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Critical Slip of Three-phase Cage Induction Motor Supplied from Limited Power Source

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Introduction

The three-phase induction motor is used in a wide variety of aplications, such as pumps, steel mills, compressors, centrifuges, separators, hoist drives and etc.

If the conventional cage induction motor is supplied from the voltage source (i. e. when the inner complex impedance of the power-supply source is significantly smaller than the input impedance of motor and the supply complex voltage of motor is constant) the performances are widely analysed according to general theory of many researchers in a number of papers and books.

But in practice there exist the cases when an induction motor is supplied from limited power source. For example it may be the electric power system such as a mobile electric power station, wind power station, diesel power plant, self-contained supply, small-scale electric power stations based on renewable and alternative energy source and etc. In this case the power of an individual induction motor may be equal to the power of the generator or transformer power or only marginally differs from it. So the large starting current may cause the considerable voltage drop and to disturb the stability of the operating load electric network.

Sometimes at unfavorable conditions the starting of a cage induction motor from low-power source may be impossible through the large voltage drop. The rational choice of the cage induction motor for electric drives is important, because the use of high-power motor increases electric energy losses.

The critical slip formulae of a cage induction motor supplying from unlimited power source are presented in many text-books of electric machines, for example [1,2].

Sometimes servomotors with high starting torque at relatively low ratio of starting current and with frequent starting are required for the electric drives such as: cranes, winches, rudders, compressors and etc. The induction motors with solid ferromagnetic or double-layer rotors for these conditions are most suitable. The critical slip of induction motor with solid ferromagnetic rotor for the borehole investigating devices supplied from limited power source and through the long cable with distributed parameters is analysed in [3,4].

Critical slip of cage induction motor operating from limited power source

The torque-slip characteristic of a cage induction motor is described easily if the critical slip s_c and maximum electromagnetic torque $T_{em,\max}$ are known. As a rule these s_c and $T_{em,\max}$ are determined taking into account the parameters L-type equivalent circuit [3,4]. But this equivalent circuit does not evaluate the parameters of supply circuit. A formula of critical slip do not taking into account the concrete electrical power source of a cage induction motor is being deduced in this paper.

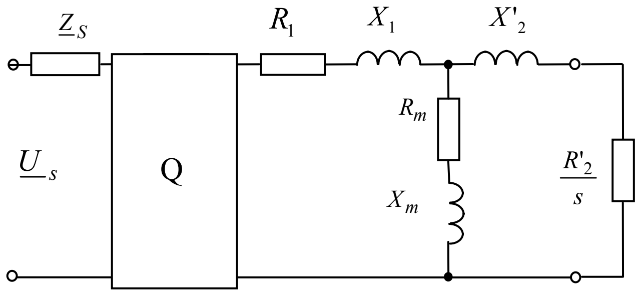

Fig. 1. Per-phase equivalent circuit of cage induction motor with the supply cable and the inner complex impedance of the power-supply source $\underline{Z}_S$: $\underline{U}_S$ is the complex voltage of power-supply source; Q is the four-pole; R_1, X_1, R_m, X_m, X'_2, R'_2 is the L-type equivalent circuit parameters of a cage induction motor; s is the slip

The complex of stator current can be expressed by applying the calculation rule of four-pole to supply circuit (Fig. 1):

$$\underline{I}_1 = \frac{\underline{U}_s}{\underline{A}_{11}\left(\underline{Z}_M + \underline{Z}_S\right) + \underline{A}_{12} + \underline{A}_{21}\underline{Z}_M\underline{Z}_S}; \quad (1)$$

where

$$\underline{Z}_M = \underline{Z}_1 + \frac{\underline{Z}_m \underline{Z}_2'}{\underline{Z}_m + \underline{Z}_2'}; \qquad \underline{Z}_2' = \frac{R_2'}{s} + jX_2';$$

$$\underline{Z}_1 = R_1 + jX_1; \qquad \underline{Z}_m = R_m + jX_m;$$

$$\underline{A}_{11} = \cosh\underline{\gamma}l; \ \underline{A}_{12} = Z_0\sinh\underline{\gamma}l; \ \underline{A}_{21} = \sinh\underline{\gamma}\,l/\underline{Z}_0;$$

$$\underline{Z}_0 = \sqrt{\frac{r_0 + j\omega_1 L_0}{g_0 + j\omega_1 C_0}}; \ \underline{\gamma} = \sqrt{(r_0 + j\omega_1 L_0)(g_0 + j\omega_1 C_0)};$$

$\omega_1 = 2\pi f_1$; f_1 is the frequency of the supply source current; r_0, g_0, L_0, C_0 is the parameters per unit length of supply cable; l is the length of supply cable.

The complex of equivalent rotor current is expressed as:

$$\underline{I}_2' = \underline{I}_1 \frac{\underline{Z}_m}{\underline{Z}_m + \underline{Z}_2'}. \qquad (2)$$

Then the electromagnetic power of motor can be written as:

$$P_{em} = m_1\left(\left|\underline{I}_2'\right|\right)^2 \frac{R_2'}{s} = m_1 \frac{R_2'}{s} \times$$

$$\times \left| \frac{\underline{U}_s \underline{Z}_m}{\underline{A}_{11}(\underline{Z}_M + \underline{Z}_s) + \underline{A}_{12} + \underline{A}_{21}\underline{Z}_M \underline{Z}_s (\underline{Z}_m + \underline{Z}_2')} \right|^2; \qquad (3)$$

where m_1 is the number of phase.

An expression for the slip at maximum electromagnetic power may be obtained by taking the derivative of (3) with respect to slip and setting of result equal to zero. The differentiation of this expression (3) is not difficult, but takes a lot place and therefore it will not be shown here. Then the critical slip is therefore given as:

$$s_c = \pm \left| \frac{R_2'\left(\underline{A}_{11}(\underline{Z}_1 + \underline{Z}_m + \underline{Z}_s) + \underline{A}_{12} + \right.}{\underline{A}_{11}\left(\underline{Z}_m(\underline{Z}_1 + \underline{Z}_s) + jX_2'(\underline{Z}_1 + \underline{Z}_m + \underline{Z}_s)\right) +}\right.$$

$$\left. \frac{\left. + \underline{A}_{21}(\underline{Z}_1 + \underline{Z}_m)\right)}{+ \underline{A}_{12}(\underline{Z}_m + jX_2') + \underline{A}_{21}\left(\underline{Z}_1\underline{Z}_m\underline{Z}_s + jX_2'\underline{Z}_s(\underline{Z}_1 + \underline{Z}_m)\right)} \right|. \qquad (4)$$

When the insulation currents of the cable are denied, then $A_{12} = R_C$ (R_C is the resistance of the cable) and the the critical slip formula (4) may now be written

$$s_c = \pm \left| \frac{R_2'\left(\underline{Z}_1 + \underline{Z}_m + \underline{Z}_s + \right.}{\left(\underline{Z}_m(\underline{Z}_1 + \underline{Z}_s) + jX_2'(\underline{Z}_1 + \underline{Z}_m + \underline{Z}_s) + \right.}\right.$$

$$\left. \frac{\left. + R_c\right)}{\left. + R_c(\underline{Z}_m + jX_2')\right)} \right|. \qquad (5)$$

If the motor is supplied from a high-capacity power-supply source and directly connected to power-supply source (i.e. $\underline{Z}_S$=0, $\underline{A}_{11}$=1, $\underline{A}_{12}$=0 and $\underline{A}_{21} = 0$ the insulation currents of the cable are not taken into account) then the equation (5) can be written as:

$$s_c = \pm \left| \frac{R_2'(\underline{Z}_1 + \underline{Z}_m)}{\underline{Z}_1\underline{Z}_m + jX_2'(\underline{Z}_1 + \underline{Z}_m)} \right|. \qquad (6)$$

After the simple transformation and taking into accont that $\underline{C}_1 = \dfrac{\underline{Z}_1 + \underline{Z}_m}{\underline{Z}_m}$ equation (6) becomes as follows:

$$s_c = \left| \frac{\underline{C}_1 R_2'}{R_1 + j(X_1 + \underline{C}_1 X_2')} \right| = \pm \frac{C_1 R_2'}{\sqrt{R_1^2 + (X_1 + C_1 X_2')^2}}. \qquad (7)$$

This formula (7) is widely used in practical and design calculations [3].

Taking into account that $\underline{C}_1 = C_1' + jC_1''$ and after the simple transformations formula (7) can be written as:

$$s_c = \pm \frac{|\underline{C}_1| R_2'}{\sqrt{(R_1 - C_1'' X_2')^2 + (X_1 + C_1' X_2')^2}}; \qquad (8)$$

where $C_1' = \left((R_m + R_1)R_m + (X_m + X_1)X_m\right)/\left(R_m^2 + X_m^2\right)$;

$$C_1'' = \left((X_m + X_1)R_m - (R_m + R_1)X_m\right)/\left(R_m^2 + X_m^2\right).$$

It is shown that the formula of critical slip (8) absolutely coincides with data presented in [4]. So, it is important to underline that equation (4) is the general case of a critical slip formula taking into account the inner complex impedance of power-supply source and parameters of the cable.

Characteristics of motor and discusion

Substituting the expression of critical slip s_c (4) in place s into equation (3) the maximum electromagnetic power is therefore given as

$$P_{em} = m_1 \frac{R_2'}{s_c} \times$$

$$\times \left| \frac{\underline{U}_s \underline{Z}_m}{\underline{A}_{11}(\underline{Z}_{Mm} + \underline{Z}_s) + \underline{A}_{12} + \underline{A}_{21}\underline{Z}_{Mm}\underline{Z}_s(\underline{Z}_m + \underline{Z}_{2m}')} \right|^2; \qquad (9)$$

where $\underline{Z}_{Mm} = \underline{Z}_1 + \dfrac{\underline{Z}_m \underline{Z}_{2m}'}{\underline{Z}_m + \underline{Z}_{2m}'}$; $\quad \underline{Z}_{2m}' = \dfrac{R_2'}{s_c} + jX_2'$.

The supply voltage of cage induction motor changes in the slip range (0-1) negligibly if the supply cable is short or the average specific resistance is small. In this case the supply voltage of motor can be written as:

$$\underline{U}_M = \underline{I}_1 \cdot \underline{Z}_M \, . \qquad (10)$$

where $\underline{Z}_M$ is the input complex impedance of a three-phase induction motor.

The electromagnetic torque of motor is expressed as:

$$T_{em} = \frac{pP_{em}}{2\pi f_1}; \qquad (11)$$

where p is the number of pole pairs.

The input power of motor can be exspressed as follows

$$\underline{S}_M = m_1 \underline{U}_M \overset{*}{\underline{I}}_1 ; \qquad (12)$$

where $\overset{*}{\underline{I}}_1$ is the conjugate complex of stator current.

The electromagnetic efficiency and power factor of the motor can be calculated:

$$\eta_{em} = \frac{P_{em}}{\mathrm{Re}(\underline{S}_M)} . \qquad (13)$$

$$\cos\varphi = \frac{\mathrm{Re}(\underline{S}_M)}{(\underline{S}_M)} . \qquad (14)$$

The other characteristics can be calculated according to (1-4) formulae.

The characteristics have been computed for the three-phase cage induction motors which rated power are: 4 kW, 45 kW, 90 kW. The parameters of equivalent circuit of these motors are known. The length 0,1 km of the supply cable will be accepted and the cross-section according to the rated current of the motor will be chosen also the parameters of the cable and the inner impedance will be chosen taking into account the impedance of generator [5,6].

The characteristics of cage induction motor such as: critical slip, maximum electromagnetic power (torque), electromagnetic power (torque), supply voltage of motor and other rated parameters (except electromagnetic efficiency and power factor) are influenced by the inner impedance of power-supply source.

The influence of the supply cable parameters (C_o, L_o) on the motor characteristics are negligible (do not exceed more than 1%).

When the inner impedance of power-supply source increases, the critical slip decreases and at all cases when the rated power of the motor increases. The ratio parameters of the motor and supply equivalent circuits are defined, the question is the following: what influence on the motor characteristics is the greater: the cable impedance or power-supply source impedance.

The decreasing of the s_c and $P_{em,\max}$ with respect to the motor size causes the change of rated parameters of the induction motor. In certain cases the motor generally could not develop rated power (90 kW motor, see Table 1).

Table 1. The main computed parameters of the three cage induction motors (2p=4) at phase voltage 220V

Motor	P_{2N}=4kW, I_{1N}=8,26A, s_c=0,047			P_{2N}=45kW, I_{1N}=80,2A, s_c=0,015			P_{2N}=90kW, I_{1N}=161A, s_c=0,014		
Length of cable, km	0	0,1		0	0,1		0	0,1	
Average cable resistance, Ω/km	0	7,7		0	0,89		0	0,44	
$\underline{Z}_S$, Ω / Parameters	0	0	(02+j0,1)	0	0	(02+j0,1)	0	0	(02+j0,1)
s_c	0,255	0,240	0,233	0,070	0,068	0,053	0,065	0,064	0,040
$U_{M(s_c)}$, V	220	205	201	220	204	170	220	204	152
$P_{em,\max}$, kW	10,3	8,89	8,46	107	91,9	61,5	217	188	92,9
$P_{em(s=0,05)}$, kW	4,52	4,29	4,22						
$P_{1(s=0,05)}$, kW	4,97	4,88	4,84						
$P_{em(s=0,02)}$, kW				59,9	55,5	45,65	126,9	117,3	79,75
$P_{1(s=0,02)}$, kW				63,3	61,4	55,5	131,8	127,6	104,7
$P_{em(s=0,03)}$, kW				80,1	72,2	55,3	168,5	151,6	90,4
$P_{1(s=0,03)}$, kW				86,1	82,9	72,5	180,0	171,2	132,2
$T_{em,\max}$, N·m	63,3	56,6	53,9	680	85	392	1381	1194	592

The study of all the computed characteristics show that the change of supply voltages, rated currents and other characteristics at the different rated power motors are relatively similar, when the length of the cable is the same, but rather more are different in cases when the motor is supplied from unlimited and limited power-supply source (depends on the impedances ratio of the rotor and the inner power-supply source). The algorithm for calculations of an induction motor characteristics may be used for checking of the suitable length and cross-section of the supply cable.

Conclusions

The critical slip expression of the cage induction motor operating from the limited power-supply source taking into account the inner complex impedance of it and other cable parameters (r_0, g_0, L_0, C_0 is the parameters per unit length of supply cable) has been derived.

The algorithm for calculations of the cage induction motor characteristics taking into account the parameters of the supply circuit have been created.

This algorithm can be successfully employed for calculation characteristics of the cage induction motor taking into account the suitable length and cross-section of the supply cable and for checking of it.

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S. Gečys. Critical Slip of Three-phase Cage Induction Motor Supplied from Limited Power Source // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 1(73). – P. 25–28.

The peculiarities and characteristics of a cage induction motor supplying from limited power source (mobile electric power station, diesel power plant, wind power station and etc.) change virtually in comparison with the case when motor is supplied from unlimited voltage source. The critical slip expression of the cage induction motor operating from the limited power-supply source taking into account the inner complex impedance of power-supply source and other cable parameters has been derived. It is shown that critical slip formula, when motor is supplied from a high-capacity power-supply source (the inner complex impedance is not taken into account) and through short cable (the cable impedance is not taken into account) absolutely coincides with well-known formula. The algorithm for calculations of the induction motor characteristics taking into account the parameters of the supply circuit and the suitable length and cross-section of the supply cable has been created. Ill.1, bibl. 6 (in English; summaries in English, Russian and Lithuanian).

С. Гячис. Критическое скольжение трехфазного асинхронного двигателя с короткозамкнутым ротором, питаемого от источника предельной мощности // Электроника и электротехника. – Каунас: Технология, 2007. – № 1(73). – С. 25–28.

Особенности и характеристики асинхронного двигателя с короткозамкнутым ротором, питаемого от источника предельной мощности (передвижная, дизельная электростанция, ветроэлектростанция и др), могут по сути дела изменятся по сравнению с тем случаем, когда двигатель питается от мощного источника питания. Выведена формула критического скольжения асинхронного двигателя, питаемого от источника предельной мощности, учитывая её внутреннее комплексное сопротивление и другие параметры кабеля. Доказано, что полученная формула критического скольжения, когда двигатель питается от мощного источника (внутреннее комплексное сопротивление не учтено) и через короткий кабель (сопротивление не учтено), полностью совпадает с общеизвестной формулой. Предложен алгоритм расчёта характеристик асинхронного двигателя, учитывая внутреннее сопротивление источника питания и определенную длину кабеля питания. Ил. 1, библ.6 (на английском языке; рефераты на английском, русском и литовском яз.).

S. Gečys. Trifazio asinchroninio narvelinio variklio, maitinamo iš ribotos galios šaltinio, kritinis slydimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 1(73) . – P. 25–28.

Asinchroninio narvelinio variklio, maitinamo iš ribotos galios šaltinio (mobiliosios, dyzelinės, vėjo elektrinių ir pan.), ypatumai ir charakteristikos iš esmės gali keistis, palyginti su atveju, kai variklis maitinamas iš galingo įtampos šaltinio. Išvesta asinchroninio narvelinio variklio, maitinamo iš ribotos galios šaltinio, kritinio slydimo formulė, atsižvelgiant į šaltinio vidinę kompleksinę varžą ir kitus maitinimo kabelio parametrus. Įrodyta, kad gautoji kritinio slydimo formulė, kai variklis maitinamas iš galingo šaltinio (paneigta šaltinio vidaus kompleksinė varža) ir per trumpą kabelį (paneigta kabelio varža), visiškai sutampa su plačiai žinoma formule. Pasiūlytas asinchroninio narvelinio variklio, maitinamo iš ribotos galios šaltinio, charakteristikų skaičiavimo algoritmas, atsižvelgiant į maitinimo šaltinio vidinę varžą, tinkamo ilgio ir skerspjūvio maitinimo kabelį. Il.1, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).