


# Numerical and experimental analysis of membrane with piezoelectric element used for synthetic jet flow control

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## 1. Introduction

Nowadays, piezoelectric applications include smart materials for vibration control, aerospace and aeronautic applications of flexible surfaces and structures [1-2]. More specifically, piezoelectric membranes are used to drive synthetic jet actuators for aeronautical flow control applications [3-4].

The study of synthetic jets (SJ) has emerged as an intriguing new field in the past few years. In fluid dynamics, synthetic jet flow is a jet flow synthesized from ambient fluid [5-6]. A jet flow is a flow of fluid where a stream of fluid mixes with the surrounding medium. Synthetic jet flow can be generated in a number of ways, for example, using electromagnetic, piezoelectric, or even a mechanical driver, such as a piston. Each installation moves a membrane up and down hundreds of times per second, sucking the surrounding fluid into a chamber and then expelling it.

Synthetic jet modules have been widely researched in controlling the airflow of aircrafts aiming to enhance their lifting and manoeuvrability, control stalls, and reduce noise [7]. The practical application of this technology faces the following problems: weight, size, response time, force, and the complexity of controlling air flows.

The actuators had to possess enough power to produce the required displacement. Previous designs involved piezoelectric membranes in generating a synthetic jet [8-9]. The main arguments for their usage are low mass, good response frequency, and cost-effectiveness. The only drawback is the space required to integrate them. A piezoelectric membrane consists of a membrane made of a flexible material onto which a piezoelectric element is bonded. The piezoelectric element excites the membrane bending the modes by contracting and extending at a defined frequency.

This article presents a numerical and experimental modal analysis of membrane with bonded piezoelectric element Sonox P502. The aim of these investigations was to compare the theoretical and experimental data that are important for the control of synthetic jet devices. As is known, in order to obtain proper control of flow, it is necessary to find the most suitable mode shapes and achieve the maximum displacement of the membrane. The experimental results of the synthetic jet flow control with

the analysed piezoelectric membrane are also presented in this paper.

Using the scanning laser vibrometer Polytec, vibration mode shapes and displacements of piezoelectric membrane were defined by means of changing control frequencies.

A numerical simulation of piezoelectric membrane with bonded element Sonox P502 was performed aiming to validate the operating principle through the modal analysis. The simulation was performed using FEM (finite element method) software ANSYS 12.1. Modal analysis of piezoelectric membrane was performed aimed at finding proper resonance frequencies and mode shapes. The results of modal analysis should show that mode shapes are similar to the experimentally identified shapes.

In order to prove the validity of the model and the reliability of the obtained results a Modal Assurance Criterion (MAC) matrix was calculated with a help of MATLAB R2011b software [10]. Analysis and evaluation of the obtained results are described in this paper. MAC was chosen because over the last 30 years the modal assurance criterion has demonstrated how this simple statistical concept can become an extremely useful tool in the field of experimental modal analysis and structural dynamics.

In order to check the suitability of the previously received data about the piezoelectric membrane, an experimental analysis of the synthetic jet generator was performed. The Constant Temperature Anemometers technique has been used for this task. It is known that the CTA (Constant Temperature Anemometers) [11-13] technique is very sensitive and can be used for the determination of frequencies and the velocity of flow. Experimental data (amplitude-frequency measurements) were carried out by Dantec equipment and are presented in this paper.

## 2. Materials and methods

### 2.1. Design of the synthetic jet generator

A synthetic jet is created at the slot by oscillation and deflection of a membrane attached to the bottom of the jet chamber. In this case, the synthetic jet chamber consists of a membrane oscillating in a circular space, and an

orifice opposite to the membrane (Fig. 1). The oscillation of a membrane generates two flows in the orifice, namely the intake and exhaust flows. The intake flow separates and forms a vortex sheet that rolls-up into a single vortex as it moves away from the orifice at its self-induced velocity. If the velocity is sufficiently high, the entrance to the chamber becomes limited and the air jet forms. Thus, a linear momentum is transferred to the flow system even if the net mass injection is equal to zero. Consequently, these jets are also called “zero net mass flux” jets [14-15]. It is important to mention that a synthetic jet is very sensitive to the parameters of the device, i.e. the sizes and forms of its cavity  $D$ ,  $H$  and orifice  $d$ ,  $h$ , displacement and velocity  $\omega$  of the brass membrane, etc.

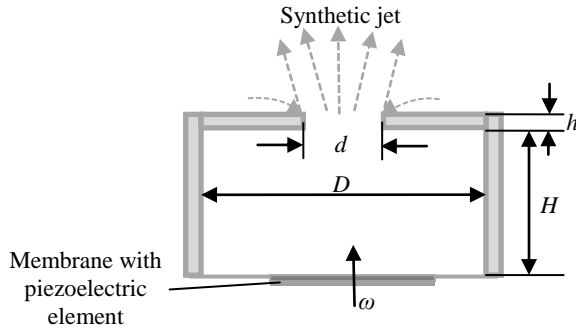


Fig. 1 The structure of the synthetic jet generator

$$s_{11}^E = s_{22}^E = 18.5p \left( \frac{m^2}{N} \right), s_{33}^E = 20.7 \left( \frac{m^2}{N} \right), s_{12}^E = -6.29p \left( \frac{m^2}{N} \right), s_{13}^E = -6.23p \left( \frac{m^2}{N} \right), s_{44}^E = s_{55}^E = 33.2p \left( \frac{m^2}{N} \right), s_{66}^E = 52.3p \left( \frac{m^2}{N} \right),$$

$$d_{31} = d_{32} = -185 \frac{pC}{N}, d_{33} = 440 \frac{pC}{N}, d_{15} = d_{24} = 560 \frac{pC}{N}, \epsilon_{11}^T = \epsilon_{22}^T = 1950 \epsilon_0, \epsilon_{33}^T = 1850 \epsilon_0. \quad (1)$$

Elements are used mainly in mixed systems where piezoelectric ceramics are used as an ultrasonic-transducer on the one hand and as a receiver on the other hand. Sonox P502 is made from specifically created high performance materials with a high rate of thermal and temporal stability. For this reason, it is extremely suitable for automotive and aircrafts industry applications in a range of temperatures from  $-40$  to  $+160^\circ\text{C}$ .

Standard electrodes are silver or nickel-gold plated. Special electrode configurations are feasible to achieve enhanced functionality. Metal plating can be applied with or without insulation margin. The standard insulation margin is  $\leq 0.3$  mm.

The piezoelectric element Sonox P502 was bonded to the brass membrane (diameter 18 mm, thickness 0.15 mm). Properties of brass: density  $8400 \text{ kg/m}^3$ , Young's modulus 100 GPa, Poisson's ratio 0.31.

## 2.2. Modal analysis in software Ansys 12.1

Aiming to perform a modal analysis and to find the natural frequencies and mode shapes of the piezoelectric membrane surface the finite element method (FEM) was used. A numerical simulation of the brass membrane with bonded piezoelectric element was performed aiming to validate the actuator's operation pattern through modal analysis. The simulation was performed using FEM software ANSYS 12.1.

In this case, the synthetic jet generator is designed as a cylinder with one membrane and one output orifice. Diameter  $D$  of the cavity is 18 mm and height  $H$  of the cavity is 0.5 mm. Diameter  $d$  and length  $h$  of output orifice are 1 and 0.5 mm respectively. The brass membrane with piezoelectric element was excited by a sinusoidal type signal, voltage  $\pm 100$  V. The synthetic jet device should be designed with respect to the frequency of the synthetic jet and the minimum power input of the actuator. The minimum power and maximum intensity of the synthetic jet can be obtained in resonant frequencies of the synthetic jet generator.

For this reason, the paper analyses the mode shapes of the piezoelectric membrane in the first three resonant frequencies. This membrane with piezoelectric element was used in the synthetic jet generator design. SJ control and appropriating working parameters depend significantly on the displacement of the membrane; thus, it is necessary to investigate the most appropriate mode shape of a piezoelectric brass membrane in different excitation frequencies. For the study, the piezoelectric element Sonox P502 (diameter 10 mm, thickness 0.5 mm) manufactured by the company *CeramTec* was selected [16]. These types of piezoelectric elements are used in aviation because of their ability to work in different environments.

The properties of piezoceramic material of Sonox P502 are [16]:

FEM model for the brass membrane with the piezoelectric element Sonox P502 was created. In the numerical simulation the edge of the membrane was fixed tightly like in the real synthetic jet generator's construction. All volumes were meshed by 3 coupled-field tetrahedral finite element SOLID 98. The material properties required for the analysis of piezoelectric materials (permittivity, piezoelectric matrix, elastic coefficient matrix, density) were entered into the program. During the analysis brass material properties were entered additionally.

## 2.3. Experimental equipment

Laser vibrometry has become more and more popular among researchers around the world. Various types of laser vibrometers can be distinguished. The simplest vibrometers allow measuring only velocity of vibrations along the laser beam in one point (manual positioning of the laser head). More sophisticated versions of vibrometers allow automatic scanning of velocity of vibration in a defined measuring mesh (scanning laser vibrometer). However, they also measure velocity along the laser beam only. The third type of vibrometers allows measuring three components of vibration velocity simultaneously (laser scanning vibrometer).

The laser vibrometer operates on the basis of the Doppler Effect. A vibrometer registers changes in the

frequency of a light beam reflecting from a vibrating surface.

A fundamental advantage of laser vibrometry is a non-contact measurement. This eliminates the detrimental effects of adding mass related to the sensor at a measured point. Another advantage of laser vibrometry is the possibility of the measurement of vibrations. This measurement technique allows registering object vibration components in a plane perpendicular to the investigated surface as well as in one parallel to it.

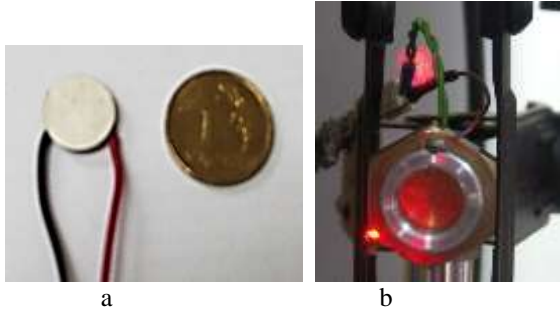


Fig. 2 Experimental equipment: a) piezoelectric element Sonox P502; b) brass membrane with bonded piezoelectric element clamped inside the chamber of the SJ generator

Laser vibrometry allows measuring vibrations in frequencies from close to 0 Hz to 24 MHz, as well as a wide range of vibration velocities – from 20 nm/s to 20 m/s.

All the mentioned advantages make laser vibrometry one of the most effective non-contact measurement techniques that allow the registering of vibrations of structures and propagations of elastic waves.

Measurements were performed using the scanning laser vibrometer Polytec® PSV 400. This vibrometer consists of three laser scanning heads, a control unit with built in signal generator, and a PC with vibrometer software. It should be mentioned that during the research 1D measurements, using one laser scanning head, were conducted. Only the vibration velocity along the laser beam (without a plane component) was recorded.

During the measurement, vibration displacements instead of velocities were extracted. It is possible to measure velocity or displacement; however, it should be underlined that the vibrometer measures velocity and calculates displacements based on velocities measured. The measured surface in some cases was covered with retroreflective tape in order to enhance the signal to noise ratio (SNR).

Experimental analysis of the dynamic characteristics of the brass membrane with this piezoelectric element Sonox P502 were carried out in order to discover what frequency would allow the achieving of the largest displacement of surfaces and what mode shapes of the piezoelectric membrane surface is at various resonance frequencies.

During the experimental analysis membrane was clamped inside the chamber of a synthetic jet generator and the piezoelectric element Sonox P502 (Fig. 2) was excited by sinusoidal voltage.

Using the laser vibrometer Polytec® PSV 400 and a computer with the specifically designed program PSV 8.8, the dependence of displacements on frequency were found. During the experiments, excitation voltages were kept constant, frequency varied, and the displacement of piezoelectric membrane surfaces was measured.

The CTA (Constant Temperature Anemometers) anemometer is today's most widely used instrument for the measurement of the structures in turbulent gas and liquid flows. The CTA technique is very sensitive and can be used for the determination of frequencies and velocity of flow. Experimental data (amplitude-frequency measurements) were carried out by Dantec equipment. The Dantec mini CTA device with probe 55P11 has a frequency limit above 10 kHz which is sufficient for this experiment. The probe was located in the fluid flow, which was coming out from the output orifice of the synthetic jet generator, in the position 1mm above the output orifice. The fluid flow velocity is slightly undervalued because of size of the probe and the distance of probe from output orifice.

### 3. Results and discussions

#### 3.1. Results of FE analysis

Using FE modal analysis, natural resonant frequencies and mode shapes of the piezoelectric membrane were found.

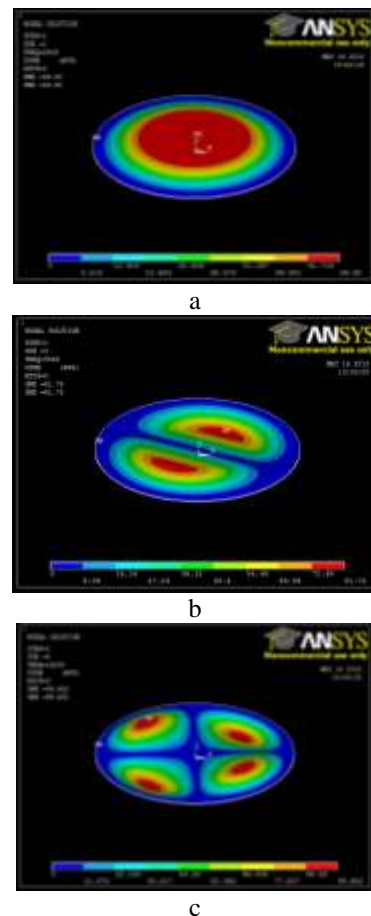


Fig. 3 Mode shapes of a membrane with the piezoelectric element: a) the first mode ( $f = 2.615$  kHz); b) the second mode ( $f = 5.06$  kHz); c) the third mode ( $f = 16.25$  kHz)

As mentioned before, maximum intensity of the synthetic jet with minimum power can be obtained in resonant frequencies of the synthetic jet generator. Consequently, the examination of the results of modal analysis disclosed that vibration modes No. 1 (2.615 Hz), No. 2 (5.06 kHz), and No. 3 (16.25 kHz) (Fig. 3) are interesting and useful for further investigation.

### 3.2. Results of experimental analysis

With the help of experimental equipment, the resonance frequencies and mode shapes of piezoelectric membrane were found. The piezoelectric element was excited by a sinusoidal voltage  $\pm 100$  V. Fig. 4 shows that the highest displacement of the surface of the piezoelectric membrane was obtained in the first resonance frequency (2.85 kHz). Maximum displacement of the surface of the piezoelectric membrane was 31 nm. And in the further resonance frequencies it was significantly lower.

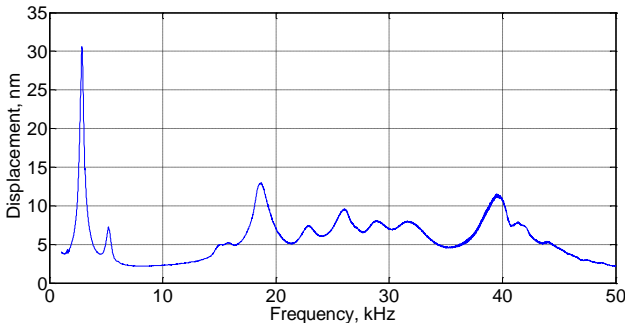


Fig. 4 Dependence of the displacement of the surface of membrane with bonded piezoelectric element Sonox P502 on frequency (voltage  $\pm 100$  V)

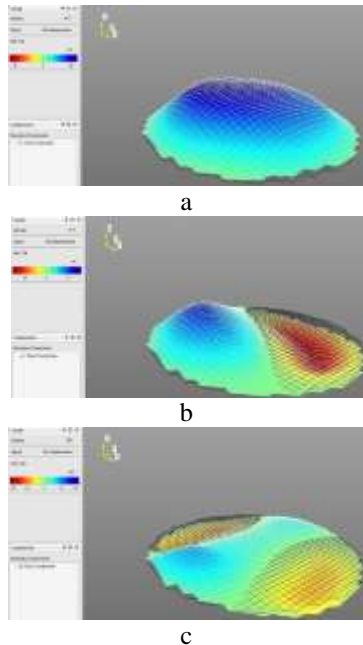


Fig. 5 Experimental shapes of the surface of membrane with piezoelectric element Sonox P502: a) frequency 2.85 kHz; b) frequency 5.22 kHz; c) frequency 15.77 kHz

Fig. 5 presented the mode shapes of the surface of piezoelectric membrane in the first three resonance frequencies. In order to improve the validity of the model it was useful to make an experimental analysis and to recognise the shapes of the surface of this membrane.

It could be interpreted that the most effective mode shape for synthetic jet generators is the first one (Fig. 5, a), since the largest displacement enables the creating of the strongest flow out of the cavity of the device. It is important to mention that the first resonance frequency and the mode shape of the membrane are mostly relevant for flow control devices. However, aiming to improve the validity of the numerical model the first three mode shapes of a membrane were analysed.

### 3.3. Comparison of the numerical and experimental results of piezoelectric membrane

One of the tasks of the study was to compare the numerical and experimental results as this was the only way to check if the results are coincident. It was very important to compare the numerical and experimental resonance frequencies.

It is important to mention that the differences of numerical and experimental resonance frequencies of membrane with bonded piezoelectric element Sonox P502 were not extremely high. Table show that the biggest difference was 8.25% in the first resonance frequency. These errors can be explained by the distinction of experimental and numerical model designs. And it should be noted that electrodes were soldered on to the piezoelectric element manually. As is known piezoelectric materials are very sensitive and their parameters can be changed by various factors. Thus, it could be concluded that solder inaccuracies can lead to errors arising between the experimentally and numerically derived data.

Table  
Comparison of the analytical and experimental resonance frequencies of piezoelectric membrane

Mode number	Analytical natural resonance frequency, kHz	Experimental resonance frequency, kHz	Error, %
1	2.615	2.85	8.25
2	5.06	5.22	3.07
3	16.25	15.77	3.04

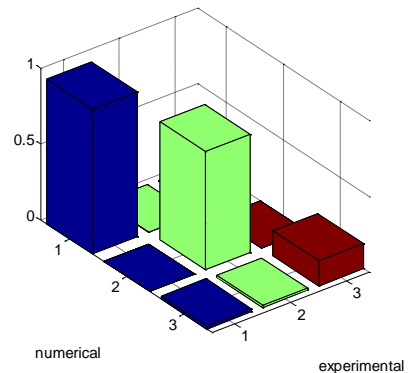


Fig. 6 Modal Assurance Criterion (MAC) matrix of experimental and analytical mode shapes of piezoelectric membrane

In order to prove the validity of the model of membrane with bonded piezoelectric element Sonox P502, a Modal Assurance Criterion (MAC) matrix (using software MATLAB R2011b), was created. A Modal Assurance Criterion (MAC) matrix is a mathematical statistical tool enabling the comparison of two vectors (analytical or experimental) to each other.

As already noted, the first resonance frequency is mostly suitable for the use in the control of a membrane in SJ generators. The best mode shape and displacement of brass membrane were achieved in this frequency. Thus, our aspiration was to get the highest possible Modal Assurance Criterion for the first mode. Fig. 6 shows, that in the first mode, MAC was 0.95, in the second – 0.78, and in the third – 0.165. The Modal Assurance Criterion takes on values from zero, which represents no consistent correspondence, to one, which represents a consistent correspondence.

Concluding the analysis of MAC, it is possible to say that the developed numerical model of membrane with bonded piezoelectric element Sonox P502 is suitable for further investigations and simulations of the design of SJ generators. Full compliance in all modes was not reached since it is difficult to create a numerical model equivalent to manually designed experiments.

#### 3.4. Experimental results of the synthetic jet control

The membrane with bonded piezoelectric element Sonox P502 under research was embedded in our produced synthetic jet generator. In order to prove the eligibility of the experimental and numerical investigation of the piezoelectric membrane experimental analysis of the synthetic jet flow control was performed.

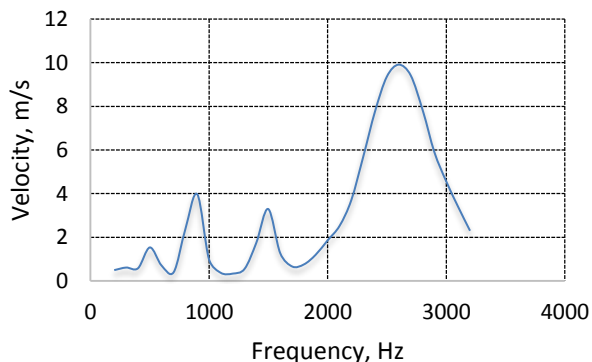


Fig. 7 Amplitude-frequency characteristic of membrane with bonded piezoelectric element Sonox P502 (voltage  $\pm 100$  V)

Fig. 7 shows the amplitude-frequency response of the real synthetic jet generator which was measured by CTA probe 55P11. The maximum operating voltage of membrane with bonded piezoelectric element Sonox P502 was set up to  $\pm 100$  V. Two local and one global maximum are visible. The highest peak ( $f = 2.597$  kHz) with output velocity (10 m/s) corresponds to the resonant frequency of membrane with bonded piezoelectric element. The resonant value of the cavity ( $f = 1.497$  kHz) is in accordance with the second value from two local maximums (from the left side).

After the final experiment it could be said that the study of the membrane with bonded piezoelectric element Sonox P502 had been carried out correctly. The discrepancy between resonant frequency (2.85 kHz) at which was found the maximum displacement (30 nm) of the investigated membrane surface and the resonant frequency (2.597 kHz) of the synthetic jet velocity was 253 Hz. Taking into the consideration that the operating resonance frequency was 2.85 kHz and the difference did not affect presented results.

#### 4. Discussions and conclusions

The performed research enables one to make following conclusions:

1. The analysis of membrane with bonded piezoelectric element Sonox P502 showed that this element is suitable for the design of the synthetic jet generators. It was found that by using this type of the piezoelectric element membrane a displacement of 31 nm is obtained, which is sufficient to generate the required synthetic jet.

2. Theoretical and experimental analysis of the piezoelectric membrane was conducted. The reliability of the results was evaluated using MAC criterion, which indicated that MAC of the first mode is 0.95. When the result exceeds 0.9 modes, it can be considered well correlated.

3. The research enabled the identification of the synthetic jet velocity of our produced generator. When the resonance frequency was 2.597 kHz (voltage  $\pm 100$  V), the SJ velocity was 10m/s. It was found that this frequency is close to the analytically and experimentally determined membrane with bonded piezoelectric element Sonox P502 resonance frequency.

#### Acknowledgements

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#### NUMERICAL AND EXPERIMENTAL ANALYSIS OF MEMBRANE WITH PIEZOELECTRIC ELEMENT USED FOR SYNTHETIC JET FLOW CONTROL

#### S u m m a r y

Synthetic jet generators have been widely analysed in controlling the airflow of aircrafts aiming to enhance aircraft lifting and manoeuvrability, control stalls, and reduce the noise level. One of the best choices is to use piezoelectric actuators in the design of such kind of the devices because of its favourable properties. This article presents a numerical and experimental modal analysis of the membrane with bonded piezoelectric element Sonox P502. The aim of this investigation was to find suitable piezoelectric actuator for controlling synthetic jet flow. And in order to check the suitability and reliability of the received data about the piezoelectric membrane, experimental analysis of the synthetic jet generator was performed. The CTA (Constant Temperature Anemometers) technique is very sensitive and was used for the determination of frequencies and velocity of flow.

**Keywords:** Piezoelectricity, piezoelectric actuator, synthetic jets, non-contact measurements.

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