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The Mathematical Model of Conductance Measurement in Growing Thin Films

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Introduction

The island stage, mono–atomic layers or self–organized derivatives are derivable producing nanostructures by vacuum technology. These layers are too thin for traditional measurements of their resistance or thickness [1,2,3]. Indirect measurement of the condensate resistance of thin layers was offered [4]. It can be used for the control and research of the condensation processes. The simplified mathematical model of this measurement method, which enabled to identify the resistance of the condensate in early condensation stages, was created [5]. But research of the electro dynamical processes in the vacuum system [6,7] disclosed to us, that the simplified mathematical model underestimated some important facts.

The mathematical model of the non–invasive measurement of the condensate resistance and the derivation of the mathematical expressions, taking into account the potential distribution in the surface of the measurement probe, are given in this paper.

The model of the measurement of condensate conductance

The probe of the conductance measurement (Fig.1) is the dielectric strip *1* in width *b* and the metallic contact areas *2* and *5* are at the ends of this strip. The distance between contact areas is *L* and the evaporated material deposits there. The layer isn't solid at the early condensation stages. Therefore the equivalent thickness of the condensate is dimensioned as *h*. The temperature of the evaporator is high during the evaporation and the evaporator emits the flow of charges i_E *3*, which flows to the probe. Not only the electrons, but also the ionized atoms of the evaporating material form this flow. The distance between the evaporator and the surface of the substrate is H_E. There is no emission of charges evaporating the materials in low temperatures, therefore the second empty evaporator of high temperature is used during such evaporation processes. The growing condensate between contact areas is reduced to the elementary elements in length *dx*.

The principled electrical scheme of the measurement model of the condensate conductance is shown in Figure 2. The contact areas are marked as *A* and *B* here. These contact areas were surfaced by conductor before the experiment and the terminals were done for the measurements of the potential u_A and u_B. The voltmeter, having inner resistance r_m, was connected to the contact area *B*. The equivalent resistivity of the thermo electrical emission current i_E was marked as ρ_E. This resistivity is set as a constant for the simplification of the model. The equation of the equivalent resistance of the charges flow channel, if we assume that the density of charges flow to the surface of the substrate between contact areas *A* and *B* is equal, can be written as:

$$r_E = \rho_E \frac{H_E}{bL}. \tag{1}$$

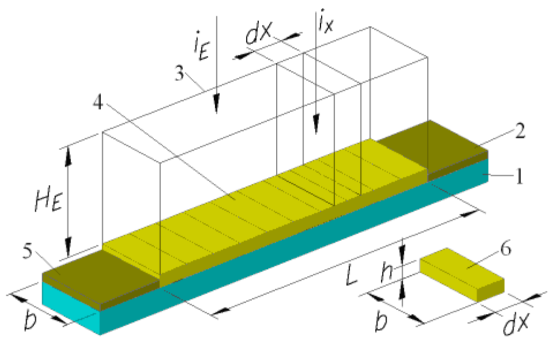

Fig. 1. The construction of the measurement probe of the condensate conductance and the division of the condensate: 1 - substrate, 2 – metallic contact area A, 3 – the flow of charges, 4 – the condensate of the conductor deposited between contact areas (the condensate on the contact areas wasn't shown), 5 - contact area B, 6 – the elementary element of the condensate

Then the equivalent resistance of the elementary charges flow channel in width *dx* will be:

$$dr_E = \rho_E \frac{H_E}{bL}\frac{L}{dx} = r_E \frac{L}{dx}. \tag{2}$$

The elementary flow of charges i_x to the node in the distance *x* from the contact area *A* will be:

$$i_x = \frac{u_x - E}{r_E L} dx . \quad (3)$$

If we assume that the equivalent thickness of the condensate material h is the constant, the resistance of the condensate between contact areas A and B can be calculated as follows:

$$r_p = \rho_p \frac{L}{bh} . \quad (4)$$

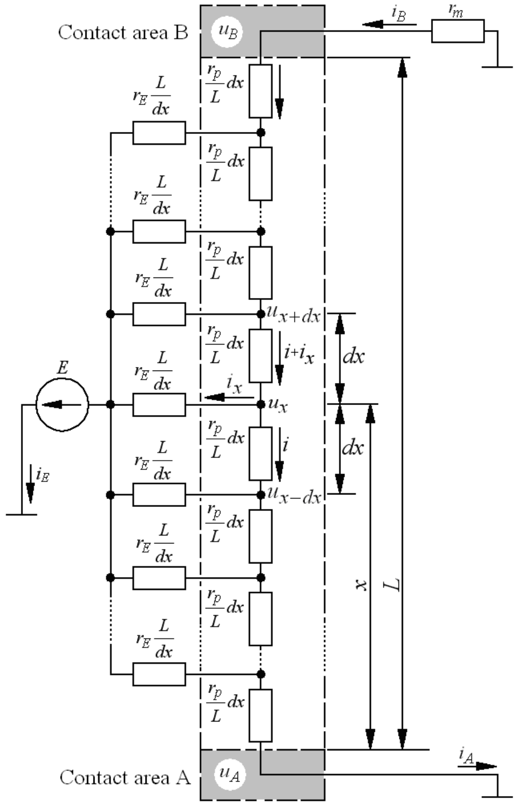

Fig. 2. The principled electrical scheme of the measurement model of the condensate conductance

The resistance of the elementary condensate element in length dx can be calculated as follows:

$$dr_p = \rho_p \frac{L}{bh} \frac{dx}{L} = \frac{r_p}{L} dx . \quad (5)$$

The voltage drop in resistance in distance between x-dx and x from the contact area A is:

$$\Delta u_{-dx} = u_{x-dx} - u_x = -i \frac{r_p}{L} dx . \quad (6)$$

The voltage drop in resistance in distance between x+dx and x from the contact area A is:

$$\Delta u_{+dx} = u_x - u_{x+dx} = -i \frac{r_p}{L} dx - (u_x - E) \frac{r_p}{r_E L^2} (dx)^2 . \quad (7)$$

The difference of the voltage drops in these adjacent resistances can be calculated as follows:

$$d^2 u_x = \Delta u_{-dx} - \Delta u_{+dx} = (u_x - E) \frac{r_p}{r_E L^2} (dx)^2 . \quad (8)$$

From the equation (8) we get the differential expression of the voltage u_x as a function of x:

$$\frac{d^2 u_x}{dx^2} - \frac{r_p}{r_E L^2} u_x = -\frac{r_p}{r_E L^2} E . \quad (9)$$

The characteristic equation of the differential equation (9) is:

$$\frac{d^2 u_x}{dx^2} - \frac{r_p}{r_E L^2} u_x = 0 . \quad (10)$$

The roots of the equation (10) are as follows:

$$\lambda_{1,2} = \pm \frac{1}{L} \sqrt{\frac{r_p}{r_E}} = \pm \frac{\beta}{L} . \quad (11)$$

The particular solution of the differential equation (9), taking into account the roots, is:

$$u_x = C_1 e^{x\lambda} + C_2 e^{-x\lambda} + C_0 . \quad (12)$$

The coefficient C_0 can be calculated from the first and second fluxions of the equation (12):

$$\begin{cases} \frac{du_x}{dx} = \lambda C_1 e^{x\lambda} - \lambda C_2 e^{-x\lambda} , \\ \frac{d^2 u_x}{dx^2} = \lambda^2 C_1 e^{x\lambda} + \lambda^2 C_2 e^{-x\lambda} . \end{cases} \quad (13)$$

The equation (12) and the expression of the second fluxion (13) were substituted into equation (10):

$$\lambda^2 \left(C_1 e^{x \cdot \lambda} + C_2 e^{-x\lambda} \right) - \\ - \lambda^2 \left(C_1 e^{x \cdot \lambda} + C_2 e^{-x\lambda} + C_0 \right) = -\lambda^2 E. \quad (14)$$

We obtained that C_0=E from the equation (14). Then the complementary solution of equation (9) will be:

$$u_x = C_1 e^{x\lambda} + C_2 e^{-x\lambda} + E . \quad (15)$$

The expressions of the coefficients C_1 and C_2 depend on the initial conditions on the contact areas A and B. Now we will explore, how the potential distributes in the condensate between contact areas, when the inner resistance of the meter is r_m=0 (i_B – mode of the current measurement) and r_m>>0 (u_B – mode of the voltage measurement).

The potential distribution in the substrate when r_m=0

In this case the contact areas A and B are connected to earth. The distance x=0 and u_x=0 in the contact area A, therefore:

$$0 = C_1 + C_2 + E . \quad (16)$$

The distance x=L and u_x=0 in the contact area B. This contact area is connected to earth too, therefore:

$$0 = C_1 e^{L\lambda} + C_2 e^{-L\lambda} + E . \quad (17)$$

From the equations (16) and (17) the system of equations for the calculation of the coefficients C_1 and C_2 was written:

$$\begin{cases} C_1 e^{L\lambda} + C_2 e^{-L\lambda} = -E, \\ C_1 + C_2 = -E. \end{cases} \quad (18)$$

From the equation (18) these coefficients are:

$$\begin{cases} C_1 = -E \dfrac{1 - e^{-L\lambda}}{e^{L\lambda} - e^{-L\lambda}}, \\ C_2 = -E \dfrac{e^{L\lambda} - 1}{e^{L\lambda} - e^{-L\lambda}}. \end{cases} \quad (19)$$

From the equation (15), knowing the expressions of the coefficients C_1 and C_2, the equation of the potential distribution u_x in the substrate in the length L was written:

$$u_x = -E \frac{e^{x\lambda} - e^{-x\lambda} + e^{(L-x)\lambda} - e^{-(L-x)\lambda}}{e^{L\lambda} - e^{-L\lambda}} + E . \quad (20)$$

Taking into account the roots (11) of the characteristic equation (10), the equation (20) can be rewritten as follows:

$$u_x = -E \frac{e^{x\frac{\beta}{L}} - e^{-x\frac{\beta}{L}} + e^{\frac{\beta}{L}(L-x)} - e^{-\frac{\beta}{L}(L-x)}}{e^{\beta} - e^{-\beta}} + E . \quad (21)$$

The meaning of the condensate resistance r_p between contact areas is in the equation (12) of the coefficient β. This resistance is infinite at the start of the process and it begins to decrease to the hundreds of ohms, when the condensate begins to grow. The potential distribution u_x in the substrate in length L was calculated according to the (21) alternating the resistance r_p from $10^{13}\Omega$ to 100Ω (Fig. 3). The potential is equal to E in all length of the substrate at the start of condensation process. The potential u_x begins to decrease, when the resistance sinks to $10^8\Omega$. But the maximum of the potential stays in the middle of condensate length.

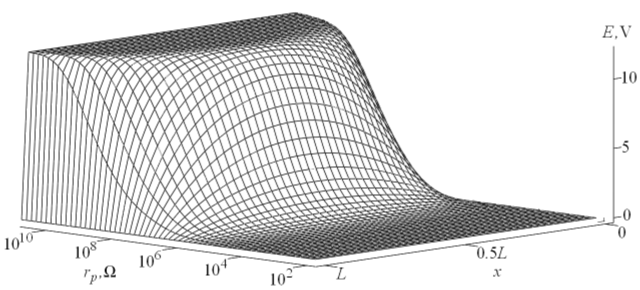

Fig. 3. The potential distribution in the substrate during the condensation process. The contact areas *A* ir *B* are connected to earth

The potential distribution in the substrate when r_m>0

In this case the contact area *A* is connected to earth and its potential u_A=0. The contact area *B* is connected to earth through the resistance r_m, therefore the potential u_B is equal to the voltage drop in resistance r_m. Taking into account that $x=L$ ir $u_x=u_L$ in the contact area *B*, equation (15) can be written as follows:

$$i_B r_m = C_1 e^{L\lambda} + C_2 e^{-L\lambda} + E . \quad (22)$$

The voltage drop in the resistance of the elementary element of the condensate is:

$$du_x = i \frac{r_p}{L} dx , \quad (23)$$

and the current through this resistance is:

$$i = \frac{du_x}{dx} \frac{L}{r_p} . \quad (24)$$

Substituting the expression of the first fluxion of u_x to the equation (24) we get:

$$i = \frac{L}{r_p} \frac{du_x}{dx} = \frac{L\lambda}{r_p} \left(C_1 e^{x\lambda} - C_2 e^{-x\lambda} \right). \quad (25)$$

The distance $x=L$ in the contact area *B*, therefore substituting the expression of the roots (13) to the equation (25) we get:

$$i = \frac{\beta}{r_p} \left(C_1 e^{\beta} - C_2 e^{-\beta} \right). \quad (26)$$

The expression of the current (26) was substituted into equation (22) and taking into account that $x=L$, we get:

$$-\frac{r_m \beta}{r_p} \left(C_1 e^{\beta} - C_2 e^{-\beta} \right) = C_1 e^{\beta} + C_2 e^{-\beta} + E . \quad (27)$$

The equation (27) we transform in such a way:

$$C_1 e^{\beta} \left(1 + \frac{r_m \beta}{r_p} \right) + C_2 e^{-\beta} \left(1 - \frac{r_m \beta}{r_p} \right) = -E . \quad (28)$$

The system of equations for the calculation of the coefficients C_1 and C_2 was written from the equations (16) and (28):

$$\begin{cases} C_1 e^{\beta} \left(1 + \dfrac{r_m \beta}{r_p} \right) + C_2 e^{-\beta} \left(1 - \dfrac{r_m \beta}{r_p} \right) = -E, \\ C_1 + C_2 = -E. \end{cases} \quad (29)$$

The solution of the equation (29) is:

$$C_1 = \frac{D_1}{D}, \quad C_2 = \frac{D_2}{D} . \quad (30)$$

The coefficients D, D_1 and D_2 calculated form the system of equations are:

$$\begin{cases} D = e^{\beta} \left(1 + \dfrac{r_m \beta}{r_p} \right) - e^{-\beta} \left(1 - \dfrac{r_m \beta}{r_p} \right), \\ D_1 = -E + e^{-\beta} \left(1 - \dfrac{r_m \beta}{r_p} \right) E, \\ D_2 = E - e^{\beta} \left(1 + \dfrac{r_m \beta}{r_p} \right) E. \end{cases} \quad (31)$$

Substituting the equations (30) and (31) to the equation (15), the expression of the potential u_x is as follows:

$$u_x = \frac{D_1}{D} e^{x\frac{\beta}{L}} + \frac{D_2}{D} e^{-x\frac{\beta}{L}} + E\,. \qquad (32)$$

The potential distribution u_x in the substrate in length L was calculated according to the equation (32) alternating the resistance r_p from $10^{13}\Omega$ to 100Ω (Fig. 4). The potential u_B of the contact area B has a typical extreme, which appears decreasing the resistance of condensate r_p $10^{10}\Omega$ to $10^5\Omega$.

The place of the potential u_B extreme depends on b, L, r_m and r_E parameters. The combination of these parameters can be chosen in such a way, that the extreme can be obtained in the proper resistance r_p.

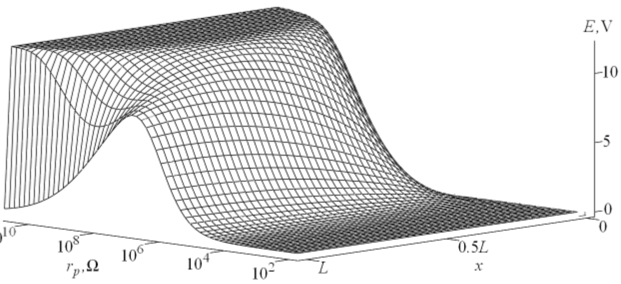

Fig. 4. The potential distribution in the substrate during the condensation process. The contact areas *A* is connected to earth and the voltmeter with the inner resistance r_m is connected to the contact area *B*

From the Figure 4 we can see that the inner resistance r_m of the connected voltmeter changes the potential distribution in the zone near the contact area *B*. Therefore we can affirm that this method of the measurement doesn't have the influence on the condensation processes. The resistance of the condensate r_p is calculated from the equation (32) using the iterative methods. There is no the right or approximate analytical expression for the calculation of the resistance r_p for the meantime.

Conclusions

The mathematical model of the non–invasive measurement of the condensate resistance, taking into account the potential distribution in the surface of the measurement probe, was created.

The width of probe and the distance between the contact areas are taken into account in this mathematical model. Therefore the mathematical model can be used projecting the probes, in which the extreme of the signal appears reaching the proper conductance of the condensate.

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The island stage, mono–atomic layers or self–organized derivatives are derivable producing nanostructures by vacuum technology. These layers are too thin for traditional measurements of their resistance or thickness. The non–invasive measurement was proposed for the measurement of these stages. The mathematical model of the non–invasive measurement of the condensate resistance, taking into account the potential distribution in the surface of the measurement probe, was created. Ill. 4, bibl. 7 (in English; summaries in English, Russian and Lithuanian).

В. Синкявичюс. Математическая модель метода измерения проводимости тонких плёнок во время их роста // Электроника и электротехника. – Каунас: Технология, 2008. – № 5(85). – С. 17–20.

При конденсации паров материала в вакууме на подложках из разных материалов можно получить островковые или моноатомные слои. Они слишком тонкие, чтобы можно было применять традиционные методы измерения проводимости. Для измерения проводимости плёнок такого состояния предложен косвенный метод измерения. Создана математическая модель этого метода измерения, учитывающая распределение потенциала на плоскости зонда. Ил. 4, библ. 7 (на английском языке; рефераты на английском, русском и литовском яз.).

V. Sinkevičius. Augančio kondensato laidžio matavimo matematinis modelis // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 5(85). – P. 17–20.

Kondensuojant medžiagos garus vakuuminės technologijos būdu ant įvairių medžiagų padėklų, gaunami saleliniai ar vienatomiai sluoksniai. Jie yra per ploni, kad būtų galima naudoti tradicinius sluoksnių laidžio matavimo būdus. Tokių būsenų kondensato laidį pasiūlyta matuoti netiesioginiu matavimo metodu. Sukurtas šio matavimo matematinis modelis, įvertinantis potencialo pasiskirstymą matavimo zondo plokštumoje. Il. 4, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).