Kaunas University of Technology
Faculty of Mechanical Engineering and Design

The Simulation of an Optimum Supply Chain Network for the Asian Products into the European Market

Master's Final Degree Project

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Kaunas, 2020

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Industrial Engineering and Management (6211EX018)

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Declaration of Academic Integrity

I confirm that the final project of mine, Bavithran Sridhar, on the topic „The Simulation of an Optimum Supply Chain Network for the Asian Products into the European Market“ is written completely by myself; all the provided data and research results are correct and have been obtained honestly. None of the parts of this thesis have been plagiarised from any printed, Internet-based or otherwise recorded sources. All direct and indirect quotations from external resources are indicated in the list of references. No monetary funds (unless required by Law) have been paid to anyone for any contribution to this project.

I fully and completely understand that any discovery of any manifestations/case/facts of dishonesty inevitably results in me incurring a penalty according to the procedure(s) effective at Kaunas University of Technology.

(name and surname filled in by hand) (signature)

Kaunas University of Technology

Faculty of Mechanical Engineering and Design

Task of the Master's final degree project

Given to the student – Bavithran Sridhar

1. Title of the project –

The Simulation of an optimum supply chain network for the Asian products into the European market

(In English)

Azijos šalių produktų optimalaus tiekimo grandinės tinklo į Europos rinką modeliavimas

(In Lithuanian)

2. Aim and tasks of the project –

To design and simulate an optimum supply chain network for configured products from Asia into the European market considering its demand fluctuation and safety stock.

Tasks:

1. To investigate into the cost structure and calculate the parameters governing the model.
2. To design a deterministic model with optimum number of distribution centres and sourcing pattern of the products with historic demand.
3. To design a stochastic model with optimum number of distribution centres and sourcing pattern of the products with normal distribution of demand.
4. To simulate sensitivity analysis with the governing parameters of the optimum supply chain network.
5. To calculate safety stock and service level for the model.

3. Initial data of the project –

N/A

4. Main requirements and conditions –

Data about the available and potential facilities, distribution centres, retail markets, cost parameters and demand. IBM CPLEX Optimization Studio to solve minimization equations of large data sets.

Project author Bavithran Sridhar

(Name, Surname) *(Signature)* *(Date)*

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Study field and area (study field group): Production and Manufacturing Engineering (E10), Engineering Sciences (E).

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Kaunas, 2020. 81 p.

Summary

This research work is carried out at X company and its primary focus is to determine an optimum supply chain network for its Asian commodities. This work aims at providing an in depth analysis and recommendations to the strategic planning department of the organization. Information required to build models, run simulations and conduct analysis were abstracted from annual business data from the X company.

After collating the required information, an outline of the potential network configuration is formulated. The problem formulation was done and the the optimum supply chain model was obtained by solving facility location problem. Various parameters influencing the design of an optimum supply chain were investigated and various cost scenarios were simulated. All the feasible designs were subjected to a sensitivity analysis and the robustness of the optimum model is checked. Recommendations were given to X company including worst case and best case scenarios from cost, material availability and operations management perspectives

Bavithran Sridhar, Azijos šalių produktų optimalaus tiekimo grandinės tinklo į Europos rinką modeliavimas. Magistro baigiamasis projektas, Doc. Dr. Inga Skiedraitė; Kauno technologijos universitetas, Mechanikos inžinerijos ir dizaino fakultetas.

Studijų kryptis ir sritis (studijų krypčių grupė): Gamybos inžinerija (E10), Inžinerijos mokslai (E).

Reikšminiai žodžiai: Tiekimo grandinės tinklo projektavimas, tinklo tvirtumas, jautrumo analizė

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Santrauka

Šis tiriamasis darbas atliktas įmonėje X ir jo pagrindinis tikslas yra nustatyti optimalų Azijos prekių tiekimo grandinės tinklą. Šiuo darbu siekiama pateikti nuodugnią analizę ir rekomendacijas organizacijos strateginio planavimo skyriui. Informacija, reikalinga modeliams kurti, modeliavimui vykdyti ir analizėms atlikti, buvo paremta iš įmonės X metinių verslo duomenimis.

Galimo tinklo konfigūracijos planas suformuluotas, surinkus ir palyginus reikalingą informaciją. Problema buvo suformuluota ir buvo gautas optimalus tiekimo grandinės modelis, išsprendžiant objekto vietos problemą. Įvairūs parametrai, turintys įtakos optimalios tiekimo grandinės kūrimui, buvo ištirti bei buvo modeliuojami įvairūs išlaidų scenarijai. Atlikta visų įmanomų konstrukcijų pažeidžiamumų analizė ir patikrintas optimalaus modelio tvirtumas. Atsižvelgiant į sąnaudas, medžiagų prieinamumą ir operacijų valdymo perspektyvas, X bendrovei buvo pateiktos rekomendacijos, įskaitant įmanomus blogiausius ir geriausius atvejus.

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List of abbreviations

SC: Supply Chain

SCN: Supply Chain Network

SCND: Supply Chain Network Design

DC: Distribution Centre

CV: Coeffecient of Variation

FLP: Facility Location Problem

SCND/R: Supply Chain Network Design/Redesign

Introduction

In the last 30 years, companies which are into distribution and manufacturing are experiencing a constantly changing and volatile business game. There are many such reasons for this change in the environment and one of the signifant driver of this volatility is the consumer behaviour and drastically increased degree of globalization. These drivers makes the industries to stay extremely competitive to each other and a more accessable global market.

Due to the increased degree of gloabalization, the consumer behaviour is also changing along with the advancements and innovations in technologies. These conditions make it compulsory for all the industries to be aware and conscious of its socio-economic and environmental responsibilities which they need to follow.

The increased rate of change in the business environment poses great challenges to the industries in the manufacturing and distribution sector. These implications also have a good side of creating new windows of opportunities to both the industry and to the consumers with enhanced accessability and collateral ownerships [1]. These changes are not just a trend but a major course or a business behaviour which creates a lasting impact on the global economy [2].

Globalization always will have an impending impact on the way the supply chains are set up. Especially the transportation costs and cost for infrastructure and communictaions started shooting up in the 1970’s and constantly grew until the mid 1990s where it attained a break even point [4]. This lays the path for the need of global strategic sourcing of goods and servies which helps the company expand its horizons. The inception of European Union, deregulatory measures in China and its accession by the World Trade Organization (WTO), declaring an open economy in the Soviet Union and Soviet liberation of the eastern European states have collectively contributed to this break even point in the global economy in the 1990s.

Strategic sourcing is a widely used methodology amongs the companies with a grlobal presence in the recent years. This helps to tackle the financial and trade restrictions and liberate the companies in the global business. This inturn provided competeive advantage against the other market share holders in the market and paved the way for continuous improvement and cost reduction strategically as well as on operations level [5].

Satisfaction of the customers, retainment of loyalty of customer i.e repeated orders from key accounts and customer delightments are the other important parameters to be focussed upon which is proven in the recent years. Any supply chain which is to be designed has to have the customer service indices considered and accounted for it to be proven optimum as cost alone is not always a unit to measure the business’ performance [1].

Impeccable order management needs an impeccable logistic network with bufferes considered at each nodal points so that the service level is always met. It’s a demanding priority which has been put on the companies to stick to the service level agreed in the market to be more customer centric with increased credibility and achieve better responsiveness and reliability [5].

Large focus has been put on the industries and the requirements to have in depth knowledge about the customer’s agreed service level and the operational needs to stay ahead on the performance curve [5]. The focus is set more towards customer retainment than just customer satisfaction [1].

Aim and tasks of this thesis

Due to this lack of visibility in the supply chain arising from lack of unambiguous data, there is an increasing trend in the freight cost of the company without any improvement in the lead time behaviour. This is a significant characteristic of a company having so much slack in its supply chain, least degree of load optimization, expensive routing pattern and high level of in-transit inventory.

The objective of this project is to recommend a minimum cost supply chain model for the Asian commodity category. This research work also covers the approximation of the normal distribution demand and recommends the optimum combination of Supplier – Distribution Centre – Retail Market supply chain network. For an optimum network also determine the required safety stock placement.

The aim is to design and simulate an optimum supply chain network for configured products from Asia into the European market considering its demand fluctuation and safety stock.

The aim is broken down into following tasks,

- To investigate into the cost structure and calculate the parameters governing the model.
- To design a deterministic model with optimum number of distribution centres and sourcing pattern of the products with historic demand.
- To design a stochastic model with optimum number of distribution centres and sourcing pattern of the products with normal distribution of demand.
- To simulate sensitivity analysis with the governing parameters of the optimum supply chain network.
- To calculate safety stock and service level for the model.

1. Company overview

This comprehensive thesis report outlines the progress of the research work at Sunrise Medical HCM B.V. This thesis cum internship program is governed vide document “STUDENT PRACTICAL TRAINING AGREEMENT – PM – 222” dated 30th May 2019 and trilaterally agreed by the following parties,

1. Sunrise Medical HCM B.V – *Host Company*
2. Kaunas University of Technology – *Sending Institution*
3. Bavithran Student – *Student/Intern*

This report illustrates about the tasks/assignments and deliverables of the aforementioned thesis work.

Sunrise Medical

Sunrise Medical is a company well known for developing, designing, production and distribution of manual wheelchairs, power wheelchairs, motorized scooters and both standard and customized seating and positioning systems worldwide. The goods produced by sunrise medicals are manufactured in their own plants based in the United States, Mexico, Germany, United Kingdom, Spain and China.

These fundamental products by Sunrise Medical are distributed over 130 countries and sold in various networks of homecare medical product dealers and are marketed under Zippie, Breezy, Sterling, JAY proprietary brands, Quickie, and Sopur. The company also employs more than 2,180 people worldwide and the global headquarters located in Malsch, Germany and the North American headquarters located in Fresno, California.

The company is providing high-quality, mobility and innovative products to help the disabled to improve their lives. Sunrise medical also develop the products according to customer requirements, value nature and to comply with all regulative commitments. Technology, teamwork, and continuous improvement through customer-focused people and processes are considered the foundation for meeting these commitments.

Sunrise Medical as a company, recorded its growth not only through expansion but also through a series of mergers and acquisitions. To cater the requirements of the specially abled people across the vast geographical area within 5 – 6 days of delivery time, requires a supply chain model which spans across the globe. In contrast, a tighter inbound supply chain and a relaxed outbound supply chain. As the company expands with acquiring new manufacturing facilities, this also translates to having a completely different set of infrastructure with its own supplier base, freight behaviour and cost structure. More over the ambiguity with the database structure, makes it nearly impossible to gather the data and unify the data and implement an optimal supply chain.

As seen in the below figure.1, is the pictorial representation of a usual supply chain network of a conventional manufacturing industry and each plant has its own pattern of routing, cost structure and level of responsiveness which will discussed in the further sections of this report.

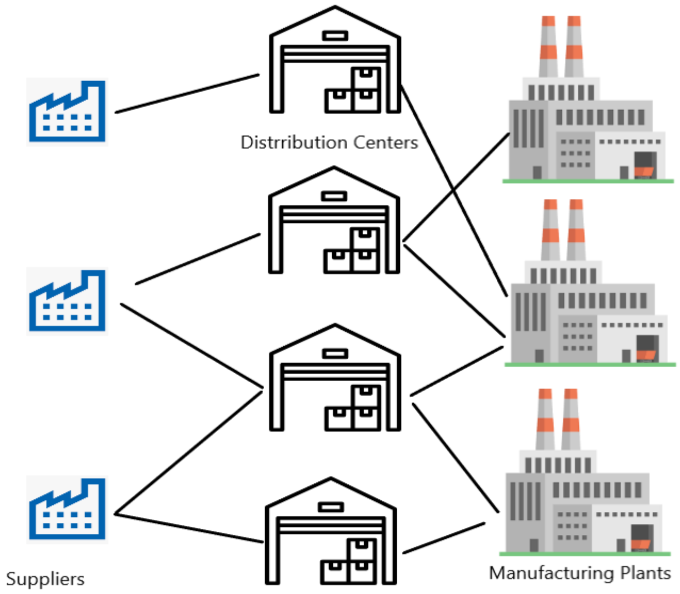


Fig. 1. Supply Chain Model

Market presence

As Sunrise Medical are widely known for their global presence and delivering all the significant markets in the geographical areas such as:

- EMEA Region.
- North America Region.
- APAC Region.

The Fig.2 below represents the worldwide presence of Sunrise Medical GmbH in the global market.

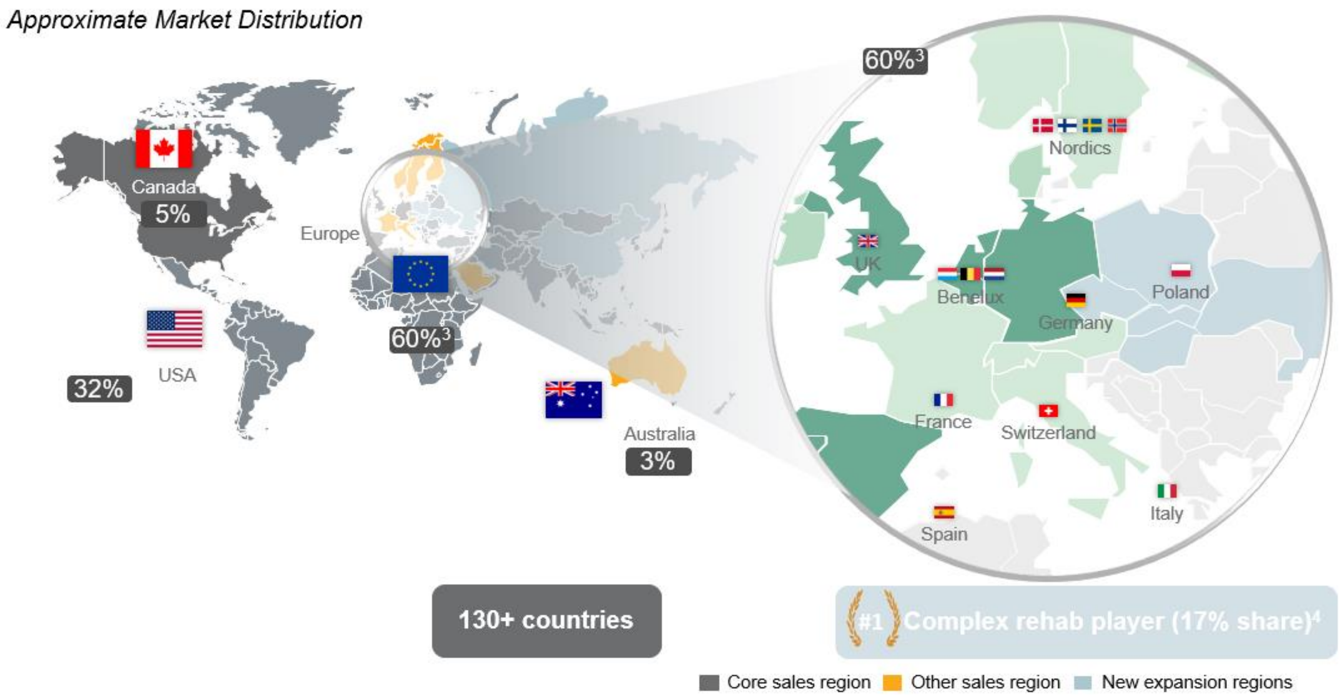


Fig. 2. Global Market Distribution of Sunrise Medical (Source : Sunrise Medical- Internal)

After a feasible market study, it was possible to state that Sunrise Medical has the maximum market share of 17% and they also noticed as a world leader in manufacturing the complex rehabilitation devices.

Product Portfolio and Market Position

Sunrise Medical's legacy product refers to the adult manual wheelchairs. Through continuous improvement and innovation and dynamic approach to improve people's lives, the company has successfully placed the following products in the market.

- Adult Manual Wheel Chair.
- Power Wheel Chairs.
- Pediatric Wheel Chairs
- Geriatric Wheel Chairs.
- Customized Seating systems.

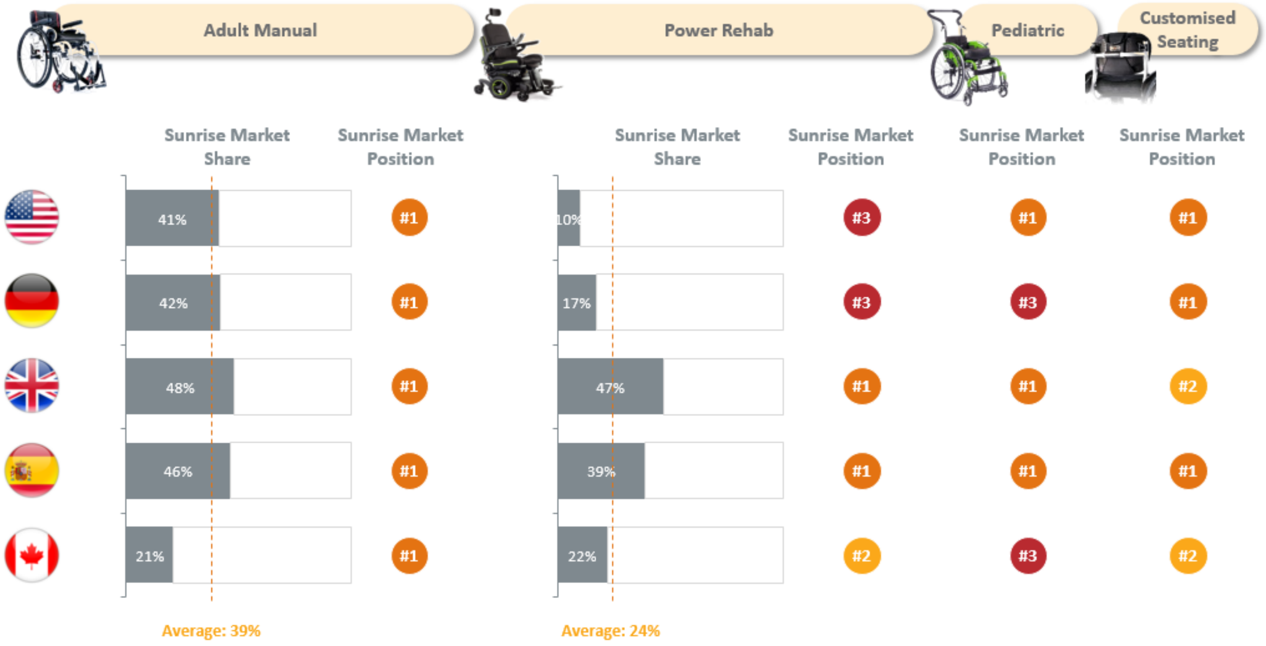


Fig. 3. Global Market Position of Sunrise Medical (Source : Sunrise Medical - Internal)

Nevertheless, Sunrise Medical secures the top 3 position in the world across its product portfolio, across the global markets.

Facilities

Sunrise Medical has its manufacturing and warehousing facilities spread across the globe to deliver the products at the most competitive lead time.

The nature of the business is more customer centered, as the customer here is not only a customer but also a patient whose day to day activity is dependent on the product. This puts immense pressure on the industry to deliver the products on the least time available and also due to high customisation requirements, the production policy is more towards make to order.

The manufacturing and warehousing facilities are available in the countries such as the United States, Mexico, the United Kingdom, the Netherlands, Spain, Germany, Norway, Poland, China and Australia.

The supplier base of these facilities are majorly based in the countries such as China, Taiwan, the US, the Netherlands, Germany, the UK, Poland, Hungary, Slovakia, Italy, Spain and Mexico.

2. Literature review of facility location problems in supply chains

2.1. Network design in a supply chain

Designing a supply chain network requires is based on four major considerations. The considerations covers determining the position of the facilities in the network, both role-wise and gerographically, the capacity it can handle and proximity to the market. In simple words asking the below four questions to prospective design will lead to an optimized supply chain [6].

- What is the role of the facility? Is it meant for manufacturing or distribution or storage or assembly?
- Where to place the facility? Is transportation and lead time considered?
- How much capacity should be allocated to the facility? Are they compatible with demand fluctuating scenarios?
- How close are the facilities to the market? Which facility is to feed which market?

An apt network of a supply chain has substantial influence on the efficiency of the supply chain because this is a deciding factor which determines the nature and purpose of the supply chain. Whether to have a high level of responsiveness or to have decreased supply chain cost are determined by the drivers of the supply chain design. The role which a facility plays is going to significantly decide the flexibility of the supply chain network. This roughly translates to fluctuations in terms of demand in the market and capacity of the supply chain. For instance, Toyota, an Asian motor company giant, until 1997 had a design where each plant is designated to supply its respective local market only. During the Asian recessions of 1990s, the idle capacity of the plants took a toll on its financial statements as they were not decined flexible enough to serve the overseas market. Only after then, Toyota added modularity and flexibility to these manufacturing facilities. Which helped them deal more competitively with fluctuating international market scenarios. This also extends to keeping the plant flexible enough with its manufacturing capabilities and so on.

Decisions concerning the location of the facility will have a prolonging impact on the performance of the supply chain because in all of the scenarios possible, it’s always an expensive option to shut down or relocate. A fine example for this again being Toyota, for its very first manufacturing facility in the US in Lexington, Kentucky state in 1988 and many more followed. The cars manufactured out of a US location seemed to be more cost competitive then the one that came out of the Japanese facilities due to two main reasons such as currency value difference between Japanese Yen and US Dollar and indirect cost of transporting the cars. Though the US plants were responsible for the local market, their capacities were designed and built in such a way that they can be altered which rules out the rigid nature of relocation option. Also, deignating a plant with more capacity will also have the company backfired with low utilization rates and a result increast cost of production. Assigning supply sources and markets to facilities has a direct effect on efficiency, as it affects the overall cost of output, inventory, and transportation incurred by the supply chain to satisfy the demand at the customer end.

These decisions are to be carefully reconsidered and re-evaluated at every stage of the planning phase so that the drivers of the supply chain are linked and there is utmost synchronicity in the supply chain with respect to the changes in market conditions, transportation disruptions and production blackouts. Of course, market distribution and sources of supply can only be changed if the facilities are dynamic enough to meet different customers and receive supply from different sources.

2.2. Supply chain and supply chain networks

The majority of the articles on the subkect of supply chain networks usually includes a pictorial representation that envisions the supply chain with a series consisting of basic materials, numerous providers, production and distributtion business along with the end customer market or retailer. For that reason, the most typical conception of this value-creating network is a straight line that can be step-by-step followed through the stages and the denomination as a supply "chain" enhances this conception. In this subchapter the distinctions and commonness in between the ideas of Supply chains and supply chain networks are discussed.

Any supply chain includes a number of entities that are linked by distribution of products, services, money, and/or information [7] from the point of raw material to the point of consumer. These value/product flow can happen upstream, towards the raw materials source, or downstream, towards the consumer end, and with a minimum three entities in the network. In this context, the end consumer becomes a part of the supply chain as same as the raw material provider whereas the distributed plays dual role facilitating both upstream and downstream flow of product/value [7]. This definition seems to validate the conception of supply chain as a direct sequence, nevertheless, an entity can be part of several supply chains and, therefore, a network structure emerges [8][9][10].

Consensus appears to exist on which main entities a SCN typically includes, since the most basic understanding of a SCN is that of interconnected SCs leading to similar or equivalent entities in the network. Providers, producing plants, distribution centers, customers, and transport resources are some of the primary entities in Supply Chain Network [11][12].

Braziotis et al. (2013) analyzes the distinction between the supply chain and conceptual supply chain networks and propose a basic classification and meaning for both the terms. According to them supply chains are "a set of primarily collaborative activities and relationships that link business in the value-creation process, in order to provide the end customer with the appropriate worth mix of items and/or services". This also highlights the value-creating process and the necessary collaborative relationships required to accomplish the common supply chain goal of offering products and/or services to suffice the end customers' requirement. On the contrary, it is explained that supply networks are a mix of active and non-active supply chain entities, where non-active members can be comprehended as extra resources to be consumed only if the requirement emerges. In this article, both principles are adjoined (as shown in Fig.4) and it is important for business to understand all network members [8].

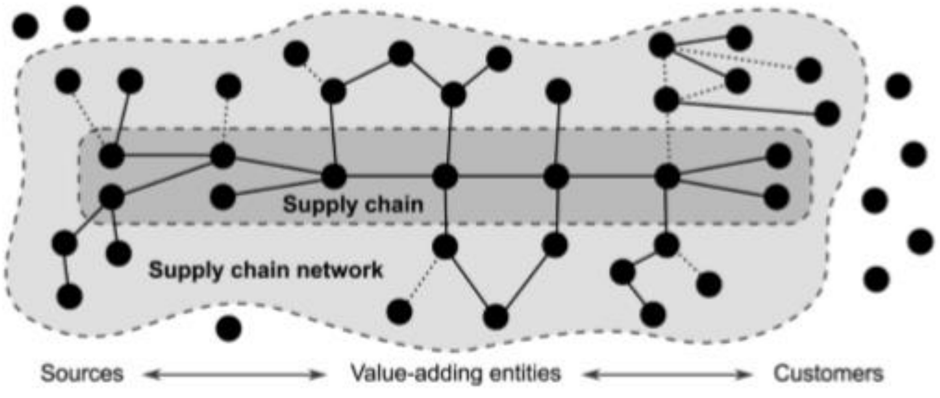
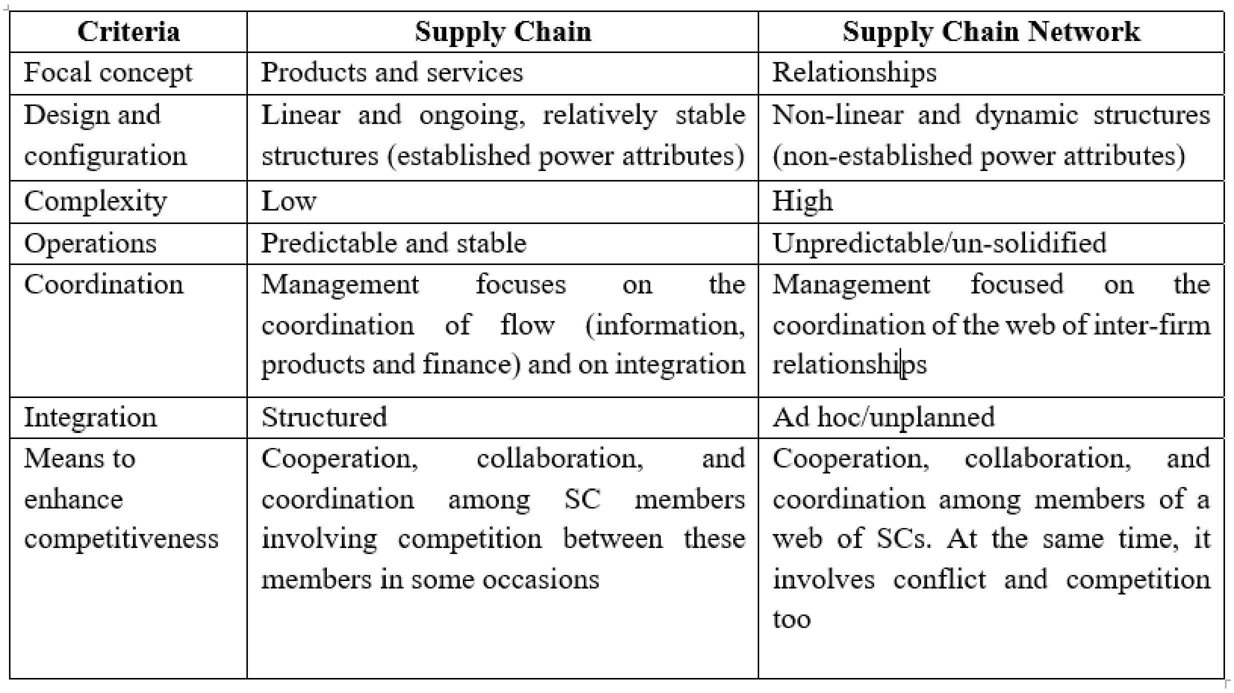


Fig. 4. Interconnectedness of SC and SCN concepts [8]

A supply chain and a suply chain network have some facets in common. In both of the concepts, strategic planning and decision-making for the management are persisting problems in addition to the need for a continual assessment of the risk-benefit allocation/distribution in the chain. One more significant element is trust as well as the level of coordination and alsocollaborative partnerships within the supply chain, including the exchange of information as well as value. Nevertheless, examining and also managing a supply chain network is a lot more complicated, which is partly because of the interconnectivity of the supply chain entities with numerous other supply chains and their entities.While supply chains function in a more secure and structured way, supply chain networks with the larger entity base tend to function in a flexible and dynamic form making it possible for the network to react faster to fluctuations. To take advantage of this wider participant base (e.g., by the reduction of supply risk or accessibility to competence for future innovation and value creation) it is essential to have a sustainable relationship with the active and non-active network entities. Nevertheless, recognizing all active and, especially, non-active entities can come to be vital to identify links in the chain that might impose a threat to the network (e.g., by unethical organisation habits) and also enable a fast reaction to these risks. In table 1 the recommended classification of SCs and SCNs based on some crucial standards are presented as well as highlights the main differences in between both ideas [8].

Table 1. Classification of SC and SCN [8]



Criteria	Supply Chain	Supply Chain Network
Focal concept	Products and services	Relationships
Design and configuration	Linear and ongoing, relatively stable structures (established power attributes)	Non-linear and dynamic structures (non-established power attributes)
Complexity	Low	High
Operations	Predictable and stable	Unpredictable/un-solidified
Coordination	Management focuses on the coordination of flow (information, products and finance) and on integration	Management focused on the coordination of the web of inter-firm relationships
Integration	Structured	Ad hoc/unplanned
Means to enhance competitiveness	Cooperation, collaboration, and coordination among SC members involving competition between these members in some occasions	Cooperation, collaboration, and coordination among members of a web of SCs. At the same time, it involves conflict and competition too

2.3. Facility location problem

In order to maximize profits or to minimize costs, a SCND/R can be developed by taking the decision factor [13] into account and by employing high quality location models that enhances the network's purpose. A brief explanation of FLP is discussed below.

2.3.1. Historical development of facility location problems

The base of the SCND/R which is the FLP is a part of the location theory research area. The location problem has been existent for centuries as old as the 17th century. French mathematician Pierre de Fermat requested scientists to discover a point in a plane that minimizes the space to each of the three present points. Even though it is not clear who proposed the first solution algorithm Scientists like Evangelista Torricelli and Bonaventura Francesco Cavalieri have been discussed in this context. Vincenzo Viviani (1659), Loria and Bassura (1919), and, Thomas Simpson (1750) came up with further solution algorithms in the future years [14].

[15] Alfred Weber was the first scientist to utilize FLP to estimate the location of a warehouse which minimizes the distance to serve the customers. He achieved this by connecting a network design problem with the Fermat problem. He explains in detail about this problem in his book 'Ueber den Standort der Industrien'. Another scientist, George pick, developed a solution method which is discussed is the basic algorithm of our model formulation as found in appendix 1. Weber remarks that the location decision is influenced by qualitative factors along with the mathematical aspects of the topic. Weber's description of this issue is more detailed and takes certain issues into account which were not explained by Carl Friedrich Launhardt who also introduced an equivalent problem that involved finding the optimal route between three points.

The competition factor was introduced to the location problem in 1929 by Harold Hotelling. This factor intends on maximizing the market share by selecting the optimal location. Rigorous development in the field of information technology and computational power has resulted in the problem solution algorithms to become more complex. Hakimi developed the base of modern location problem solutions in 1964. He indicates and proves that the p-median problem and p-center problem are NP-hard problems [14]. Hakimi is often quoted regarding the location problems and several researchers agree that his work encouraged an interest in location theory [16]. Multiple variations of the location problem have been proposed and designed since the 1960s which made the location theory one of the most covered fields in the operation research domain [17].

2.3.2. Definition and Classification

The FLP is defined as determining a location for numerous facility types such as warehouses, production plants or distribution centres in a network that reduces the total network costs or total distance to serve all the customers [13]. Depending on a researcher's model, there is a variation in the cost types considered but generally variable costs for "transportation, capacity expansion, and penalties" and "fixed costs for opening and closing facilities" are taken into consideration [13]. The original model only searched for a solution where the distance between points were minimized but recent models take various strategic decision factors such as "procurement, production, distribution and customer demand satisfaction" into account.

A basic classification is mandatory to keep a record of the developments in this area of research as there are a wide array of existing models. A brief introduction and classification of the basic FLPs by Owen and Daskin are illustrated in Fig.5. There is a basic differentiation in between static-deterministic, dynamic-deterministic, and staticstochastic models. The decision factors including demand and traveling distance are assumed as static in the static-deterministic models. Also, only one point in time is taken into account. Dynamic-deterministic models consider uncertain charasteristics such as multi-period planning horizon and thereby making it mandatory to determine locations that can be used and adjusted even when the conditions change in course of time. On the other hand, static-stochastic models consider uncertainty supported by incomplete knowledge of input parameters like location or demand quantity [17].

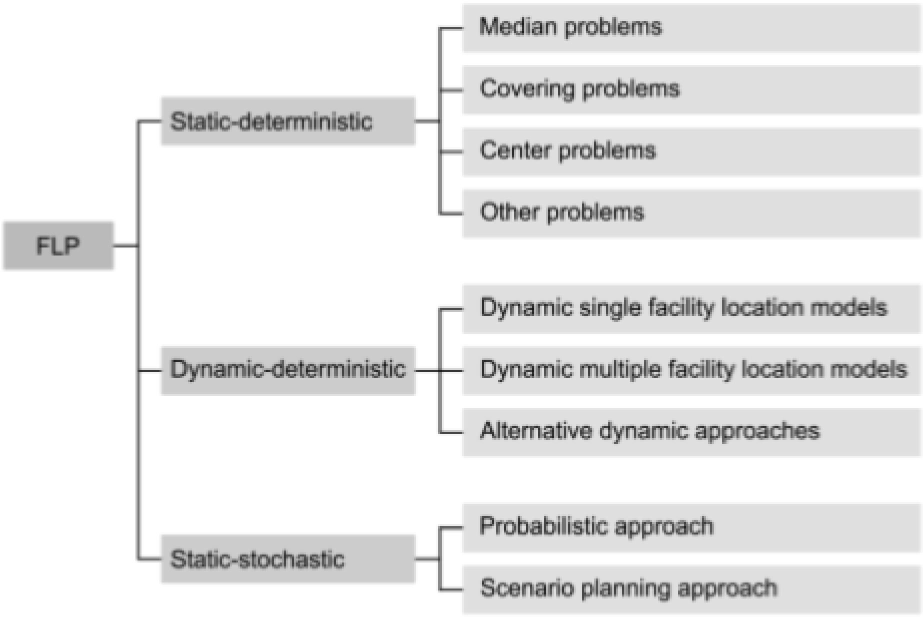


Fig. 5. Classification o FLP models [17]

There are median, covering, center and other problems in the deterministic models. The average distance travelled needs to be minimized in case of median problems. In some locations, there are weights and in that case a weighted average distance is calculated.

A covering problem is used for public service locations including ambulances, fire, or, police stations where a customer has to be served within a maximum amount of distance or time. These demands are covered when such requirements are completed. These problems are classified into location set covering problems and maximal covering problems. The first problem is targeted at reducing the total cost to attain the desired coverage level. The disadvantage in the formulation of this problem is that the demand quantity is not considered which results in multiple locations and expensive solutions where the acceptable distance is minimal. But, this formulation can be used to examine the number of facilities required for a guaranteed service level. On the other hand, the second problem considers the obtainable resources to reach as many customers possible in the desired time.

Nevertheless, if a customer needs to travel a distance more than the desired distance or if they have to wait longer than the desired time, then this solution does not cover that. This is covered in the center problems where all requirements and demands have to be fulfilled. Based on the requirements, the facilities can be located anywhere in the network or on the nodes of the network. If the facility can be located anywhere it is known as the absolute center problem and if it has to be located on the nods of the network then it is called as the vertex center problem. The first problem provides a better solution than the latter due to the enhanced number of locations to select from. Considering additional factors and constraints, numerous formulations have been derived. For example, the basic formula has been extended by adding flows between facilities to the decision making criteria to formulate the location allocation problem. Formulations that aim at maximising multiple objectives have been developed. These cases find optimum solutions using integer linear programming and heuristics [17].

Dynamic-deterministic problems can be categorized into dynamic-single, multiple facility location models and alternative dynamic approaches. The main goal is to reduce the costs over the planning horizon. Relocating facilities is allowed in many single facility location models. These models try to estimate the time period during when the relocation should occur and the costs these relocations incur differ for each case. However in case of multiple facility location models, the p-median problem and dynamic extensions to location-allocation have been established to obtain real world occurrences. Multiple period planning horizon and multiple objectives are used to further discuss the covering problem. Every model mentioned above is formulated in such a way where every time period is examined independently in an iterative process [17].

A stochastic location problem is solved by either using the probabilistic planning approach or the scenario planning approach. The probabilistic approach uses randomly generated numbers to estimate the uncertainty factors. The random numbers follow some probability distributions. There are two types of proposed models in this approach including standard formulations and queueing models. The standard formulation adapts static deterministic models to include uncertainty. Hakimi proved that if an optimal solution for a deterministic network problem is located on the node of a network then the same can be applied for a stochastic problem foundation also. The covering problem is also included in the stochastic assumptions of deterministic problems.This explains how the covered demand is evaluated when an assigned resource is already in use.The remaining aspects of the FLP are evaluated by combining the queuing theory with previous formulations to form the other part of probabilistic approach.If the demand arrival, location and service time is unknown then the optimal distribution of facilities to minimize the queueing time which are used in covering problems and center problems are examined in the queueing model [17].

In order to find a solution that performs in all situations, many different situations are examined and defined in the scenario planning approach. Due to the estimation of demands and other factors being substituted by numerous scenarios many contigency plans are developed in few models in this approach.If the scenarios are described in words , it is a qualitative approach and if the scenarios are described using numbers then it is a quantitative approach. The quantitative approach comes first in the FLP. Three basic configurations that have to be optimized exist in this approach. Thus the model is able to demonstrate best and worst case scnarios. This planning approach has been deployed on various problem formulations mentioned before [17].

2.4. Factors affecting the facility location problems

Considering the diversity in the facility location modelling, its approach is also based on multiple factors in order to determine their corresponding solutions. In this Chapter, the reviewed publications are analysed based on their aim, data generation and their approach to the solutions. The parameters can be divided as economic, environmental, social and competitive factors, in which the first three categories can be merged to evolve, sustainable SCND/R research area and the economic-competitive factors are analysed in the competitive SCND/R research area. The relationship between these factors is visualized in Fig.6.

The similarities observed among these publications are the Origin location and the demand nodes are assumed to be constant. Apart from which, another set of potential locations are given alongside the newly established locations to decrease the computational effort for determining the appropriate solution. Mixed-integer linear programming, Lagrange relaxation, other heuristics and metaheuristics, like (branch and bound, tabu search, neighbourhood search, and greedy heuristic) are few of the criteria for solution approaches. Some of the publications rely on "the fuzzy set theory" [18] or "simulation" [19], for their solution approach. Adding to this, the key focus is to enhance the existing solution algorithms and developing more for the SCN's forward flow. Although, the closed-loop SCN'S research and their models are growing [20], involving non-economic factors like, environmental) or social factors are meagre [21]. The proposal of new models range from, models that focus majorly on economic factors to the complex ones replicating the global SCNs, in terms of production, capacity, financial factors, etc [22]. Alternatively, the existing developments denote a steady growth in the research of sustainable SCND/R, addressing the facility location, vehicle routing and inventory problem to facilitate realistic models.

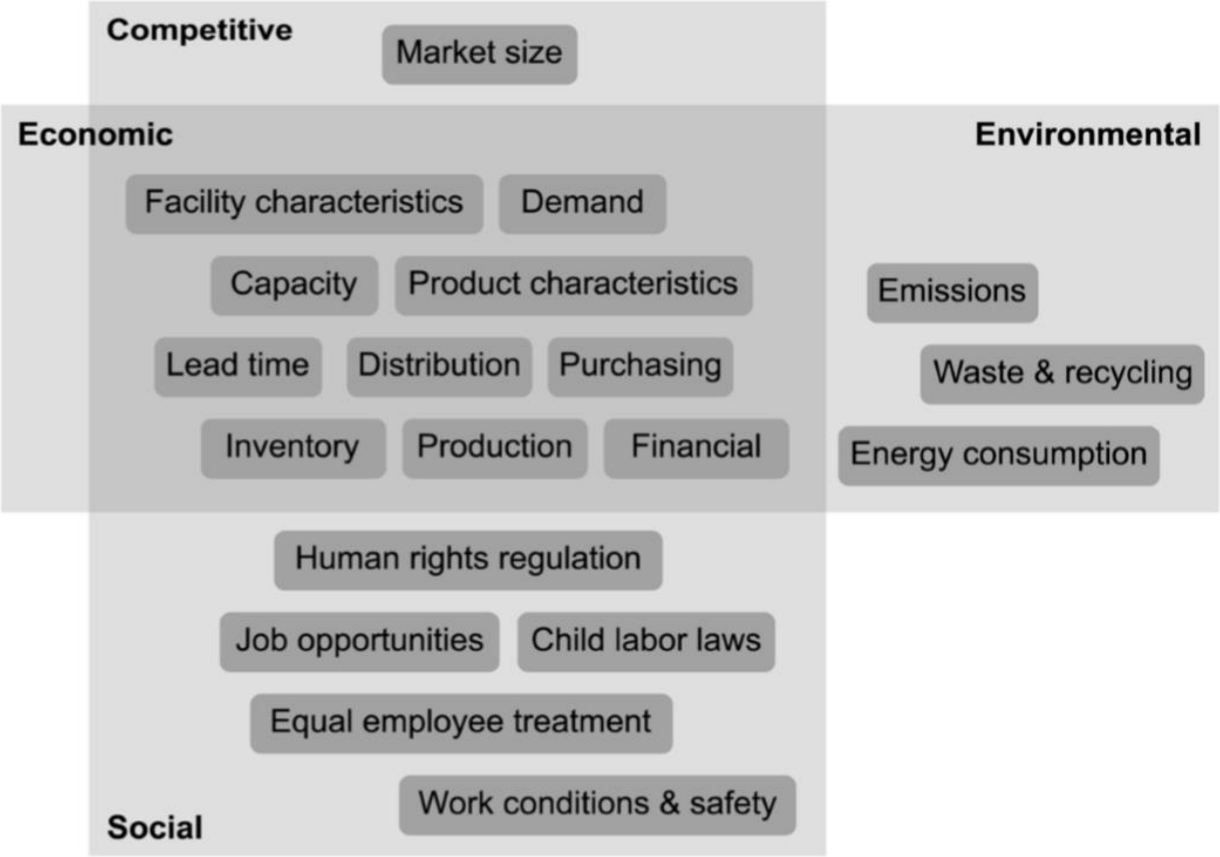


Fig. 6. Interrelationship of factor classifications and selected factors [20]

The primary aim of solution proposals is to minimize the cost [23], in few cases it is limiting the travel distance and in one case, it was observed to minimize the open service. Some aim for maximizing the profit [19][20]; or many other reasons. Mininimizing the costs and lead times, Operating costs and failures, Greenhouse gas emissions and costs. The dissatisfaction of not so desirable facilities in waste collection and the corresponding cost minimization, Impact on environment and Maximizing the positive social impact are few of the objectives focused on. Furthermore, minimizing the cost seems to be the common objective in all the above cases. [20][24][19].

In all these publications, the solution proposals had to undergo extensive computational tests for their performance evaluation and behaviour. The data collection for the evaluation were determined using multiple methods, like case studies where values or forecasts, are known, some of which included predefined problem sets from preceding papers [24][21], or generating the data, randomly based on different distributions or just normal distribution. Adding to this, the data generation methods were combined, for applications in some cases [25].

Based on the Model's objective and existing conditions of the market, multiple factors are considered. In the consecutive sub-chapters various deciding factors are analysed and are, classified according to their similarities.

2.4.1. Economic factors

Every model analysis takes demand quantity into consideration as a basis to build any SCN. The demand quantity is a variable. Although, few researchers assume it to be known and partially invariable, estimating the quantity[25][20][21][24], while few others recognise the demand as the variable it is and its behaviour is uncertain. Furthermore, the deciding criteria for the demand also depends on the permission to divide the orders and assigning them to the respective distribution centres where there is a need to serve the customers. However, majority of the publications believe that the fulfilment of the demand is satisfied from a single location and [25] has prohibited this from their model, explicitly.As far as, the closed-loop SCNs are concerned, it is divided as Demand for new and remanufactured products. Sometimes, few models that are examined consider interchange ability of the products and not separate the demands.

The FLP can be divided as incapacitated and capacitated problem formulations. The incapacitated models have no capacity limit for the modelled variable, which enables them to be solved easily by limiting the realistic representation of the problem. On the contrary, the capacitated models its various factors have an upper limit that is either not allowed to exceed or provoke additional costs when exceeded. For, Areas with multiple themes their factors can be modelled by restricting their capacity.For example, inventory space, production, supplier or the transportation capacity can be confined alongside the existing budget for investments and operations [25].

Another case is the lead time, which is often defined as the difference between the time of order placement and the time of receipt of the order in supply management. The time between the beginning of a process or project and the appearance of its results. In the model analysis [13] it was considered in the purchasing and delivery processes or as a combination of multiple. The times were assumed to be constant or estimated in most of the cases studied above. In the distribution and purchasing processes these are generally uncertain, considering the various factors that influences the process's time. The order fulfilment time referred in this thesis can be associated with lead times by definition. Hence, both denominations are used interchangeable [13][26].

In group distribution and transportation, costs are based on the travel distance to the customer transportation time or product quantity. All these parameters are uncertain , resulting in uncertain distribution and transportation .Furthermore, few models consider their mode of travel via air,ocean and road. Some of the newer models involve routing options wherethe geographically closer customers demands are met using the same vehicle but these formulations have not been analysed in detail. [21]-[25]

The inventory group includes service level requirements. The service level is nothing but the percentage of successfully completed orders and is usually targeted to a minimum value that has to be satisfied in SCNs. It is included in the group inventory because in a practical scenario the inventory level will be agitated to guarantee a higher level of service. In addition to that, only two of the examined models contain service level to be included as a part of their formulation and [26] defines it as the requirement to be achieved at the retailers.Eventhough, the service level is trivial in the studied facility location models and [13] notifies it as a factor to be worked on and suggests to utilise it instead of satisfying the overall demand of each period. Another factor in the inventory group includes the model formulation's positioning during stock-outs. In the examined papers, only one formulation allows them explicitly and includes penalty costs during their occurrence . Two of the other formulations includes penalty costs for dissatisfaction of customers' demands [24], the reason being facility failures or not stating explicitly. In order to avoid stock-outs [26] introduces a safety stock, which also triggers the additional costs. Some of the publications consider the standard inventory costs per product and time. The optimal inventory level is however, uncertain and therefore, it is subjective in the inventory area of operations research [13].

The third group purchasing involves decision making in terms of how and where to procure essential products and services for the own operations of focal company. Most formulations consider these factors as invariable, but include them during their assumptions of the model. Only single sourcing is considered as the sourcing option, while few formulations include purchasing costs. Supplier's location, ordered quantity, introducing penalty costs for delayed delivery are considered only by [20]

The production area is defined by cost factors majorly, for example, Cost per unit for manufacturing processing and handling also remanufacturing of products In addition to this , the output level of production of their competent facility location model is also considered [20][22].

Financial parameters are considered in formulations containing models taht represent international SCNs. Some are relevant for decision making of national locations. For example, In Germany, its companies' trade taxes differ based on its regions, offering tax saving opportunities. In the examined publications [27]considers the currency exchange rate and [223 takes into account the loan amount, country oriented depreciation rates and corporate income taxes. In addition to this [13] includes duties and transfer costs as a part of financial factor.

The basic product representation involves the type of the product, sales and the decision making if the formulation contains single or if it is a whole of its constituent products.

The basic facility is represented as a certain facility location model assumption, like the quantity and size of the facilities. Also, multiple costs(fixed and variable)for commencing ,operating costs and closing facilities [20]-[27].

2.4.2. Competetive factors

The market size for the new market is a key deciding factor to detect the market sharing potential available for the new entrant. [27] Considers this as a random variable because of the uncertainty of the market's potential. Also includes the product cost, the service level and the distance towards its customers as competitive factors. These parameters were discussed previously and will be discussed in other subchapters. It illustrates that all the factors are inter-related.

2.4.3. Environmental factors

Facility location model formulations, that takes into account the environmental issues are usually rare despite, the steady growth in the literature on closed-loop and reverse SCNs. The varying GHG emissions considered are expressed as the standardized CO2 equivalent measure. However, only one of the analysed model [21] considers emissions for the design of a cold SCN, that is subject to coolant leakage (HFC gas) and CO2 emissions during the distribution processes per product unit. [28] Suggests the usage of carbon credit exchange systems as part of the strategic decision making to enable better precautionary measures of GHG emissions.

[20] Addresses the environmental issues by proposing environmental impact measures for new opening locations, product handling, disposal and utilizing the remanufactured products. The Impact measures mentioned above are factors dependant on the quantity of demand and are produced according to functional usage in their computational experiments.

Additionally, energy consumption and waste generation as parameters that are considered while modelling the environmental impact. The energy consumption relies on the size of the facility and the technologies that are chosen for the production and distribution of products and services. In order to assess and quantify the environmental impact, the lifecycle assessment (LCA) method is opted, however for general formulations GHG emissions are prevalent [28].

2.4.4. Social factors

Facility location models that consider social factors are even rarer than models that include environmental issues. The key reason could be because of social factors are expressed qualitatively and are difficult to quantify, in order to implement them into mathematical model formulations. The qualitative factors include, adherence to human rights, child labour laws and regulations, work conditions and equality in treating the employees irrespective of their gender and ethnicity.

In all the publications that are analysed, one of the model formulations included social impact i.e. social impact of lost days because of work safety reasons and so included job opportunities in their model. Their computational experiments consist of values generated randomly through uniform distribution [20][28].

2.5. Decision making under uncertainity

Based on the time a decision influences the network the SCND/R decisions can be structured into three level. They are strategic decisions, tactical decisions and operational decisions [6][9][11][13][29]. The timeframes for the mentioned decisions can be defined throughout literature as relatively uniform. If a decision required more than one year, then it is a long-term decision and are made on the strategic level. If the decision has a mid-term duration, then it needs to be reevaluated every year and it belongs to the tactical level. If the decisions are day to day and short term, then it belongs to the operational level [29].

As the strategic decisions are fixed for a long duration and will result in high costs if changed, they have the highest impact on cost performance. Typically strategic decisions cover selection of strategic supply partners, distribution service providers, subcontractors, determining the location of facilities, quantity and size. A strategic decision also includes whether to create or buy a product and what should be offered to the customers [6][9][11][29].

Distribution strategies and decisions involving inventory policies, production and purchasing schedules are included in the tactical level decisions. This also includes how the production plants are assigned to suppliers, how the distribution centers are assigned to production plants and determines the product flow structure by knowing how distribution centers are assigned to demand. The operational level which takes the current customer needs and environmental changes into account is the most flexible. This includes daily production scheduling, truck load decisions, vehicle routing and material requirement planning.

The decisions in various decision-making levels are mentioned in the Fig.7. Here, the strategic level is placed at the bottom of the pyramid in contrast to other organization level visualizations. This is done to show that all decisions are limited by the strategic decisions. Some graphical visualizations lead us to the belief that each decision level is distinct. However, this is not possible, and all the layers are not separable or autonomous. This is because tactical and operational level decisions are taken into account at the strategic level and the decisions made in the previous levels affect the other levels. This is very explicit for strategic decisions as the decisions made pertaining to facilities for example can influence and affect future capacity, inventory and routing decisions. [29]

Manzini and Bindi as well as Klibi *et.al.*, discuss that the problems related to decisions are treated individually and originate in the FLP which will be discussed later. [11]

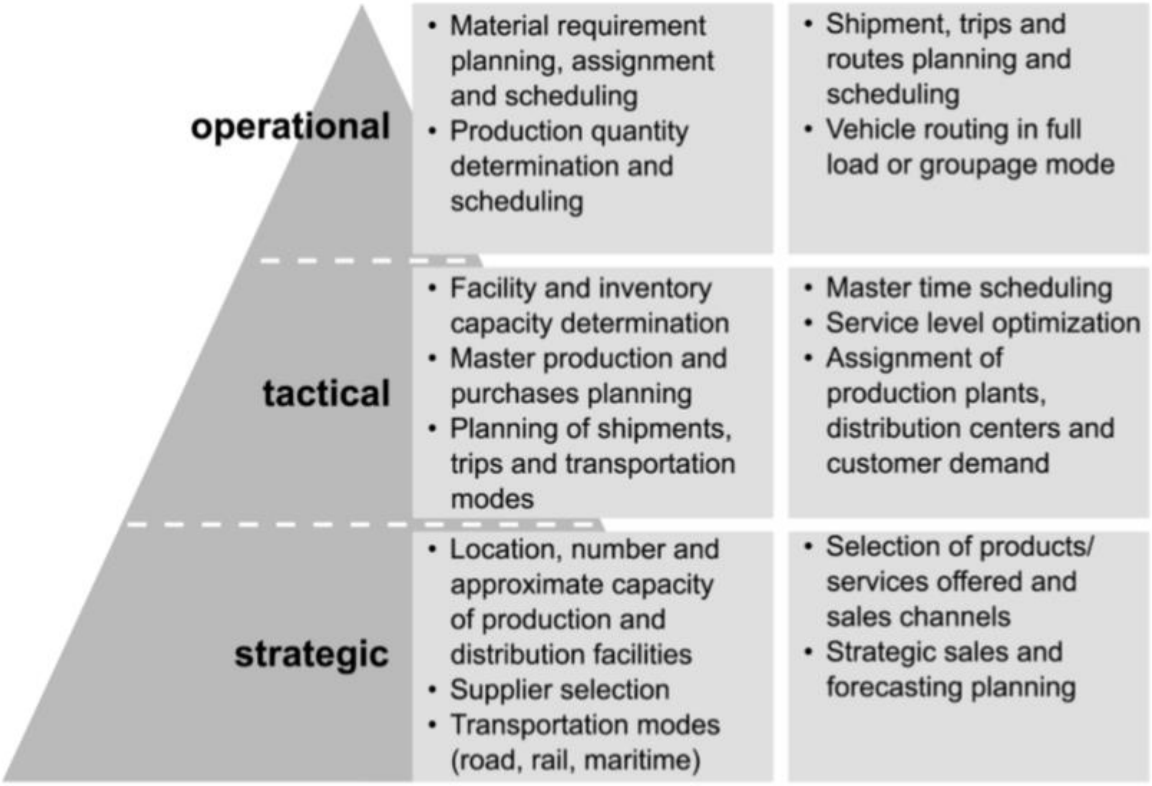


Fig. 7. Decision made on different levels [29]

3. Methodology

In this section of the thesis, the formulation of the model, estimation of the parameters which influence the design are discussed in detail.

3.1. Model formulation

To model the Asian commodity supply chain, a set of suppliers ($i \in I$), set of potential distribution centers (j ∈ J) and a set of sell outs or retailers (k ∈ K), as depicted in the below figure. The model will which is about to be built will map the most optimum supply chain network of these entities.

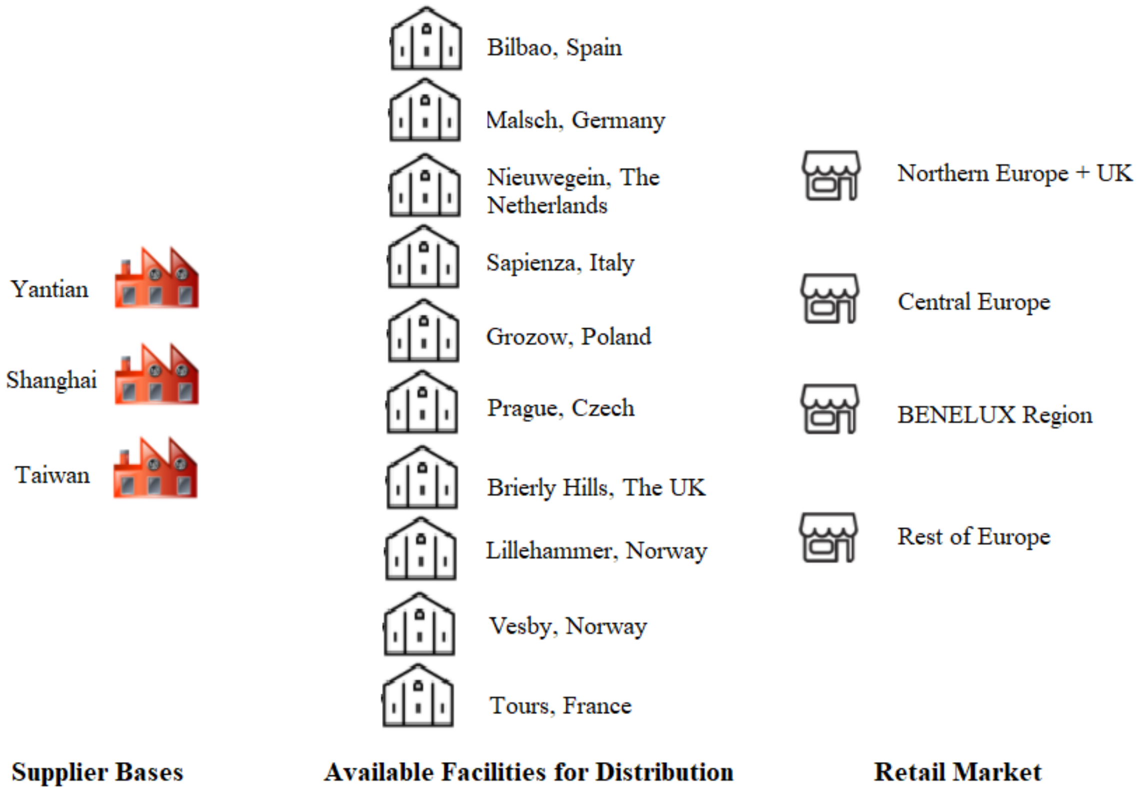


Fig. 8. Outline of Network configuration

3.2. Estimation of influencing parameters

To begin with formulating the model, the first step is to estimate the various parameters that influence the supply chain. Annual business data in the period July 2018 to July 2019 is taken for estimating the following parameters.

Freight Cost

The cost incurred to move the goods from one location to another location. Considering the multi-echelon supply chain, the transportation cost is split into two parts. Inbound and outbound transport and also this is a common book keeping practice amongs any manufacturing industry.

$$Freight\ Cost = \frac{Freight\ Invoice\ value}{Goods\ in\ transit}$$

$c_{ij}^{1} -$ Unit transportation cost from Supplier *i* to DC *j* in €/unit

$c_{jk}^{2} -$ Unit transportation cost from DC *j* to retail market *k* in €/unit

Inventory Cost

Each potential distribution centre is a facility of the company and accounts to its respective cost structure. The complex cost structure of a facility is broken down into cost incurred to store and handle goods per year like the following.

$$Inventory\ cost = \frac{Total\ cost\ of\ handling\ or\ storage}{Storage\ or\ handling\ consumption\ per\ selling\ unit}$$

$f_f -$ Fixed cost for setting up storage at a potential site *j* in (€/year)

$g_f -$ Variable cost for handling storage at a potential site *j* in (€/year)

$h_j^1 -$ Annual unit cost of holding inventory at a potential DC *j* in (€ per unit/year)

$h_k^2 -$ Annual unit cost of holding inventory at retail market *k* in (€ per unit/year)

Pipeline cost

Pipeline cost is defined as the cost incurred by the goods which are in transit which could have been possibly sold. There are various methods to calculate this in terms of lead time or demand or Minimum order quantity. This thesis calculates the pipeline inventory cost based on the demand. In simpler terms of calculating the sale value of average demand during the average transit time.

$$Pipeline\ inventory\ cost = No\ of\ units\ in\ transit\ \times Selling\ Price$$

$\theta_j^1 -$ Annual unit pipeline inventory cost from supplier *i* to DC *j* in (€ per unit/year)

$\theta_k^2 -$ Annual unit pipeline inventory cost from DC *j* to retail market *k* in (€ per unit/year)

Demand

The demand at the retail market’s end is estimated with the historical sales data in these regions.Apart from the deterministic demand, considering X days per year, the normal distribution of demand is estimated with the following indices,

μ_k – mean demand at retail market *k* per in (Units/day)

σ_k^2 - Variance of demand at retail market *k* in (Units/day)^2

Service Level Parameters

The service level in this product based industry is estimated on the basis of time i.e the number of days guaranteed by an entity within which the agreed service will be delivered. These information are collected from the customer service department and the purchasing team and the values are weighted averaged.

t_{ij}^{1} - Order processing time of DC *j* if it is served by supplier *i* in (days)

t_{jk}^{2} - Order processing time of retail market *k* if it is served by DC *j* in (days)

r_k- Maximum service level guaranteed to retail market *k* in (days)

si_j- Maximum service level guaranteed of supplier *i* in (days)

λ_j^1 – Safety factor of DC *j*

λ_k^2 – Safety factor of retailer k

L_k - Net lead time of customer demand zone k

N_j – Net lead time of DC *j*

S_j - Guaranteed service time of DC j to its successive customer demand zones

Conditions and decision variables

In addition to the influencing parameters, certain conditions, constraints and network conditions are to be established as follows,

X_{ij} – Return 1 if DC j is served by supplier base i and 0 otherwise.
Y_j - Return 1 if a DC is installed a potential site j and 0 otherwise.
Z_{jk} – Return 1 if market zone k is served by DC j and 0 otherwise.

3.3. Formulating a facility location problem

In the formulated supply chain network, the task is to find the optimal number and location of distribution centers and determine which suppliers should serve each distribution centre, and which distribution centres should serve each retail market. The total cost C includes fixed and variable costs related to installing the storage and handling at the potential distribution centre, transportation costs and annual pipeline inventory cost through the network.

3.3.1. Deterministic approach

In a first problem formulation, the demand is assumed to be deterministic. The demand at each retail market k is assumed to be fixed and equal to the mean demand.

$$min \sum_{j}^{J} Y_j f_j \ + \sum_{j}^{J} \sum_{k}^{K} g_j q_{jk}^2 + \sum_{i}^{I} \sum_{j}^{J} c_{ij}^1 \, q_{ij}^1 + \sum_{j}^{J} \sum_{k}^{K} c_{jk}^2 \, q_{jk}^2 + \sum_{i}^{I} \sum_{j}^{J} (\theta_j^1 \, t_{ij}^1 \, q_{ij}^1)/x + \sum_{j}^{J} \sum_{k}^{K} (\theta_k^2 \, t_{jk}^2 \, q_{jk}^2)/x$$

Above equation (1) is subject to below contraints

Demand at retail market to be satisfied by $\sum_j^J q_{jk}^2 \geq \ \mu_k x \ , \forall k;$ (2)

Demand at Distribution centre to be satisfied by $\sum_i^I q_{ij}^2 \geq \ \sum_k^K q_{jk} \ , \forall j$; (3)

Shipment of goods from supplier to DC is only possible if link between supplier and DC by $q_{ij}^1 \ \leq M * \ X_{ij} \ , \forall i, j$; (4)

Shipment of goods from DC to retailer is only possible if link between DC and retail market by $q_{jk}^2 \ \leq M * \ Z_{jk} \ , \forall j, k$; 5)

Enforce single sourcing from supplier to DC by $\sum_i^I X_{ij} \leq \ 1 \ , \forall j$; (6)

Enforce single sourcing from DC to retail market by $\sum_j^J Z_{jK} \leq \ 1 \ , \forall k$; (7)

Transport from supplier to DC is only possible if DC is open by $Y_j \ \geq X_{ij} \ , \forall i, j$; (8)

Transport from DC to retail market is only possible if DC is open by $Y_j \ \geq Z_{jk} \ , \forall i, k$;
....(9)

This is the mathematical representation of the deterministic facility location problem. This formulation minimizes the total costs. At the end of every mathematical formulation, the constraints are elucidated. Equations (2) and (3) both make sure that demand at respectively the retailer and at the DC is fulfilled, while equations (6) and (7) enforce single sourcing to both retailers as well as to our chosen distribution centers. Furthermore, (4) and (5) make sure that when goods are shipped from or to a DC, that Dc is effectively open. Similarly, (7) and (8) state that when there is transportation to or from a certain distribution center, that DC must be opened.

3.3.2. Stochastic approach

In this stochastic approach to the facility location problem where the demand is normally distributed with (n=2 and n=3) discrete points, the task is to minimize the expected total cost and make the model adaptable for different scenarios ($s \ \in S$)

$$
\begin{aligned}
min \sum_{j}^{J} Y_j f_j &+ \sum_{j}^{J}\sum_{k}^{K}\sum_{s}^{S} g_j q_{jks} p_s \\
&+ \sum_{i}^{I}\sum_{j}^{J}\sum_{s}^{S} C_{ij}^{1} q_{ijs} p_s \\
&+ \sum_{j}^{J}\sum_{k}^{K}\sum_{s}^{S} c_{jk}^{2} q_{jks} p_s + \sum_{i}^{I}\sum_{j}^{J}\sum_{s}^{S} \frac{\theta_j^1 t_{ij}^1 q_{ijs} p_s}{x} + \sum_{j}^{J} \frac{\sum_k^K \sum_s^S \left(\theta_k^2 t_{jk}^2 q_{jks} p_s\right)}{x}
\end{aligned}
$$

(10) Subject to below constraints

Demand at retail market to be satisfied by $\sum_j^J q_{jks} \geq \mu k x, \forall k, s$; (11)

Demand at Distribution centre to be satisfied by $\sum_i^I q_{ijs} \geq \sum_k^K q_{jks}, \forall j, s$; (12)

Shipment of goods from supplier to DC is only possible if link between supplier and DC by $q_{ijs} \leq X_{ij}, \forall j, k, s$; (13)

Shipment of goods from DC to retailer is only possible if link between DC and retail market by $q_{jks} \leq Z_{jk}, \forall j, k, s$; (14)

Enforce single sourcing from supplier to DC by $\sum_i^I X_{ij} \leq 1, \forall j$; (15)

Enforce single sourcing from DC to retail market by $\sum_j^J Z_{jk} \leq 1, \forall k$; (16)

Transport from supplier to DC is only possible if DC is open by $Y_j \geq X_{ij}, \forall i, j$; (17)

Transport from DC to retailer is only possible if DC is open by $Y_j \geq Z_{jk}, \forall j, k$; (18)

The reasoning process behind the different constraints stays the same. Equations (15)-(18) are completely the same as the last 4 equations from the previous scenario, so they should not need further ado. Moreover, (11)-(14) have the function as their counterparts in the deterministic model, but enable the model to encompass different scenarios in order to minimize the total expected cost of these scenarios.

3.3.3. Minimizing the maximum scenario cost

After determining the minimum total expected cost out of a stochastic model with 2 and 3 discrete points, further scenarios are simulated. Constraining the expected total cost from the previous model (10) as a fixed upper threshold, other decision variables are once again iterated to check what will be the maximum cost scenario which will actually define the worst case threshold of the model.

$$\min C \tag{19}$$

Subject to below constraints,

Demand at retail market to be satisfied by $\sum_j^J q_{jks} \geq \mu_k x, \forall k, s$; (20)

Demand at Distribution centre to be satisfied by $\sum_i^I q_{ijs} \geq \sum_k^K q_{jks}, \forall j, s$; (21)

Shipment of goods from supplier to DC is only possible if link between supplier and DC by $q_{ijs} \leq X_{ij}, \forall i, j, s$; (22)

Shipment of goods from DC to retailer is only possible if link between DC and retail market by $q_{jks} \leq Z_{jk}, \forall j, k, s$; (23)

Enforce single sourcing from supplier to DC by $\sum_i^I X_{ij} \leq 1, \forall j$; (24)

Enforce single sourcing from DC to retail market by $\sum_j^J Z_{jk} \leq 1, \forall k$; (25)

Transport from supplier to DC is only possible if DC is open by $Y_j \geq X_{ij}, \forall i, j$; (26)

Transport from DC to retailer is only possible if DC is open by $Y_j \geq Z_{jk}, \forall j, k$; (27)

$$\sum_j^J Y_j f_j + \sum_j^J \sum_k^K g_j q_{jks}^2 + \sum_i^I \sum_j^J c_{ij}^1 q_{ijs}^1 + \sum_j^J \sum_k^K c_{jk}^2 q_{jks}^2 + \sum_i^I \sum_j^J (\theta_j^1 t_{ij}^1 q_{ijs}^1)/x + \sum_j^J \sum_k^K (\theta_k^2 t_{jk}^2 q_{jks}^2)/x \leq c, \ \forall s$$

; key contraint to minimize the maximum scenario cost (28)

This model is almost completely the same as the mathematical formulation of the previous model. Equation (28) was added to the formulation. This is a key constraint to minimize the maximum scenario.

3.4. Safety stock placement model

Safety stock is defined as the buffer stock stored at any nodal location of the supply chain to cover the demand during the replenishment time.

$$Replenishment\ time = Order\ processing\ time\ at\ predeccessor + Transit\ time$$

$$Safety\ stock = Repleshment\ time \times Demand\ of\ goods\ during\ replenishment\ time$$

To estimate the safety stock, the optimum network configuration of facility location problem is adopted. Now the focus on minimising total cost is relaxed as the aim now is to determine the cost of safety stock to be maintained at the distribution centres and retail markets.

In contrast to all the other models formulated in the previous sections of this thesis, safety stock placement model alone has a square root function which has the need for approximation. Earlier, the normal distribution of demand was intentional to account for the uncertainity in demand. But to estimate cost of safety stock, its is neccessary to approximate this square root funtion. Though there are various methods to linearize a square root function, considering the complexity of the model piecewise linearization method is used. According to [30]-[32] piecewise linearization by 10 pieces is more accurate than conventional linearisation like secant method.

$$\min(\sum_{j}^{J} h_j \lambda_j \sqrt{\sum_{k}^{K} \sigma_k^2 Z_{jk} \, (\sum_{p}^{P} b_{jp}^1 v_{jp}^1 + \, m_{jp}^1 u_{jp}^1))} + \sum_{k}^{K} h_k \lambda_k \sqrt{\sigma_k^2 (\sum_{p}^{P} b_{kp}^2 v_{kp}^2 + m_{kp}^2 u_{kp}^2)}$$

(29) Subject to,

$$N_j = \sum_{i}^{I} X_{ij} SI_i + \sum_{i}^{I} X_{ij} t_{ij}^1 - \, S_j \ , \ \forall j \tag{30}$$

$$L_k = \sum_{j}^{J} Z_{jk} S_j + \sum_{j}^{J} Z_{jk} t_{jk}^2 - \, S_k \ , \ \forall k \tag{31}$$

$$S_k \ \leq \ r_k \ , \forall k \tag{32}$$

$$m_{jp}^1 = (\sqrt{x_{j(p+1)}^1} - \sqrt{x_{jp}^1} \ / \ (x_{j(p+1)}^1 - \, x_{jp}^1) \ , \ \forall j, p \tag{33}$$

$$m_{kp}^2 = (\sqrt{x_{k(p+1)}^2} - \sqrt{x_{kp}^2} \ / \ (x_{k(p+1)}^2 - \, x_{kp}^2) \ , \ \forall k, p \tag{34}$$

$$b_{jp}^1 = \sqrt{b_{jp}^1 - \left(m_{jp}^1 x_{jp}^1\right)} \ , \ \forall j, p \tag{35}$$

$$b_{kp}^2 = \sqrt{b_{kp}^2 - \left(m_{kp}^2 x_{kp}^2\right)} \ , \ \forall k, p \tag{36}$$

$$\sum_{p}^{P} v_{jp}^1 = 1, \forall j \tag{37}$$

$$\sum_{p}^{P} v_{kp}^2 = 1, \forall j \tag{38}$$

$$\sum_{p}^{P} u_{jp}^1 = N_j, \forall j \tag{39}$$

$$\sum_{p}^{P} u_{kp}^{2} = L_k, \forall k \tag{40}$$

$$x_{jp}^{1} v_{jp}^{1} \leq u_{jp}^{1}, \forall j, p \tag{41}$$

$$x_{j(p+1)}^{1} v_{jp}^{1} \geq u_{jp}^{1}, \forall j, p \tag{42}$$

$$x_{kp}^{2} v_{kp}^{2} \leq u_{kp}^{2}, \forall k, p \tag{43}$$

$$x_{k(p+1)}^{2} v_{kp}^{2} \geq u_{kp}^{2}, \forall k, p \tag{44}$$

Where,

- Equations (30)-(31) denote respectively the net lead time or replenishment time of the DC and the net lead time of the retail market.
- Equation (32) states the constraint that guaranteed service time by retailers cannot exceed the maximum allowed service time.
- The other constraints all make piecewise linearisation possible where (33)-(34) define the slope and of the piecewise linearisation and (35)-(36) define the intercept. Next, (38)-(37) state that the factor to multiply with the intercept must sum to 1 while factors to multiply with slope must sum to lead time is given by equations (39)-(35). Finally, (41)-(44) represent the slope-multiplying-factor to be within the right segment.

4. Results and discussions

This section of the thesis discusses about the results and the inferences made on the results. The models which were discussed on detail previously are formulated and are run in IBM's CPLEX Optimization Studio to obtain the results. Throughout this section, the following code of reference is to be followed,

Table 2. Code of reference to entities of the SC

Supplier Base Code (i)	**Supplier Bases**	**DC Code (j)**	**DC Locations**	**Retail Market Code (k)**	**Retail Market Locations**
1	Yantian, China	1	Bilbao, Spain	1	Northern Europe
2	Shanghai, China	2	Malsch, Germany	2	Central Europe
3	Taiwan	3	Nieuwegein, Netherlands	3	BENELUX
		4	Sapienza, Italy	4	Rest of Europe
		5	Grozow, Poland		
		6	Prague, Czech		
		7	Brierly Hills, UK		
		8	Lillehammer, Norway		
		9	Vesby, Norway		
		10	Tours, France		

4.1. Estimation of influencing parameters

From the annual business data for the fiscal year 2018-2019, the influencing parameters are estimated as shown in the below tables.

Transportation parameters

The average transit time and the average cost per route is taken estimated from the freight database. This data is then tabulated as per DC locations *j*, the supplier bases *i* and retail markets *k*.

Table 3. Transportation costs and time from Supplier basees to DCs

Transit time from Supplier to DC in days	**i1**	**i2**	**i3**	Transportation cost per unit from Supplier to DC in EUR	**i1**	**i2**	**i3**
j1	4	2	3	**j1**	0.39	0.42	0.32
j2	1	3	5	**j2**	0.20	0.20	0.18
j3	6	2	3	**j3**	0.60	0.43	0.21
j4	4	5	7	**j4**	0.50	0.35	0.27
j5	6	3	4	**j5**	0.12	0.14	0.28
j6	2	5	2	**j6**	0.28	0.19	0.36
j7	4	2	5	**j7**	0.58	0.25	0.34
j8	7	2	2	**j8**	0.45	0.22	0.29
j9	3	5	3	**j9**	0.34	0.57	0.41
j10	4	2	4	**j10**	0.26	0.23	0.19

Table 4. Transportation costs and time from DCs to retail markets

Transit time from DC to Retail Market in days	**k1**	**k2**	**k3**	**k4**
j1	3	3	1	1
j2	2	2	3	2
j3	1	2	3	2
j4	2	3	1	2
j5	3	2	2	3
j6	2	3	1	2
j7	3	3	2	2
j8	2	2	1	3
j9	1	2	3	2
j10	3	3	2	1
Transportation cost per unit from DC to Retail Market in EUR	**k1**	**k2**	**k3**	**k4**
j1	2.40	1.82	1.38	0.97
j2	2.15	0.98	2.10	1.07
j3	2.40	1.40	0.98	2.10
j4	0.92	0.98	1.15	1.20
j5	0.90	1.60	1.80	0.60
j6	2.10	0.90	2.10	1.10
j7	0.88	2.10	1.87	1.80
j8	2.10	0.86	1.90	0.95
j9	1.35	0.93	1.80	1.10
j10	1.05	2.10	1.82	1.80

Inventory parameters

The costs related to setting up and maintaining storage and cost of goods in pipeline are estimated from the annual inventory database by filetering out the Asian products alone.

Table 5. Inventory costs pertaining to DC

Fixed inventory cost per year in EUR		Variable inventory cost in EUR per unit per year		Inventory holding cost @ DC in Eur per Unit/year		Pipeline inventory cost from Supplier to DC (EUR per Year per unit)	
j1	33,130	**j1**	0.09	**j1**	169	**j1**	0.99
j2	22,952	**j2**	0.01	**j2**	214	**j2**	1
j3	51,497	**j3**	0.08	**j3**	299	**j3**	0.97
j4	68,691	**j4**	0.08	**j4**	129	**j4**	1.1
j5	80,869	**j5**	0.07	**j5**	90	**j5**	0.9
j6	24,757	**j6**	0.01	**j6**	164	**j6**	1
j7	25,334	**j7**	0.07	**j7**	218	**j7**	2
j8	36,050	**j8**	0.03	**j8**	215	**j8**	0.81
j9	29,676	**j9**	0.07	**j9**	88	**j9**	0.93
j10	26,360	**j10**	0.06	**j10**	109	**j10**	0.83

Table 6. Inventory related costs pertaining to retailer

Inventory holding cost @ Retailer Market in (EUR per Year per unit)	**k1**	**k2**	**k3**	**k4**
	407	484	383	328
Pipeline inventory cost from DC to retail market (EUR per Year per unit)	**k1**	**k2**	**k3**	**k4**
	1.2	0.85	0.94	1.12

Service level parameters and demand

The agreed service level with the suppliers are extracted from the purchasing department and the service level with the retail markets are extracted from the customer service department along with the demand information from the regional sales team and tabulated as shown below.

Table 7. Demand information at the retail market

Mean Demand at Retailer market	**k1**	**k2**	**k3**	**k4**
Units/day	120	150	160	210
Variance of demand at retailer market (units/day)^2	**k1**	**k2**	**k3**	**k4**
	2,500	1,800	1,000	100
Max service level to retail zone days	**k1**	**k2**	**k3**	**k4**
	3	0	1	0

The guaranteed service time at supplier *i* is always kept 0 days to maintain the just in time concept. Also the safety factors at all the DCs *j* and the retailer markets *k* is always 1.96 to satisfy 97.5% level of service.

4.2. Optimum supply chain network

The first part of the thesis aims at determining the optimal number and location of the distribution centers (DC's) and the optimized supply chain network configuration, i.e. which plant serves which DC and which DC serves which retail market. In the following, different solutions for this task is presented by considering both a deterministic and stochastic problem formulation. Obviously, only one design can be implemented in practice.

Based on a comparison between the different approaches, mentioned further in this reading, it is recommend to implement the design displayed in Figure below, in which DC 2,3 and 7 are opened, i.e. the facilities at Germany, the Netherlands and the UK are recommended to be utilized. Also for the considered demand pattern and cost structure, supplier bases 2 and 3 are sufficient and suitable for the chain i.e supplier bases in the districts of Yantian, Shanghai and Taiwan. It is important to note that these districts in China are protected Special Economic Zones where there are numerous constraints in material movement from one district to another.

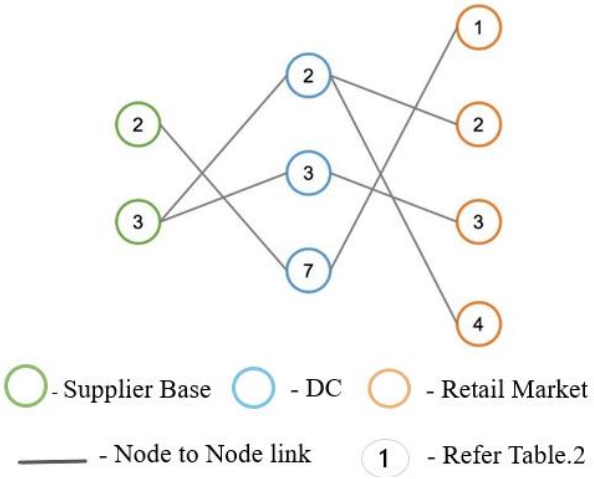


Fig. 9. The optimal network design

4.3. Deterministic model

The deterministic problem formulation as discussed in previous sections of this thesis is formulated and the above the network configuration as shown in Fig.9. is obtained as result. The objective function consists of a combination of the fixed investment costs, the variable investment costs, the transportation costs from the plants to the DCs as well as the transportation costs from DCs to retail markets. Furthermore, pipeline inventory in both stages, i.e. Supplier to DC and DC to retail market is taken into account as well when minimizing the total cost of the supply chain network.

When solving the problem formulation mentioned above, an optimal objective function value is obtained, i.e. the minimal total network design cost of € 391,602. Table.8 provides an overview of the different cost components and the optimal network design, in which DCs in Germany, The Netherlands and The UK are opened. It is important to note that the transportation costs from DCs to retailers and the fixed investment cost constitute the largest fraction of the total network design cost.

Table 8. Costs of the optimal network design of a deterministic model

Cost category	Amount	Weightage
Fixed investment costs	€ 99 783	**2**
Variable investment costs	€9 152	**4**
Transportation costs from supplier to DC	€46 866	**3**
Transportation costs from DC to retailer	€231 446	**1**
Annual pipeline inventory to DC	€2 746	**5**
Annual pipeline inventory to retailer	€1 609	**6**
Total	**€ 391,602**	

Fig. 10. Optimal network design of a deterministic model

4.4. Sensitivity analysis

The proposed supply chain network minimizes the costs related to transportation, pipeline inventory and storage investment in a DC. The relative impact of changes in cost parameters on the total cost of the supply chain is shown in Fig.11 and Fig.12 under the optimised network. For instance: if the unit transportation cost from DC 2 to retailer 4 increases with 50%, then -ceteris paribus- the total costs of the optimised network will increase by 10%. The same reasoning can be applied for the other cost parameters. Furthermore, Fig.11 and Fig.12 clearly visualises that investment cost and unit transportation cost have a considerable influence on the total cost. Pipeline inventory cost however has a neglectable impact.

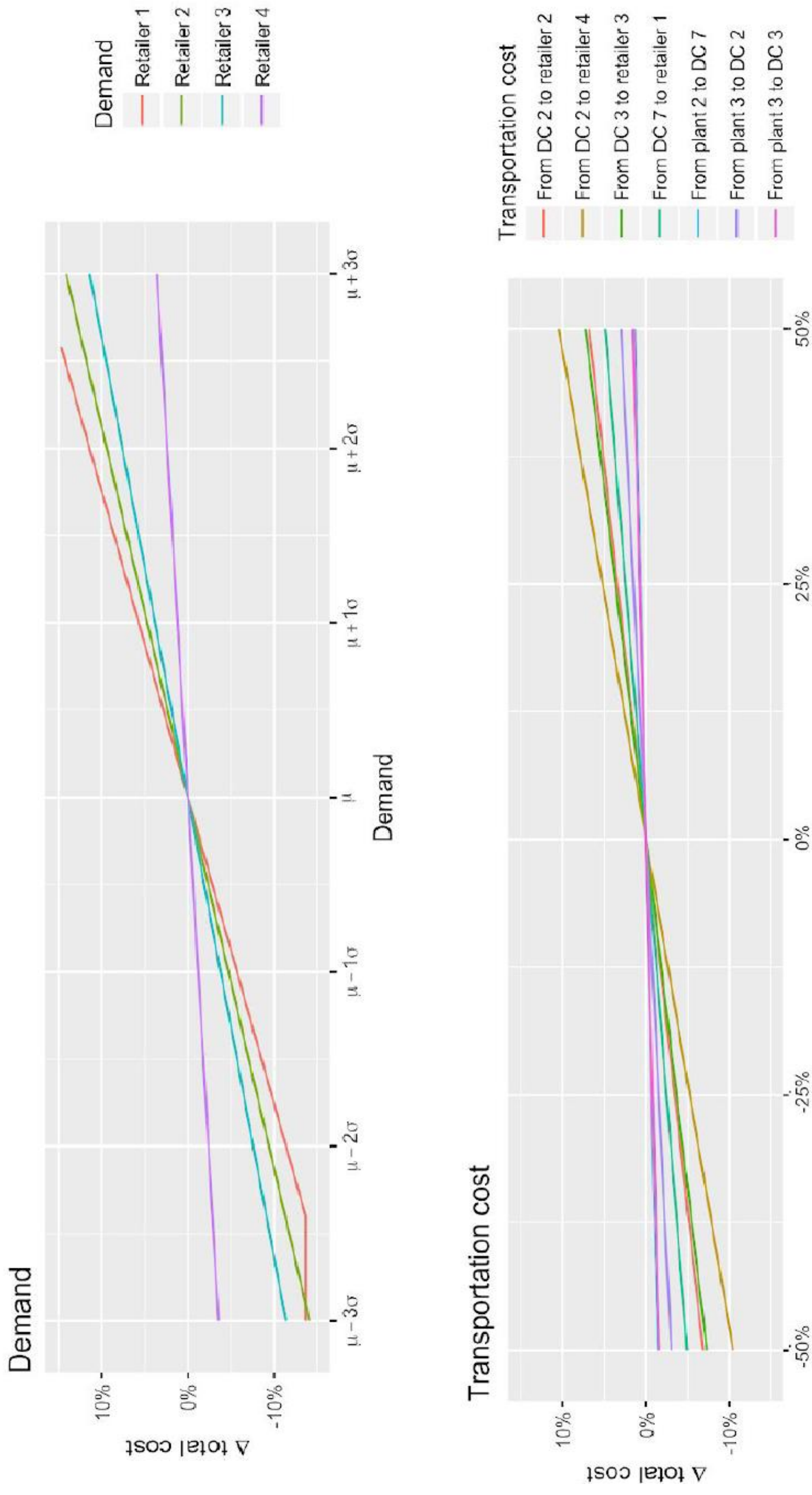


Fig. 11. Sensitivity Analysis showing the relative impact of changes in cost parameters in the optimal network design – Demand and Transportation

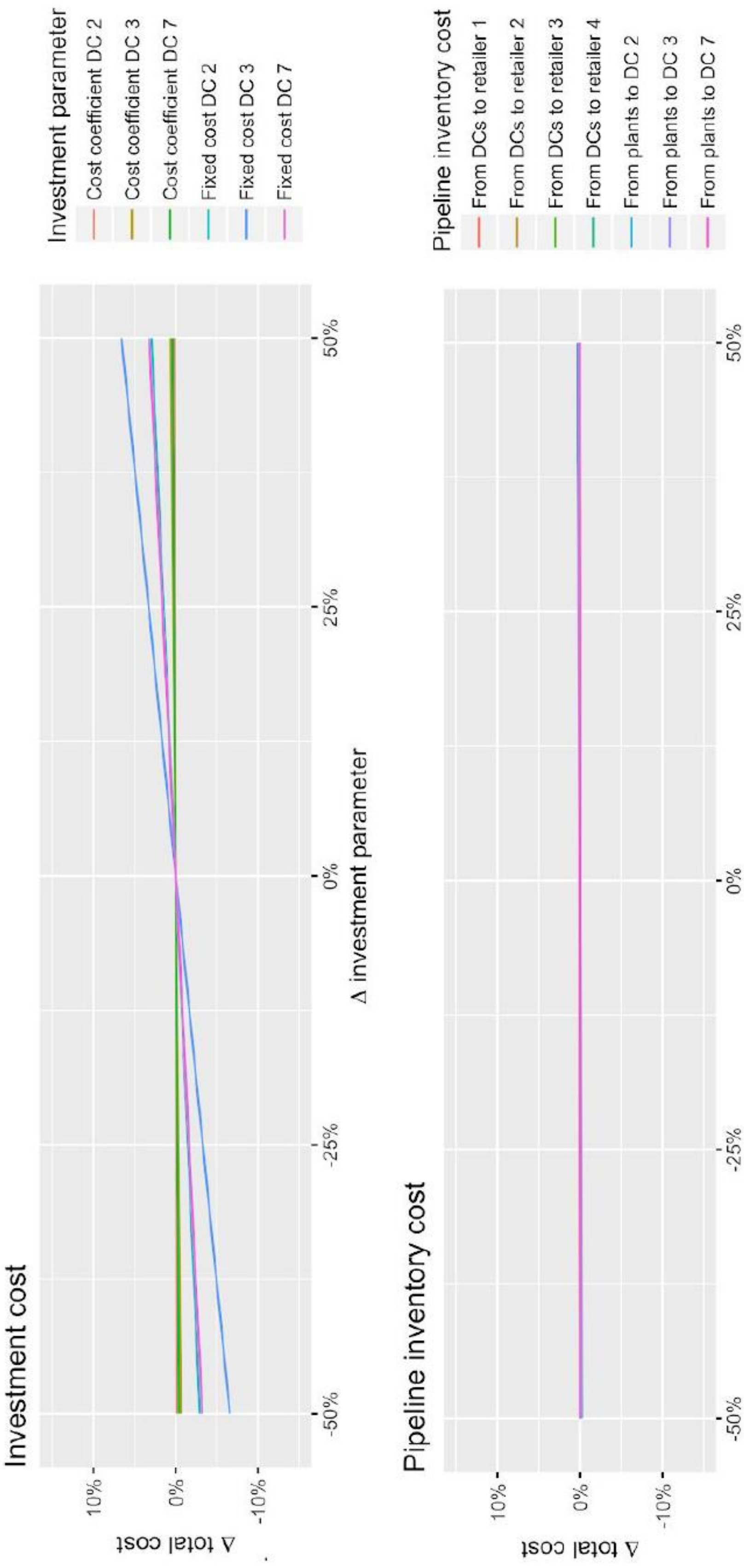


Fig. 12. Sensitivity Analysis showing the relative impact of changes in cost parameters in the optimal network design – Investment cost and Pipeline inventory cost.

This analysis also helps to determine the robustness of the current network design for changes in these influential cost parameters in more detail. It would be interesting to analyse by how much a certain cost parameter has to change in order for the network design to switch its optimal configuration. Fig.13. visualises the relative change in cost parameter value required to change the optimal network design.

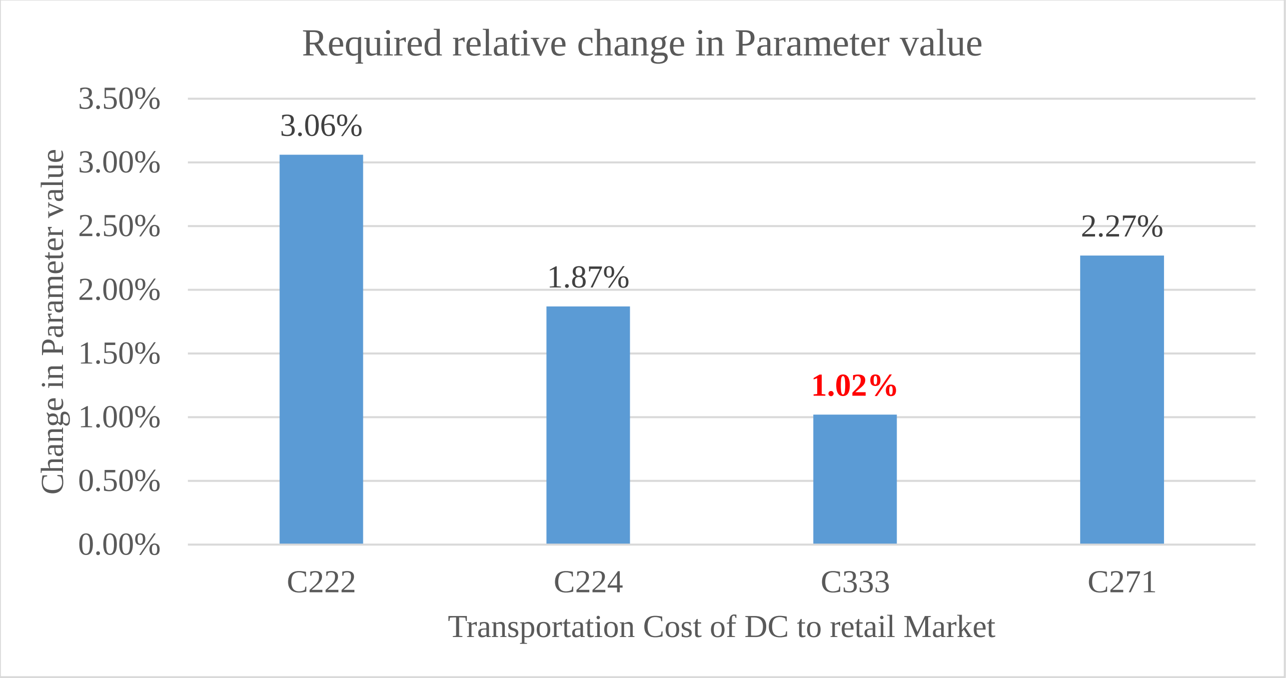


Fig. 13. Maximum increase in transportation cost parameters in the optimal network design

Fig.13 illustrates the maximal increase in transportation cost parameters in order for the current network design to remain the optimal one. As such, valuable information is provided on the robustness of the network design. In the optimal network design, DC 2 delivers to retail market 2 and 4, DC 3 transports to retailer 3 and DC 7 ships goods to retaile market 1. However, even a 1.02% increase$_{33}$ in the cost C_{33}^{2} will cause the optimal network design to include only DC 2 and DC 10, instead of DC 2,3 and 7. The same reasoning can be followed for the other cost parameters.

The fixed investment cost represents the second largest fraction of the total cost. Fig.14 illustrates the maximal increase in the fixed investment cost parameter, before another network configuration becomes the optimal solution.

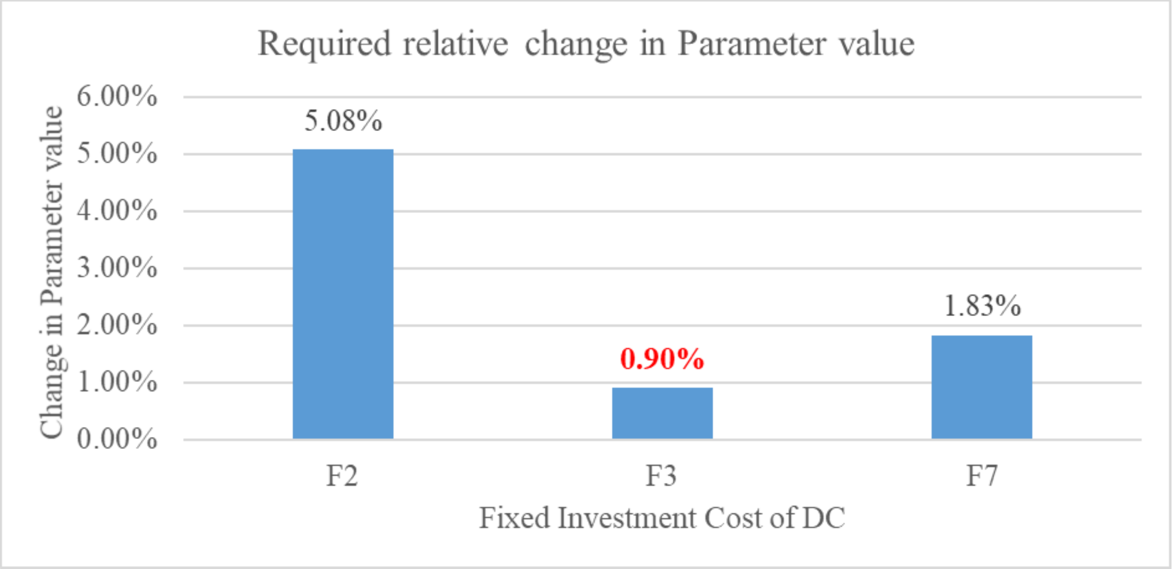


Fig. 14. Maximum increase in fixed investment cost parameters in the optimal network design

Note that an increase of less than 1% will already cause the optimal network configuration to open DCs other than DC 3. As a result, the proposed deterministic network design will become suboptimal in case of a minor increase in the yearly fixed investment cost for DC 3. The fixed investment cost for DC 2, on the other hand, can increase up to about 5% before other DCs become optimal. Subsequently, it‘s analysed that the cost parameters that currently are not part of the optimal solution of the deterministic model. Fig.13 illustrates by how much the transportation cost of a certain DC to a certain retailer should decrease in order to become part of the network design. Note that only the outliers are plotted. The cost parameters on the left hand side of the bar chart can decrease by more than 50% before changing the optimal network design. In other words, the current optimal network design is very robust for changes in these cost parameters.

To conclude, table 9 summarizes the impact of parameters on total costs.

Table 9. Summary of the sensitivity analysis

High impact	**Medium impact**	**Low impact**
Demand at retailers Transportation cost	Fixed investment cost	Variable investment cost Pipeline inventory cost

4.5. Stochastic model with cost scenarios

Minimizing the expected cost

In this model, the assumption of deterministic demand is relaxed. Demand at retailer k now follows a normal distribution. The stochastic problem formulation to minimize the expected cost as discussed in methodology is formulated and run. It is important to note that the objective function still minimizes the same set of costs as in the deterministic problem model. However, costs are now minimized over a weighted set of scenarios, which were defined using a discrete approximation of demand at the retailer market.

Minimizing the maximum scenario cost

Similar to the above formulation for minimizing the expected cost, the objective function still minimizes the same set of costs as in the deterministic problem formulation. However, the optimal network design is now selected considering only the maximum scenario cost. In other words, the network design that results in the lowest costs for the worst-case scenario is selected as the optimal network design.

Minimizing the expected cost with two discrete points (N=2)

When solving the scenario-based problem by making use of 2 discrete points, the expected total cost (problem formulated in b-i) is minimized when DC's 2,3 and 7 are opened. The minimized total cost is € 391,502. The resulting network configuration can be consulted in table 10

Table 10. Costs of optimum network design of a stochastic model with minimum expected cost with N=2

Cost category	**Amount**
Fixed investment costs	€99 783
Variable investment costs	€9 052
Transportation costs from supplier to DC	€46 866
Transportation costs from DC to retailer	€231 446
Annual pipeline inventory to DC	€2 746
Annual pipeline inventory to retailer	€1 609
Total	€391 502

Fig. 15. Optimum network design of a stochastic model with minimum expected cost with N=2

Minimizing the maximum scenario cost with two discrete points (N=2)

The maximum scenario total cost is minimized when DC's 2,3 and 7 are opened. The minimized total cost is € 451,688. The resulting network configuration can be consulted in table 11.

Table 11. Costs of optimum network design of a stochastic model with maximum scenario cost with N=2

Cost category	**Amount**
Fixed investment costs	€99 783
Variable investment costs	€11 484
Transportation costs from supplier to DC	€57 298
Transportation costs from DC to retailer	€277 882
Annual pipeline inventory to DC	€3 299
Annual pipeline inventory to retailer	€1 973
Total	€451 688

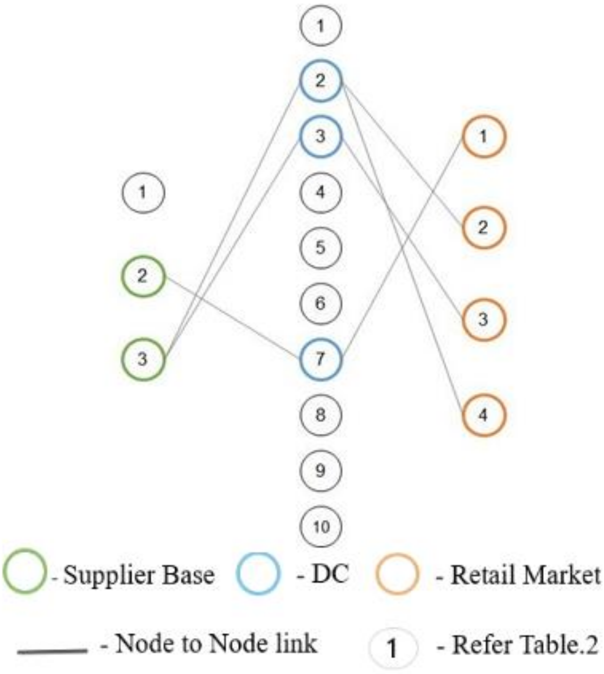


Fig. 16. Optimum network design of a stochastic model with maximum scenario cost with N=2

Minimizing the expected cost with three discrete points (N=3)

When solving the scenario-based problem by making use of 3 discrete points, the expected total cost (problem formulated in b-i) is minimized when DC's 2 and 10 are opened. The minimized total cost is € 385,111 The resulting network configuration can be consulted in table 12.

Table 12. Costs of optimum network design of a stochastic model with minimum expected cost with N=3

Cost category	**Amount**
Fixed investment costs	€49 312
Variable investment costs	€7 297
Transportation costs from supplier to DC	€42 209
Transportation costs from DC to retailer	€282 189
Annual pipeline inventory to DC	€2 675
Annual pipeline inventory to retailer	€1 429
Total	€ 385 111

Fig. 17. Optimum network design of a stochastic model with minimum expected cost with N=3

Minimizing the maximum scenario cost with three discrete points (N=3)

The maximum scenario total cost is minimized when DC’s 3,6 and 7 are opened. The minimized total cost is € 495,296. The resulting network configuration can be consulted in table 13.

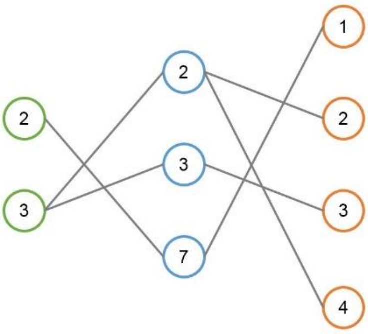
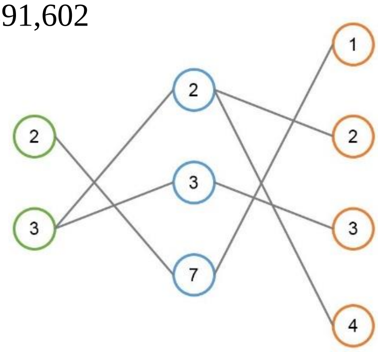
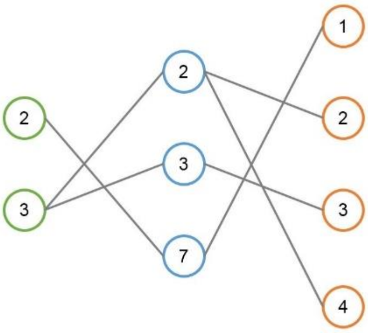
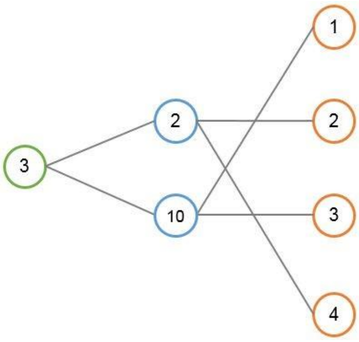
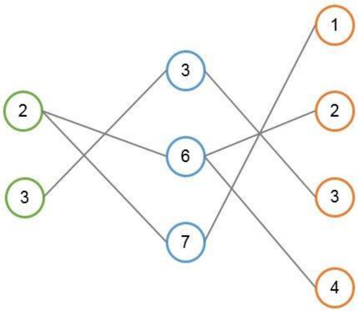
Table 13. Costs of optimum network design of a stochastic model with maximum scenario cost with N=3

Cost category	**Amount**
Fixed investment costs	€101 588
Variable investment costs	€13 209
Transportation costs from supplier to DC	€66 576
Transportation costs from DC to retailer	€307 790
Annual pipeline inventory to DC	€3 704
Annual pipeline inventory to retailer	€2 429
Total	€495 296

Fig. 18. Optimum network design of a stochastic model with maximum scenario cost with N=3

Below table 14 provides an overview of the different network designs and the corresponding total costs.

Table 14. Costs and representation of the different optimal network design

<table>
<tr><td colspan="2">Deterministic network design</td></tr>
<tr><td colspan="2">min cost = € 391,602
design 1:
</td></tr>
<tr><td colspan="2">Stochastic network designs</td></tr>
<tr><td colspan="2">Discrete points N = 2</td></tr>
<tr><td>Expected total cost
min cost = € 391,602
design 2:
</td><td>Maximum scenario total cost
min cost = € 451,688
design 3:
</td></tr>
<tr><td colspan="2">N = 3</td></tr>
<tr><td>Expected total cost
min cost = € 385,111
design 4:
</td><td>Maximum scenario total cost
min cost = € 495,296
design 5:
</td></tr>
<tr><td colspan="2">◯ - Supplier Base ◯ - DC ◯ - Retail Market
—— - Node to Node link ① - Refer Table.2</td></tr>
</table>

For N=2, both stochastic problem formulations lead to the exact same optimal network design as the deterministic problem. This can explained by the fact that the problem with two discrete points (N=2) is very similar to the deterministic problem (where N=1). As one can see, the optimal network design does change when a problem formulation with three discrete points (N=3) is used. This can be explained by the fact that more scenarios are used (higher N) and therefore, a more accurate discretization of a continuous distribution is made.

4.6. Practical implementation recommendation

In practice, demand is seldom deterministic. Therefore, decisions concerning the optimal network design are taken in an uncertain environment. As a result, actual total network costs will probably differ from the minimized total cost for a certain design. Obviously, it is not cost-efficient to open and close DCs each time a different demand occurs. Therefore, it is advisable to work with fixed DC locations, as indicated by the optimal network design, but adjust the configuration (i.e. supplier-DC and DC-retailer relationships) according to the actual demand realizations. In order to decide which DC locations, it’s useful to compare the mean and CV (Coefficient of Variation) for each design over the different scenarios we defined, as illustrated in table 15. A design with in general a low (mean) total cost and a low CV would be the prefered design.

The table below is constructed by solving the problem for the different demand inputs denoted as D_k and by replacing the decision variable Y by a fixed parameter**,** indicating the selected DC locations for each design.

Table 15. Overview of the minimal cost of the network designs considering different scenarios.

Scenarios	**Design 1, 2 and 3**	**Design 4**	**Design 5**
D_k = {70, 108, 128, 200}	€ 331,316	€ 320,660	€ 333,373
D_k= {170, 192, 192, 220}	€ 451,688	€ 463,268	€ 451,962
D_k = {33, 77, 105, 193}	€ 287,319	€ 268,634	€ 290,040
D_k = {120, 150, 160, 210}	€ 391,602	€ 391,964	€ 392,668
D_k = {207, 223 ,215, 227}	€ 495,684	€ 515,294	€ 495,296
mean	€**391,602**	€**391,954**	€**392,668**
variance	**$5.79x10^9$**	**$8.12x10^9$**	**$5.62x10^9$**
range (max - min)	**€ 208,365**	**€ 246,660**	**€ 205,256**
CV	**14,789**	**20,717**	**14,312**

Based on table 15, we would recommend to implement the first network design in which DC 2,3 and 7 are opened. It has the lowest expected value over all scenarios and appears to be rather robust when looking at the CV. Only design 5 has a slightly lower CV and range, but the expected value (mean total cost) is higher.

Normally, stochastic scenario-based problems are expected to deliver better solutions, since different demand scenarios are considered in the models. Obviously, the higher the number of scenarios anticipated by the model, the better the discrete approximation of the continuous demand distribution and hence, the better the solutions. Considering practical reasons of computational times and complexity, only a limited amount of scenarios (N) can be considered in practice. However, for the specific case of the assignment, the deterministic network design appears to be the better solution. Based on both the robust performance and its simplicity, we decided to use the deterministic network design as the optimal one.

4.7. Safety Stock Placement

In this model, the assumption of deterministic demand is relaxed as well. Demand at retailer *k* follows a normal distribution. Due to this demand uncertainty, safety stocks are required to hedge against stockouts. Safety stocks can be held both at DCs and at retail markets. In the hierarchical model, it is needed to determine the optimal quoted service times and the levels of safety stock that should be maintained at each DC and retail market, assuming the optimal network design under deterministic demand. At the same time, its required to ensure a service level α of 97,5 % both at the distribution centers and at the retailers. This means that the model disregards the minimization of investment, transportation and pipeline inventory costs and focuses solely on the minimization of total safety stock holding cost, which will be a sum of the holding cost at the DCs and the holding cost at the retail markets.

It's important to note that the objective function is nonlinear as it includes square roots of decision variables. 10 linear pieces were used to approximate the square root. The more pieces used, the closer this type of linearisation will resemble the square root and the better the solution will be. As computational time and model complexity could be reduced, it's decided to use an approximation of the square root by 10 linear pieces.

Table 16. Safety Stock level of the optimum network configuration.

	DC	**Retailer**
Quoted service time in days	[0 0 **3** 0 0 0 0 0 0 0]	[**3** 0 **1** 0]
Net lead time in days	[0 **5** 0 0 0 0 **2** 0 0 0]	[0 **2 5 2**]
Safety stock level in Units	[0 **191** 0 0 0 0 **139** 0 0 0]	[0 **118 139 28**]

In the optimal network design under deterministic demand, the 2nd, 3rd and 7th DC opens. The quoted service times of the 2nd and 7th DC are zero, which means these DCs fulfill demand from stock. As a result, both DCs have a positive net lead time and safety stocks are required. The 3rd DC on the other hand, does quote a positive service time to its customers. This DC fulfills uncertain demand on order and therefore does not hold safety inventory. Furthermore, the total amount of safety stock held at the DCs exceeds the the total amount of safety stock held at the retailers.

Table 17. Costs inclusive of safety stock of the optimal network design

Cost category	**Amount**
Fixed investment costs	€ 99 783
Variable investment costs	€9 052
Transportation costs from supplier to DC	€46 866
Transportation costs from DC to retailer	€23 446
Annual pipeline inventory to DC cost	€2 745
Annual pipeline inventory to retailer cost	€1 608
Annual safety stock inventory at DC	€71 095
Annual safety stock inventory at retailer	€119 060
Total	**€581 657**

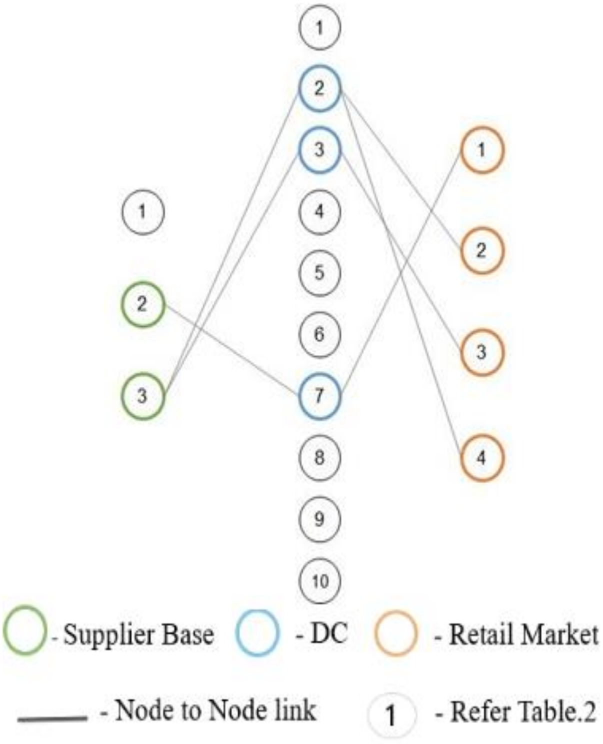


Fig. 19. Deterministic model chosen for safety stock placement

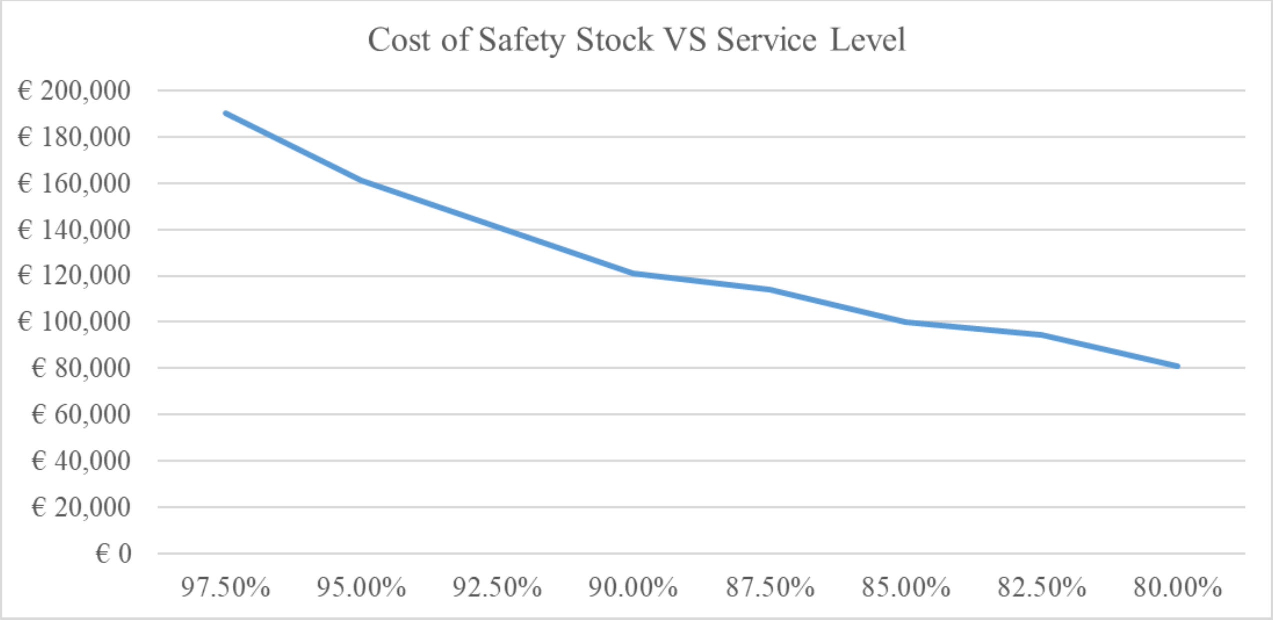


Fig. 20. The relationship between service level and safety stock level

Lastly, the relationship between the service level and the total safety stock holding cost in the network is analysed. As illustrated in Figure 20, the higher the service level, the higher the total safety stock holding cost. Obviously, higher service levels require higher safety stock levels, since it's required to be able to meet demand in more cases. To make sure demand is met 97.5% of the time, it's required to keep enough stock. More stock implies a higher cost of holding the stock and hence, a higher total cost of safety stock in the network.

Conclusions

1. Various influencing parameters and supply chain network constraints are studied, analysed and estimated to build a supply chain network model with realtime active and inactive entities. The active drivers are found to be the key performance indicators of the models.

2. With the cuurent available data and available potential sites of distribution centres, supplier bases and retail market, an optimum network configuration is determined. A supply chain network is successfully designed by solving the facility location problem. The model is tested with both deterministic and stochastic demand approach with 2 and 3 discrete points to achieve better approximation. The deterministic model holds a total cost of € 391,602.

3. The formulated models were successfully subjected to minimization functions of total cost with 2 and 3 discrete points. Also, different scenarios of demand fluctuations were simulated to estimate the impact and performance of the model with respect to worst case and best-case scenarios respectively. The model with 2 discrete points allowing more room for fluctuation in demand, is considered the realistic model has a total expected cost of € 391,502 and a maximum scenario i.e. worst case scenario cost of € 451,688. However, when the demand forecast is considered to be reliable, the model with 3 discrete points can be used to simulate.

4. With various configuration of the networks developed, an in-depth sensitivity analysis is successfully performed to estimate and analyse the effect of change/fluctuation in one influencing parameter over another parameter and eventually impacting the total cost of the network. Increase in unit tansportaion cost from DC2 to retail market 4 by 50% will increase the total cost of supply chain network by 10%. The change in investment cost by 50% will not increase the total cost of supply chain more than 7%. Whilst, the pipeline inventory cost has no effect on the total supply chain cost.

5. In addition to the sensitivity analysis, robustness of the optimum network configuration is also checked by determining the threshold in the key drivers of the network which when exceeded results in stock out and eventually disproves the optimum network configuration. The most critical link in the network is found to be the transportation cost from DC 3 to retail market 3, where an increase in cost of 1.02% will make the model no more optimum. Similarly, the investment cost of DC 3 if increased by 0.90% will make disprove the optimum network configuration.

6. The required safety stock and for the quoted service level is estimated successfully with analytics explaining the co-relation of cost of safety stock and the quoted service level. A safety stock worth of € 119,060 is considered optimum for the model to meet its servce level. After comparing and analysing all the scenarios, recommendations on practical implementations is provided to the strategic planning team with solid data evidence.

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Appendix

Appendix 1. Mathematical Progamming code in IBM CPLEX Optimization Studio

Declaring data sets for deterministic approach

```
/*********************************************
* OPL 12.8.0.0 Data
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

nb_suppliers = 3;
nb_districns = 10;
nb_retailers = 4;

cost_investment = [33130, 22952, 51497, 68691, 80869, 24757, 25334, 36050, 29676, 26360]; //f(j)

cost_demand = [0.09, 0.01, 0.08, 0.08, 0.07, 0.01, 0.07, 0.03, 0.07, 0.06]; //g(j)


cost_transportation_supply = [ [0.39, 0.20, 0.60, 0.50, 0.12, 0.28, 0.58, 0.45, 0.34, 0.26],
[0.42, 0.20, 0.43, 0.35, 0.14, 0.19, 0.25, 0.22, 0.57, 0.23],
[0.32, 0.18, 0.21, 0.27, 0.28, 0.36, 0.34, 0.29, 0.41, 0.19] ];

cost_transportation_delivery = [ [2.40, 1.82, 1.38, 0.97],
[2.15, 0.98, 2.10, 1.07],
[2.40, 1.40, 0.98, 2.10],
[0.92, 0.98, 1.15, 1.20],
[0.90, 1.60, 1.80, 0.60],
[2.10, 0.90, 2.10, 1.10],
[0.88, 2.10, 1.87, 1.80],
[2.10, 0.86, 1.90, 0.95],
[1.35, 0.93, 1.80, 1.10],
[1.05, 2.10, 1.82, 1.80] ];

time_supply = [ [4, 1, 6, 4, 6, 2, 4, 7, 3, 4],
[2, 3, 2, 5, 3, 5, 2, 2, 5, 2],
[3, 5, 3, 7, 4, 2, 5, 2, 3, 4] ];

time_delivery = [ [3, 3, 1, 1],
[2, 2, 3, 2],
[1, 2, 3, 2],
[2, 3, 1, 2],
[3, 2, 2, 3],
[2, 3, 1, 2],
[3, 3, 2, 2],
[2, 2, 1, 3],
[1, 2, 3, 2],
[3, 3, 2, 1] ];

cost_inventory_supply = [0.99, 1, 0.97, 1.1, 0.9, 1, 2, 0.81, 0.93, 0.83];
```

```
cost_inventory_delivery = [1.20, 0.85, 0.94, 1.12];
demand_mean = [120, 150, 160, 210];
demand_variance = [2500, 1800, 1000, 100];
```

Model formation for deterministic approach

```
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/


//declare parameters
int nb_suppliers = ...;
int nb_districns = ...;
int nb_retailers = ...;

//declare ranges
range supplier = 1..nb_suppliers;
range districn = 1..nb_districns;
range retailer = 1..nb_retailers;

//declare costs, times and demand
int cost_investment[districn] = ...;
float cost_demand[districn] = ...;
float cost_transportation_supply[supplier][districn] = ...;
float cost_transportation_delivery[districn][retailer] = ...;
float cost_inventory_supply[districn] = ...;
float cost_inventory_delivery[retailer] = ...;

int time_supply[supplier][districn] = ...;
float time_delivery[districn][retailer] = ...;

int demand_mean[retailer] = ...;
int demand_variance[retailer] = ...;

int M = 10000000;

//declare decision variables
dvar boolean open[districn];
dvar boolean supply[supplier][districn];
dvar boolean deliver[districn][retailer];
dvar int+ quantity_deliver[districn][retailer];
dvar int+ quantity_supply[supplier][districn];

//objective function: minimize annual costs per ton
minimize

//fixed investment costs
sum(j in districn) (open[j] * cost_investment[j]) +

//variable investment costs
sum(j in districn, k in retailer) (cost_demand[j] * quantity_deliver[j][k]) +

//transportation costs from supplier to DC
sum(i in supplier, j in districn) (cost_transportation_supply[i][j] * quantity_supply[i][j]) +
```

```

//transportation costs from DC to retailer
sum(j in districn, k in retailer) (cost_transportation_delivery[j][k] * quantity_deliver[j][k]) +

//annual pipeline inventory costs from supplier to DC
sum(i in supplier, j in districn) (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j] / 365) +

//annual pipeline inventory costs from DC to retailer
sum(j in districn, k in retailer) (cost_inventory_delivery[k] * time_delivery[j][k] * quantity_deliver[j][k] / 365);


subject to
{
//demand must be satisfied for each retailer
forall(k in retailer) (sum(j in districn) (quantity_deliver[j][k]) >= demand_mean[k] * 365);

//demand must be satisfied for each DC
forall(j in districn) (sum(i in supplier) (quantity_supply[i][j]) >= sum(k in retailer) (quantity_deliver[j][k]));

//shipment of goods from supplier to DC is only possible if link between supplier and DC
forall(i in supplier, j in districn) (quantity_supply[i][j] <= M * supply[i][j]);

//shipment of goods from DC to retailer is only possible if link between DC and retailer
forall(j in districn, k in retailer) (quantity_deliver[j][k] <= M * deliver[j][k] );

//enforce single sourcing from supplier to DC
forall(j in districn) (sum(i in supplier) (supply[i][j]) <= 1);

//enforce single sourcing from DC to retailer
forall(k in retailer) (sum(j in districn) (deliver[j][k]) <= 1);

//transport from supplier to DC is only possible if DC is open
forall(i in supplier, j in districn) (open[j] >= supply[i][j]);

//transport from DC to retailer is only possible if DC is open
forall(j in districn, k in retailer) (open[j] >= deliver[j][k]);
}


float cost1 = sum(j in districn) (open[j] * cost_investment[j]);
float cost2 = sum(j in districn, k in retailer) (cost_demand[j] * quantity_deliver[j][k]);
float cost3 = sum(i in supplier, j in districn) (cost_transportation_supply[i][j] * quantity_supply[i][j]);
float cost4 = sum(j in districn, k in retailer) (cost_transportation_delivery[j][k] * quantity_deliver[j][k]);
float cost5 = sum(i in supplier, j in districn)
```

```
(cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j] / 365);
float cost6 = sum(j in districn, k in retailer) (cost_inventory_delivery[k] * time_delivery[j][k] * quantity_deliver[j][k] / 365);

execute
{
writeln(cost1 + " (fixed investment cost)")
writeln(cost2 + " (variable investment cost)")
writeln(cost3 + " (transportation cost from supplier to DC)")
writeln(cost4 + " (transportation cost from DC to retailer)")
writeln(cost5 + " (pipeline inventory cost from supplier to DC)")
writeln(cost6 + " (pipeline inventory cost from DC to retailer)")
}

```

Data declaration for Stochastic Approach (N=2)

```
/*********************************************
* OPL 12.8.0.0 Data
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

 nb_suppliers = 3;
nb_districns = 10;
nb_retailers = 4;

cost_investment = [33130, 22952, 51497, 68691, 80869, 24757, 25334, 36050, 29676, 26360]; //f(j)

cost_demand = [0.09, 0.01, 0.08, 0.08, 0.07, 0.01, 0.07, 0.03, 0.07, 0.06]; //g(j)


cost_transportation_supply = [ [0.39, 0.20, 0.60, 0.50, 0.12, 0.28, 0.58, 0.45, 0.34, 0.26],
[0.42, 0.20, 0.43, 0.35, 0.14, 0.19, 0.25, 0.22, 0.57, 0.23],
[0.32, 0.18, 0.21, 0.27, 0.28, 0.36, 0.34, 0.29, 0.41, 0.19] ];

cost_transportation_delivery = [ [2.40, 1.82, 1.38, 0.97],
[2.15, 0.98, 2.10, 1.07],
[2.40, 1.40, 0.98, 2.10],
[0.92, 0.98, 1.15, 1.20],
[0.90, 1.60, 1.80, 0.60],
[2.10, 0.90, 2.10, 1.10],
[0.88, 2.10, 1.87, 1.80],
[2.10, 0.86, 1.90, 0.95],
[1.35, 0.93, 1.80, 1.10],
[1.05, 2.10, 1.82, 1.80] ];

```

```
time_supply = [ [4, 1, 6, 4, 6, 2, 4, 7, 3, 4],
[2, 3, 2, 5, 3, 5, 2, 2, 5, 2],
[3, 5, 3, 7, 4, 2, 5, 2, 3, 4] ];

time_delivery = [ [3, 3, 1, 1],
[2, 2, 3, 2],
[1, 2, 3, 2],
[2, 3, 1, 2],
[3, 2, 2, 3],
[2, 3, 1, 2],
[3, 3, 2, 2],
[2, 2, 1, 3],
[1, 2, 3, 2],
[3, 3, 2, 1] ];

cost_inventory_supply = [0.99, 1, 0.97, 1.1, 0.9, 1, 2, 0.81, 0.93, 0.83];
cost_inventory_delivery = [1.20, 0.85, 0.94, 1.12];
//demand_mean = [120, 150, 160, 210];
demand_variance = [2500, 1800, 1000, 100];

demand_mean = [ [70, 170],
[108, 192],
[128, 192],
[200, 220] ];

probability = [0.50, 0.50]; //2 discrete points

```

Data declaration for Stochastic Approach (N=3)

```
/*********************************************
* OPL 12.8.0.0 Data
* Author: Bavithran Srihdar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

nb_suppliers = 3;
nb_districns = 10;
nb_retailers = 4;

cost_investment = [33130, 22952, 51497, 68691, 80869, 24757, 25334, 36050, 29676, 26360]; //f(j)

cost_demand = [0.09, 0.01, 0.08, 0.08, 0.07, 0.01, 0.07, 0.03, 0.07, 0.06]; //g(j)


cost_transportation_supply = [ [0.39, 0.20, 0.60, 0.50, 0.12, 0.28, 0.58, 0.45, 0.34, 0.26],
[0.42, 0.20, 0.43, 0.35, 0.14, 0.19, 0.25, 0.22, 0.57, 0.23],
[0.32, 0.18, 0.21, 0.27, 0.28, 0.36, 0.34, 0.29, 0.41, 0.19] ];

cost_transportation_delivery = [ [2.40, 1.82, 1.38, 0.97],
```

```
[2.15, 0.98, 2.10, 1.07],
[2.40, 1.40, 0.98, 2.10],
[0.92, 0.98, 1.15, 1.20],
[0.90, 1.60, 1.80, 0.60],
[2.10, 0.90, 2.10, 1.10],
[0.88, 2.10, 1.87, 1.80],
[2.10, 0.86, 1.90, 0.95],
[1.35, 0.93, 1.80, 1.10],
[1.05, 2.10, 1.82, 1.80] ];

time_supply = [ [4, 1, 6, 4, 6, 2, 4, 7, 3, 4],
[2, 3, 2, 5, 3, 5, 2, 2, 5, 2],
[3, 5, 3, 7, 4, 2, 5, 2, 3, 4] ];

time_delivery = [ [3, 3, 1, 1],
[2, 2, 3, 2],
[1, 2, 3, 2],
[2, 3, 1, 2],
[3, 2, 2, 3],
[2, 3, 1, 2],
[3, 3, 2, 2],
[2, 2, 1, 3],
[1, 2, 3, 2],
[3, 3, 2, 1] ];

cost_inventory_supply = [0.99, 1, 0.97, 1.1, 0.9, 1, 2, 0.81, 0.93, 0.83];
cost_inventory_delivery = [1.20, 0.85, 0.94, 1.12];
//demand_mean = [120, 150, 160, 210];
demand_variance = [2500, 1800, 1000, 100];

demand_mean = [ [33, 120, 207],
[77, 150, 223],
[105, 160, 215],
[193, 210, 227] ];

probability = [0.16, 0.66, 0.16]; //3 discrete points

```

Model formulation for Total expected cost in Stochastic approach for N=2

```
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

//declare parameters
int nb_suppliers = ...;
int nb_districns = ...;
int nb_retailers = ...;

//declare ranges
range supplier = 1..nb_suppliers;
range districn = 1..nb_districns;
range retailer = 1..nb_retailers;

```

```
//declare scenarios
range scenario = 1..2;

//declare costs, times and demand
int cost_investment[districn] = ...;
float cost_demand[districn] = ...;
float cost_transportation_supply[supplier][districn] = ...;
float cost_transportation_delivery[districn][retailer] = ...;
float cost_inventory_supply[districn] = ...;
float cost_inventory_delivery[retailer] = ...;

int time_supply[supplier][districn] = ...;
float time_delivery[districn][retailer] = ...;

int demand_variance[retailer] = ...;
int demand_mean[retailer][scenario] = ...; //now,
demand depends on the scenario
float probability[scenario] = ...;

int M = 10000000;

//declare decision variables
dvar boolean open[districn];
dvar boolean supply[supplier][districn];
dvar boolean deliver[districn][retailer];
dvar int+ quantity_deliver[districn][retailer][scenario];
dvar int+ quantity_supply[supplier][districn][scenario];

//objective function: minimize annual costs per ton
minimize

//fixed investment costs
sum(j in districn) (open[j] * cost_investment[j]) +

//variable investment costs
sum(j in districn, k in retailer, s in scenario)
 (cost_demand[j] * quantity_deliver[j][k][s] * probability[s]) +

//transportation costs from supplier to DC
sum(i in supplier, j in districn, s in scenario)
(cost_transportation_supply[i][j] * quantity_supply[i][j][s] *
probability[s]) +

//transportation costs from DC to retailer
sum(j in districn, k in retailer, s in scenario)
(cost_transportation_delivery[j][k] * quantity_deliver[j][k][s] *
probability[s]) +

//annual pipeline inventory costs from supplier to DC
sum(i in supplier, j in districn, s in scenario)
(cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i]
[j][s] * probability[s] / 365) +

//annual pipeline inventory costs from DC to retailer
sum(j in districn, k in retailer, s in scenario)
(cost_inventory_delivery[k] * time_delivery[j][k] *
quantity_deliver[j][k][s] * probability[s] / 365);

```

```

subject to
{
//demand must be satisfied for each retailer
forall(k in retailer, s in scenario) (sum(j in districn) (quantity_deliver[j][k][s]) >= demand_mean[k][s] * 365);

//demand must be satisfied for each DC
forall(j in districn, s in scenario) (sum(i in supplier) (quantity_supply[i][j][s]) >= sum(k in retailer) (quantity_deliver[j][k][s]));

//shipment of goods from supplier to DC is only possible if link between supplier and DC
forall(i in supplier, j in districn, s in scenario) (quantity_supply[i][j][s] <= M * supply[i][j]);

//shipment of goods from DC to retailer is only possible if link between DC and retailer
forall(j in districn, k in retailer, s in scenario) (quantity_deliver[j][k][s] <= M * deliver[j][k] );

//enforce single sourcing from supplier to DC
forall(j in districn) (sum(i in supplier) (supply[i][j]) <= 1);

//enforce single sourcing from DC to retailer
forall(k in retailer) (sum(j in districn) (deliver[j][k]) <= 1);

//transport from supplier to DC is only possible if DC is open
forall(i in supplier, j in districn) (open[j] >= supply[i][j]);

//transport from DC to retailer is only possible if DC is open
forall(j in districn, k in retailer) (open[j] >= deliver[j][k]);
}


float cost1 = sum(j in districn) (open[j] * cost_investment[j]);
float cost2 = sum(j in districn, k in retailer, s in scenario) (cost_demand[j] * quantity_deliver[j][k][s] * probability[s]);
float cost3 = sum(i in supplier, j in districn, s in scenario) (cost_transportation_supply[i][j] * quantity_supply[i][j][s] * probability[s]);
float cost4 = sum(j in districn, k in retailer, s in scenario) (cost_transportation_delivery[j][k] * quantity_deliver[j][k][s] * probability[s]);
float cost5 = sum(i in supplier, j in districn, s in scenario) (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j][s] * probability[s] / 365);
float cost6 = sum(j in districn, k in retailer, s in scenario) (cost_inventory_delivery[k] * time_delivery[j][k] * quantity_deliver[j][k][s] * probability[s] / 365);


execute
{
writeln(cost1 + " (fixed investment cost)")
```

```
writeln(cost2 + " (variable investment cost)")
writeln(cost3 + " (transportation cost from supplier to DC)")
writeln(cost4 + " (transportation cost from DC to retailer)")
writeln(cost5 + " (pipeline inventory cost from supplier to DC)")
writeln(cost6 + " (pipeline inventory cost from DC to retailer)")
}




```

Model formulation for Total expected cost in Stochastic approach for N=3

```
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

//declare parameters
int nb_suppliers = ...;
int nb_districns = ...;
int nb_retailers = ...;

//declare ranges
range supplier = 1..nb_suppliers;
range districn = 1..nb_districns;
range retailer = 1..nb_retailers;

//declare scenarios
range scenario = 1..3;

//declare costs, times and demand
int cost_investment[districn] = ...;
float cost_demand[districn] = ...;
float cost_transportation_supply[supplier][districn] = ...;
float cost_transportation_delivery[districn][retailer] = ...;
float cost_inventory_supply[districn] = ...;
float cost_inventory_delivery[retailer] = ...;

int time_supply[supplier][districn] = ...;
float time_delivery[districn][retailer] = ...;

int demand_variance[retailer] = ...;
int demand_mean[retailer][scenario] = ...; //now, demand depends on the scenario
float probability[scenario] = ...;

int M = 10000000;

//declare decision variables
dvar boolean open[districn];
dvar boolean supply[supplier][districn];
dvar boolean deliver[districn][retailer];
```

```
dvar int+ quantity_deliver[districn][retailer][scenario];
dvar int+ quantity_supply[supplier][districn][scenario];

//objective function: minimize annual costs per ton
minimize

//fixed investment costs
sum(j in districn) (open[j] * cost_investment[j]) +

//variable investment costs
sum(j in districn, k in retailer, s in scenario)
(cost_demand[j] * quantity_deliver[j][k][s] * probability[s]) +

//transportation costs from supplier to DC
sum(i in supplier, j in districn, s in scenario)
(cost_transportation_supply[i][j] * quantity_supply[i][j][s] *
probability[s]) +

//transportation costs from DC to retailer
sum(j in districn, k in retailer, s in scenario)
(cost_transportation_delivery[j][k] * quantity_deliver[j][k][s] *
probability[s]) +

//annual pipeline inventory costs from supplier to DC
sum(i in supplier, j in districn, s in scenario)
(cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i]
[j][s] * probability[s] / 365) +

//annual pipeline inventory costs from DC to retailer
sum(j in districn, k in retailer, s in scenario)
(cost_inventory_delivery[k] * time_delivery[j][k] *
quantity_deliver[j][k][s] * probability[s] / 365);


subject to
{
//demand must be satisfied for each retailer
forall(k in retailer, s in scenario) (sum(j in districn)
 (quantity_deliver[j][k][s]) >= demand_mean[k][s] * 365);

//demand must be satisfied for each DC
forall(j in districn, s in scenario) (sum(i in supplier)
(quantity_supply[i][j][s]) >= sum(k in retailer)
(quantity_deliver[j][k][s]));

//shipment of goods from supplier to DC is only possible if
link between supplier and DC
forall(i in supplier, j in districn, s in scenario)
(quantity_supply[i][j][s] <= M * supply[i][j]);

//shipment of goods from DC to retailer is only possible if
link between DC and retailer
forall(j in districn, k in retailer, s in scenario)
(quantity_deliver[j][k][s] <= M * deliver[j][k] );

//enforce single sourcing from supplier to DC
forall(j in districn) (sum(i in supplier) (supply[i][j]) <= 1);

```

```
//enforce single sourcing from DC to retailer
forall(k in retailer) (sum(j in districn) (deliver[j][k]) <= 1);

//transport from supplier to DC is only possible if DC is open
forall(i in supplier, j in districn) (open[j] >= supply[i][j]);

//transport from DC to retailer is only possible if DC is open
forall(j in districn, k in retailer) (open[j] >= deliver[j][k]);
}


float cost1 = sum(j in districn) (open[j] * cost_investment[j]);
float cost2 = sum(j in districn, k in retailer, s in scenario) (cost_demand[j] * quantity_deliver[j][k][s] * probability[s]);
float cost3 = sum(i in supplier, j in districn, s in scenario) (cost_transportation_supply[i][j] * quantity_supply[i][j][s] * probability[s]);
float cost4 = sum(j in districn, k in retailer, s in scenario) (cost_transportation_delivery[j][k] * quantity_deliver[j][k][s] * probability[s]);
float cost5 = sum(i in supplier, j in districn, s in scenario) (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j][s] * probability[s] / 365);
float cost6 = sum(j in districn, k in retailer, s in scenario) (cost_inventory_delivery[k] * time_delivery[j][k] * quantity_deliver[j][k][s] * probability[s] / 365);


execute
{
writeln(cost1 + " (fixed investment cost)")
writeln(cost2 + " (variable investment cost)")
writeln(cost3 + " (transportation cost from supplier to DC)")
writeln(cost4 + " (transportation cost from DC to retailer)")
writeln(cost5 + " (pipeline inventory cost from supplier to DC)")
writeln(cost6 + " (pipeline inventory cost from DC to retailer)")
}




```

Data declarion for Minimizing total scenario cost in Stochastic approach for N=2

```
/*********************************************
* OPL 12.8.0.0 Data
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

nb_suppliers = 3;
nb_districns = 10;
nb_retailers = 4;
```

```

cost_investment = [33130, 22952, 51497, 68691, 80869, 24757, 25334, 36050, 29676, 26360]; //f(j)

cost_demand = [0.09, 0.01, 0.08, 0.08, 0.07, 0.01, 0.07, 0.03, 0.07, 0.06]; //g(j)


cost_transportation_supply = [ [0.39, 0.20, 0.60, 0.50, 0.12, 0.28, 0.58, 0.45, 0.34, 0.26],
[0.42, 0.20, 0.43, 0.35, 0.14, 0.19, 0.25, 0.22, 0.57, 0.23],
[0.32, 0.18, 0.21, 0.27, 0.28, 0.36, 0.34, 0.29, 0.41, 0.19] ];

cost_transportation_delivery = [ [2.40, 1.82, 1.38, 0.97],
[2.15, 0.98, 2.10, 1.07],
[2.40, 1.40, 0.98, 2.10],
[0.92, 0.98, 1.15, 1.20],
[0.90, 1.60, 1.80, 0.60],
[2.10, 0.90, 2.10, 1.10],
[0.88, 2.10, 1.87, 1.80],
[2.10, 0.86, 1.90, 0.95],
[1.35, 0.93, 1.80, 1.10],
[1.05, 2.10, 1.82, 1.80] ];

time_supply = [ [4, 1, 6, 4, 6, 2, 4, 7, 3, 4],
[2, 3, 2, 5, 3, 5, 2, 2, 5, 2],
[3, 5, 3, 7, 4, 2, 5, 2, 3, 4] ];

time_delivery = [ [3, 3, 1, 1],
[2, 2, 3, 2],
[1, 2, 3, 2],
[2, 3, 1, 2],
[3, 2, 2, 3],
[2, 3, 1, 2],
[3, 3, 2, 2],
[2, 2, 1, 3],
[1, 2, 3, 2],
[3, 3, 2, 1] ];

cost_inventory_supply = [0.99, 1, 0.97, 1.1, 0.9, 1, 2, 0.81, 0.93, 0.83];
cost_inventory_delivery = [1.20, 0.85, 0.94, 1.12];
//demand_mean = [120, 150, 160, 210];
demand_variance = [2500, 1800, 1000, 100];

demand_mean = [ [70, 170],
[108, 192],
[128, 192],
[200, 220] ];

probability = [0.50, 0.50];

```

Model formulation for Minimizing total scenario cost in Stochastic approach for N=2

```
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 01 May 2020 at 13:25:28
*********************************************/
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

//declare parameters
int nb_suppliers = ...;
int nb_districns = ...;
int nb_retailers = ...;

//declare ranges
range supplier = 1..nb_suppliers;
range districn = 1..nb_districns;
range retailer = 1..nb_retailers;

//declare scenarios
range scenario = 1..2;

//declare costs, times and demand
int cost_investment[districn] = ...;
float cost_demand[districn] = ...;
float cost_transportation_supply[supplier][districn] = ...;
float cost_transportation_delivery[districn][retailer] = ...;
float cost_inventory_supply[districn] = ...;
float cost_inventory_delivery[retailer] = ...;

int time_supply[supplier][districn] = ...;
float time_delivery[districn][retailer] = ...;

int demand_variance[retailer] = ...;
int demand_mean[retailer][scenario] = ...; //now, demand depends on the scenario
float probability[scenario] = ...;

int M = 10000000;

//declare decision variables
dvar boolean open[districn];
dvar boolean supply[supplier][districn];
dvar boolean deliver[districn][retailer];
dvar int+ quantity_deliver[districn][retailer][scenario];
dvar int+ quantity_supply[supplier][districn][scenario];
dvar float+ cost;

//objective function: minimize annual costs per ton
minimize cost ;


subject to
{
//demand must be satisfied for each retailer
forall(k in retailer, s in scenario) (sum(j in districn)
```

```
(quantity_deliver[j][k][s]) >= demand_mean[k][s] * 365);

//demand must be satisfied for each DC
forall(j in districn, s in scenario) (sum(i in supplier)
(quantity_supply[i][j][s]) >= sum(k in retailer)
(quantity_deliver[j][k][s]));

//shipment of goods from supplier to DC is only possible if
link between supplier and DC
forall(i in supplier, j in districn, s in scenario)
(quantity_supply[i][j][s] <= M * supply[i][j]);

//shipment of goods from DC to retailer is only possible if
link between DC and retailer
forall(j in districn, k in retailer, s in scenario)
(quantity_deliver[j][k][s] <= M * deliver[j][k] );

//enforce single sourcing from supplier to DC
forall(j in districn) (sum(i in supplier) (supply[i][j]) <= 1);

//enforce single sourcing from DC to retailer
forall(k in retailer) (sum(j in districn) (deliver[j][k]) <=
1);

//transport from supplier to DC is only possible if DC is open
forall(i in supplier, j in districn) (open[j] >= supply[i][j]);

//transport from DC to retailer is only possible if DC is open
forall(j in districn, k in retailer) (open[j] >= deliver[j]
[k]);

//MFLP key constraint to maximize the maximum scenario
forall(s in scenario)(
sum(j in districn) (open[j] * cost_investment[j]) +

sum(j in districn, k in retailer) (cost_demand[j] *
quantity_deliver[j][k][s]) +

sum(i in supplier, j in districn)
(cost_transportation_supply[i][j] * quantity_supply[i][j][s]) +

sum(j in districn, k in retailer)
(cost_transportation_delivery[j][k] * quantity_deliver[j][k][s]) +

sum(i in supplier, j in districn) (cost_inventory_supply[j] *
time_supply[i][j] * quantity_supply[i][j][s] / 365) +

sum(j in districn, k in retailer) (cost_inventory_delivery[k] *
time_delivery[j][k] * quantity_deliver[j][k][s] / 365) <= cost);
}


float cost1 = sum(j in districn) (open[j] * cost_investment[j]);
float cost2 = sum(j in districn, k in retailer) (cost_demand[j] *
quantity_deliver[j][k][2]);
float cost3 = sum(i in supplier, j in districn)
(cost_transportation_supply[i][j] * quantity_supply[i][j][2]);
float cost4 = sum(j in districn, k in retailer)
```

```
 (cost_transportation_delivery[j][k] * quantity_deliver[j][k][2]);
float cost5 = sum(i in supplier, j in districn)
 (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i]
 [j][2] / 365);
float cost6 = sum(j in districn, k in retailer)
 (cost_inventory_delivery[k] * time_delivery[j][k] *
 quantity_deliver[j][k][2] / 365);
float total = cost1 + cost2 + cost3 + cost4 + cost5 + cost6;

execute
{
writeln(cost1 + " (fixed investment cost)")
writeln(cost2 + " (variable investment cost)")
writeln(cost3 + " (transportation cost from supplier to
DC)")
writeln(cost4 + " (transportation cost from DC to retailer)")
writeln(cost5 + " (pipeline inventory cost from supplier
to DC)")
writeln(cost6 + " (pipeline inventory cost from DC to
retailer)")
writeln()
writeln(total + " (total)")
}

```

Data declarion for Minimizing total scenario cost in Stochastic approach for N=3

```
/*********************************************
* OPL 12.8.0.0 Data
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

nb_suppliers = 3;
nb_districns = 10;
nb_retailers = 4;

cost_investment = [33130, 22952, 51497, 68691, 80869, 24757, 25334,
 36050, 29676, 26360]; //f(j)

cost_demand = [0.09, 0.01, 0.08, 0.08, 0.07, 0.01, 0.07, 0.03,
 0.07, 0.06]; //g(j)


cost_transportation_supply = [ [0.39, 0.20, 0.60, 0.50, 0.12, 0.28,
 0.58, 0.45, 0.34, 0.26],
[0.42, 0.20, 0.43, 0.35, 0.14, 0.19,
 0.25, 0.22, 0.57, 0.23],
[0.32, 0.18, 0.21, 0.27, 0.28, 0.36,
 0.34, 0.29, 0.41, 0.19] ];

cost_transportation_delivery = [ [2.40, 1.82, 1.38, 0.97],
[2.15, 0.98, 2.10, 1.07],
[2.40, 1.40, 0.98, 2.10],
[0.92, 0.98, 1.15, 1.20],
[0.90, 1.60, 1.80, 0.60],
[2.10, 0.90, 2.10, 1.10],
```

```
[0.88, 2.10, 1.87, 1.80],
[2.10, 0.86, 1.90, 0.95],
[1.35, 0.93, 1.80, 1.10],
[1.05, 2.10, 1.82, 1.80] ];

time_supply = [ [4, 1, 6, 4, 6, 2, 4, 7, 3, 4],
[2, 3, 2, 5, 3, 5, 2, 2, 5, 2],
[3, 5, 3, 7, 4, 2, 5, 2, 3, 4] ];

time_delivery = [ [3, 3, 1, 1],
[2, 2, 3, 2],
[1, 2, 3, 2],
[2, 3, 1, 2],
[3, 2, 2, 3],
[2, 3, 1, 2],
[3, 3, 2, 2],
[2, 2, 1, 3],
[1, 2, 3, 2],
[3, 3, 2, 1] ];

cost_inventory_supply = [0.99, 1, 0.97, 1.1, 0.9, 1, 2, 0.81, 0.93, 0.83];
cost_inventory_delivery = [1.20, 0.85, 0.94, 1.12];
//demand_mean = [120, 150, 160, 210];
demand_variance = [2500, 1800, 1000, 100];

demand_mean = [ [33, 120, 207],
[77, 150, 223],
[105, 160, 215],
[193, 210, 227] ];

probability = [0.16, 0.66, 0.16];

```

Model formulation for Minimizing total scenario cost in Stochastic approach for N=3

```
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 01 May 2020 at 13:25:28
*********************************************/
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 01 Apr 2020 at 00:10:21
*********************************************/

//declare parameters
int nb_suppliers = ...;
int nb_districns = ...;
int nb_retailers = ...;

//declare ranges
range supplier = 1..nb_suppliers;
range districn = 1..nb_districns;
range retailer = 1..nb_retailers;

```

```
//declare scenarios
range scenario = 1..3;

//declare costs, times and demand
int cost_investment[districn] = ...;
float cost_demand[districn] = ...;
float cost_transportation_supply[supplier][districn] = ...;
float cost_transportation_delivery[districn][retailer] = ...;
float cost_inventory_supply[districn] = ...;
float cost_inventory_delivery[retailer] = ...;

int time_supply[supplier][districn] = ...;
float time_delivery[districn][retailer] = ...;

int demand_variance[retailer] = ...;
int demand_mean[retailer][scenario] = ...; //now, demand depends on the scenario
float probability[scenario] = ...;

int M = 10000000;

//declare decision variables
dvar boolean open[districn];
dvar boolean supply[supplier][districn];
dvar boolean deliver[districn][retailer];
dvar int+ quantity_deliver[districn][retailer][scenario];
dvar int+ quantity_supply[supplier][districn][scenario];
dvar float+ cost;

//objective function: minimize annual costs per ton
minimize cost ;


subject to
{
//demand must be satisfied for each retailer
forall(k in retailer, s in scenario) (sum(j in districn) (quantity_deliver[j][k][s]) >= demand_mean[k][s] * 365);

//demand must be satisfied for each DC
forall(j in districn, s in scenario) (sum(i in supplier) (quantity_supply[i][j][s]) >= sum(k in retailer) (quantity_deliver[j][k][s]));

//shipment of goods from supplier to DC is only possible if link between supplier and DC
forall(i in supplier, j in districn, s in scenario) (quantity_supply[i][j][s] <= M * supply[i][j]);

//shipment of goods from DC to retailer is only possible if link between DC and retailer
forall(j in districn, k in retailer, s in scenario) (quantity_deliver[j][k][s] <= M * deliver[j][k] );

//enforce single sourcing from supplier to DC
forall(j in districn) (sum(i in supplier) (supply[i][j]) <= 1);

//enforce single sourcing from DC to retailer
```

```
forall(k in retailer) (sum(j in districn) (deliver[j][k]) <= 1);

//transport from supplier to DC is only possible if DC is open
forall(i in supplier, j in districn) (open[j] >= supply[i][j]);

//transport from DC to retailer is only possible if DC is open
forall(j in districn, k in retailer) (open[j] >= deliver[j][k]);

//key constraint to maximize the maximum scenario
forall(s in scenario)(
sum(j in districn) (open[j] * cost_investment[j]) +

sum(j in districn, k in retailer) (cost_demand[j] * quantity_deliver[j][k][s]) +

sum(i in supplier, j in districn) (cost_transportation_supply[i][j] * quantity_supply[i][j][s]) +

sum(j in districn, k in retailer) (cost_transportation_delivery[j][k] * quantity_deliver[j][k][s]) +

sum(i in supplier, j in districn) (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j][s] / 365) +

sum(j in districn, k in retailer) (cost_inventory_delivery[k] * time_delivery[j][k] * quantity_deliver[j][k][s] / 365) <= cost);
}


float cost1 = sum(j in districn) (open[j] * cost_investment[j]);
float cost2 = sum(j in districn, k in retailer) (cost_demand[j] * quantity_deliver[j][k][3]);
float cost3 = sum(i in supplier, j in districn) (cost_transportation_supply[i][j] * quantity_supply[i][j][3]);
float cost4 = sum(j in districn, k in retailer) (cost_transportation_delivery[j][k] * quantity_deliver[j][k][3]);
float cost5 = sum(i in supplier, j in districn) (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j][3] / 365);
float cost6 = sum(j in districn, k in retailer) (cost_inventory_delivery[k] * time_delivery[j][k] * quantity_deliver[j][k][3] / 365);
float total = cost1 + cost2 + cost3 + cost4 + cost5 + cost6;

execute
{
writeln(cost1 + " (fixed investment cost)")
writeln(cost2 + " (variable investment cost)")
writeln(cost3 + " (transportation cost from supplier to DC)")
writeln(cost4 + " (transportation cost from DC to retailer)")
writeln(cost5 + " (pipeline inventory cost from supplier to DC)")
writeln(cost6 + " (pipeline inventory cost from DC to retailer)")
writeln()
```

```
writeln(total + " (total)")
}

```

Data declarion for Safety Stock Placement

```
/*********************************************
* OPL 12.8.0.0 Data
* Author: Bavithran Sridhar
* Creation Date: 07 May 2020 at 16:21:49
*********************************************/
nb_suppliers = 3;
nb_districns = 10;
nb_retailers = 4;

cost_inventory_dc = [169, 214, 299, 129, 90, 164, 218, 215, 88, 109];
cost_inventory_retailer = [407, 484, 383, 328];

time_supply = [ [4, 1, 6, 4, 6, 2, 4, 7, 3, 4],
[2, 3, 2, 5, 3, 5, 2, 2, 5, 2],
[3, 5, 3, 7, 4, 2, 5, 2, 3, 4] ];

time_delivery = [ [3, 3, 1, 1],
[2, 2, 3, 2],
[1, 2, 3, 2],
[2, 3, 1, 2],
[3, 2, 2, 3],
[2, 3, 1, 2],
[3, 3, 2, 2],
[2, 2, 1, 3],
[1, 2, 3, 2],
[3, 3, 2, 1] ];

demand_mean = [120, 150, 160, 210];
demand_variance = [2500, 1800, 1000, 100];

safety_factor = 1.96;
service_time_DC_in = [0, 0, 0]; //SI(i)
service_time_retailer_out_max = [3, 0, 1, 0]; //r(k)

cost_investment = [33130, 22952, 51497, 68691, 80869, 24757, 25334, 36050, 29676, 26360]; //f(j)
cost_demand = [0.09, 0.01, 0.08, 0.08, 0.07, 0.01, 0.07, 0.03, 0.07, 0.06]; //g(j)
cost_transportation_supply = [ [0.39, 0.20, 0.60, 0.50, 0.12, 0.28, 0.58, 0.45, 0.34, 0.26],
[0.42, 0.20, 0.43, 0.35, 0.14, 0.19, 0.25, 0.22, 0.57, 0.23],
[0.32, 0.18, 0.21, 0.27, 0.28, 0.36, 0.34, 0.29, 0.41, 0.19] ];
cost_transportation_delivery = [ [2.40, 1.82, 1.38, 0.97],
[2.15, 0.98, 2.10, 1.07],
[2.40, 1.40, 0.98, 2.10],
[0.92, 0.98, 1.15, 1.20],
[0.90, 1.60, 1.80, 0.60],
[2.10, 0.90, 2.10, 1.10],
```

```
[0.88, 2.10, 1.87, 1.80],
[2.10, 0.86, 1.90, 0.95],
[1.35, 0.93, 1.80, 1.10],
[1.05, 2.10, 1.82, 1.80] ];
cost_inventory_supply = [0.99, 1, 0.97, 1.1, 0.9, 1, 2, 0.81, 0.93, 0.83];
cost_inventory_delivery = [1.20, 0.85, 0.94, 1.12];

open = [0, 1, 1, 0, 0, 0, 1, 0, 0, 0];
quantity_deliver = [[0, 0, 0, 0]
[0 54750 0 76650]
[0, 0, 58400, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]
[43800, 0, 0, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]];
quantity_supply = [[0, 0, 0, 0 ,0 ,0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 43800, 0, 0, 0]
[0, 131400, 58400, 0, 0, 0, 0, 0, 0, 0]];
deliver = [[0, 0, 0, 0]
[0, 1, 0, 1]
[0, 0, 1, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]
[1, 0, 0, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]
[0, 0, 0, 0]];
supply = [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 1, 0, 0, 0]
[0, 1, 1, 0, 0, 0, 0, 0, 0, 0]];
```

Model formulation for Safety Stock Placement

```
/*********************************************
* OPL 12.8.0.0 Model
* Author: Bavithran Sridhar
* Creation Date: 07 May 2020 at 16:21:49
*********************************************/
//declare parameters
int nb_suppliers = ...;
int nb_districns = ...;
int nb_retailers = ...;

//declare ranges
range supplier = 1..nb_suppliers;
range districn = 1..nb_districns;
range retailer = 1..nb_retailers;

//declare costs, times and demand
float cost_inventory_dc[districn] = ...;
float cost_inventory_retailer[retailer] = ...;

```

```
int time_supply[supplier][districn] = ...;
float time_delivery[districn][retailer] = ...;

int demand_mean[retailer] = ...;
int demand_variance[retailer] = ...;

float safety_factor = ...;
int service_time_DC_in[supplier] = ...; //SI(i)
int service_time_retailer_out_max[retailer] = ...; //r(k)

//declare costs to calculate total cost
int cost_investment[districn] = ...;
float cost_demand[districn] = ...;
float cost_transportation_supply[supplier][districn] = ...;
float cost_transportation_delivery[districn][retailer] = ...;
float cost_inventory_supply[districn] = ...;
float cost_inventory_delivery[retailer] = ...;

//Declare deterministic network configuration
int deliver[districn][retailer] = ...;
int supply[supplier][districn] = ...;
int open[districn] = ...;
int quantity_deliver[districn][retailer] = ...;
int quantity_supply[supplier][districn] = ...;

//declare decision variables for safety stock placement
dvar int+ net_lead_time_dc[districn]; //N(j) lead time of DC
dvar int+ net_lead_time_retailer[retailer]; //L(k) lead time of retailer
dvar int+ service_time_DC_out[districn]; //S(j) Guaranteed service time of DC(j) to retailers
dvar int+ service_time_retailer_out[retailer]; // Guaranteed service time of retailer k to customer

//define upper bounds to linearize model
float net_lead_time_dc_upper[districn]; //NU(j)
float net_lead_time_retailer_upper[retailer]; //LU(k)

execute
{
var max;
var totalsum;

//upperbound for net lead time of DC
for(var j in districn)
{
max = 0;
for(var i in supplier)
{
if(service_time_DC_in[i] + time_supply[i][j] > max)
{
max = service_time_DC_in[i] + time_supply[i][j];
}
}

net_lead_time_dc_upper[j] = max;
}
```

```

//upperbound for net lead time of retailer
for(var k in retailer)
{
max = 0;
for(var j in districn)
{
if(net_lead_time_dc_upper[j] + time_delivery[j][k] - service_time_retailer_out_max[k] > max)
if(net_lead_time_dc_upper[j] + time_delivery[j][k] - service_time_retailer_out_max[k] > max)
{
max = net_lead_time_dc_upper[j] + time_delivery[j][k] - service_time_retailer_out_max[k];
}
}

net_lead_time_retailer_upper[k] = max;
}
}

//declare decision variables for piecewise linearisation
int nb_pieces = 10;
range piece = 1..nb_pieces;
range piece1 = 1..(nb_pieces+1);
dvar float+ u1[districn][piece];
dvar float+ u2[retailer][piece];
dvar boolean v1[districn][piece];
dvar boolean v2[retailer][piece];
dvar float slope1[districn][piece];
dvar float intercept1[districn][piece];
dvar float slope2[retailer][piece];
dvar float intercept2[retailer][piece];

//define bounds for piecewise linearisation
float bound1[districn][piece1];
float bound2[retailer][piece1];
execute
{
for(var j in districn)
{
for(var p in piece1)
{
bound1[j][p] = ((p-1) / nb_pieces) * net_lead_time_dc_upper[j];
}
}

for(var k in retailer)
{
for(var p in piece1)
{
bound2[k][p] = ((p-1) / nb_pieces) * net_lead_time_retailer_upper[k];
bound2[k][p] = ((p-1) / nb_pieces) * net_lead_time_retailer_upper[k];
}
}
```

```
}

//objective function: minimize annual costs per ton
minimize

//annual safety stock inventory costs at DC
sum(j in districn) (cost_inventory_dc[j] * safety_factor * sqrt(sum(k in retailer) (demand_variance[k] * deliver[j][k])) * sum(p in piece) (intercept1[j][p] * v1[j][p] + slope1[j][p] * u1[j][p])) +

//annual safety stock inventory costs at retailer
sum(k in retailer) (cost_inventory_retailer[k] * safety_factor * sqrt(demand_variance[k]) * sum(p in piece) (intercept2[k][p] * v2[k][p] + slope2[k][p] * u2[k][p]));

subject to
{
//net_lead_time_supply[districn] is greater than its replenishment lead time minus its guaranteed service time to its retailer
forall(j in districn) (net_lead_time_dc[j] == sum(i in supplier) (supply[i][j] * service_time_DC_in[i]) + sum(i in supplier) (time_supply[i][j] * supply[i][j]) - service_time_DC_out[j]);

//net_lead_time_delivery[retailer] is greater than its replenishment lead time minus service_time_DC_out_max[retailer]
forall(k in retailer) (net_lead_time_retailer[k] == sum(j in districn) (deliver[j][k] * service_time_DC_out[j]) + sum(j in districn) (time_delivery[j][k] * deliver[j][k]) - service_time_retailer_out[k]);

//guaranteed service time by retailers cannot exceed maximum allowed
forall(k in retailer) (service_time_retailer_out[k] <= service_time_retailer_out_max[k]);

//define slope of piecewise linearisation
forall(p in piece, j in districn) (slope1[j][p] == ((sqrt(bound1[j][p+1]) - sqrt(bound1[j][p])) / (bound1[j][p+1] - bound1[j][p])));
forall(p in piece, j in districn) (slope1[j][p] == ((sqrt(bound1[j][p+1]) - sqrt(bound1[j][p])) / (bound1[j][p+1] - bound1[j][p])));
forall(p in piece, k in retailer) (slope2[k][p] == ((sqrt(bound2[k][p+1]) - sqrt(bound2[k][p])) / (bound2[k][p+1] - bound2[k][p])));

//define intercept of piecewise linearisation
forall(p in piece, j in districn) (intercept1[j][p] == sqrt(bound1[j][p]) - slope1[j][p] * bound1[j][p]);
forall(p in piece, k in retailer) (intercept2[k][p] == sqrt(bound2[k][p]) - slope2[k][p] * bound2[k][p]);

//factor to multiply with intercept must sum to 1
forall(j in districn) (sum(p in piece) (v1[j][p]) == 1);
forall(k in retailer) (sum(p in piece) (v2[k][p]) == 1);
```

```

//factors to multiply with slope must sum to x
forall(j in districn) (sum(p in piece) (u1[j][p]) == net_lead_time_dc[j]);
forall(k in retailer) (sum(p in piece) (u2[k][p]) == net_lead_time_retailer[k]);

//Force the slope-multiplying-factor to be within the right segment
forall(p in piece, j in districn) (bound1[j][p] * v1[j][p] <= u1[j][p]);
forall(p in piece, j in districn) (bound1[j][p+1] * v1[j][p] >= u1[j][p]);

forall(p in piece, k in retailer) (bound2[k][p] * v2[k][p] <= u2[k][p]);
forall(p in piece, k in retailer) (bound2[k][p+1] * v2[k][p] >= u2[k][p]);
}

float cost1 = sum(j in districn) (open[j] * cost_investment[j]);
float cost2 = sum(j in districn, k in retailer) (cost_demand[j] * quantity_deliver[j][k]);
float cost3 = sum(i in supplier, j in districn) (cost_transportation_supply[i][j] * quantity_supply[i][j]);
float cost4 = sum(j in districn, k in retailer) (cost_transportation_delivery[j][k] * quantity_deliver[j][k]);
float cost5 = sum(i in supplier, j in districn) (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j] / 365);
float cost5 = sum(i in supplier, j in districn) (cost_inventory_supply[j] * time_supply[i][j] * quantity_supply[i][j] / 365);
float cost6 = sum(j in districn, k in retailer) (cost_inventory_delivery[k] * time_delivery[j][k] * quantity_deliver[j][k] / 365);
float cost7 = sum(j in districn) (cost_inventory_dc[j] * safety_factor * sqrt(sum(k in retailer) (demand_variance[k] * deliver[j][k])) * net_lead_time_dc[j] / sqrt(net_lead_time_dc_upper[j]));
float cost8 = sum(k in retailer) (cost_inventory_retailer[k] * safety_factor * sqrt(demand_variance[k]) * net_lead_time_retailer[k] / sqrt(net_lead_time_retailer_upper[k]));
float cost9 = sum(j in districn) (cost_inventory_dc[j] * safety_factor * sqrt(sum(k in retailer) (demand_variance[k] * deliver[j][k])) * sum(p in piece) (intercept1[j][p] * v1[j][p] + slope1[j][p] * u1[j][p]));
float cost10= sum(k in retailer) (cost_inventory_retailer[k] * safety_factor * sqrt(demand_variance[k]) * sum(p in piece) (intercept2[k][p] * v2[k][p] + slope2[k][p] * u2[k][p]));
float cost11= sum(j in districn) (cost_inventory_dc[j] * safety_factor * sqrt(net_lead_time_dc[j])) * sqrt(sum(k in retailer) (demand_variance[k] * deliver[j][k]));
float cost12= sum(k in retailer) (cost_inventory_retailer[k] * safety_factor * sqrt(demand_variance[k]) * sqrt(net_lead_time_retailer[k]));


```

```
execute
{
writeln(cost1 + " (fixed investment cost)")
writeln(cost2 + " (variable investment cost)")
writeln(cost3 + " (transportation cost from supplier to DC)")
writeln(cost4 + " (transportation cost from DC to retailer)")
writeln(cost5 + " (pipeline inventory cost from supplier to DC)")
writeln(cost6 + " (pipeline inventory cost from DC to retailer)")
writeln()
writeln(cost11 + " (annual safety stock inventory costs at DC)")
writeln(cost7 + " (annual safety stock inventory costs at DC approximated by secant )")
writeln(cost9 + " (annual safety stock inventory costs at DC approximated by piecewise)")
writeln(cost12 + " (annual safety stock inventory costs at retailer)")
writeln(cost8 + " (annual safety stock inventory costs at retailer approximated by secant)")
writeln(cost10 + " (annual safety stock inventory costs at retailer approximated by piecewise)")
writeln()
writeln("Total safety stock holding cost: " + (cost11 + cost12))
writeln("Total cost: " + (cost1 + cost2 + cost3 + cost4 + cost5 + cost6 + cost9 + cost10))
}
```