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The Influence of Dielectric Admixtures on Capacitance of Plane Capacitor

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Introduction

The dielectric of plane capacitor can have admixtures. Field distribution and capacitance of capacitor with dielectric admixtures are different in comparison with capacitor containing isotropic dielectric. The admixtures are undesirables usually and its shape can be diverse. We approximate the different particle shapes by ellipsoid. This shape allows to generalize bodies of very different form. The limiting cases of ellipsoid are sphere, disc, cylinder, lamella and other. The influence of dielectric ellipsoidal particle on uniform electrostatic field is investigated in [1,2,3], supposing that orientation of particle and uniform field is fixed. The admixture particles can have any orientation with the same probability. Therefore it is important to know the mean characteristics of field inside and outside particle.

The results obtained in this paper can be generalised for problems, related with necessity of evaluation the influence of different non-uniformities on homogeneous media. Such problems arise, for example, when the influence of admixtures on electric strength of transformer oil, the measurement uncertainty in electromagnetic flow meters, the properties of electrorheological fluids, the corona discharge are investigated.

The electric flux density inside dielectric ellipsoid in a uniform electrostatic field

We suppose, that media and the particles of admixtures are homogeneous, and the uniform field $\boldsymbol{E}_0$ was created, before the ellipsoidal particle was placed into this field. We use the global and local rectangular coordinate systems. The global coordinate system x, y, z is related with capacitor. The axes of local coordinate system q, r, s coincide with ellipsoid axes (see Fig.1).

In local coordinate system the equation of ellipsoid of common shape is:

$$q^2/a^2 + r^2/b^2 + s^2/c^2 = 1, \tag{1}$$

where a,b,c are the lengths of ellipsoid semiaxes. We obtain different forms of particles varying the ratios a/b and b/c.

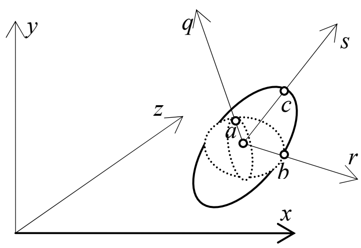

Fig. 1. Global x, y, z and local q, r, s coordinate systems

The components of electric flux density D_{pq}, D_{pr}, D_{ps} were expressed in matrix form [2]:

$$\left[D_{pq}, D_{pr}, D_{ps}\right]^T = \boldsymbol{A}\cdot\left[D_{0q}, D_{0r}, D_{0s}\right]^T,\ \boldsymbol{A} = \begin{bmatrix} A_a & 0 & 0 \\ 0 & A_b & 0 \\ 0 & 0 & A_c \end{bmatrix}, \tag{2}$$

where A_a, A_b and A_c – the shape coefficients:

$$\begin{cases} A_a = \dfrac{(\varepsilon_p/\varepsilon_m)}{1 + L_a[(\varepsilon_p/\varepsilon_m) - 1]},\ A_b = \dfrac{(\varepsilon_p/\varepsilon_m)}{1 + L_b[(\varepsilon_p/\varepsilon_m) - 1]}, \\ A_c = \dfrac{(\varepsilon_p/\varepsilon_m)}{1 + L_c[(\varepsilon_p/\varepsilon_m) - 1]}, \end{cases} \tag{3}$$

ε_m and ε_p - permittivities of media and particle, correspondingly, L_a, L_b, L_c – depolarisation factors [1]:

$$L_a = \frac{abc}{2}\int_0^\infty \frac{dt}{\sqrt{(t+a^2)^3(t+b^2)(t+c^2)}}, \tag{4}$$

$$L_b = \frac{abc}{2}\int_0^\infty \frac{dt}{\sqrt{(t+a^2)(t+b^2)^3(t+c^2)}}, \tag{5}$$

$$L_c = \frac{abc}{2}\int_0^\infty \frac{dt}{\sqrt{(t+a^2)(t+b^2)(t+c^2)^3}}. \tag{6}$$

The components of electric flux density can be related in local and global coordinate systems this way:

$$[D_x, D_y, D_z]^{\mathrm{T}} = \boldsymbol{h}[D_q, D_r, D_s]^{\mathrm{T}}, [D_q, D_r, D_s]^{\mathrm{T}} = \boldsymbol{h}^{\mathrm{T}}[D_x, D_y, D_z]^{\mathrm{T}}, \quad (7)$$

where $\boldsymbol{h}$ is 3×3 quadratic matrix. When the local coordinate system is rotated about axes x, y and z of global system by angles ψ, ν and φ, the elements $\boldsymbol{h}$ are the following [2]:

$$\begin{cases} h_{11} = \cos\nu\cos\varphi,\ h_{12} = -\sin\varphi\cos\nu,\ h_{13} = \sin\nu,\ h_{21} = \cos\psi\sin\varphi + \\ + \sin\psi\sin\nu\cos\varphi,\ h_{22} = \cos\psi\cos\varphi - \sin\psi\sin\nu\sin\varphi, \\ h_{23} = -\sin\psi\cos\nu,\ h_{31} = \sin\psi\sin\varphi - \cos\psi\sin\nu\cos\varphi, \\ h_{32} = \sin\psi\cos\varphi + \cos\psi\sin\nu\sin\varphi,\ h_{33} = \cos\psi\cos\nu. \end{cases} \quad (8)$$

The components of electric flux density D_{px}, D_{py} and D_{pz} inside ellipsoid and the components of uniform field D_{0x}, D_{0y}, D_{0z}.are related by equation:

$$[D_{px}, D_{py}, D_{pz}]^{\mathrm{T}} = \boldsymbol{K}_{\mathrm{p}} \cdot [D_{0x}, D_{0y}, D_{0z}]^{\mathrm{T}}, \quad (9)$$

where $\boldsymbol{K}_{\mathrm{P}}$ is the shape matrix

$$\boldsymbol{K}_p = \boldsymbol{h} \cdot \boldsymbol{A} \cdot \boldsymbol{h}^{\mathrm{T}}. \quad (10)$$

The mean value of electric flux density inside ellipsoidal particle with any orientation

Let the relative volume concentration of randomly oriented particle be k. We express the mean value of electric flux density $\overline{D}_p$, supposing that the uniform electric field $\boldsymbol{D}_0$ =[0, D_{y0}, 0]$^{\mathrm{T}}$, directed by y axis, was created in the homogeneous dielectric of capacitor.

We can calculate the vector $\boldsymbol{D_p}$ inside particle for fixed particle position from (9):

$$\boldsymbol{D_p} = \begin{vmatrix} (h_{11}h_{21}A_a + h_{12}h_{22}A_b + h_{13}h_{23}A_c) \cdot D_{y0} \\ (h_{21}^2 A_a + h_{22}^2 A_b + h_{23}^2 A_c) \cdot D_{y0} \\ (h_{21}h_{31}A_a + h_{22}h_{32}A_b + h_{23}h_{33}A_c) \cdot D_{y0} \end{vmatrix}, \quad (11)$$

We obtain any particle position by rotation of local coordinate system with respect to global system. When we rotate the local coordinate system about any global axis by angle π/2, the semiaxes a, b, c will be oriented by other axes, than it is showed in Fig. 1. This exchange corresponds to exchange of coefficients A_a, A_b and A_c in matrix $\boldsymbol{A}$. The exchange of any axes or any shape coefficients has the same probability. The mean value of matrix $\boldsymbol{A}$ is:

$$[\overline{\boldsymbol{A}}] = \frac{1}{3}(A_a + A_b + A_c) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (12)$$

The mean value $\overline{\boldsymbol{D}}_{\boldsymbol{p}}$ will be:

$$\left\{ \overline{\boldsymbol{D}}_{\boldsymbol{p}} = \begin{vmatrix} (h_{11}h_{21} + h_{12}h_{22} + h_{13}h_{23})(1/3)(A_a^\varepsilon + A_b^\varepsilon + A_c^\varepsilon) \cdot D_{y0} \\ (h_{21}^2 + h_{22}^2 + h_{23}^2)(1/3)(A_a^\varepsilon + A_b^\varepsilon + A_c^\varepsilon) \cdot D_{y0} \\ (h_{21}h_{31} + h_{22}h_{32} + h_{23}h_{33})(1/3)(A_a^\varepsilon + A_b^\varepsilon + A_c^\varepsilon) \cdot D_{y0} \end{vmatrix} \right. . \quad (13)$$

Evaluating the values elements of matrix h (8), we obtain

$$\begin{cases} h_{21}^2 + h_{22}^2 + h_{23}^2 = 1, \quad h_{11}h_{21} + h_{12}h_{22} + h_{13}h_{23} = 0, \\ h_{21}h_{31} + h_{22}h_{32} + h_{23}h_{33} = 0. \end{cases} \quad (14)$$

We can write from equations (13) and (14):

$$\overline{\boldsymbol{D}}_p = (1/3) \cdot (A_a + A_b + A_c) \cdot \boldsymbol{D}_0, \quad (15)$$

therefore, the mean direction of electric flux density vector inside particle coincides with uniform field direction, and its value is independent on angles ν, φ and ψ.

The mean value of electric flux density outside ellipsoidal particle

When the ellipsoidal body is placed into uniform field with electric flux density $\boldsymbol{D}_0 = \boldsymbol{e}_x D_{0x} + \boldsymbol{e}_y D_{0y} + \boldsymbol{e}_z D_{0z}$, the electric flux density outside body $\boldsymbol{D}_{\mathbf{m}}$ will vary too: $\boldsymbol{D}_{\mathrm{m}} = \boldsymbol{e}_x D_{\mathrm{m}x} + \boldsymbol{e}_y D_{\mathrm{m}y} + \boldsymbol{e}_z D_{\mathrm{m}z}$. We name the vector $\boldsymbol{D}' = \boldsymbol{D}_{\mathrm{m}} - \boldsymbol{D}_0$ as distortion field. The distortion field $\boldsymbol{D}'$ is investigated in [3]. It was expressed by distortion field $\boldsymbol{D}'_\infty$ near metallic particle with the electrical properties analogical to dielectric particle with $\varepsilon_{\mathrm{p}} \to \infty$:

$$\boldsymbol{D}' = [D'_x, D'_y, D'_z]^{\mathrm{T}} = \boldsymbol{K}_{\boldsymbol{\kappa}} \boldsymbol{D}'_\infty = \boldsymbol{K}_{\boldsymbol{\kappa}} \cdot [D'_{x\infty}, D'_{y\infty}, D'_{z\infty}]^{\mathrm{T}}, \quad (16)$$

where $\boldsymbol{K}_{\boldsymbol{\kappa}}$ is quadratic matrix 3×3:

$$\boldsymbol{K}_{\boldsymbol{\kappa}} = \boldsymbol{h} \cdot \boldsymbol{\kappa} \cdot \boldsymbol{h}^{\mathrm{T}}, \quad (17)$$

$$\boldsymbol{\kappa} = \begin{bmatrix} \kappa_a & 0 & 0 \\ 0 & \kappa_b & 0 \\ 0 & 0 & \kappa_c \end{bmatrix}, \quad \kappa_a = \frac{L_a[(\varepsilon_p/\varepsilon_m) - 1]}{L_a[(\varepsilon_p/\varepsilon_m) - 1] + 1}, \quad (18)$$
$$\kappa_b = \frac{L_b[(\varepsilon_p/\varepsilon_m) - 1]}{L_b[(\varepsilon_p/\varepsilon_m) - 1] + 1}, \kappa_c = \frac{L_c[(\varepsilon_p/\varepsilon_m) - 1]}{L_c[(\varepsilon_p/\varepsilon_m) - 1] + 1}.$$

If the probability of any orientation is the same, the matrix $\boldsymbol{K}_{\boldsymbol{\kappa}}$ turns into coefficient $\overline{K}_\kappa$ expressed by analogy with (15):

$$\overline{K}_\kappa = (1/3)(\kappa_a + \kappa_b + \kappa_c). \quad (19)$$

The mean values $\boldsymbol{D}'$ and $\boldsymbol{D}'_\infty$ are related by equation:

$$\overline{\boldsymbol{D}}' = (1/3)(\kappa_a + \kappa_b + \kappa_c)\overline{\boldsymbol{D}}'_\infty. \quad (20)$$

The mean directions of electric field strength $\boldsymbol{E}$ and flux density $\boldsymbol{D}$ coincide with y axis in capacitor dielectric (Fig.2). The mean values of other components and its increments are equal to zero:

$$\overline{D}_x = \overline{D}_z = \overline{D}'_x = \overline{D}'_z = \overline{E}_x = \overline{E}_z = E'_x = E'_z = 0.$$

To evaluate the mean value $\overline{E}_y$ let a particle of capacitor dielectric admixtures is projected to the electrode plane. We obtain the plane figure of area S_p (see Fig. 2). Let this figure move from lower to upper electrode parallel to it. We obtain the cylinder C with the particle inside it. Let all electrode surface with area S is divided into M

elementary areas ΔS_{y0}, so, that some number K of areas ΔS_{y0} could fill the area S_p fully.

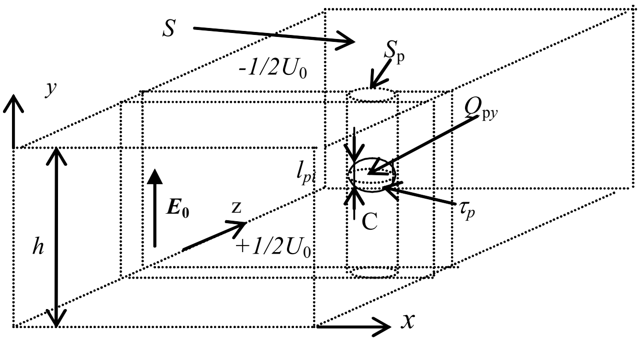

Fig. 2. The admixture particle inside capacitor dielectric

We study the electric field strength $\boldsymbol{E}$ inside cylinder C. The field is absent inside metallic particle. It varies outside particle, only. The mean height of particle by y direction is equal to l_{pi} in surroundings of area ΔS_{0i} (see Fig. 2). We form the cylinders C_i of base ΔS_{0i} analogically to cylinder C. The cylinder C is composed of K cylinders C_i. The electric field is distributed along length h-l_{pi} in any cylinder C_i. The mean field value $\overline{E}_{Ci}$ in any cylinder, evaluating, that $l_{pi}<<h_{m}$, is:

$$\overline{E}_{Ci} = \frac{U_0}{h - l_{pi}} \cong \frac{U_0}{h}\cdot(1+\frac{l_{pi}}{h}) = E_0\cdot(1+\frac{l_{pi}}{h})\,. \qquad (21)$$

We express the mean value E_{Ci} increment in comparison with E_0 and obtained expression multiply and divide to ΔS_{y0}:

$$\overline{E}'_{\infty Ci} = \overline{E}_{Ci} - E_0 = E_0\frac{l_{pi}}{h} = E_0\frac{l_{pi}\cdot\Delta S_{0i}}{\Delta S_{y0}\cdot h}\,. \qquad (22)$$

The relative mean value of electric field increment can be calculated in all capacitor dielectric volume:

$$\delta_{\infty m} = \frac{\overline{E}'_\infty}{E_0} = \lim_{M\to\infty}\left(\frac{\sum_{i=1}^{M} l_{pi}\cdot\Delta S_{0i}}{M\cdot\Delta S_0\cdot h}\right) = \frac{\sum_{i=1}^{K} l_{pi}\cdot\Delta S_{0i}}{M\cdot\Delta S_0\cdot h} = \frac{\tau_p}{\tau_0}\,. \quad (23)$$

This expression is obtained evaluating that $l_{pi}\neq 0$ in cylinder C, only. The permittivity ε_m is constant outside admixture particles. Therefore, the relative increments of electric field strength and flux density are the same: $\delta_m = \overline{E}'_\infty / E_0 = \overline{D}'_\infty / D_0 = k$ and

$$\overline{D}'_\infty = \overline{D}'_{y\infty} = kD_0\,. \qquad (24)$$

The electric flux density increment inside capacitor dielectric with admixtures

The uniform electric field E_0 and flux density $D_0=\varepsilon_m E_0$ is created in homogeneous dielectric, when the voltage U_0 is connected to capacitor plates. If the dielectric has admixtures of volume concentration k and permittivity ε_p, the mean value of electric flux density increment $\overline{D}'_p$ inside admixtures is expressed by (15). The mean value of electric flux density increment $\overline{D}'$ outside admixtures from (19), (20) and (24) is:

$$\overline{D}' = \overline{K}_\kappa \overline{D}'_\infty = (1/3)\cdot(\kappa_a+\kappa_b+\kappa_c)\cdot k\cdot D_0\,. \qquad (25)$$

The relative variation of electric flux density in all dielectric volume

$$\delta_D = \frac{\overline{D}_p - D_0}{D_0}k + \frac{\overline{D}'}{D_0}(1-k) = \frac{1}{3}[(A_a-1)+$$
$$+(A_b-1)+(A_c-1)]k + (\kappa_a+\kappa_b+\kappa_c)k(1-k) \cong$$
$$\cong \frac{(\varepsilon_p/\varepsilon_m)-1}{3}\cdot\{\frac{1}{1+L_a[(\varepsilon_p/\varepsilon_m)-1]}+$$
$$+\frac{1}{1+L_b[(\varepsilon_p/\varepsilon_m)-1]}+\frac{1}{1+L_c[(\varepsilon_p/\varepsilon_m)-1]}\}\cdot k. \qquad (26)$$

When the charge q is distributed with density σ on the capacitor plate of area S and voltage U_0 is connected to the plates, the capacitor capacitance is:

$$C = \frac{q}{U_0} = \frac{\sigma S}{U_0} = D\frac{S}{U_0}\,. \qquad (27)$$

The relative variation of capacitor capacitance δ_C is equal to relative variation of electric flux density:

$$\delta_C = \frac{C - C_0}{C_0} = \frac{(S/U_0)\cdot(D-D_0)}{(S/U_0)\cdot D_0} = \delta_D\,. \qquad (28)$$

If the permittivity of admixtures ε_p is less a capacitor dielectric permittivity ε_m, the variation δ_D is negative and capacitor capacitance decrease. The variation of $\varepsilon_p/\varepsilon_m$ in interval [0,1] corresponds to variation of δ_D in interval [0,–1,5k]. It depends on particles shape very weakly in this case.

If $\varepsilon_p>\varepsilon_m$, the δ_D value is positive. The maximal value $\delta_{D\max}$ of δ_D is obtained for $\varepsilon_p\to\infty$, i.e., if admixtures are metallic. It is:

$$\delta_{D\max} = \frac{1}{3}(\frac{1}{L_a}+\frac{1}{L_b}+\frac{1}{L_c})\cdot k\,. \qquad (30)$$

This expression strongly depends on particle shape. The values L_a, L_b, L_c and the ratio $K_{n\max}=\delta_{D\max}/k$ were calculated from (4)-(6) and from (33) using Matlab and supposing, that $c>b>a$. The ratios $e=b/a$ and $g=c/b$ were varied with step 0,05 in intervals [0,05; 1]. Some part of obtained results is presented in Fig. 3 and 4. The equations $e=g=1$, $L_a=L_b=L_c=1/3$ and $K_{n\max}=3$ are correct for a sphere.

The value $K_{n\max}$ quickly increases for elongate and plane particles. If the ratios e and g decrease from 1 to 0,3, i.e. 3,3 times, the value of $K_{n\max}$ increases proportional, i. e. from 3 to 9,64. If e and g decrease from 0,3 to 0,05, i.e. additionally 6 times, the value $K_{n\max}$ increases from 9,6 to 806, i.e. even 85 times.

The inequations $1/L_c>>1/L_b>>1/L_a$ are correct and $K_{n\max}\approx 1/3L_c$, if $e\leq 0,3$ and $g\leq 0,3$. We can orientate the admixture particles especially, that the longest ellipsoid axis could be directed perpendicular to capacitor electrode plane. The K_{nmax}=1/L_c in this case, i. e. the K_{nmax} value increases 3 times in comparison with randomly orientated particles. The capacitance of capacitor with such dielectric increases ≈24 times, if the longest axes of ellipsoidal particles with e=g=0,05 are orientated perpendicular to electrode plane and the particles concentration is k=1%.

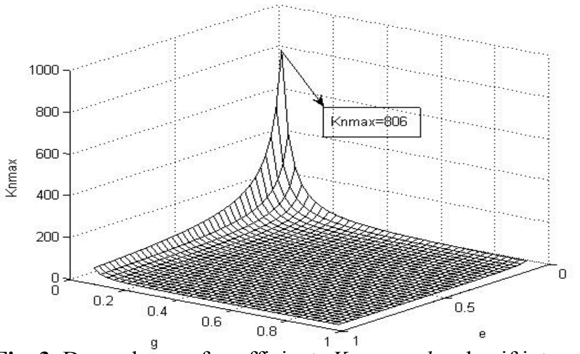

Fig. 3. Dependence of coefficient K_{nmax} on d and e, if interval of d and e variation is [0,05; 0,95]

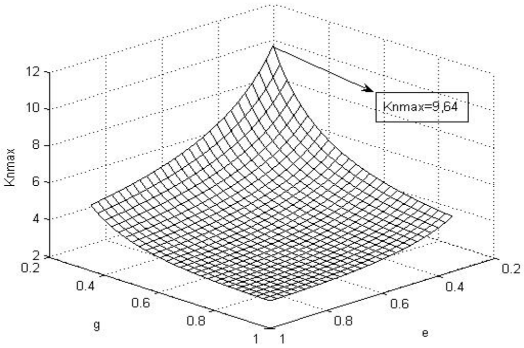

Fig. 4. Dependence of coefficient K_{nmax} on d and e, if interval of d and e variation is [0,3; 0,95]

The capacitor electric strength decreases if metallic particles in capacitor dielectric appear. But the electric strength decreases not more than 2 times, if the particles are not longer than the half of distance between electrodes.

Conclusions

1. The influence of dielectric admixtures on capacitor capacitance was investigated approximating the particle shape by ellipsoid. Dependence of field distribution upon particles shape and orientation is shown for several practical cases.
2. The capacitor capacitance increase especially, if the admixtures are composed of elongate and plane particles, orientated perpendicular to electrode plane.

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The field distribution and capacitance of capacitor vary, when the admixtures are in the dielectric layer of capacitor. Influence of admixture particle shape is evaluated approximating the particle shape by ellipsoid. The expressions are obtained for calculation of electric flux density inside and outside particle and for calculation of relative capacitance variation. The ratio of relative capacitance variation with admixture concentration is calculated for different particle shape using MATLAB. Capacitance increases especially when the admixtures consist of very elongate and plane metallic particles oriented in direction perpendicular to capacitor plates. The electric strength decreases. Obtained results can be used for other problems, when the influence of admixtures to homogeneous medium must be evaluated. Ill. 4, bibl. 3 (in English; summaries in English, Russian and Lithuanian).

Ю. А. Вирбалис, С. Жебраускас. Оценка влияния примесей диэлектрика на емкость конденсатора // Электроника и электротехника. – Каунас: Технология, 2007. – № 3(75). – С. 69–72.

Если в диэлектрике плоского конденсатора есть примеси, в диэлектрике меняется распределение поля и емкость конденсатора. Влияние формы частиц примесей на емкость конденсатора оценена, апроксимируя форму частицы эллипсоидом. Подучены выражения для расчета плотности электрического потока внутри и вне частицы и выражение для расчета относительного изменения электрической емкости. При помощи MATLAB рассчитано отношение относительного изменения емкости с объемной концентрацией примесей для частиц разной формы. Особенно емкость конденсатора возрастает если примеси составляют очень длинные и плоские частицы, которые ориентированы перпендикулярно плоскости электродов. Одновременно понижается электрическая прочность конденсатора. Полученные резуьтаты можно применять для решения других задач, если нужно оценить влияние различных примесей на свойства однородной среды. Ил. 4, библ. 3 (на литовском языке; рефераты на английском, русском и литовском яз.).

J. A.Virbalis, S. Žebrauskas. Dielektriko priemaišų įtaka plokščiojo kondensatoriaus elektrinei talpai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 3(75). – P. 69–72.

Jei plokščiojo kondensatoriaus dielektrike yra priemaišų, pasikeičia jo lauko pasiskirstymas ir kondensatoriaus talpa. Į priemaišų dalelių formos įtaką talpai atsižvelgta aproksimuojant dalelių formą elipsoidu. Gautos išraiškos elektrinio srauto tankiui dalelių viduje be išorėje ir talpos santykiniam pokyčiui apskaičiuoti. Naudojant MATLAB apskaičiuotas talpos pokyčio santykis su priemaišų koncentracija esant įvairių formų dalelėms. Ypač kondensatoriaus talpa išauga, jei priemaišas sudaro labai pailgos ir plokščios metalinės dalelės, orientuotos elektrodų plokštumai statmena kryptimi. Kartu sumažėja kondensatoriaus elektrinis atsparumas. Gautus rezultatus galima taikyti ir kitiems uždaviniams spręsti, kai būtina nustatyti įvairių priemaišų įtaką vienalytei terpei. Il. 4, bibl. 3 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).