form of

$$\hat{y}_{i+1} = \alpha y_i + (1-\alpha)\hat{y}_i$$

where y_i is the actual, known series value for time period i, $\hat{y}i$ is the forecast value of the variable Y for time period i, $\hat{y}i+1$ is the forecast value for time period $i+1$ and α is the smoothing constant [2]. The forecast $\hat{y}i+1$ is based on weighting the most recent observation yi with a weight α and weighting the most recent forecast $\hat{y}i$ with a weight of $1-\alpha$.

To get the algorithm started, we need an initial forecast, an actual value and a smoothing constant. Since $\hat{y}1$ is not known, we can:

1. Set the first estimate equal to the first observation. Thus we can use $\hat{y}1 = y1$.

2. Use the average of the first five or six observations for the initial smoothed value.

Smoothing constant α is a selected number between zero and one, $0 < \alpha < 1$. When $\alpha = 1$, the original and smoothed version of the series are identical. At the other extreme, when $\alpha = 0$, the series is smoothed flat.

Rewriting the model (1) to see one of the neat things about the SES model

$$\hat{y}_{i+1} - \hat{y}_i = \alpha\,(y_i - \hat{y}_i), \qquad (2)$$

change in forecasting value is proportionate to the forecast error. That is

$$\hat{y}_{i+1} = \hat{y}_i + \alpha\, e_i\,, \qquad (3)$$

where residual ei = yi – ŷi is forecast error for time period i. So, the old forecast plus a correction for the mistake in the previous forecast is the exponential smoothing prediction. (Aczel & Sounderpandian 2006; Ostertagová 2011).

We get model (1) by continuing to substitute prior forecasted values back to the data‘s starting point.:

$$\hat{y}_{i+1} = \alpha\, y_i + (1-\alpha)\,[\alpha y_{i-1} + (1-\alpha)\,\hat{y}_{i+1}] = \alpha y_i + \alpha(1-\alpha) y_{i-1} + (1-\alpha)^2\,\hat{y}_{i+1},$$

$$\hat{y}_{i+1} = \alpha\, y_i + \alpha\,(1-\alpha)\, y_{i-1} + \alpha(1-\alpha)^2\, y_{i-2} + (1-\alpha)^3\,\hat{y}_{i-2},$$

$$\hat{y}_{i+1} = \alpha\, y_i + \alpha\,(1-\alpha)\, y_{i-1} + \alpha(1-\alpha)^2\, y_{i-2} + \alpha\,(1-\alpha)^3\, y_{i-3}\,(1-\alpha)^4\,\hat{y}_{i-3},$$

In its most basic form, the forecast equation is

$$\hat{y}_{i+1} = \alpha y_i + \alpha (1-\alpha) y_{i-1} + \alpha(1-\alpha)^2 y_{i-2} + \cdots + \alpha (1-\alpha)^{i-2} y_2 + \alpha (1-\alpha)^{i-1} y_1 = = \alpha \sum_{k=0}^{i-1} (1-\alpha) k y_{i-k}$$

where $\hat{y}_{i+1}$ is the forecast value of the variable Y at time period i +1 from knowledge of the actual series values y_i, y_{i-1}, y_{i-2} and so on back in time to the first known value of the time series, y_1 (Brown & Meyer, 1961; Montgomery, Zhang, Johnson & Gardiner, 1992). Therefore, $\hat{y}_{i+1}$ is the weighted moving average of all past observations.

The set of weights that were utilized to create the forecast $\hat{y}_{i+1}$ is α, $\alpha(1-\alpha)$, $\alpha(1-\alpha)^2$,....

As we progress through the series, each number has a lesser impact on the forecast, as the weights decline exponentially approaching zero. The weights‘ exponential fall approaching zero is seen (Aczel & Sounderpandian 2006).

Following the specification of the model, its performance characteristics should be tested or validated by comparing its forecast to historical data for the process it was created to forecast. We can use the error measures such as MAPE (Mean Absolute Percentage Error), MSE (Mean Square Error) or RMSE (Root Mean Square Error):

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|ei|}{y} . 100\%, \quad (5)$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} ei^2, \quad (6)$$

$$RMSE = \sqrt{MSE} \quad (7)$$

The error measure chosen has a significant impact on the findings reached about which of a set of forecasting methods is the most accurate. The value of is a function of the speed at which older answers are dampened (smoothed). When the smoothing constant is near 1, damping is quick; when it is near 0, damping is slow (Zhang et al. 1992). If we want our forecasts to be reliable and stochastic fluctuation smoothed, we should use a small α. A greater value is essential if we desire a quick answer. The MSE or RMSE is commonly used as a criterion for determining a suitable smoothing constant. For instance, by assigning α values from 0.1 to 0.99, we select the value that produces the smallest MSE or RMSE (Aczel & Sounderpandian 2006).

2.4. Autoregressive Integrated Moving Average (ARIMA) Model:

The time-series Autoregressive Integrated Moving Average (ARIMA) model has been used to forecast social, political, economic, engineering, foreign exchange, stock, and other problems since it was initially proposed by Box and Jenkins in 1976. It is assumed that a time series‘ future values have a clear and obvious functional relationship with its current, past, and white noise values. This model has the benefit of precise predicting in a short time period; nevertheless, it has the limitation of requiring at least 50, and ideally 100, data. Furthermore, this model employs the concept of

measurement error to account for variations between estimators and observations, yet the data in this model are precise values that are free of measurement errors (Box, Jenkins, & Reinsel 2013).

The ARIMA model is a stochastic process modeling framework (Box, Jenkins, & Reinsel 2013) defined by six parameters (p, d, q) and (P, D, Q), where the first triple defines the model concerning trend and noise component, and the second vector is optional and defines a model for the seasonal component. Where P or p is the order of the AR(p) process, D or d is the order of integration (needed for the transformation into a stationary stochastic process), and Q or q is the order of the MA(q) process (Herbst, Huber, Kounev, & Amrehn 2014). The difficulty in defining parameters is a flaw in this model. In general, different unit-root tests and the Akaike information criteria (AIC) are used to choose candidates. To address this issue, a methodology for automatic model selection is proposed in the R forecast package's auto.arima() function (Hyndman & Khandakar 2008). This function fits an ARIMA model to the time series data and calculates point forecasts using the best parameters.

ARIMA is a statistical analysis model that employs time series data to better comprehend a data collection and forecast future trends. It‘s a type of regression analysis that determines how strong one dependent variable is in relation to other variables that change. The purpose of the model is to forecast future events by looking at the disparities between values in a series rather than actual values. The concept of stationarity and the technique of differencing time series are both used in this model. A stationary time series' properties are independent of the time at which it is observed. Patterns or seasonality in time series, on the other hand, are not stationary. At different times, the trend and seasonality will change the value of the time series. A white noise series, on the other hand, is stationary; it should seem the same at any point in time, regardless of when it is examined.

2.5. Decomposition model:

The decomposition model is a forecasting method based on the assumption that if the underlying causes of a data trend can be defined and forecasted separately, the forecast can be improved. Decomposition is the process of breaking down data into its constituent bits. The following are the two models commonly used for the decomposition of a time series into its components.

i. **Additive model**; Accoding to the additive model, the time series can be expresed as

$$Y_t = T_t + S_t + C_t + I_t$$

Where Y_t is the time series value at time t, and T_t, S_t, C_t, I_t represents the trend, seasonal, cyclical and random variation at time t respetively. The additive model assumes that all the four components of the time series operate independently of each other so that none of these components has any effect on the other. Additive model is preferable when the time series has roughly the same variablity through the length of the series. Also, additive model is approprite if the magnitude of the seasonal fluctuation does not vary with the level of the series.

ii. **Multiplicative model;** The time series can be expressed as

$$Y_t = T_t + S_t + C_t + I_t$$

Where Y_t is the time series value at time t, and T_t, S_t, C_t, I_t represents the trend, seasonal, cyclical and random variation at time t respetively. This model assumes that the four components of the time series are due to different causes but they are not necessary independently and they can affect each other. When the variabiity of the time series increases with the level such that the values of the series become large as the trend increases, a multiplicative model is appropriate.

2.6. BATS model

Exponential smoothing methods developed by Holt-Winters (HW) are commonly used to anticipate time series with a single seasonal pattern (additive or multiplicative) and produce good results. However, this framework has two flaws: models cannot be evaluated using maximum likelihood, and prediction intervals cannot be calculated.

State Space Models with Multiple Sources of Error (SSM, Harvey, 1989) and innovative State Space Models with a Single Source of Error are two superior ideas (ETS, Hyndman et al, 2008). The above-mentioned exponential smoothing methods have been investigated using state space models. The SS models include those that underpin Holt Winters‘ additive and multiplicative approaches.

Taylor (2003) adds a second seasonal component to the HW technique by extending it over a seasonal pattern. It can be difficult to estimate the parameters and seed values when the number of seasonal components is large. In addition, if the seasonal period is extended, the resulting model is likely to be over-parametrized. When one seasonal period is a multiple of the other, this becomes easier. White noise is assumed to be serially uncorrelated in the exponential smoothing model. In actuality, this assumption isn’t always correct because it might sometimes behave like an AR (1) process.

2.7. Judgmental forecasting

In statistical modeling, econometric models and time-series models are two types of forecasting models. (Kmenta 2015). Econometric and time-series models are applicable when there are historical data. As a result, in situations where there is a sense of continuity between the past, present, and future, time-series and econometric techniques are crucial. These quantitative methods, on the other hand, may be of limited utility when projections cannot simply be made from historical data but must be produced through a cognitive process, or when the environment or the business undergoes a transformation. Judgmental forecasting is crucial in this scenario. When events change rapidly, Makridakis et al. (1983) found that judgmental forecasts are preferred over statistically determined projections because they are easier to adjust to.

Professional forecasters' performance is influenced by three factors: (1) the environment about which forecasts are formed, (2) the information system that provides data about the environment to the forecaster, and (3) the forecaster's cognitive system. It is important to recognize that judgmental forecasting is subjective and comes with limitations. Judgmental forecasts can be

inconsistent and biased by personal political reasons (Goodwin 2007). However, implementing systematic and well-structured approaches can curb these limitations and markedly improve forecast accuracy (Lawrence, Goodwin, O'Connor, Onkal 2006 As a result, both historical trend-based data and competent judgments based on growing experience and understanding are required in an efficient forecasting process.

2.7.1 Judgmental Adjustments

Judgmental adjustments are useful for situations where historical data are available and are used to generate statistical forecasts. It is common for practitioners to then apply judgmental adjustments to these forecasts. Fildes and Goodwin (2007) surveyed 149 companies and found that over 75% integrated judgment in their forecasting process and that the adjustments to statistical forecasts reduced Mean Absolute Percentage Error (MAPE) by 3.6%. These adjustments can potentially provide advantages: they provide an avenue for incorporating factors that may not be accounted for in the statistical model, such as promotions, large sporting events, holidays, or recent events that are not yet reflected in the data. However, these advantages come with biases and limitations if methodical strategies are not implemented. Hence, when carrying out judgmental adjustments, the following are best practices to be followed:

i. Ensure the panelists have visibility to the statistical baseline forecasts when making the adjustments
ii. Document all overrides made and justify.
iii. Track accuracy of the adjusted and the baseline to understand where you are adding or destroying values.

2.7.2 Delphi Method

The Delphi method is one of the most widely utilized techniques for judgmental forecasting. This technique uses aggregation of forecasts from a group of individuals as the forecast value (Rowe 2007). The purpose of this research was to develop a novel method of predicting that used both historical data and human judgment in a formal forecasting model. The Delphi method generally involves the following stages:

i. A panel of experts is organized and a facilitator is appointed.
ii. Forecasting tasks are set and distributed to the panelists.
iii. Each panelist returns their initial forecast along with their justifications.
iv. These forecasts/justifications are compiled and summarized in order to provide feedback.
v. Feedback is provided to the panelists, who in return revise their forecasts and resubmit. This step may be iterated until a satisfactory level of consensus emerges.
vi. Final forecasts are obtained by aggregating the panelists' forecasts.

2.8. Model Evaluation

In empirical modeling, the Akaike Information Criterion (AIC) (Akaike 1974) and Bayesian information criterion (BIC) (Schwartz 1978) are commonly used to determine the best among

competing candidate's models in approximating the unknown true model. In this subsection, these goodness of fit measures are briefly described, as well diagnostic measures such as Mean Absolute Percentage Error (MAPE) which is specifically used to confirm the adequacy of any other complex model over Naïve model and L-Jung Box for testing significance of autocorrelation values of residual.

Akaike Information Criterion (AIC)

The performance of time series model can be assessed through the Akaike Information criterion given by:

$$AIC = 2p - 2\mathcal{L} \qquad (8)$$

Where $\mathcal{L}$ is the log-likelihood value, p is the number of parameters. The advantage of this measure is that it involves a penalty term $2p$, which discourages overfitting of the model. Thus, the best model, among the competing models. Is the model with the lowest AIC value.

A little modification is required for finite sample sizes. Assuming that the model is univariate, linear, and has normally distributed residuals, the corrected AIC is defined as:

$$AIC_c = AIC + \frac{2p(p+1)}{n-p-1} \qquad (9)$$

Where, n is the sample size, p is the number of parameter and AIC is as defined in (8).

Bayesian Information Criterion (BIC)

The Bayesian Information criterion (BIC) is defined as:

$$BIC = 2\log(n) - 2\mathcal{L} \qquad (10)$$

where $\mathcal{L}$ is the log-likelihood value, p is the number of parameters and n is the sample size. The advantage of BIC is that it involves a penalty term $2\log(n)$, which discourages overfitting of the model. For this measure, the smaller the BIC, the better the model.

L-Jung Box Statistic

The L-Jung Box test is used to test whether or not observations collected over time are time random and independent. Thus, in this case, we test the hypothesis:

$$H_o: The\ autocorrelations\ up\ to\ lag\ m\ are\ all\ zero$$

Versus

$$H_1: Autocorrelation\ of\ one\ or\ more\ lags\ differ\ from\ zero$$

The L-Jung Box statistic, a function of the accumulated squared sample autocorrelations, r_i^2, up to any specified time lag m, as a function of m, is defined as:

$$Q_m = n(n+2)\sum_{k=1}^{m}\frac{r_k^2}{n-k} \qquad (11)$$

Where n is the sample size, r_k, is the sample autocorrelation at lag k, and m is the number of lags been tested. We reject H_o at α level of significance, if $Q > \chi^2_{1-\alpha,m}$. Where $\chi^2_{1-\alpha,m}$ is the $1-\alpha$ quantile of the chi-squared distribution with m degree of freedom.

2.9. Data analysis and software

The first step of this project was to gather the desired data upon which we will base our study. The dataset used for this study is the historical quarterly performance indicators for pension accumulation funds which cover the year 2017 to 2021, from 18 different management companies. The data of each company have been extracted from the official website of Bank of Lithuania (https://www.lb.lt/en/pf-performance-indicators#ex-1-1). The data were thoroughly examined and checked for errors. The following three quantitative variables were extracted from the lot: (1) number of quarterly participants; (2) Percentage investment risk and (3) Net Asset Value.

After gathering all the data, the next step was to import it to Rstudio in order to be able to manipulate it using the R language. The data were formerly on excel worksheet, so the data were imported to R development environment to perform some data wrangling tasks such as splitting data into groups, data sampling and then aggregating data into summary statistics in order to come up with clear, understandable, up-to-date and reliable information aimed at achieving objectives of the research study. The study used descriptive statistics to analyze the data to establish patterns and relationships. The various methods for analyzing the data included tables, forecasting plots and narrative. The data was split into two periods: (1) training from January 2017 to December 2020, and (2) testing from January 2020 to December 2021. During the training period, this training data set was utilized to change each of the statistical models and estimate the parameters of the temporal components (seasonality, linear trend, and stochastic effect). During the test period, this test data was compared to values estimated by statistical models (ARIMA, BATS, and Naïve) in order to examine the predictive power of these models.

For the judgmental forecasting, google form was used to present the plot of the Number of participants along with the numerical values, to the participants who then used their judgment of the patterns depicted in the plots to provide forecasts for a year. Once the forecasts produced by the participant are considered satisfactory, then the participant can submit. The actual experiment consisted of three rounds, and in each round, the participants were provided with different information regarding previous forecasts derived from all the participants using their Gmail accounts. The purpose of this was to investigate how opinion of others affect the judgmental forecasts.

2.10. Chapter two summary

One of the most important problems facing pensioners when it comes to fund priority is developing a credible model that will rate the various options (what form of pension fund should be invested?). Typically, investors use a variety of criteria to make their investment decisions, some of which are contradictory. The factors for choosing a pension fund were highlighted in this chapter. The use of multi-criteria decision analysis (MCDA) models with a single decision maker (the potential investor) is then used. Specifically, we selected the Analytic Hierarchy Process (AHP). Prototypes of the real problem are used to demonstrate the models. Lastly, we evaluate the predictive ability of these models using the ARIMA, BATS, and Naïve methods.

3. Results

3.1 Introduction

This section presents the results obtained from the analysis described in the previous section. The summary statistics and the model building arising from the data analysis involving Lithuania Pension Portfolio. Discussion and conclusions are also all spelled out in details.

Table 4. The summary descriptive statistics of the data. The mean, median, maximum, minimum, 1st and 3rd quartile values of the variables are presented.

Series	1st Quartile	Mean	Median	Minimum	Maximum	3rd Quartile
Number of participants (1000)	774.40	3812.10	1762.10	81.00	27243.20	4056.00
Investment Risk (%)	3.113	6.496	6.413	1.009	11.328	9.930
Net Asset Value (million Euro)	1092237	6905216	2949825	95000	33112730	19401914
Fund Fees (%)	1.425	1.730	1.650	1.250	2.600	1.650

In order to have a quick overview of the summary statistics of the variables under study, we first provide in Table 4, the mean, median, minimum, maximum, 1st and 3rd quartile values for the four variables. From the results in the Table 4 it can be observed that the minimum number of participants, from the periods considered, is 81, 000 while the maximum number of the participants is 27,243,200. Also, the minimum and maximum values for the Percentage Investment Risk are, approximately, 1.01% and 11.33% respectively. The Table indicates that the minimum Net Asset value of the participants is €95 billion and the maximum Net Asset Value is £33, 112, 730 (million). It can also be observed that the minimum and maximum of percentage Fund Fees are 1.43% and 2.6%. Another important feature that can be observed in the Table is the fact that the maximum and median values of percentage Fund Fees tallied. This is not surprising since there's no significant variation in the values of this series. Hence, little to no attention was given to this variable in the subsequent analysis.

Table 5. Pension plans by Rank

Ranking	Pension Plan	Ranking	Pension Plan
1	Luminor Pension 2 plus	10	INVL_III_EQUITY
2	SEB Pension 2 plus	11	INVL Apdairus
3	INVL STABILO III	12	Luminor employee pension 2 plus
4	INVL_Extremo_III	13	Swedbank Supplementary Pension fund
5	Luminor Pension 1 plus	14	Luminor employee pension 1 plus
6	SEB Pension 1 plus	15	Swedbank Pension fund 100
7	Luminor Pension 3 plus	16	Swedbank Pension fund 60
8	INVL Medio III	17	SEB Pension 50
9	INVL Drasus	18	Swedbank Pension fund 30

The results presented in the Table 5 show the ranks of the various pension plans available to the participants. From the results, it is observed that the Luminors Pension 2+ is the top rated by the participants, followed by SEB pension 2+. The Swedbank Pension Fund30 is rated the lowest. As a result, further analysis was carried on Luminors Pension fund 2+.

3.2 Univariate Time-series Analysis Results of the Pension Portfolio

The following presents the outputs of univariate Time series analysis on the three series: Number of participants, Percentage Investment Risk and Net Assets Value using ARIMA model with order p, d, q and P, D, Q.

Table 6. Model for Forecasting Number of Participants

Best Model	Sar1	Standard error	Ljung-Box Sig.	MAPE	MAPE(Naive)	AICc
$\mathbf{ARIMA(0,2,0)x(1,0,0)_4}$	0.5904	0.1908	0.9453	0.8698	2.2606	205.53

From the results in Table 6, it is observed that the model that best fitted the Number of Participants is $\mathrm{ARIMA(0,2,0)x(1,0,0)_4}$. This is evident from the L-Jung Box test results which gave p-value =0.9453 >0.05 (significance level), which implies that the residuals produced by this model are independently distributed. In addition to that, the standard error of the parameter estimate produced by the model is very close zero (0.1908.

The Naïve models are the benchmark to compare with more complex models adopted in terms of performance (Hyndman and Athanasopoulos 2011). This step was carried out for the $\mathrm{ARIMA(0,2,0)x(1,0,0)_4}$ model to confirm its adequacy for forecasting the number of participants. Since the Naïve model produced MAPE of value 2.26% > the MAPE(ARIMA(0,2,0)(1,0,0)[4]) = 0.87%, we can conclude that the $\mathrm{ARIMA(0,2,0)x(1,0,0)_4}$ model correctly fit the number of participants data and hence can be used for forecasting the number of participants.

Table 7. Model for Forecasting Investment Risk

Best Model	Standard error	Ljung-Box Sig.	MAPE	MAPE(Naive)	AICc
ARIMA(0,1,0)	0.0436	0.3422	1.2843	6.3042	-3.67

The results in the Table 7 above indicate that the model that best fitted the percentage investment risk is ARIMA(0,1,0), a null model with difference of order 1. The standard error produced by this model is 0.0436, which indicates that the estimated values are very close to the actual series value. As such, the model is a good fit to the Percentage Investment risk.

Table 8. Model for Forecasting Net Assets Value

Best Model	Drift	Standard error	Ljung-Box Sig.	MAPE	MAPE(Naive)	AICc
ARIMA(0,1,0)	1286613.7	353833.1	0.0567	3.1983	10.4536	503.54

From the results in the Table 8, it can be concluded that the model that best fitted the percentage investment risk is ARIMA(0,1,0), a null model with difference of order 1. The presence of drift in this model indicates the average Net asset value. However, since the standard error of this model is extremely large, adopting this model for forecasting will lead to a very large confidence interval for prediction, leading to large forecasting errors.

The plots of graphical display of how these models perform with respect to the forecasts against the time at which they were obtained are presented in Fig. 2 to Fig. 9.

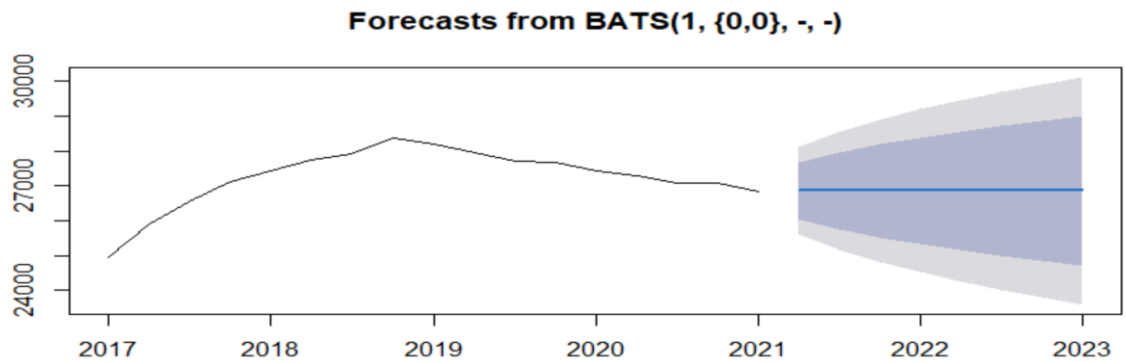

Fig. 2. Plots of forecast values produced by $BATS(1,\{0,0\},-,-)$

Fig. 2 shows the original series of the number of participants plotted against the time it was collected (2017 – 2021), the eight forecasted values which cover the year 2021 to 2023 using the BATS(1,{0,0},-,-) model, as well as the prediction intervals. The graph generally shows a downward trend after the sudden shift that occurred in Q4 of the year 2018. The large and rapidly increasing prediction intervals show that the number of participants could start increasing or decreasing at any time, while the point forecasts reduces over time.

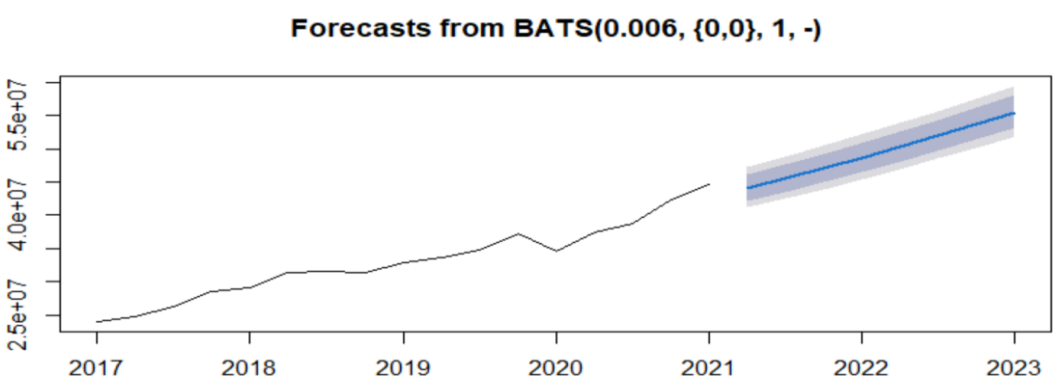

Fig. 3. Forecast values produced by $\mathrm{BATS}(0.006,\{0,0\},1,-)$ for Net Assets Value

Fig. 3 depicts the original series of the net assets value plotted against the time it was collected (2017-2021), the 8 forecasted values produced by BATS (0.006,{0,0},1,-) model which cover the

year 2021 to 2023, as well as the prediction interval. The graph generally shows an upward trend. This indicates that the net assets value continues to be on the increase after the year 2021.

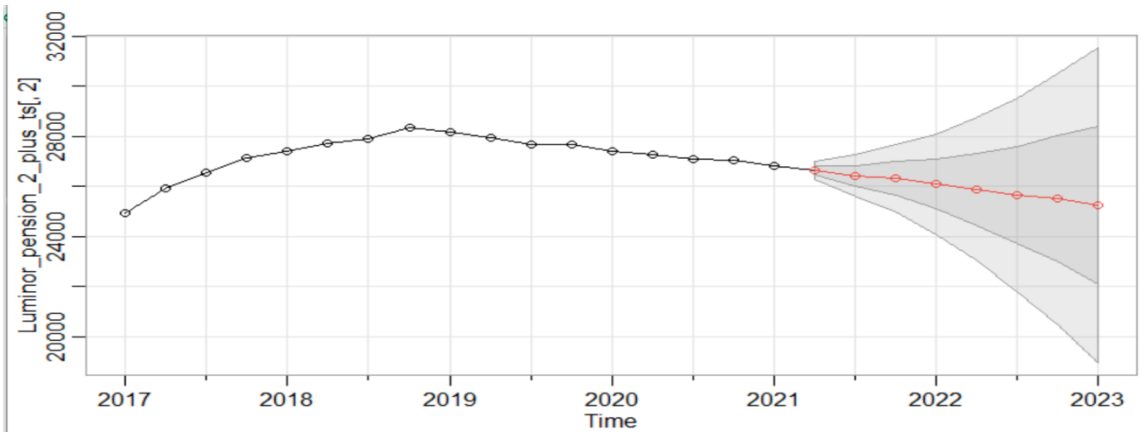

Fig. 4. Forecast values produced by $\mathrm{ARIMA}(0,2,0)\mathrm{x}(1,0,0)_4$ for Number of Participants

The plot above shows the original series of the number of participants plotted against the time it was collected (2017 – 2021), the 8 forecasted values which cover the year 2021 to 2023 using the $\mathrm{ARIMA}(0,2,0)\mathrm{x}(1,0,0)_4$, model, as well as the prediction interval. The graph generally shows a downward trend after the sudden shift that occurred in Q4 of the year 2018. The large and rapidly increasing prediction intervals show that the number of participants could start increasing or decreasing at any time, while the point forecasts reduces as the time grows. This confirms the results obtained by the BATS model in Fig. 2. This is not surprising, since if you are not paying more people pension, the number of applicants will reduce.

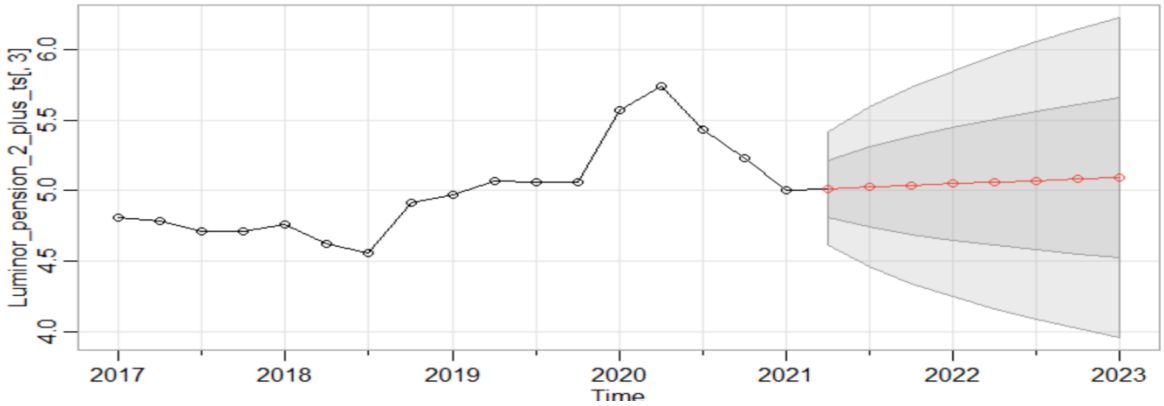

Fig. 5. Forecast values produced by $ARIMA(0,2,0)x(1,0,0)_4$ for Percentage Investment Risk

The plot above depicts the original series of the percentage investment risk against the time it was collected (2017 – 2021), as well as the 8 forecasted values which cover the year 2021 to 2023. The graph generally shows a cyclic and upward trend from Q2 of 2018 to 2020, followed by a sudden shift that occurred in Q1 of the year 2021. This indicates that the percentage investment risk was very high in the Q1 of 2021. The graph, however, indicate that percentage investment risk is

expected to be either increasing or decreasing due to the large prediction interval while the point forecast trend downward.

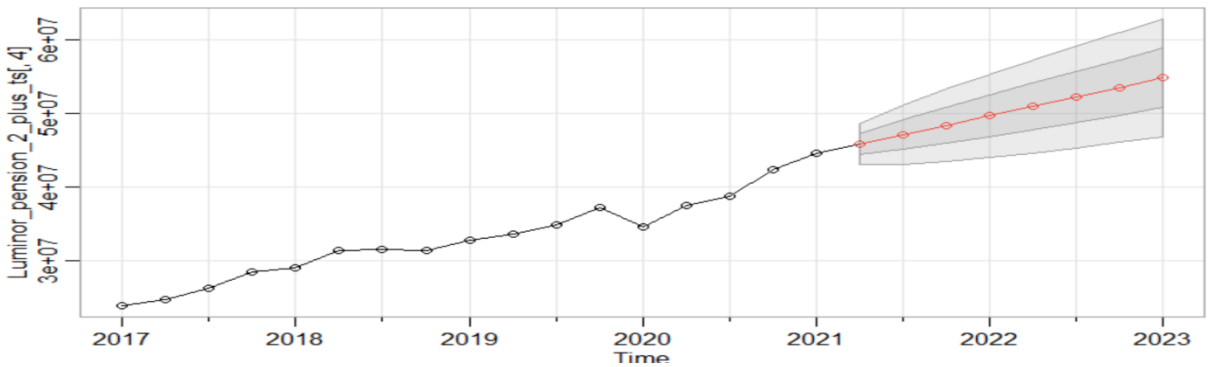

Fig. 6. Forecast values produced by $\mathrm{ARIMA}(0,2,0)\mathrm{x}(1,0,0)_4$ for Net Assets Value

The plot above shows the original series of the net assets value plotted against the time it was collected (2017 – 2021), the eight forecasted values which cover the year 2021 to 2023, as well as the prediction intervals. The graph generally shows an upward trend which is an indication that net assets value will continue to increase even after the year 2021.

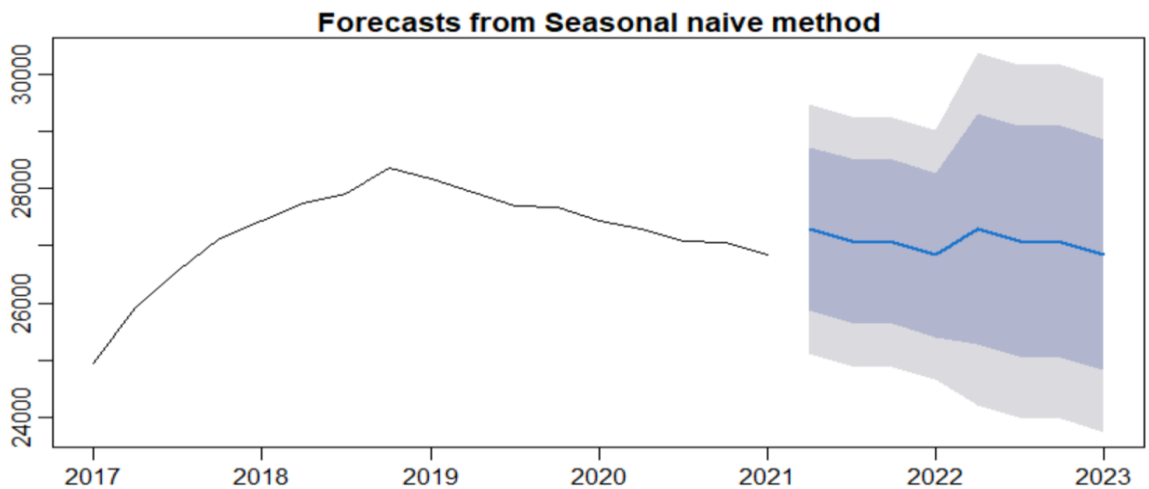

Fig. 7. Forecast values produced by Seasonal Naïve Method for Number of Participants

The plot above shows the original series of the number of participants plotted against the time it was collected (2017 – 2021), seasonal Naïve forecasted values which cover the year 2021 to 2023 using the seasonal Naïve model. Also, the prediction interval is provided by the graph. The graph generally shows a downward trend after the sudden shift that occurred in Q4 of the year 2018. The large and rapidly increasing prediction intervals show that the number of participants could start increasing or decreasing at any time, while the point forecasts trend downwards.

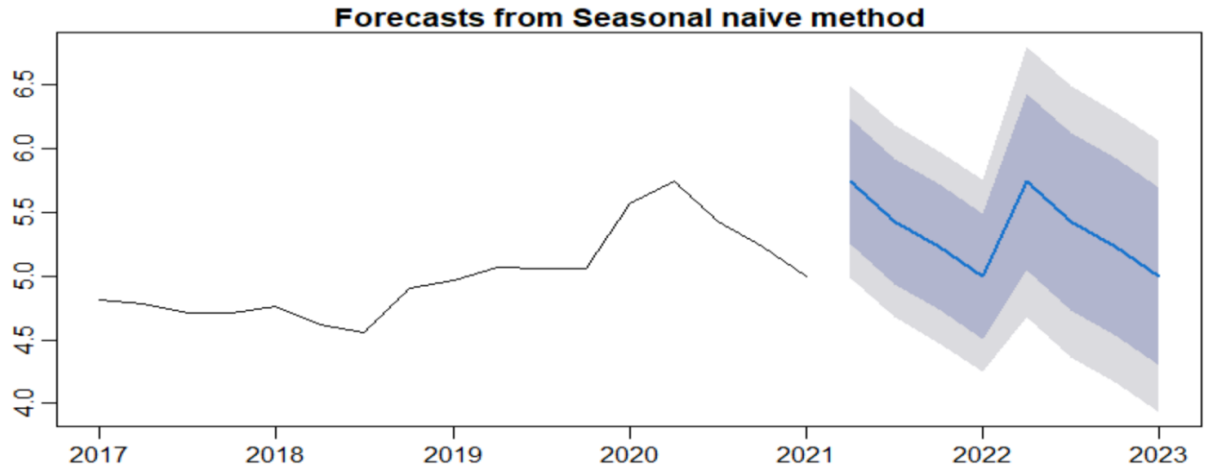

Fig. 8. Forecast values produced by Seasonal Naïve Percentage Investment Risk

The plot above depicts the original series of the percentage investment risk against the time it was collected (2017 – 2021), as well as the eight forecasted values which cover the year 2021 to 2023, using the seasonal naïve method. The graph generally shows a cyclic and upward trend from the year 2017 to 2020, followed by a sudden shift that occurred in Q1 of the year 2021. This indicates that the percentage investment risk goes very high in the Q1 of 2021. The forecasted values, however, indicate that percentage investment risk is expected to be increasing and decreasing year after year.

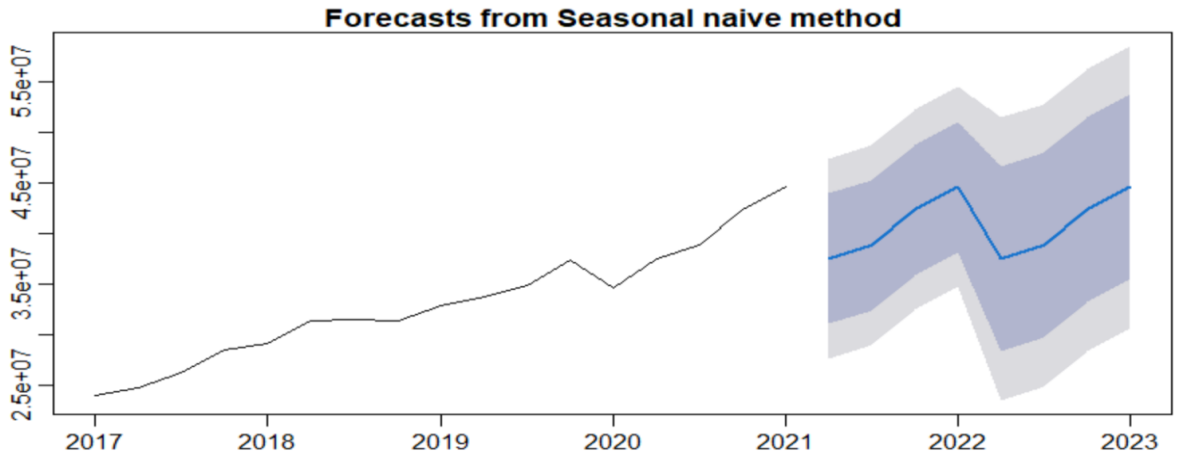

Fig. 9. Forecast values produced by Seasonal Naïve for Net Assets Value

The plot above depicts the original series of the net assets value against the time it was collected (2017 – 2021), as well as the 8 seasonal Naïve forecasted values which cover the year 2021 to 2023, and the prediction interval. The graph generally shows an upward trend followed by a sudden shift that occurred. This indicates that the percentage investment risk goes very high in the Q1 of 2021. The forecasted values, however, indicate that net assets value is expected to be increasing and damped.

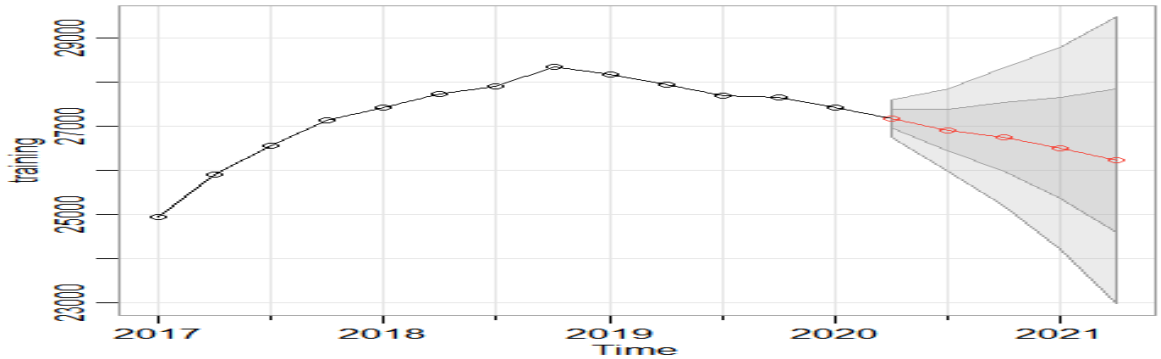

Fig. 10. Forecast values produced by $\mathrm{ARIMA}(0,2,0)(1,0,0)_4$ for Number of Participants

To compare the forecasts through the suggested model, $\mathrm{ARIMA}(0,2,0)(1,0,0)_4$, to the actual values of the process, the last five values were withheld. The plot above shows the original series of the number of participants plotted against the time, as well as the five forecasted values for the withheld data, using $\mathrm{ARIMA}(0,2,0)(1,0,0)_4$. The forecasts follow the recent trend in the data, because of the double differencing. The large and rapidly increasing prediction intervals show that the number of participants could start increasing or decreasing at any time, While the point forecasts tend to be negative, the prediction intervals allow the data to increase with time.

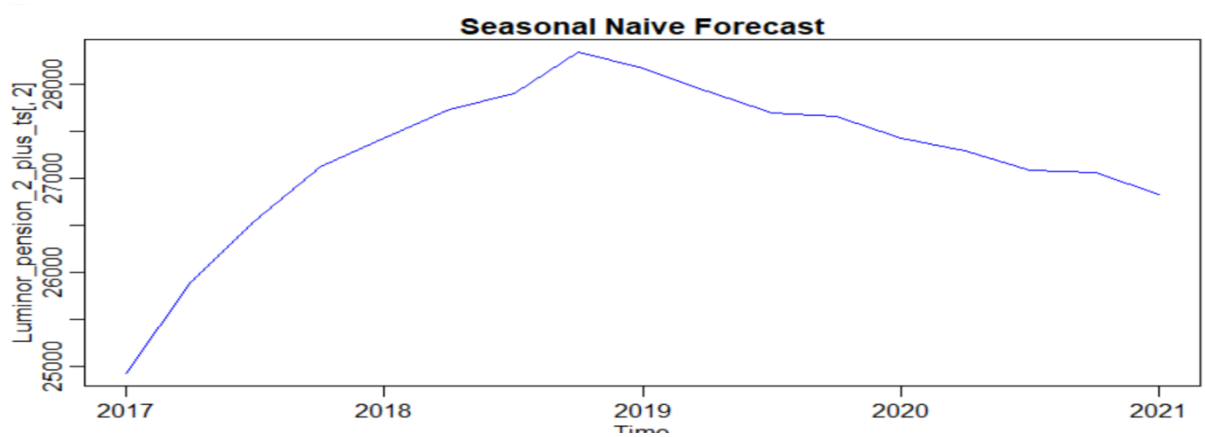

Fig. 11. Time series plot of Number of Participants

The plot above depicts the original series of the number of participants plotted against the time it was collected. The graph shows an upward trend starting from the year 2017 to the Q4 of 2018, then followed by a sudden shift in direction, and the trend decreases downwards. This confirms the results presented in Fig. 9.

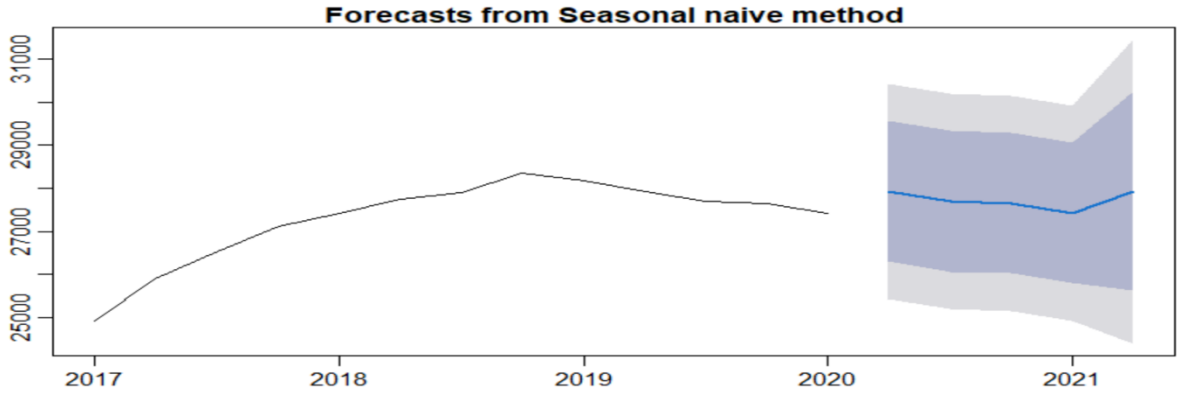

Fig. 12. Forecast values produced by Seasonal Naïve for Number of Participants

The plot above depicts the original series of the number of participants plotted against the time it was collected, the 8 forecasted values which cover the year 2020 to 2021. The graph generally shows a downward trend after the sudden shift that occurred in Q4 of the year 2018. However, the large and rapidly increasing prediction intervals show that the number of participants could start increasing or decreasing at any time, While the point forecasts tend to be negative, the prediction intervals allow the data to rise during the forecast period.

3.3 Judgmental Forecasting Results

The behavioral experiment was introduced to groups of undergraduates and postgraduates studying at various universities. The experiment was posted to several relevant groups on facebook and whatsapp, as explained in section 2.9. Five participants started the tasks, however, only three of them completed the tasks. This behavioral study of judgmental forecasting was limited to students, which is not uncommon in the literature (Deck and Smith 2013). Table 9 presents the distribution of participants across roles and gender (columns).

Table 9. Distribution of gender of the forecasters by student status (n = 7)

Status	Male	Female	Total
Undergraduate	3	1	4
Postgraduate	1	0	1
Total	4	1	5

Table 9 shows that majority of the forecasters were male (80%), same is observed for the student status, where majority were undergraduate and also accounts for % of the total number of forecasters in the study.

Table 10. Forecasts of Number of Participants by Judgment (1000) for Q2 2021 – Q1 2022

Round 1 Forecast						
Quarter	**A**	**B**	**C**	**D**	**E**	**Average**
Q2 2021	2800	2600	2800	2500	3000	2740
Q3 2021	2790	2400	2700	2400	2800	2618
Q4 2021	2750	2350	2675	2300	2700	2555
Q1 2022	2750	2550	2650	2200	2550	2540
Round 2 Forecast						
Q2 2021	2800	2750	2800	2700	2900	2790
Q3 2021	2790	2650	2700	2400	2800	2668
Q4 2021	2750	2650	2675	2300	2700	2615
Q1 2022	2750	2550	2650	2250	2600	2560
Round 3 Forecast						
Q2 2021	2800		2800		2900	2733
Q3 2021	2790		2700		2800	2700
Q4 2021	2750		2675		2700	2600
Q1 2022	2650		2650		2600	2550

Focusing on the first four rows of Table 10, it can be observed that the initial forecasts varied across the participants and the average of these are highly inflated. The forecasts are compared within the inputs of the forecasters until a high level of consensus was reached. This can be inferred from the last four rows, where the inputs of the participants were approximately the same. The averages of these forecasts were then taken as the forecast values. The graph below presents how judgmental forecasts compare to those of ARIMA model.

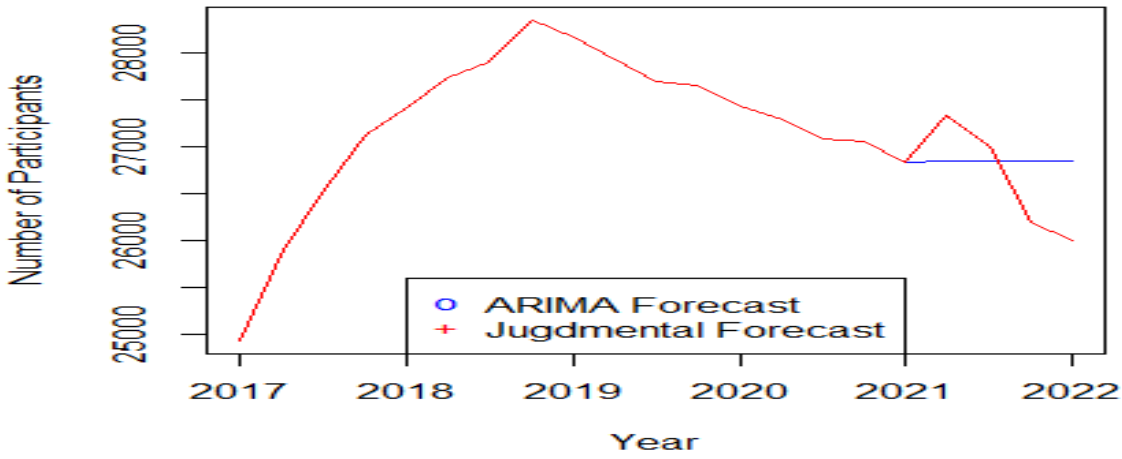

Fig. 13. Comparison between ARIMA model forecasts and Judgmental forecasts

Fig. 13 shows how these forecasts obtained by judgment compare with those obtained by a statistical model, in this case ARIMA model. It can be inferred from the graph that both the judgmental forecasting and statistical method were able to capture the downward trend in the Number of participants over time. However, the forecasts produced by ARIMA seem more sensible and follow the path of the actual number of participants.

3.4 Discussion

The various pension accumulation system, in Lithuania, includes three Pillars which are: SODRA (a state social security pension), the second pillar allows accumulating additional funds for old age while the third Pillar is independently accumulated pension based on an investment life insurance. In section two, we first reviewed various methods for analyzing time-dependent data, in details, and then implemented them on the secondary data which span a total of 3 years (2017 – 2021). However, due to the unavailability of data, the total sample size used is 17 observations, which calls for further validation with larger sample sizes. Nonetheless, we strongly believe in the results obtained in this study and can serve as a basis for future studies.

The analysis on Number of participants indicate that the ARIMA (0, 2, 0) (1, 0, 0) [4] model is relatively accurate. This is confirmed by the L-Jung Box test results which gave p-value =0.9453 >0.05 (significance level). Then, we have the matter of statistical test. The standard error of the parameter estimate produced by the model is very close zero (0.1908). This is not surprising, since the MAPE of this model is significantly lower than the one of Naïve, as presented in Table 6.

Also, looking at the results for the percentage, the model that best fitted the percentage investment risk is ARIMA (0, 1, 0), a null model with difference of order 1. The standard error produced by this model is 0.0436, which indicates that the estimated values are very close to the actual series value. However, the results on net assets values indicate a cause for caution. The best model for

this data is ARIMA (0, 1, 0) and it's accompany standard error is extremely large (2.137x10^12). The same was observed for the standard error of the parameter estimate. Hence, adopting this model for forecasting will lead to a very large confidence interval for prediction, leading to large forecasting errors. Nevertheless, forecasting can still be done using this model but its results should be used with proper care.

3.5 Conclusion

As established in chapter one, this work aimed at analyzing and assessing pension portfolio in Lithuania using a multi-criteria decision-making analysis. This study provided a number of findings, some of which are consistent with our expectation. In Lithuania, the pension accumulation system includes three pillars. The first pillar (SODRA) is the state social security pension, the second pillar allow accumulating additional funds for old age and the third pillar is independently accumulated pension based on an investment life insurance agreement. The purpose of which is to enable a person to secure a well-off old age and to independently accumulate his/her own pension. The private corporate entities, more specifically investment corporations founded by banks or insurance firms, handle second and third pillar pension funds. This indicates that pension funds are contractual in Lithuania.

From the results in Table 8, it can be concluded that Luminors Pension 2+ is the most sought after out of the 18 available pension plans in the states of Lithuania. Also, the results in this study showed that both BATS and ARIMA models were able to describe the process that produced the data. However, ARIMA model was preferred for forecasting the number of participants, Net Assets Value and Percentage Investment Risk because it is more reliable and easier to interpret based on fuzzy logic.

The results in Table 9 to the Table 12 showed that $ARIMA(0,2,0)(1,0,0)_4$, $ARIMA(0,1,0)$ and $ARIMA(0,1,0)$ best fitted the number of participants, Percentage Investment Fund and Net Assets Value respectively. These models were adjusted with the training time series from Q1 of 2017 to Q1 of 2020 and were tested against the actual data from Q2 of 2020 to Q1 of 2021. Thus, given the sensible standard error and allowable mean absolute percentage of error (MAPE) produced by these models, it can be concluded that the forecasts produced by these models are sensible and reliable. Therefore, it is strongly recommended that these models should aid in decision making for Pension Portfolio management to meet the needs of the people of Lithuania.

Finally, the results from the behavioral experimental study in Fig.13 and Table 13 concluded that given a systemic approach is adopted, judgmental forecasts can be used in place of statistical forecasts when the sample size is small to provide a useful overview of the scenario of the decision makers. However, this concern of small sample could be viewed as a possible limitation of this study. This shortage of historical data is understanding because in 2019, there was a change in the pension regime plus the COVID-19 pandemic. Future research can, however, build upon the results in this study when there are more historical data on pension portfolio selection.

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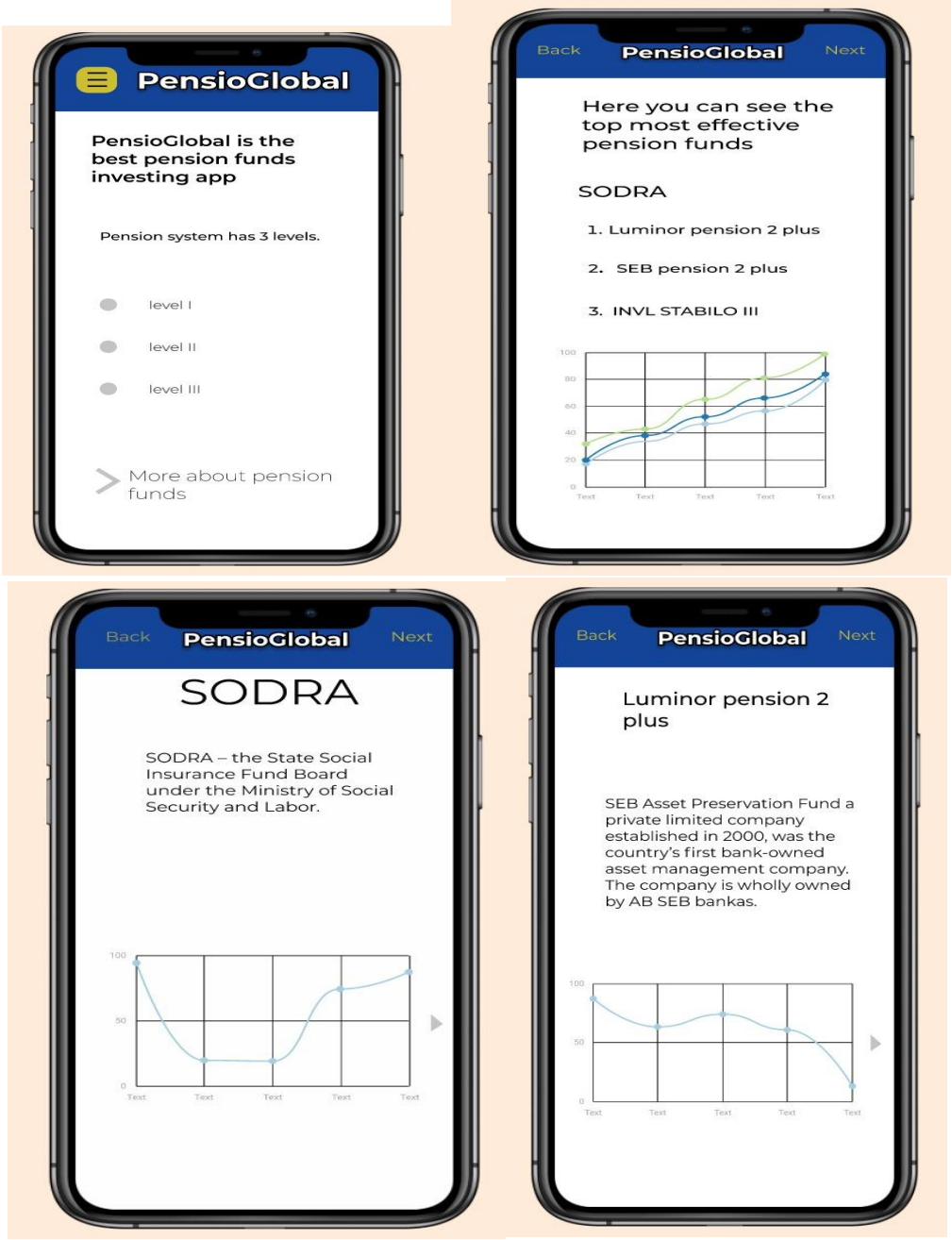
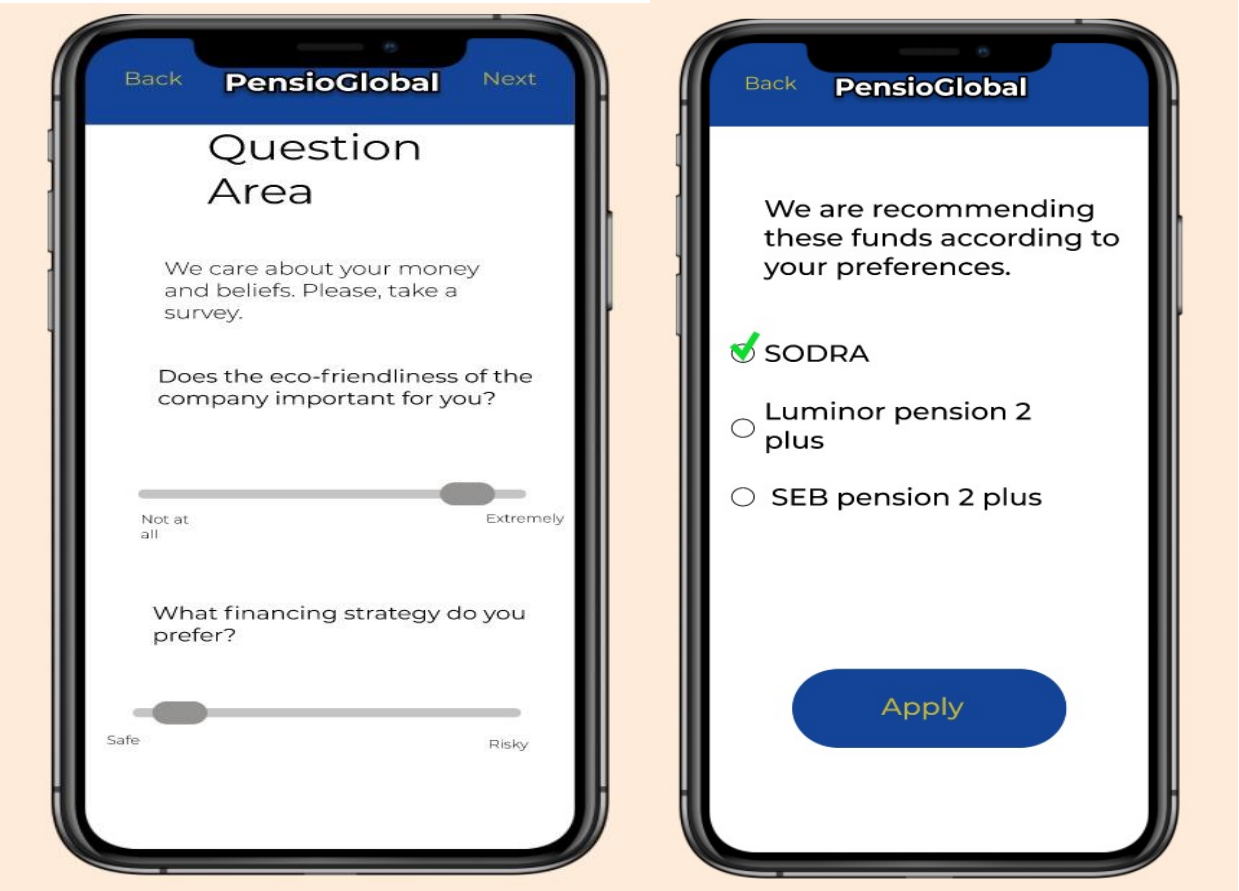
Appendixes

1. Prototype


PensioGlobal
PensioGlobal is the best pension funds investing app
Pension system has 3 levels.
level I
level II
level III
More about pension funds
Back
PensioGlobal
Next
Here you can see the top most effective pension funds
SODRA
1. Luminor pension 2 plus
2. SEB pension 2 plus
3. INVL STABILO III
Back
PensioGlobal
Next
SODRA
SODRA – the State Social Insurance Fund Board under the Ministry of Social Security and Labor.
Back
PensioGlobal
Next
Luminor pension 2 plus
SEB Asset Preservation Fund a private limited company established in 2000, was the country's first bank-owned asset management company. The company is wholly owned by AB SEB bankas.


Back
PensioGlobal
Next
Question Area
We care about your money and beliefs. Please, take a survey.
Does the eco-friendliness of the company important for you?
Not at all
Extremely
What financing strategy do you prefer?
Safe
Risky
Back
PensioGlobal
We are recommending these funds according to your preferences.
SODRA
Luminor pension 2 plus
SEB pension 2 plus
Apply

2. R Codes

R codes
library(readxl) library(forecast) library(MLmetrics) library(xts) library(astsa) library(MCDA) library(MCDM) library(dplyr) ## DATA PREPARATION ## P3P_data <- read_xlsx("C:/Users/User/Documents/Master_Thesis/Data_file_Master_3.xlsx", sheet = "Sheet1") # Data file values = seq(from = as.Date("2017-03-31"), to = as.Date("2021-03-31"), by = 'quarter') # Setting dates SEB_pension_2_plus <- P3P_data[3:19,] # Extracting specific fund data

```
cols <- P3P_data[2,2:7]                                                    # Selecting where
column names are

colnames(SEB_pension_2_plus) <- c("Date", cols[1,])                                  # Giving
correct column names to extracted data

SEB_pension_2_plus[, 2:7] <- sapply(SEB_pension_2_plus[, 2:7], as.numeric)                  #
Converting numeric data to numeric type

SEB_pension_2_plus$Date <- values                                          # Adding date
column to data

SEB_pension_2_plus$Name <- rep("SEB_pension_2_plus", 17)                             #
Setting pension fund name as a separate column (will be used later for aggregation)

SEB_pension_2_plus_ts <- ts(SEB_pension_2_plus, start=c(2017, 1), end=c(2021, 1),
frequency=4)  # Converting data to time series (used for forecasting). Time series for every
fund ends with _ts



INVL_Extremo_III <- P3P_data[25:41,]                                       # The same
steps are repeated for every fund below (because I was to lazy to make a loop for that)

colnames(INVL_Extremo_III) <- c("Date", cols[1,])

INVL_Extremo_III[, 2:7] <- sapply(INVL_Extremo_III[, 2:7], as.numeric)

INVL_Extremo_III$Date <- values

INVL_Extremo_III$Name <- rep("INVL_Extremo_III", 17)

INVL_Extremo_III_ts <- ts(INVL_Extremo_III, start=c(2017, 1), end=c(2021, 1),
frequency=4)



INVL_STABILO_III <- P3P_data[46:62,]

colnames(INVL_STABILO_III) <- c("Date", cols[1,])

INVL_STABILO_III[, 2:7] <- sapply(INVL_STABILO_III[, 2:7], as.numeric)

INVL_STABILO_III$Date <- values

INVL_STABILO_III$Name <- rep("INVL_STABILO_III", 17)
```

```
INVL_STABILO_III_ts <- ts(INVL_STABILO_III, start=c(2017, 1), end=c(2021, 1), frequency=4)


INVL_Medio_III <- P3P_data[67:83,]

colnames(INVL_Medio_III) <- c("Date", cols[1,])

INVL_Medio_III[, 2:7] <- sapply(INVL_Medio_III[, 2:7], as.numeric)

INVL_Medio_III$Date <- values

INVL_Medio_III$Name <- rep("INVL_Medio_III", 17)

INVL_Medio_III_ts <- ts(INVL_Medio_III, start=c(2017, 1), end=c(2021, 1), frequency=4)


SEB_pension_1_plus <- P3P_data[88:104,]

colnames(SEB_pension_1_plus) <- c("Date", cols[1,])

SEB_pension_1_plus[, 2:7] <- sapply(SEB_pension_1_plus[, 2:7], as.numeric)

SEB_pension_1_plus$Date <- values

SEB_pension_1_plus$Name <- rep("SEB_pension_1_plus", 17)

SEB_pension_1_plus_ts <- ts(SEB_pension_1_plus, start=c(2017, 1), end=c(2021, 1), frequency=4)


Luminor_employee_pension_1_plus <- P3P_data[109:125,]

colnames(Luminor_employee_pension_1_plus) <- c("Date", cols[1,])

Luminor_employee_pension_1_plus[, 2:7] <- sapply(Luminor_employee_pension_1_plus[, 2:7], as.numeric)

Luminor_employee_pension_1_plus$Date <- values

Luminor_employee_pension_1_plus$Name <- rep("Luminor_employee_pension_1_plus", 17)

Luminor_employee_pension_1_plus_ts <- ts(Luminor_employee_pension_1_plus, start=c(2017, 1), end=c(2021, 1), frequency=4)
```

```
Luminor_employee_pension_2_plus <- P3P_data[130:146,]

colnames(Luminor_employee_pension_2_plus) <- c("Date", cols[1,])

Luminor_employee_pension_2_plus[, 2:7] <- sapply(Luminor_employee_pension_2_plus[, 2:7], as.numeric)

Luminor_employee_pension_2_plus$Date <- values

Luminor_employee_pension_2_plus$Name <- rep("Luminor_employee_pension_2_plus", 17)

Luminor_employee_pension_2_plus_ts <- ts(Luminor_employee_pension_2_plus, start=c(2017, 1), end=c(2021, 1), frequency=4)


Luminor_pension_1_plus <- P3P_data[151:167,]

colnames(Luminor_pension_1_plus) <- c("Date", cols[1,])

Luminor_pension_1_plus[, 2:7] <- sapply(Luminor_pension_1_plus[, 2:7], as.numeric)

Luminor_pension_1_plus$Date <- values

Luminor_pension_1_plus$Name <- rep("Luminor_pension_1_plus", 17)

Luminor_pension_1_plus_ts <- ts(Luminor_pension_1_plus, start=c(2017, 1), end=c(2021, 1), frequency=4)


Luminor_pension_2_plus <- P3P_data[172:188,]

colnames(Luminor_pension_2_plus) <- c("Date", cols[1,])

Luminor_pension_2_plus[, 2:7] <- sapply(Luminor_pension_2_plus[, 2:7], as.numeric)

Luminor_pension_2_plus$Date <- values

Luminor_pension_2_plus$Name <- rep("Luminor_pension_2_plus", 17)

Luminor_pension_2_plus_ts <- ts(Luminor_pension_2_plus, start=c(2017), end=c(2021), frequency=4)
```

```
Luminor_pension_3_plus <- P3P_data[193:209,]

colnames(Luminor_pension_3_plus) <- c("Date", cols[1,])

Luminor_pension_3_plus[, 2:7] <- sapply(Luminor_pension_3_plus[, 2:7], as.numeric)

Luminor_pension_3_plus$Date <- values

Luminor_pension_3_plus$Name <- rep("Luminor_pension_3_plus", 17)

Luminor_pension_3_plus_ts <- ts(Luminor_pension_3_plus, start=c(2017, 1), end=c(2021, 1),
frequency=4)


INVL_III_EQUITY <- P3P_data[256:272,]

colnames(INVL_III_EQUITY) <- c("Date", cols[1,])

INVL_III_EQUITY[, 2:7] <- sapply(INVL_III_EQUITY[, 2:7], as.numeric)

INVL_III_EQUITY$Date <- values

INVL_III_EQUITY$Name <- rep("INVL_III_EQUITY", 17)

INVL_III_EQUITY_ts <- ts(INVL_III_EQUITY, start=c(2017, 1), end=c(2021, 1),
frequency=4)


Swedbank_supplementary_pension_fund <- P3P_data[278:294,]

colnames(Swedbank_supplementary_pension_fund) <- c("Date", cols[1,])

Swedbank_supplementary_pension_fund[, 2:7] <-
sapply(Swedbank_supplementary_pension_fund[, 2:7], as.numeric)

Swedbank_supplementary_pension_fund$Date <- values

Swedbank_supplementary_pension_fund$Name <-
rep("Swedbank_supplementary_pension_fund", 17)

Swedbank_supplementary_pension_fund_ts <- ts(Swedbank_supplementary_pension_fund,
start=c(2017, 1), end=c(2021, 1), frequency=4)
```

```
INVL_Drasus <- P3P_data[300:316,]

colnames(INVL_Drasus) <- c("Date", cols[1,])

INVL_Drasus[, 2:7] <- sapply(INVL_Drasus[, 2:7], as.numeric)

INVL_Drasus$Date <- values

INVL_Drasus$Name <- rep("INVL_Drasus", 17)

INVL_Drasus_ts <- ts(INVL_Drasus, start=c(2017, 1), end=c(2021, 1), frequency=4)


INVL_Apdairus <- P3P_data[321:337,]

colnames(INVL_Apdairus) <- c("Date", cols[1,])

INVL_Apdairus[, 2:7] <- sapply(INVL_Apdairus[, 2:7], as.numeric)

INVL_Apdairus$Date <- values

INVL_Apdairus$Name <- rep("INVL_Apdairus", 17)

INVL_Apdairus_ts <- ts(INVL_Apdairus, start=c(2017, 1), end=c(2021, 1), frequency=4)


Swedbank_Pension_fund_30 <- P3P_data[342:358,]

colnames(Swedbank_Pension_fund_30) <- c("Date", cols[1,])

Swedbank_Pension_fund_30[, 2:7] <- sapply(Swedbank_Pension_fund_30[, 2:7], as.numeric)

Swedbank_Pension_fund_30$Date <- values

Swedbank_Pension_fund_30$Name <- rep("Swedbank_Pension_fund_30", 17)

Swedbank_Pension_fund_30_ts <- ts(Swedbank_Pension_fund_30, start=c(2017, 1),
end=c(2021, 1), frequency=4)


Swedbank_Pension_fund_60 <- P3P_data[363:379,]

colnames(Swedbank_Pension_fund_60) <- c("Date", cols[1,])

Swedbank_Pension_fund_60[, 2:7] <- sapply(Swedbank_Pension_fund_60[, 2:7], as.numeric)
```

```
Swedbank_Pension_fund_60$Date <- values

Swedbank_Pension_fund_60$Name <- rep("Swedbank_Pension_fund_60", 17)

Swedbank_Pension_fund_60_ts <- ts(Swedbank_Pension_fund_60, start=c(2017, 1),
end=c(2021, 1), frequency=4)



Swedbank_Pension_fund_100 <- P3P_data[384:400,]

colnames(Swedbank_Pension_fund_100) <- c("Date", cols[1,])

Swedbank_Pension_fund_100[, 2:7] <- sapply(Swedbank_Pension_fund_100[, 2:7],
as.numeric)

Swedbank_Pension_fund_100$Date <- values

Swedbank_Pension_fund_100$Name <- rep("Swedbank_Pension_fund_100", 17)

Swedbank_Pension_fund_100_ts <- ts(Swedbank_Pension_fund_100, start=c(2017, 1),
end=c(2021, 1), frequency=4)



SEB_Pension_50 <- P3P_data[405:421,]

colnames(SEB_Pension_50) <- c("Date", cols[1,])

SEB_Pension_50[, 2:7] <- sapply(SEB_Pension_50[, 2:7], as.numeric)

SEB_Pension_50$Date <- values

SEB_Pension_50$Name <- rep("SEB_Pension_50", 17)

SEB_Pension_50_ts <- ts(SEB_Pension_50, start=c(2017, 1), end=c(2021, 1), frequency=4)



#a <- summary(INVL_Extremo_III)
```

```
################################################ Preparation and aggregation of
data for fund ranking) ##############################


FULL_TABLE <- rbind(INVL_Apdairus, INVL_Drasus, INVL_III_EQUITY,
INVL_Extremo_III, INVL_Medio_III, INVL_STABILO_III,

        Luminor_pension_1_plus, Luminor_pension_2_plus, Luminor_pension_3_plus,
Luminor_employee_pension_1_plus,

        Luminor_employee_pension_2_plus, SEB_pension_1_plus,
SEB_pension_2_plus, SEB_Pension_50,

        Swedbank_Pension_fund_100, Swedbank_Pension_fund_30,
Swedbank_Pension_fund_60, Swedbank_supplementary_pension_fund)


# I don't know by which aggregation you need to collect data for every fund. So these are a
few aggregated data sets (mean value for fund or last value for fund, etc. You choose, but for
analysis I'm using mean value)


FULL_TABLE_mean <- aggregate(FULL_TABLE[, 2:7], list(FULL_TABLE$Name), mean,
na.rm = TRUE) # Average velues per fund


FULL_TABLE_min <- aggregate(FULL_TABLE[, 2:7], list(FULL_TABLE$Name), min,
na.rm = TRUE) # Min values per fund


FULL_TABLE_max <- aggregate(FULL_TABLE[, 2:7], list(FULL_TABLE$Name), max,
na.rm = TRUE) # Max values per fund


FULL_TABLE_sum <- aggregate(FULL_TABLE[, 2:7], list(FULL_TABLE$Name), sum,
na.rm = TRUE) # Total values per fund
```

```
FULL_TABLE_last <- aggregate(FULL_TABLE[, 2:7], list(FULL_TABLE$Name), last,
na.rm = TRUE) # Last values (2021 Q1) per fund


summary(FULL_TABLE_mean)


rownames(FULL_TABLE_mean) <- FULL_TABLE_mean$Group.1


################################################## PENSION FUND RANKING
(use MCDA or MCDM or both) ##################################


# MCDA


normalizationTypes <- c("percentageOfMax","percentageOfMax")


names(normalizationTypes) <- c("g1","g2")


nPT <- normalizePerformanceTable(FULL_TABLE_mean[,c(2,4)],normalizationTypes)


w <- c(0.5,0.5)

names(w) <- colnames(FULL_TABLE_mean[,c(2,4)])

ws<-weightedSum(FULL_TABLE_mean[,c(2,4)],w)

rankings <- rank(-ws)

rankings


# MCDM
```

```
d <- as.matrix(FULL_TABLE_mean[,c(2,4)])

w <- c(0.5,0.5)

cb <- c('max','max')

lambda <- 0.5

v <- 0.5


AB <- matrix(c(	81.0000,	27243.1765	,	95000.0, 33112730.0),nrow = 2,ncol=2)  # <-- I don't understand these

CD <- matrix(c(	27243.1765,	27243.1765, 33112730.0, 33112730.0),nrow = 2,ncol=2) #


MetaRanking(d,w,cb,lambda,v,AB,CD) # All ranking methods in MCDM package, doesn't fully work, because RIM doesn't work.


MMOORA(d, w, cb)


RIM(d, w, AB, CD) # Doesn't work


TOPSISLinear(d, w, cb)


TOPSISVector(d, w, cb)


VIKOR(d, w, cb, v)


WASPAS(d, w, cb, lambda)
```

```
# Example of RIM, that works


#d <- matrix(c(30,40,25,27,45,0,9,0,0,15,2,1,3,5,2,3,3,1,3,2,3,2,3,3,3,2,2,2,1,4),

#           nrow = 5, ncol = 6)

#w <- c(0.2262,0.2143,0.1786,0.1429,0.119,0.119)

#AB = matrix(c(23,60,0,15,0,10,1,3,1,3,1,5),nrow = 2,ncol = 6)

#CD = matrix(c(30,35,10,15,0,0,3,3,3,3,4,5),nrow = 2,ncol = 6)

#RIM(d,w,AB,CD)




########################################### FORECASTING (only the top rated fund)
#######################################################


#data=AirPassengers


# Tbats model (test)


tbats_model = tbats(Luminor_pension_2_plus_ts[,2])

tbats_forecast = forecast(tbats_model, h=8)

plot(tbats_forecast)

MAPE(tbats_forecast$mean, validation) * 100
```

```
# I don't know for which column exactly forecasting needs to be done, so [,2], [,3] and [,4] means first 3 numerical columns for every fund.

# Here forecasting is only done for Luminor_pension_2_plus fund (because it ranked first), if you want to forecast other funds, you basically only need to change the name of the fund


#################### ARIMA forecasting


# Forecasting Number of Participants


arima_optimal = auto.arima(Luminor_pension_2_plus_ts[,2]) # arima_optimal finds the optimal configuration of ARIMA model, use these results to set parameters for sarima.for() function

arima_optimal


sarima_forecast_1 = sarima.for(Luminor_pension_2_plus_ts[,2], n.ahead=8, p=0,d=2,q=0,P=1,D=0,Q=0,S=4) # Set these numbers based from the result from arima_optional. Here, for example is ARIMA(0,2,0)(1,0,0)[4]

                                                                    # n.ahead=8 also means forecasting for 2 years (8 quarters). But you can set it to whatever you want

Box.test(arima_optimal$resid, lag=5, type="Ljung-Box")


# Forecasting Investment Risk


arima_optimal = auto.arima(Luminor_pension_2_plus_ts[,3])

arima_optimal
```

```
sarima_forecast_2 = sarima.for(Luminor_pension_2_plus_ts[,3], n.ahead=8, p=0,d=1,q=0)

# Forecasting NET asset value

arima_optimal = auto.arima(Luminor_pension_2_plus_ts[,4])
arima_optimal

sarima_forecast_3 = sarima.for(Luminor_pension_2_plus_ts[,4], n.ahead=8, p=0,d=1,q=0)



#################### Naive forecast

# Naive forecast Number of Participants
naive_1 = snaive(Luminor_pension_2_plus_ts[,2], h=8)
plot(naive_1)

naive_2 = snaive(Luminor_pension_2_plus_ts[,3], h=8)
plot(naive_2)

naive_3 = snaive(Luminor_pension_2_plus_ts[,4], h=8)
plot(naive_3)
```

```
# Comparing a few models by dividing time series into training and validation and checking
MAPE (if you need that of course)


# Arima MAPE checking


training=window(Luminor_pension_2_plus_ts[,2], start = c(2017), end = c(2020))

validation=window(Luminor_pension_2_plus_ts[,2], start = c(2020))


arima_optimal = auto.arima(training)

arima_optimal


sarima_forecast_4 = sarima.for(training, n.ahead=8, p=0,d=2,q=0,P=1,D=0,Q=0,S=4)


MAPE(sarima_forecast_4$pred, validation) * 100 # MAPE for ARIMA



# Naive forecast MAPE checking


naive = snaive(training, h=h=length(validation))

MAPE(naive$mean, validation) * 100          # MAPE for Naive


plot(Luminor_pension_2_plus_ts[,2], col="blue", main="Seasonal Naive Forecast", type='l')

lines(naive$mean, col="red", lwd=2)

plot(naive)
```

```
######################## VARIABLE IMPORTANCE
##############################################################################


names(Luminor_pension_2_plus)

y <- "Net asset value(EUR millions)"

df <- Luminor_pension_2_plus[,c(2,3,4,7)]

colnames(df) <- c( "No_of_participants", "Investment_risk", "Net_asset_value", "Fund_fees")

dfy <- df[,c(3)]

dfx <- df[,c(1,2,4)]

dfx <- as.data.frame(dfx)

dfy <- as.data.frame(dfy)


library(randomForest)

require(randomForest)

fit <- randomForest(Net_asset_value ~ ., data=df, ntree=500,

                    importance = TRUE, na.action = na.omit)


# Print regression model

print(fit)


plot(fit)


importance(fit)


varImpPlot(fit,type=1)
```

```
################### Multivariate forecasting / Vector Autoregressions
##########################################################################

library(dynlm)



LUM_Net_asset_value <- ts(Luminor_pension_2_plus_ts[,4],

            start = c(2017, 1),

            end = c(2021, 1),

            frequency = 4)


LUM_No_of_participants <- ts(Luminor_pension_2_plus_ts[,2],

                    start = c(2017, 1),

                    end = c(2021, 1),

                    frequency = 4)


LUM_Investment_risk <- ts(Luminor_pension_2_plus_ts[,3],

                      start = c(2017, 1),

                      end = c(2021, 1),

                      frequency = 4)



# Estimate both equations using 'dynlm()'
```

```
VAR_EQ1 <- dynlm(LUM_Net_asset_value ~ L(LUM_Net_asset_value, 1:2) +
L(LUM_No_of_participants, 1:2),
            start = c(2017, 1),
            end = c(2021, 1))


VAR_EQ2 <- dynlm(LUM_No_of_participants ~ L(LUM_Net_asset_value, 1:2) +
L(LUM_No_of_participants, 1:2),
            start = c(2017, 1),
            end = c(2021, 1))


VAR_EQ3 <- dynlm(LUM_Investment_risk ~ L(LUM_Net_asset_value, 1:2) +
L(LUM_Investment_risk, 1:2),
            start = c(2017, 1),
            end = c(2021, 1))


VAR_EQ4 <- dynlm(LUM_Net_asset_value ~ L(LUM_Net_asset_value, 1:2) +
L(LUM_Investment_risk, 1:2),
            start = c(2017, 1),
            end = c(2021, 1))


# rename regressors for better readability
names(VAR_EQ1$coefficients) <- c("Intercept","nav_t-1",
                        "nav_t-2", "nop_t-1", "nop_t-2")
names(VAR_EQ2$coefficients) <- names(VAR_EQ1$coefficients)


names(VAR_EQ3$coefficients) <- c("Intercept","nav_t-1",
```

```
                                "nav_t-2", "ir_t-1", "ir_t-2")


names(VAR_EQ3$coefficients) <- names(VAR_EQ3$coefficients)


library(lmtest)

coeftest(VAR_EQ1, vcov. = sandwich)


library(car)

library(vars)

linearHypothesis(VAR_EQ1,

                hypothesis.matrix = c("nav_t-1", "nav_t-2"),

                vcov. = sandwich)


linearHypothesis(VAR_EQ2,

                hypothesis.matrix = c("nop_t-1", "nop_t-2"),

                vcov. = sandwich)


linearHypothesis(VAR_EQ3,

                hypothesis.matrix = c("ir_t-1", "ir_t-2"),

                vcov. = sandwich)


#linearHypothesis(VAR_EQ4,

#                 hypothesis.matrix = c("nav_t-1", "nav_t-2"),

#                 vcov. = sandwich)
```

```
VAR_data <- window(ts.union(LUM_Net_asset_value, LUM_No_of_participants), start =
c(2017, 1), end = c(2021, 1))


VAR_data_2 <- window(ts.union(LUM_Net_asset_value, LUM_Investment_risk), start =
c(2017, 1), end = c(2021, 1))


library(vars)

# estimate model coefficients using `VAR()`

VAR_est <- VAR(y = VAR_data, p = 2)

VAR_est


VAR_est_2 <- VAR(y = VAR_data_2, p = 2)

VAR_est_2


forecasts <- predict(VAR_est)

forecasts


forecasts2 <- predict(VAR_est_2)

forecasts2




plot(forecasts)

plot(forecasts2)
```