

ELEKTRONIKA IR ELEKTROTECHNIKA

Modeling of Efficiency of Dynamic Electronic Systems

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Introduction

With each year more and more complex tasks are entrusted for electronic systems (ES) of technical, energetic, biologic or educational purpose. Higher versatility, precision (more optimal decisions) adaptivity, flexibility and dynamic stability is expected from them, even when their environment, inner states and user requirements are changing rapidly. ES performance results often depend on how it manages to satisfy user expectations. These results are often characterized using the concept of efficiency [1] – degree by which the needs are satisfied, and for systems of technical purpose – task accomplishment probability. Values of this probability are random and non-stationary.

There still is a lack of models of such ES and their efficiency dynamics, their evolution research and control methods.

ES dynamics

ES dynamics (variation in time) is conditioned by various dynamic factors (factors, determining changing states of the system), including various dynamic loads (loads, which create impacts of variable degree in the

system) [2]. Systems changing in time are considered [3,4] as dynamic systems. Variation of general, functional and economical efficiency, also dynamic sensitivity and dynamic stability are characteristic to them. Dynamic sensitivity is understood as a variation degree of dependence of ES states or (and) output parameter values on values of parameters of dynamic factors. Dynamic stability is understood as ES ability to recreate initial state or state which is close to initial after some disturbance.

When distribution densities of values of dynamic factor parameters from the set $\{D_i\}$ (i=I,...N) at any time t are $\{f_i(l_i,t)\}$, then momentary values of ES efficiency are E_m E_M $\binom{l_i'}{l_i'} \pm \Delta l_i$ (here l_i' – the selected value of the i-th parameter, and Δl_i – small change of this parameter), and probability

$$P_{EM}\left(\left\{i \pm \Delta l_{i}\right\}\right) = \prod_{i=1}^{N} \int_{l_{i}-\Delta l_{i}}^{l_{i}+\Delta l_{i}} f_{i}(l_{i},t) dl_{i}. \tag{1}$$

Static character of ES efficiency is determined by the random character of $\{l'_i\}$, and dynamics is determined by the variation of $\{f_i(l_i,t)\}$.

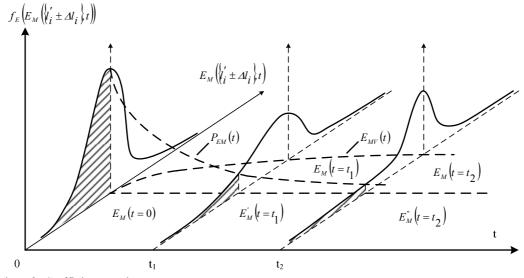


Fig. 1. Dynamics of ES efficiency values

Thus ES efficiency is characterized by distribution of its values, the density of which is $f_E\left(E_M\left(v_i'\pm\Delta l_i\right)t\right)$ and is variable (Fig. 1).

It's natural, that the dynamics illustrated in Fig. 1 may be characteristic only for intellectual ES or for other ES, developed using other measures. As a rule, the efficiency of ES of the lower level decreases. But this fact does not change the peculiarities of efficiency dynamics evaluation.

For illustrative purposes let's assume, that momentary values of efficiency are distributed according the law of normal distribution. Then $E_M(t=0)$, $E_M(t=t_I)$ and $E_M(t=t_2)$ are mathematical means of momentary efficiencies in these moments of time, and since

$$E_{M}(t=0) = E_{M}(t=t_{1}) = E_{M}(t=t_{2}),$$
 (2)

then $P_{EM}(t=0) \neq P_{EM}(t=t_1) \neq P_{EM}(t=t_2)$. Dynamics of this probability is illustrated by the graph of $P_{EM}(t)$.

But as in the case of momentary values of ES quality, values of momentary efficiency do not carry a sufficient amount of information [3]. It is not sufficient to know, that in initial time (t=0) the efficiency of ES was $E_M(t=0)$. As we have mentioned before, it can increase later, remain constant or decrease. It is necessary to know the dynamics of efficiency and to create its general estimate for the time duration of interest (e.g. $(t=0)\div(t=t_2)$).

In case of normal distribution we can guarantee with probability $P_{\rm EM}=0.5$, that during the indicated period of time the generalized efficiency of dynamic ES will be not less than

$$E_{0,5}(0 \div t_2) = \frac{\int_0^{t_2} E_{MV}(t)dt}{t_2}.$$
 (3)

In practice it is often required to know the possibilities to maintain the selected (e.g. $E_M(t=0)$) level of efficiency. In this case the generalized probability of such efficiency

$$P_{EM(t=0)} = \frac{\int_{0}^{t_2} \int_{0}^{E_M(t=0)} f_E\left(E_M\left(\left(t_i' \pm \Delta l_i\right) t\right)\right) dE_M \cdot dt}{t_2}.$$
 (4)

Since the dynamics of ES is determined by the variation of parameters influenced by it $(l_{i\min} < l_i < l_{i\max})$, then it is necessary to know the generalized values of these probabilities during the selected period of time.

Probability of the value $l_{i \min} < l_i \le l_{i \min} + \Delta l_i$

$$P_{\Delta li} = \frac{\int\limits_{0}^{t_2} \int\limits_{0}^{\Delta li} f_i(l_i, t) dl_i \cdot dt}{t_2}, \tag{5}$$

of the value $l_i^* < l_i \le l_i^* + \Delta l_i$ -

$$P_{l_{i}^{*}} = \frac{\int_{0}^{t_{2}} \int_{l_{i}^{*}}^{t_{1}} f_{i}(l_{i}, t) dl_{i} \cdot dt}{t_{2}};$$
(6)

and of the value $l_{i \text{max}} - \Delta l_i < l_i \leq l_{i \text{max}}$

$$P_{l_{i_{\max}}} = \frac{\int_{0}^{t_{i_{\max}}} \int_{l_{i_{\max}}-Ali}^{l_{i_{\max}}} f_{i}(l_{i},t)dl_{i} \cdot dt}{t_{2}};$$
 (7)

here $l_{i\min}$ and $l_{i\max}$ – minimal and maximal values of parameter L_i , formed during period of time $0 \div t_2$. By using (5)–(7) formulas it is possible to form a generalized distribution of the i-th parameter values (its density $f_{ai}(l_i)$) and to determine its mathematical mean l_{mi} . The dynamics coefficient of the value (argument) of the i-th parameter of the impact:

$$k_{di} = \int_{l_{min}}^{l_{max}} |l_{mi} - l_i| f_{ai}(l_i) dl_i.$$
 (8)

The dynamics ES with its external factors, for which the set of values of \underline{L} parameters – $\{l_i\}$, and output results \underline{Y} , in one or other way determining (characterizing) the efficiency $E(\underline{Y})$, can be illustrated using scheme presented in Fig. 2.

$$\underline{D} \quad \underline{L} \qquad \xrightarrow{l_{l}(t)} \qquad \underline{M} \qquad ES \qquad \underbrace{y_{l}(t)} \qquad \underbrace{M} \qquad \underline{H} \qquad \underbrace{V} \qquad \underbrace{y_{M}(t)} \qquad \underbrace{Y} \qquad \underbrace$$

Fig. 2. Scheme of dynamic ES

In this scheme the set of factors \underline{D} characterized using parameters \underline{L} , values of which are alternating, influence the ES (its operation algorithms $-\underline{A}$, processes $-\underline{H}$ and decision making models $-\underline{M}$). Also inside dynamic ES the set of inner factors \underline{V} is active, including the standard (legal) ones. The dynamic ES influenced in this manner outputs the set of results \underline{Y} , which can be characterized by the values of parameters $\{y_i(t)\}$. Thus

$$y_1(t) = f_{v_1}(\{l_i(t)\}, \{m_i(t)\}, \{a_i(t)\}, \{v_i(t)\}, \{h_i(t)\}, t),$$
(9)

here $m_i(t)$, $a_i(t)$, $v_i(t)$ and $h_i(t)$ – values of *i*-th parameters of models, algorithms, inner factors and processes in time *t*. The first coefficient of dynamics of ES (1st function performed by it) in respect of $l_1(t)$ is

$$ky_{1(1)}^{(t)} = \frac{\int_{y_{1\min}(t)}^{y_{1\max}(t)} |y_{m1}(t) - y_1(t)| fy_{1a}(y_1(t)) dy_1}{k_{d1}};$$
(10)

here $y_{1max}(t)$ and $y_{1min}(t)$ — maximal and minimal mean of the values of the 1st parameter of the ES performance result in time t, when it is influenced by $l_1(t)$; $y_{m1}(t)$ — mathematical mean of y_l ; $f_{y1a}(y_1(t))$ — distribution density of y_1 values; k_{d1} — the dynamics coefficient of $l_1(t)$. After calculating all values of dynamics coefficients determined by $\{l_i(t)\}$, under some one particular scenario of implementation of the sets \underline{M} , \underline{A} , \underline{H} and \underline{V} , e.g. M_1 , A_1 , H_1 and V_1 , the matrix is formed:

In this matrix the inter-relations of $\{l_i\}$ values and possibilities of systematic impact are ignored.

The relation between ES agent (argument) and its decision (function) can be demonstrated using elementary graphical model of their relation (Fig. 3).

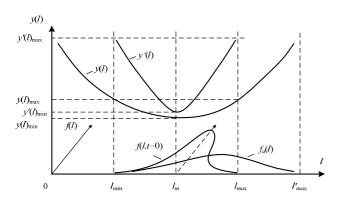


Fig. 3. Influence of the model on the coefficient of dynamics of the ES function

When t=0, then the coefficient of the argument dynamics

$$k_{d} = \int_{-L}^{l_{\text{max}}} \left| l_{m} - l \right| f(l, t = 0) dl , \qquad (12)$$

and of the ES result when its dependence y(l) is present

$$k_{y} = \frac{\int_{\min}^{l_{\max}} (y(l) - y(l)_{\min}) f(l, t = 0) dl}{k_{d}}$$
 (13)

When the objective function is changed into y'(l), we obtain that

$$k'_{y} = \frac{\int_{\min}^{l_{\max}} (y'(l) - y'(l)_{\min}) f(l, t = 0) dl}{k_{d}}$$
 (14)

It is obvious, that

$$k_{v}^{\prime} > k_{v} . \tag{15}$$

L during time $0 \div t_2$ is generalized, the other density of the distribution $f_a(l)$ is obtained (see (5)–(7) and Fig. 3). Then the coefficient of the argument dynamics

When the dynamics of the value l of the parameter

$$k_d = \int_{l_{\min}}^{l_{\max}} l'_m - l \left| f_a(l) dl \right|, \tag{16}$$

here l'_m – mathematical mean of $f_a(l)$. The coefficient of dynamics of the ES result (function) during this period will be found as

$$k_{ya} = \frac{\int_{\min}^{l_{\min}} (y(l) - y(l)_{\min}) f_a(l) dl}{k_d}$$
(17)

It is obvious that

$$k_{,,} < k_{,,,,} \tag{18}$$

and furthermore

$$k'_{v} < k'_{va}; \tag{19}$$

here k'_{ya} – the coefficient of dynamics of the ES result (during $0 \div t_2$ time), when the objective function – y'(l).

Analysis of the efficiency of dynamic ES

When evaluating the efficiency of the dynamic ES, let's use the matrix \underline{K} ((11) expression). From Fig. 3. it can be seen that each element of the matrix (e.g. $ky_{(i)j}$) in this case symbolizes the entirety of several sets ($\{ky_{(i)j_{M_s}}\}$, $\{ky_{(i)j_{H_s}}\}$ and $\{ky_{(i)j_{V_s}}\}$), which is formed due to the variants of models (\underline{M}), algorithms (\underline{A}), processes (\underline{H}) and other inner factors (\underline{V}). By assessing the priorities of each component of each set and using weight coefficients ($\{\eta_{M_s}\}$, $\{\eta_{A_s}\}$, $\{\eta_{H_s}\}$ bei $\{\eta_{V_s}\}$), we obtain partial coefficients of dynamics of j-th function (due to variation of the i-th argument). This is illustrated by expressions (20)–(23).

$$ky_{(i)jM} = \eta_{M_1}ky_{(i)jM_1} + \eta_{M_2}ky_{(i)jM_2} + \dots + \eta_{M_s}ky_{(i)jM_s} + \dots + \eta_{M_B}ky_{(i)jM_B};$$
(20)

$$ky_{(i)jA} = \eta_A ky_{(i)jA_1} + \eta_A ky_{(i)jA_2} + \dots + \eta_{A_r} ky_{(i)jA_r} + \dots + \eta_{A_R} ky_{(i)jA_C};$$
(21)

$$ky_{(i)jH} = \eta_{H_1}ky_{(i)jH_1} + \eta_{H_2}ky_{(i)jH_2} + \dots + \eta_{H_s}ky_{(i)jH_s} + \dots + \eta_{H_B}ky_{(i)jH_G};$$
(22)

$$ky_{(i)jV} = \eta_{V_1}ky_{(i)jV_1} + \eta_{V_2}ky_{(i)jV_2} + \dots + \eta_{V_n}ky_{(i)jV_n} + \dots + \eta_{V_n}ky_{(i)jV_n};$$
(23)

here: $ky_{(i)jM_s}$, $ky_{(i)jA_s}$, $ky_{(i)jH_s}$ and $ky_{(i)jV_s}$ – coefficients of dynamics of the function, formed after selecting the s-th model, algorithm, process or other factor; B, C, G and R numbers of the components of the respective sets (models, algorithms, ...).

Here when creating one of the expressions (20)-(23), elements (components) of other sets are left as they were used when creating the \underline{K} matrix (see formula (11)). When writing expression (20), only elements of the set Mchange, and A_1 , H_1 and V_1 remain the same.

If all inner factors of ES dynamics are not interrelated, then the generalized value of $ky_{(i)}$ calculated as

$$ky_{(i)ja} = \frac{ky_{(i)jM} \cdot ky_{(i)jA} \cdot ky_{(i)jH} \cdot ky_{(i)jV}}{k^3 y_{(i)j}}$$
(24)

This expression spans only four sets of the inner factors (M, A, H) and V, but the evaluation principle by itself is suitable for any number of the sets. If priorities of the inner factor components of dynamic ES are selected so, that, for example

$$p(M_1) > p(M_2) > \dots > p(M_R)$$
 (25)

and each following component will be used only then, when the earlier selected component will be not suitable (e.g. will be broken), then (e.g. in the set M)

$$\eta_{M1} = P(M_1); \tag{26}$$

$$\eta_{M2} = [1 - P(M_1)]P(M_2);$$
(27)

$$\eta_{M3} = [1 - P(M_1)][1 - P(M_2)]P(M_3);$$
(28)

$$\eta_{MB} = \prod_{s=1}^{B-1} [1 - P(M_s)] P(M_B), \tag{29}$$

here $P(M_s)$ – probability that the s-th inner model of dynamic ES will be operational in any time.

When after failure (disturbance) models are recreated, then

$$y_{1\min,a} = y_{m1a} - ky_{(1)1a} \cdot k_{d1} - \dots - ky_{(i)1a} \cdot k_{di} - \dots - ky_{(N)1a} \cdot k_{dN};$$
(33)

$$y_{1\max,a} = y_{m1a} + ky_{(1)1a} \cdot k_{d1} + \dots + ky_{(i)1a} \cdot k_{di} + \dots + ky_{(N)1a} \cdot k_{dN};$$
(34)

$$y_{M \min a} = y_{mMa} - k y_{(1)Ma} \cdot k_{d1} - \dots - k y_{(i)Ma} \cdot k_{di} - \dots - k y_{(N)Ma} \cdot k_{dN}$$
 (35)

$$y_{M \max, a} = y_{mMa} + k y_{(1)Ma} \cdot k_{d1} + \dots + k y_{(i)Ma} \cdot k_{di} + \dots + k y_{(N)Ma} \cdot k_{dN};$$
(36)

here $y_{1 \min,a}$ and $y_{1 \max,a}$ - minimal and maximal value of the generalized value of the 1^{st} function (Y_1 parameter); y_{m1a} - generalized mathematical mean of parameter Y_1 values; other indexes are analogous.

Degree of satisfying needs of the parameter Y_1 values (efficiency of dynamic ES according to this

$$P(M_s) = \frac{\sum_{v=1}^{O_D} t_{M_s v}}{\sum_{v=1}^{O_D} t_{M_s v} + \sum_{d=1}^{O_A} t_{a M_s d}};$$
(30)

here $t_{M_s v}$ – the duration of recreation of the v-th operation interval of the Ms model; t_{aM_sd} - duration of the d-th recreation of the Ms model; O_D – number of operation intervals during the analyzed period of time; O_A – number of recreations during the analyzed period of time.

Weight coefficients of components of other sets of inner factors of dynamic ES are calculated analogously.

It is not hard to tell, that $P(M_s)$ and probabilities analogous to it $P(A_s)$, $P(H_s)$ and $P(V_s)$, and at the same time η_{M_s} , η_{A_s} , η_{H_s} and η_{V_s} are functions of time – $P\big(M_s,t\big), \quad P\big(A_s,t\big), P\big(H_s,t\big), \quad P\big(V_s,t\big), \quad \eta_{M_s}(t)\,, \quad \eta_{A_s}(t)\,,$ $\eta_{H_s}(t)$ and $\eta_{V_s}(t)$. Variation of $P(M_s,t)$ is determined by non-stationarities of the average operation duration between adjacent failures (disturbances) and average recreation duration of the s-th module. Therefore

$$ky_{(i)ja}(t) = \frac{ky_{(i)jM}(t) \cdot ky_{(i)jA}(t) \cdot ky_{(i)jH}(t) \cdot ky_{(i)jV}(t)}{k^3 y_{(i)j}}$$
(31)

By using formulas (20)–(23), (26)–(30) and (24) and by considering the dynamics of the magnitudes, we form the generalized matrix $||ky_{(i)ja}(t)||$, or \underline{K}_a , which will correct and alter the matrix K. When using the generalized matrix \underline{K}_a , it is possible to analyze the efficiency of dynamic ES further.

The momentary matrixes are distinguished from it: $\underline{K}_{a(t=0)}$, ..., $\underline{K}_{a(t=t_2)}$. These matrixes differ only in the fact that in the first one

$$f_{ai}(l_i) = f_i(l_i) \tag{32}$$

i.e. only initial distribution density of the values l_i is used to form it, in the second one - only the final, and in the matrix \underline{K}_a – generalized $(f_{ai}(l_i))$.

By using the assumption of additive character of the changes of the function values it is possible to write, that the generalized (during duration $0 \div t_2$) values of the functions are

$$y_{M \min, a} = y_{mMa} - k y_{(1)Ma} \cdot k_{d1} - \dots - k y_{(i)Ma} \cdot k_{di} - \dots - k y_{(N)Ma} \cdot k_{dN};$$
(35)

$$+ \dots + k y_{(i)Ma} \cdot k_{di} + \dots + k y_{(N)Ma} \cdot k_{dN}; \tag{50}$$

parameter) is determined by the peculiarities of its technical purpose. Assume that

$$E_{ES_1}(y_1) = f_{E1}(y_1), \tag{37}$$

here $E_{ES_1}(y_1)$ – efficiency of dynamic ES, when value of parameter Y_1 equals y_1 . By using this dependence values of $E_{ES_1}(y_{1\min,a})$ and $E_{ES_1}(y_{1\max,a})$ are found and intermediate values (in interval $y_{1\min,a} \div y_{1\max,a}$) are calculated. Generalized efficiency of dynamic ES in respect of parameter Y_1

$$E_{ES_{1a}}(y_{1}) = \frac{\int_{y_{1 \min.a}}^{y_{1 \min.a}} f_{E1}(y_{1}) \cdot f_{y_{1}a}(y_{1}) dy_{1}}{\int_{y_{1 \min.a}}^{y_{1 \max.a}} f_{y_{1}a}(y_{1}) dy_{1}}.$$
(38)

Analogously by using momentary matrixes $(\underline{K}_{a(t=0)},...,\underline{K}_{a(t=t_i)},...,\underline{K}_{a(t=t_2)})$ it is possible to calculate the momentary efficiencies of dynamic ES in respect of this parameter.

After calculating $E_{ES_{1a}}$, ... $E_{ES_{Ma}}$ and their dynamics, there remains to calculate efficiency and its dynamics of entire dynamic ES in time interval $(0 \div t_2)$. For this purpose it falls to form model of efficiency of entire ES using practical results –

$$E_{ES}^*(t) = f_*(E_{ESia}, t)$$
(39)

and to find momentary efficiencies $E_{ES}^*(t=0)$, ..., $E_{ES}^*(t=t_i)$, ..., $E_{ES}^*(t=t_2)$ by using this model. The most generalized estimate will be obtained by using the following formula:

$$E_{ES}^{(0)} = \frac{\int_{ES}^{t_2} E_{ES}^*(t)dt}{t_2}$$
 (40)

It is necessary to note, that there are some cases when the function variations are not of additive character,

when influence of $\{l_i\}$ values depends on their combination, when $\{y_i\}$ values are dependent on each other, etc. But these are the objects of the further research.

Conclusions

By attempting to create models of efficiency of dynamic ES there is inevitable sense of a lack of educational material.

Modeling of efficiency of dynamic ES is particularly burdened by the random character of external and internal factors and instability of this character.

Modeling of states of dynamic ES can be performed by creating the generalized models of impacts and consequences formed due to their dynamics. But in this case also problems of evaluation of impact inter-relations remain unsolved.

When evaluating efficiency of dynamic ES, it is necessary to consider the variation of their inner models, algorithms, processes and other factors and its influence on the efficiency.

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A problem of efficiency modeling of dynamic electronic systems (ES) was formulated. Indexes of dynamics and efficiency of these ES were selected. Dynamics of ES efficiency values was analyzed. It was offered to evaluate the variation of ES parameter values by using coefficients of dynamics. The influence of the model used in system on the dynamics of its subsystems was demonstrated. Analysis of efficiency of dynamic ES was performed, beginning with the efficiency of separate functions, ending with the generalized efficiency estimate for entire system. When looking for generalized estimate for entire ES, a transition from its momentary efficiency to medium efficiency during the analyzed period of time is made. Several still unsolved problems of efficiency modeling of dynamic ES were indicated. Ill. 3, bibl 4 (in English; summaries in English, Russian and Lithuanian).

П. Балайшис, Д. Эйдукас, Р. Гужаускас, А. Валиневичус. Эффективность моделирования динамических электронных систем // Электроника и электротехника. – Каунас: Технология, 2008. – № 6(86). – С. 55–59.

Сформулирована проблема моделирования эффективности динамических электронных систем (ЭС). Подобраны показатели динамики указанных систем и их эффективности. Исследована динамика значений показателей (ЭС) предлагается оценивать коэффициентами динамики. Показано влияние в системе используемой модели на динамику её функций и завершая обобщенной оценкой для всей системы. При определении обобщенной оценки для всей (ЭС) от моментной её эффективности переходят к усредненной за весь исследуемый период. Приведено несколько еще нерешенных проблем моделирования эффективности динамических (ЭС). Ил. 3, библ. 4 (на английском языке; рефераты на английском, русском и литовском яз.).

P. Balaišis, D. Eidukas, R. Gužauskas, A. Valinevičius. Dinaminių elektroninių sistemų efektyvumo modeliavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 6(86). – P. 55–59.

Suformuluota dinaminių elektroninių sistemų (ES) efektyvumo modeliavimo problema. Parinkti šių ES dinamikos ir efektyvumo rodikliai. Išnagrinėta ES efektyvumo verčių dinamika. ES rodiklių verčių kitimą siūloma vertinti dinamikos koeficientais. Parodyta sistemoje naudojamo modelio įtaka jos dinamikai. Atlikta dinaminės ES efektyvumo analizė, pradedant atskirų funkcijų efektyvumu ir baigiant visai sistemai apibendrintu įverčiu. Ieškant visai ES apibendrinto įverčio, nuo momentinio jos efektyvumo pereinama prie vidutinio efektyvumo per tiriamąjį laikotarpį. Nurodyta keletas dar neišspręstų dinaminių ES efektyvumo modeliavimo problemų. Il. 3, bibl. 4 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).