

Non-destructive Identification of Direction of Orthotropy of Paperboard

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The eigenmodes of bending vibrations of packaging materials as of a plate for various directions of orthotropy are analyzed. The obtained results are used in the process of identification of machine direction and cross direction of paperboard.

One dimensional model of tension of physically non-linear packaging materials is proposed. The displacements obtained by using this model are compared with the linear ones.

Experimental investigations of longitudinal uniaxial tension of paperboard used for the production of packages were performed. Uniaxial tension was applied in the machine direction as well as in the cross direction of the paperboard. Mechanical characteristics and stress – strain relationships have been determined and are analyzed. Correspondence between the experimental and numerical results in the initial stage of deformations is determined.

Keywords: packaging materials, machine direction, cross direction, plate bending, vibrations, eigenmodes, method of identification, one dimensional model, tension of paperboard, physical nonlinearity, experimental investigation, experimental setup, stress – strain curve, mechanical characteristics.

1. INTRODUCTION

Various materials are used for the production of various polygraphic products. Among those materials paperboard can be noted as being most often used. Materials used for packaging are to have good qualities of exploitation during the processes of transportation and long term keeping of goods while experiencing various changes and effects of the environment. The previously mentioned polygraphic materials, which are most often used, are orthotropic and can be noted for their different qualities and characteristics. This is important in the process of production of packages and other polygraphic materials (during printing, packaging and similar activities) [1 – 3].

Mechanical behaviour of paperboard under the action of various loadings was already investigated in the papers [4–9]. But the estimation of the effects of orthotropic properties of the materials used for packaging to their physical – mechanical qualities is missing there.

The purpose of this paper is to investigate the vibrations and physically non-linear tension as well as the eigenmodes of packaging materials, which are used for production of packages and to propose the model that could be used in the process of identification of machine direction and cross direction of polygraphic materials.

This presented work is the sequel of previous work [10]. In the previous investigation [10] it was assumed that the paperboard is tensioned in its plane (both symmetric and asymmetric loadings were analyzed) and that the machine direction and the cross direction of the paperboard coincide with the axes of coordinates. In this paper it is assumed that there is no tension applied to the paperboard,

but the machine direction and the cross direction not necessarily coincide with the axes of coordinates. For this case the procedure of identification of the angle that the machine direction and the cross direction make with the axes of coordinates is a much more complicated problem. In this paper recommendations for the identification of this angle are provided.

2. ANALYSIS OF VIBRATIONS OF PAPERBOARD

2.1. Model for the analysis of vibrations of paperboard

Further x , y and z denote the axes of the system of coordinates. The plate bending element has three nodal degrees of freedom: the transverse displacement of the paperboard w and the rotations Θ_x and Θ_y about the axes of coordinates x and y . x' and y' denote the machine direction and the cross direction of the paperboard. Matrix of elastic constants has the form:

$$[D] = \begin{bmatrix} \frac{E_x}{1-\nu_{x'y'}\nu_{y'x'}} & \frac{E_y\nu_{x'y'}}{1-\nu_{x'y'}\nu_{y'x'}} & 0 \\ \frac{E_x\nu_{y'x'}}{1-\nu_{x'y'}\nu_{y'x'}} & \frac{E_y}{1-\nu_{x'y'}\nu_{y'x'}} & 0 \\ 0 & 0 & \frac{E_x E_y}{E_x + E_y + E_x\nu_{y'x'} + E_y\nu_{x'y'}} \end{bmatrix}, \quad (1)$$

where E_x and E_y are the modulus of elasticity of the paperboard, $\nu_{x'y'}$ and $\nu_{y'x'}$ are the Poisson's ratios of the paperboard and:

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$$E_{x'} = E_{y'} \frac{v_{x'y'}}{v_{y'x'}}. \quad (2)$$

Strains in the plane x_0y are related by the following matrix:

$$[T] = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}, \quad (3)$$

where:

$$\begin{cases} s = \sin \theta, \\ c = \cos \theta, \end{cases} \quad (4)$$

where θ is an angle between the coordinate systems x_0y and $x'y_0$.

The mass matrix of the paperboard has the form:

$$[M] = \int [N]^T \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix} [N] dx dy, \quad (5)$$

where ρ is the density of the material of the paperboard, h is the thickness of the paperboard and:

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_1 & 0 & \dots \\ 0 & 0 & N_1 & \dots \end{bmatrix}, \quad (6)$$

where N_i are the shape functions of the finite element.

The stiffness matrix of the paperboard has the form:

$$[K] = \int \left([B]^T [T] \frac{h^3}{12} [D] [T] [B] + [\tilde{B}]^T \right).$$

$$\left. \frac{E_{x'} E_{y'} h}{(E_{x'} + E_{y'} + E_{x'} v_{y'x'} + E_{y'} v_{x'y'}) 1.2} [\tilde{B}] \right) dx dy, \quad (7)$$

where:

$$[B] = \begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & \dots \\ 0 & -\frac{\partial N_1}{\partial y} & 0 & \dots \\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \dots \end{bmatrix}, \quad (8)$$

$$[\tilde{B}] = \begin{bmatrix} \frac{\partial N_1}{\partial y} & -N_1 & 0 & \dots \\ \frac{\partial N_1}{\partial x} & 0 & N_1 & \dots \end{bmatrix}. \quad (9)$$

2.2 Results of analysis of vibrations of paperboard

The square piece of paperboard is analyzed. On the lower and upper boundaries all the generalized displacements are assumed equal to zero. It is assumed that the modulus of elasticity $E_{y'} = 0.34 \cdot 10^9$ Pa, Poisson's ratio $v_{x'y'} = 0.4$, Poisson's ratio $v_{y'x'} = 0.14$, thickness of the paperboard $h = 0.0001$ m, density of the material of the paperboard $\rho = 785$ kg/m³. The first eigenmode when

$\theta = \frac{\pi}{8}(i-1)$ for $i = 1, 2, \dots, 5$ is presented in Fig. 1, the

second eigenmode is presented in Fig. 2, ..., the sixth eigenmode is presented in Fig. 6.

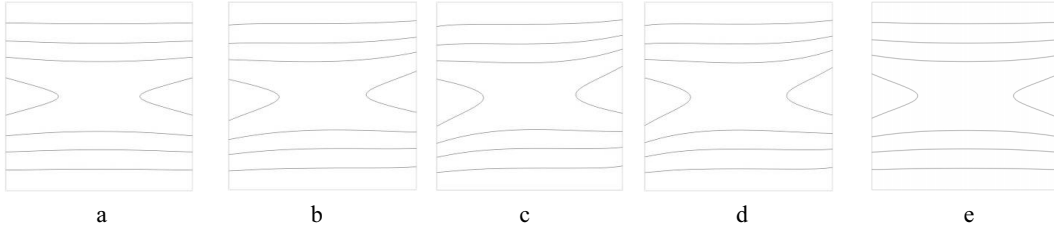


Fig. 1. The first eigenmode when $\theta = \frac{\pi}{8}(i-1)$ for $i = 1, 2, \dots, 5$

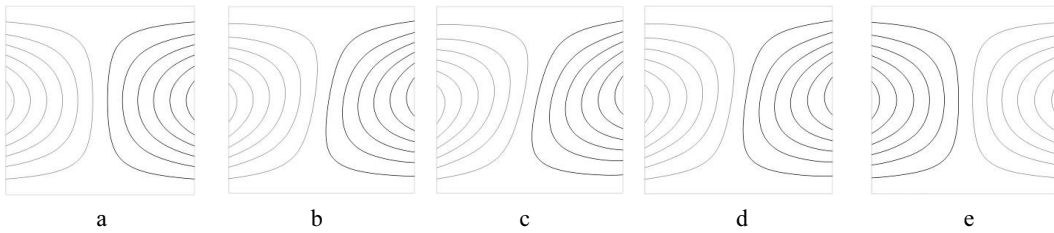


Fig. 2. The second eigenmode when $\theta = \frac{\pi}{8}(i-1)$ for $i = 1, 2, \dots, 5$

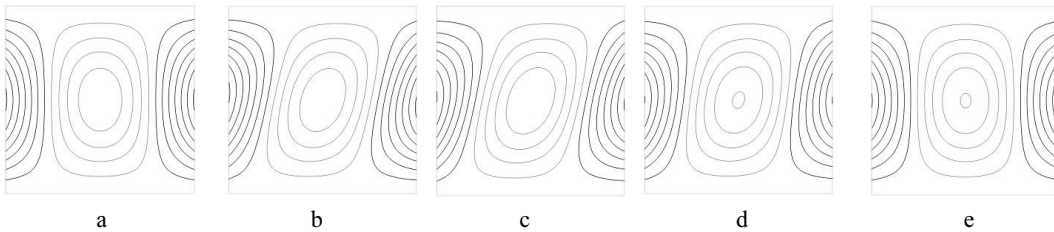


Fig. 3. The third eigenmode when $\theta = \frac{\pi}{8}(i-1)$ for $i = 1, 2, \dots, 5$

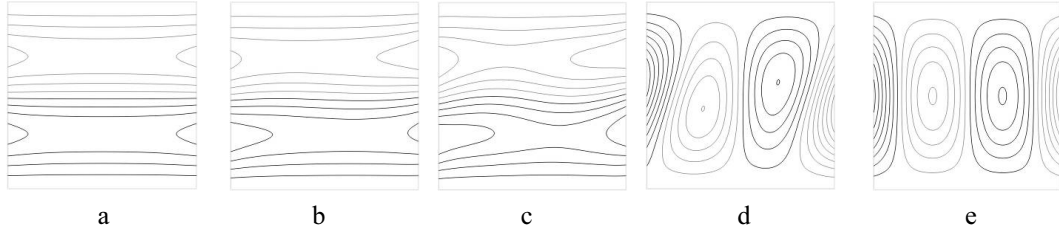


Fig. 4. The fourth eigenmode when $\theta = \frac{\pi}{8}(i-1)$ for $i = 1, 2, \dots, 5$

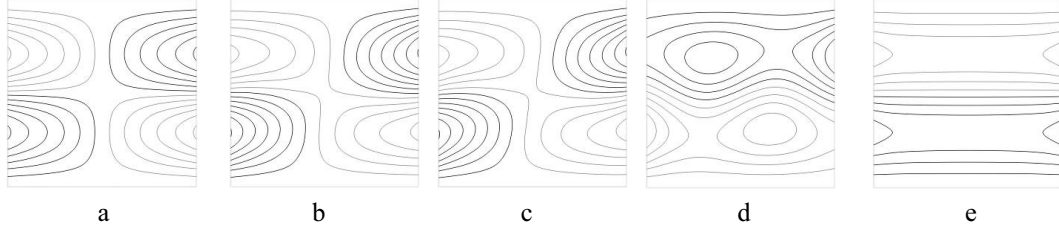


Fig. 5. The fifth eigenmode when $\theta = \frac{\pi}{8}(i-1)$ for $i = 1, 2, \dots, 5$

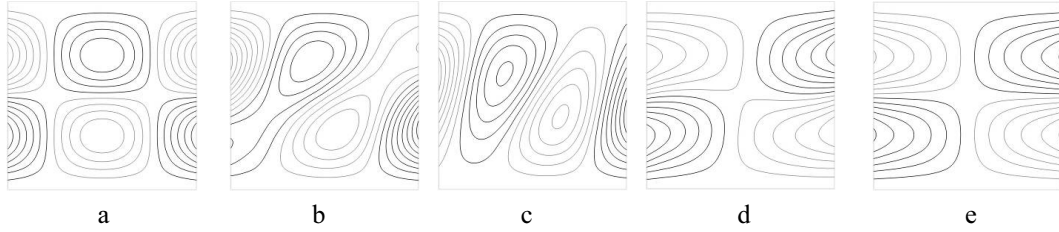


Fig. 6. The sixth eigenmode when $\theta = \frac{\pi}{8}(i-1)$ for $i = 1, 2, \dots, 5$

From the presented results it is seen that the eigenmodes of the analyzed paperboard with various machine directions and cross directions have differences. Thus from the eigenmodes it is possible to identify the machine direction and the cross direction of the paperboard.

3. ANALYSIS OF PHYSICALLY NON-LINEAR TENSION OF PAPERBOARD

3.1. Model for the analysis of physically non-linear tension of paperboard

One dimensional model is used for the solution of this problem. The structure coincides with the x axis of the system of coordinates and the finite element has one nodal degree of freedom: the longitudinal displacement of the paperboard u .

The longitudinal strain is expressed as:

$$\varepsilon_x = [B]\{\delta\}, \quad (10)$$

where $\{\delta\}$ is the vector of nodal displacements and:

$$[B] = \begin{bmatrix} \frac{dN_1}{dx} & \dots \end{bmatrix}, \quad (11)$$

where N_i are the shape functions of the one dimensional finite element.

The longitudinal stress is expressed as:

$$\sigma_x = \frac{E}{1-\nu^2} (1+b\varepsilon_x^2) \varepsilon_x, \quad (12)$$

where E is the modulus of elasticity of the paperboard, ν is the Poisson's ratio of the paperboard and b is the Duffing parameter of the paperboard.

Thus the stiffness matrix of the one dimensional physically non-linear model of the paperboard has the form:

$$[K] = \int [B]^T \left[\frac{E}{1-\nu^2} h (1+b\varepsilon_x^2) \right] [B] dx, \quad (13)$$

where h is the thickness of the paperboard.

3.2. Results of analysis of physically non-linear tension of paperboard

The piece of paperboard of length equal to 0.2 m is analyzed. On the left side the nodal displacement is assumed equal to zero. The load is applied to the node on the right side of the paperboard. It is assumed that the modulus of elasticity $E = 0.6 \cdot 10^9$ Pa, Poisson's ratio $\nu = 0.3$, thickness of the paperboard $h = 0.0001$ m, Duffing parameter for the physically non-linear model $b = -300$.

The stress-strain laws for the linear and the physically non-linear problems are shown in Fig. 7.

Displacements of the paperboard are determined when the applied force $F = 2000 \frac{i}{8}$ N, $i = 1, 2, \dots, 8$. Results for

a linear structure are presented in Fig. 8, a, and for a physically non-linear one in Fig. 8, b.

From the presented results the effect of physical non-linearity of Duffing type to the displacements of the paperboard is clearly seen. Physical non-linearity increases the displacements of the paperboard. The advantage of the one dimensional model is in its ability to represent all the results in a single graphical figure.

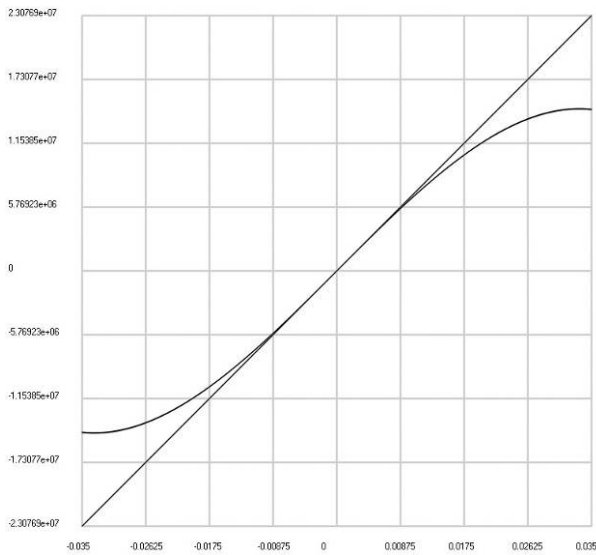


Fig. 7. The stress–strain laws for the linear and the physically non-linear problems

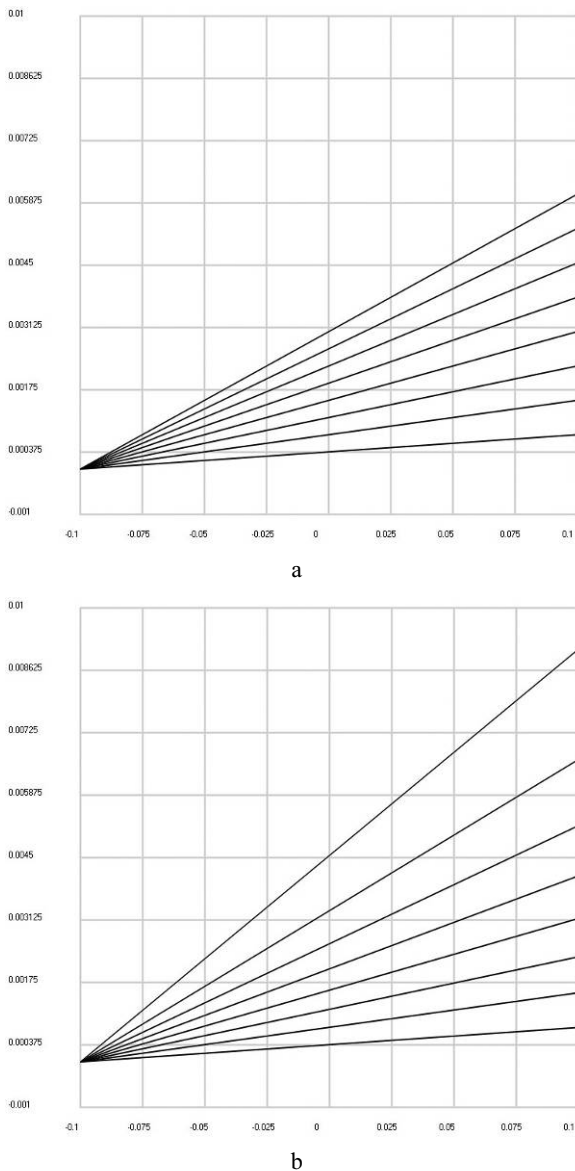


Fig. 8. Displacements of the paperboard: a – for a linear structure; b – for a physically non-linear structure

4. EXPERIMENTAL INVESTIGATIONS OF TENSION OF PAPERBOARD

4.1. Description of the method and devices of experimental investigations of paperboard

In order to be able to compare the obtained numerical results with the experimental ones the investigations of longitudinal uniaxial tension of paperboard used for the production of packages were performed. Uniaxial tension was applied in the machine direction as well as in the cross direction of paperboard.

The method of investigations of tension experiments of the paper and the devices used in the investigations were already described in reference [11]. Set of physical parameters used for one dimensional model that was applied for the analysis of tension of orthotropic materials was also determined from these experiments.

4.2. Results of analysis of experimental investigations of paperboard

In Table 1 and in Fig. 9 the results of tension of paperboard in the machine direction as well as in the cross direction are presented. Here the results obtained in reference [11] are used.

When analyzing the character of variation of the curves of the stress–strain relationships for the paperboard Kromopak (see Fig. 9) it was noticed that in the obtained relationships two regions may be determined: the first is the region where the direction of production of paperboard has very small influence to the values of strains and stresses and the second is the region where the direction of production of paperboard has substantial influence to the mentioned mechanical characteristics.

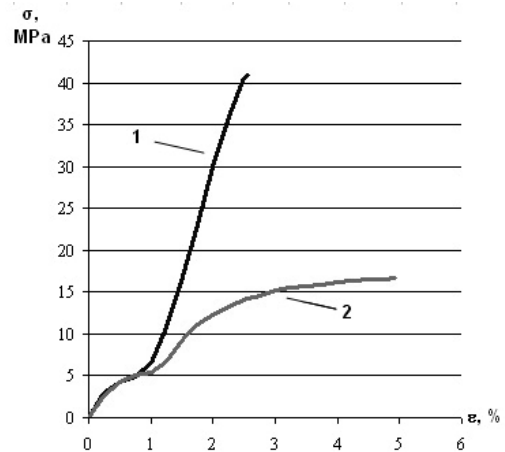


Fig. 9. The stress–strain relationships of paperboard of Kromopak type: 1 – in the machine direction ($E = 1.6$ GPa); 2 – in the cross direction ($E = 0.33$ GPa)

When performing the comparison of the obtained results of experimental and numerical investigations of paperboard it is determined that in the initial stage of deformation results obtained by using the numerical model qualitatively agree with the results obtained experimentally.

Note that presented results are in further developments. The authors are continuing their research in this field.

Table 1. Results of tension experiment of paperboard Kromopak (MD – machine direction, CD – cross direction).

Paper specimen description	Elongation Δl_0 , mm	Fracture force F_f , N	Fracture stress σ_f , MPa	Fracture strain ε_f , %
Paperboard Kromopak (MD)	2.56	197.3	40.98	2.56
Paperboard Kromopak (CD)	4.94	80.15	16.65	4.94

CONCLUSIONS

The model for the analysis of vibrations of paperboard for various machine directions and cross directions is proposed. The eigenmodes of bending vibrations as of a plate are calculated. From the presented results it is seen that the eigenmodes of the analyzed paperboard depend on the machine directions and the cross directions. Thus from the eigenmodes it is possible to identify the machine direction and the cross direction of the analyzed paperboard.

One dimensional model of tension of physically non-linear paperboard with cubic nonlinearity of Duffing type is proposed. From the presented results the effect of physical non-linearity of Duffing type to the displacements of the paperboard is determined. The advantage of the one dimensional model is in its ability to represent all the results in a single graphical image.

In the initial stage of deformation the obtained results of experimental investigations of paperboard correspond to the results obtained by using the numerical model.

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