Kaunas University of Technology
Faculty of Civil Engineering and Architecture

Research of Composite Steel-Concrete Floor Vibrations

Master's Final Degree Project

Mabast Salih Ali
Project author

Lect. Dr. Tadas Zingaila
Supervisor

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Master's Final Degree Project
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Mabast Salih Ali
Project author

Lect. Dr. Tadas Zingaila
Supervisor

Lect. Dr. Nerijus Adamukaitis
Reviewer

Kaunas, 2020

Kaunas University of Technology
Faculty of Civil Engineering and Architecture
Mabast Salih Ali

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Task of the master's final degree project

Topic of the project	Research of Composite Steel-Concrete Floor Vibrations
Requirements and conditions (title can be clarified, if needed)	
Supervisor	
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Summary

Developments in lightweight steel-concrete composite floors with long spans are resulting in structures with low natural frequencies. Potentially, these floors are sensitive to vibration issues. Vibrations are mostly caused by human activities on the floor. However, vibrations due to mechanical systems can also cause problems.

This thesis analyses approaches on how to manually calculate natural frequency and dynamic response of slim composite floors by using available design guidelines. Composite slim steel-concrete floors with hollow-core slabs are chosen in the analysis because they are one of the most popular on construction market nowadays. Additionally, Finite Element Method software (i.e. SCIA Engineer and Autodesk Robot Structural Analysis) is used to determine the dynamic properties of these floors. Moreover, the author has created nomographs to calculate the natural frequency of floor elements much easier with less calculation. Finally, all results from the manual and numerical calculations, as well as nomographs are compared and concluded.

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Santrauka

Didelių tarpatramių, lengvos kompozitinės plienbetoninės perdangos pasižymi turinčios žemą savųjų svyravimų dažnį, ir potencialiai yra jautrios vibracijų poveikiui. Dažniausiai pasitaikantis vibracijų šaltinis, tai žmogaus veikla ant perdangos. Tačiau mechaniškai sukeltos vibracijos taip pat gali sukelti problemų.

Šiame darbe analizuojami esami projektavimo rekomendacijose pateikiami skaičiavimo metodai, kuriais rankiniu būdu galima apskaičiuoti liaunų kompozitinių perdangų savųjų svyravimų dažnį ir dinaminį atsaką,. Atliekant analizę, pasirinktos liaunos kompozitinės plienbetoninės perdangos su kiaurymėtomis plokštėmis, kadangi šiais laikas jos yra vienos iš populiariausių statybų rinkoje. Kompozitinių perdangų dinaminėms savybėms nustatyti, taip pat naudojama programinė įranga „baigtinių elementų metodas“ (t.y. „SCIA Engineer“ ir „Autodesk Robot Structural Analysis“). Be to, autorius sukūrė nomogramas, kad būtų galima lengviau apskaičiuoti perdangos elementų savųjų svyravimų dažnį, atliekant mažiau skaičiavimų. Galiausiai, palyginami ir apibendrinami visi rankinio skaičiavimo, skaitiniai ir nomogramų rezultatai.

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1. Introduction

Extreme floor vibration due to human activity/machines is an essential serviceability condition to be considered in building design. It has turned out to be a greater problem as new rhythmic activities, thinner slabs, less structural damping, and long-span floor structures have become more common. Indeed, the problems concerned with floor vibration are not new. It can be dated back to 1828 when Tredgold [1] wrote that girders must have enough depth to prohibit unpleasant vibrations which can cause shaking in all objects in the room.

From what Tredgold has said, it can be realized that vibration is directly proportional to stiffness. Stiffer structural elements can perform better when subjected to vibration. Nowadays, structural engineers and architects design long and slim members because they are more economical and aesthetic. These slender structural members have poor stiffness and are strongly adaptable to potent vibration.

In the traditional method of designing structures, The Ultimate Limit State (ULS) is used to determine the dimensions of the floor components. The concept of Serviceability Limit State (SLS) was to limit the maximum deflection of the floor and to avoid cracks of the finishing materials. Limitations were applied to the dimensions of members to fulfil the serviceability requirements without any criteria for vibration. Recently, human rhythmic activities, such as aerobics and high-impact dancing magnified the vibration problems in the buildings.

Nowadays, the vibration serviceability has become a design concern due to the following reasons:

- Longer spans. The new building practices allow longer spans with a much lighter construction.
- Less mass. Open plans and paperless offices play a significant role in reducing the mass of non-structural components.
- Technology advances. New devices in medical, scientific, and micro-manufacturing fields have higher precision. They are extremely sensitive to vibration.

A prominent example of the vibration of slender structures due to human activities can be London's Millennium footbridge (Figure 1.1). When the bridge was opened to the public in June 2000, excessive lateral vibration could be felt. This vibration caused pedestrians to feel uncomfortable and adhere to handrails [2].

The vibration of a structure can be measured by its vibration cycle repetitions per unit time which is known as *Frequency*. Every structure has a natural (fundamental) frequency f_0 and harmonics (secondary) frequencies.

A simple model of a guitar string shows mass, stiffness, and frequency relations clearly. When a string is plucked (with a mass) and left to vibrate freely, the fundamental sound frequency of that string can be heard. The tension in the string defines the stiffness. The tighter the string is, the higher the frequency is and vice versa. On the other hand, the thicker strings have lower sound frequency and thinner ones have a higher sound frequency. On a strong pluck, the sound produced is higher, but the sound frequency does not change. The string vibrates with its self-exiting frequency and gradually its movement dampens and stops vibrating after a time. A string can be also vibrated when another string is plucked but with a remarkably similar frequency to the one under investigation. This phenomenon is called resonance. This model can be exactly related to the mechanical vibration of the structural elements. Every structural element has its specific stiffness and cross-section which determines its fundamental frequency. The vibration of a structure occurs when there is a moving mass acting on that structure. The source of this moving mass can be human activity or machinery. When the intensity of the vibrating mass increases, the natural frequency of the structural member stays the same, but the vibration acceleration increases which sometimes causes discomfort to the human being. The human activities can be a source for a harmonic dynamic load where the impact of the load repeats in a defined time interval, such as walking, jogging, dancing...etc. When the frequency of the human activity matches the structure's frequency, the vibration acceleration of the structural member increases due to resonance action.

Fig. 1.1. Millennium footbridge, London [2]

To prohibit the unpleasant vibration of a structure, it is important to calculate the natural frequency of the structure and the frequency of human activity. These two frequencies should not be the same to restrain resonance. It is also quite helpful to understand how humans respond to vibration. The human response is compared to the predefined floor acceleration limit which is known as the Response Factor.

Composite floors can be defined as slender structural members. This type of floor is designed to be slim by using high strength materials to increase the span and reduce the thickness. In most cases, these composite floors have one-way spanning behaviour unlike traditional two-way spanning reinforced concrete. This behaviour makes it even more sensitive to vibration. There are two types of vibration, namely forced vibration and free vibration. A machine with out-of-balance mass causes forced vibration. Free vibration happens due to occupants’ activity. The termination of forced vibration is easily reached by isolating the vibrating machine from the floor. The main problem is vibration from the activity of the building occupants through daily usage.

1.1. Scope of Research

Composite floors can be seen in different forms. The most popular two forms are corrugated sheets within situ concrete topping and precast slabs within situ concrete topping. In both conditions, supplying shear connectors (nail-like structures welded to the top of the steel beam flanges) is necessary. The reinforcement in precast slabs should represent the total amount of positive reinforcement required for the composite floor. On the other hand, the corrugated sheet can compensate for the required positive reinforcement. If not, normal reinforcement bars can be supplied to reach the required amount. Figure 1.2 shows typical models of composite corrugated sheet and steel beams.

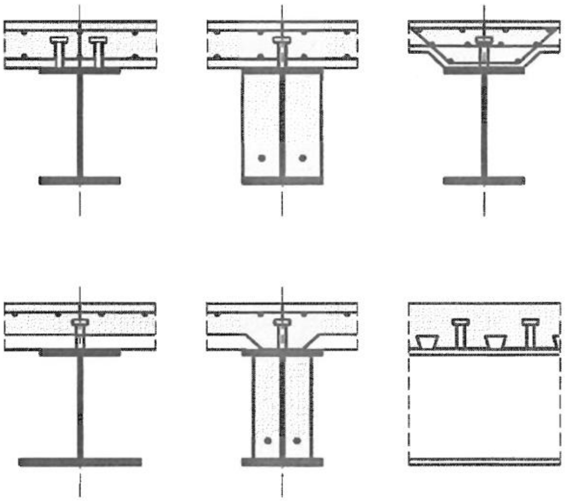


Fig. 1.2. Typical cross-sections of composite beams [3]

The combination of beams with prestressed slabs and concrete topping is not mentioned in Eurocode. However, the procedures of design are similar. Additionally, this kind of composite floor is extremely popular all-over European countries. Varieties of this type of composite floor are demonstrated in SCI P287 book of the Steel Construction Institute (Figure 1.3).

The chosen composite floors of this thesis consist of steel beams with hollow-core slab units and optionally concrete topping. This type is selected among several types of composite floors because of the lack of literature about this kind of floor. In chapter 4, the stiffness of composite slim floors is calculated as well as dynamic properties such as natural frequency and Response Factor. In addition to manual calculation, finite element analysis software such as SCIA Engineer and Autodesk Robot Structural Analysis are used. The author of the thesis also created several nomographs that are presented in Appendix A to define the natural frequency of floor members with less calculation.

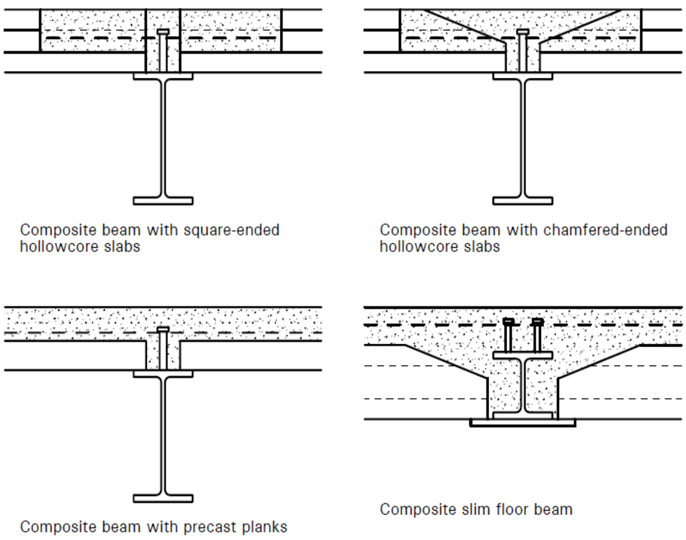


Fig. 1.3. Different composite applications of steel and precast concrete [4]

1.2. Need for Research

The EN 1994: Eurocode 4 refers to EN 1990 for the vibration of composite structures and only the following are mentioned:

1. Floor vibrations should not cause discomfort;
2. The function of the structure should be determined;
3. Natural frequency should be higher than the applied one;
4. If the natural frequency was lower, refined analysis is needed;
5. Sources of vibration should be considered.

Other than the recommendations above, the Eurocode does not supply a clear method to calculate the dynamic properties of the floors. There are other guidelines such as AISC/CISC DG11 and SCI P354 to calculate the natural frequency and Response Factor of the floors. There are several examples in both guidelines but again, there is no vivid example on how to analyse shallow floor slabs using hollow-core units.

1.3. Terminology

Vibration: The oscillation of a system about its equilibrium position. There are two types of vibrations: Free and forced vibrations. Free vibration is the oscillation of a system with its fundamental frequency. Forced vibration is the excitation of a system with an external source with a different frequency from the fundamental frequency. The system vibrates with the frequency of the external source.

Cycle: One complete cycle of a frequency. It is the return of a system to a given point and direction.

Period: Time needed for completion of one cycle of vibration (Figure 1.4).

Amplitude: The magnitude of the vibration cycles. It can be measured as displacement, velocity, and acceleration (Figure 1.4).

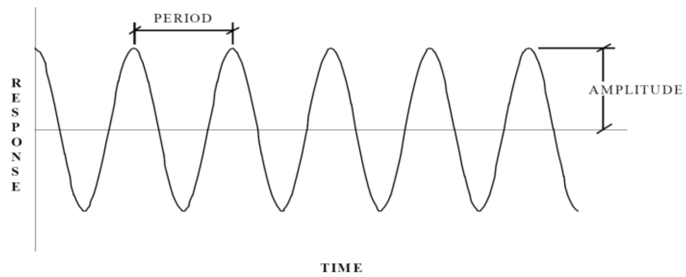


Fig. 1.4. Amplitude and period definition [5]

Fundamental Frequency: The lowest frequency in which the system tends to vibrate freely. In Figure 1.5, number 1 shows the fundamental frequency of the beam while numbers 2 and 3 show harmonic frequencies.

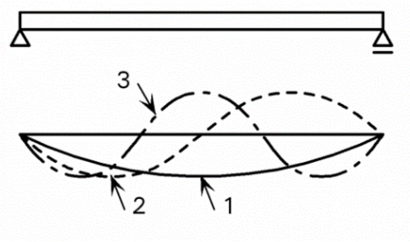


Fig. 1.5. Fundamental and harmonic frequencies of a typical beam [6]

Damping: Loss in vibration magnitude, usually due to friction. If the damping was proportional to velocity, then it is called viscous damping (Figure 1.6).

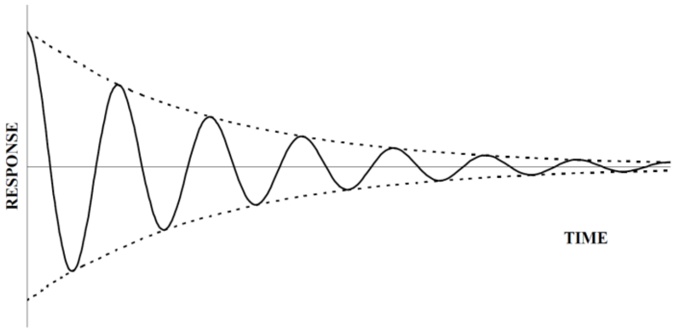


Fig. 1.6. Viscous damping [5]

Dynamic loads: A load whose magnitude changes with time. Dynamic loads are classified as harmonic, periodic, transient and impulsive loads (Figure 1.7).

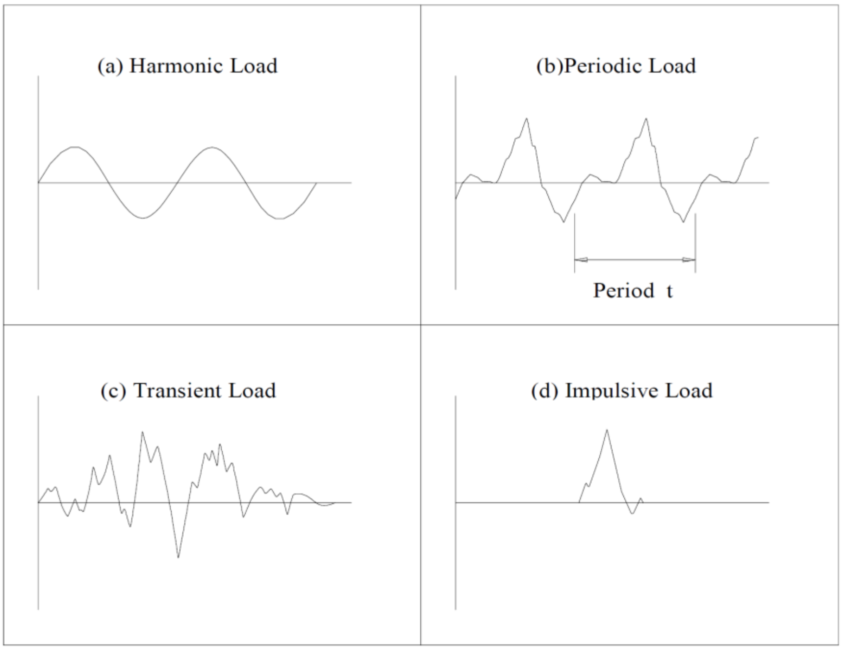


Fig. 1.7. Types of dynamic loads [5]

2. Literature Review

Traditional reinforced concrete floors perform well about vibration serviceability because of their heavyweight. With the current technology, it is possible to construct long-span floors with slender cross sections by using prestressed and high strength concrete. These floors are more sensitive to vibrations due to their lightweight and low stiffness.

Numerous researchers published books and articles about the vibration of the floors. Their literatures are reviewed in this chapter. Further studies about human-induced loads are also reviewed. Then, the design parameters such as fundamental frequency and damping are also showed.

This chapter sums up the earlier researches, current information and knowledge gaps and the contribution of this research for showing calculation methods for shallow composite floors.

2.1. Dynamic Loads on Structures

Throughout construction history, before designing a structure, engineers tried to predict possible dynamic loads acting on the structure in its service life span. Dynamic loads acting on a structure may cause unpleasant motion or even failure of the members of the structure. The recent incidents were swaying of Millennium Bridge in London, Tsunami in Asia, earthquakes in Iran and many other incidents. The vibration due to dynamic loads acting on a building can be classified into externally induced vibrations and internally induced vibrations. The vibrations caused by wind, earthquake, and traffic are classified as externally induced vibrations while vibrations due to human activities are classified as internally induced vibrations.

Wind loads are the most common type of naturally occurring dynamic loads. Wind load exerts a lateral load on structures. It should be considered in designing tall buildings and skyscrapers. Earthquake-induced dynamic loads are also naturally occurring dynamic loads. The earthquake shock releases a high amount of energy enough to vibrate structures on the earth's surface. Traffic induced vibrations are caused by moving cars and machines. This vibration may cause cracks or even structural damage and failure of a structure. It can be a source of complaints of people living in buildings with traffic-induced vibrations [7].

Human-induced dynamic loads come from human activities inside the buildings. Unlike most of the residential houses, the buildings for activities have longer spans with fewer columns to have more area for activities. These buildings include gymnasiums, sports halls, concert halls and structures like footbridges. Most of the time, this load is not strong enough to cause structural failure, but it causes disturbance of the residents.

2.1.1. Types of Dynamic Loads

Dynamic loads can be classified into harmonic, periodic, transient and impulsive (Figure 1.7). Harmonic or sinusoidal loads take place due to rotating machinery. Periodic loads repeat at systematic intervals. It occurs from human activity or rotating machinery. Transient loads are the result of people's movement including walking and running. Furthermore, loads induced by wind, earthquake and water waves can also be classified as Transient loads. Impulsive loads occur due to an impact force such as a single jump or heel-drop test.

2.1.2. Human-induced loading

Dynamic loads generated by human activities can be considered as periodic loads such as walking, running, and dancing. Table 2.1 shows the average values of step frequency, velocity, and step length for common human activities.

Table 2.1. Common step frequency, velocity, and step length [8]

Activity	Step frequency (Hz)	velocity (m/s)	Step length (m)
Slow walk	1.7	1.1	0.60
Normal walk	2.0	1.5	0.75
Fast walk	2.3	2.2	1.00
Slow running	2.5	3.3	1.30
Fast running	>3.2	5.5	1.75

For establishing a mathematical method of walking force, it is assumed the paces are perfectly periodic and Fourier series can be used to represent the loading function [8]. The general form of loading function $F(t)$ can be expressed as:

$$F(t) = P\left(1 + \sum_{i=1}^{3-4} \alpha_i \sin\left(2\pi f_p t + \phi_i\right)\right) \tag{2.1}$$

where P is the weight of a walking person, f_p is step frequency, α_i and ϕ_i are dynamic coefficient and phase lag for the i-th component of the excitation, respectively. Bachmann and Ammann suggested the first three harmonics with $\alpha_1 = 0.4 - 0.5$, $\alpha_2 = \alpha_3 = 0.1$ and $\phi_1 = 0$ and $\phi_2 = \phi_3 = \pi/2$. Different suggested values for α_i and ϕ_i can be found in different articles and design guides.

2.2. Natural Frequency and Deflection of Floors

A floor can be expressed as a single degree of freedom (SDOF) system with a force $F(t)$ acting on it as shown in Figure 2.1. Its equation of motion can be expressed as:

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{2.2}$$

where m, c and k are parameter constants [9]. The free vibration of an elastic body is called natural vibration which its frequency is known as natural frequency. Natural vibration occurs under no influence of external force. The natural frequency of simple harmonic motion in a mass-spring (SDOF) system can be expressed as the ratio of stiffness k to the mass m.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{2.3}$$

Stiffness is the resistance of an elastic body to deflection under an applied load. This can be expressed as:

$$k = \frac{mg}{\delta} \tag{2.4}$$

where δ is the mid-span deflection and g is the gravitational acceleration constant (9.81 m/sec^2). Substituting Equation 2.4 in Equation 2.3 and simplifying,

$$f_n \approx \frac{15.76}{\sqrt{\delta}} \tag{2.5}$$

where δ is in mm and f_n in Hertz.

In 1964, Huang modified Equation 2.3 by multiplying k_n factor for different support conditions and mode number. In Table 2.2, natural frequencies for 3 support conditions under different loading conditions are shown. The main equation is

$$f_n = \frac{k_n}{2\pi}\sqrt{\frac{EIg}{wl^4}} \tag{2.6}$$

where k_n is mode and support condition factor, E is the modulus of elasticity, I is the second moment of area, g is gravitational acceleration, w is uniform load on member and l is the length of the member.

From Table 2.2, it can be noticed for the first frequency mode, in all conditions of both ends simply supported, both ends fixed and one end fixed, after substituting k_n value in the equations of frequency for uniform load, both three equations can be rewritten as

$$f_n \approx \frac{18}{\sqrt{\delta}} \tag{2.7}$$

Another approach is to encounter average deflection (75% of member deflection) in Equation 2.3 [10]. This assumption is also quite similar to Equation 2.7. This equation is used by many authors and researchers because it is simple and at the same time no coefficients needed for different support types.

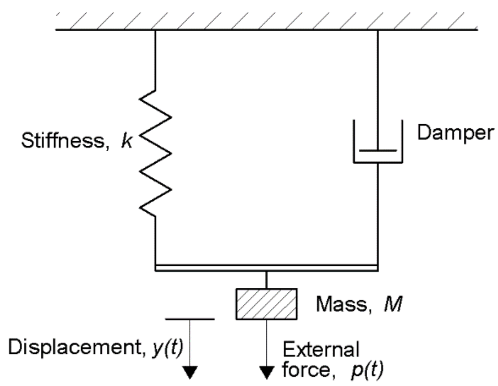


Fig. 2.1. SDOF model

Table 2.2. The natural frequency of members with k_n coefficients [11]

Case no. and description		Natural frequencies		
1. Uniform beam; both ends simply supported	1a. Centre load W, beam weight negligible	$f_1 = \frac{6.93}{2\pi}\sqrt{\frac{EIg}{Wl^3}}$		
	1b. Uniform load w per unit length including beam weight	$f_n = \frac{k_n}{2\pi}\sqrt{\frac{EIg}{wl^4}}$	*Mode*	k_n
			1	9.87
			2	39.5
			3	88.8
			4	158
			5	247
	1c. Uniform load w per unit length plus a centre load W	$f_1 = \frac{6.93}{2\pi}\sqrt{\frac{EIg}{Wl^3 + 0.486wl^4}}$		
2. Uniform beam; both ends fixed	2a. Centre load W, beam weight negligible	$f_1 = \frac{13.86}{2\pi}\sqrt{\frac{EIg}{Wl^3}}$		
	2b. Uniform load w per unit length including beam weight	$f_n = \frac{k_n}{2\pi}\sqrt{\frac{EIg}{wl^4}}$	*Mode*	k_n
			1	22.4
			2	61.7
			3	121
			4	200
			5	299
	2c. Uniform load w per unit length plus a centre load W	$f_1 = \frac{13.86}{2\pi}\sqrt{\frac{EIg}{Wl^3 + 0.383wl^4}}$		
3. Uniform beam; left end fixed, right end free (cantilever)	1a. Centre load W, beam weight negligible	$f_1 = \frac{1.732}{2\pi}\sqrt{\frac{EIg}{Wl^3}}$		
	3b. Uniform load w per unit length including beam weight	$f_n = \frac{k_n}{2\pi}\sqrt{\frac{EIg}{wl^4}}$	*Mode*	k_n
			1	3.52
			2	22.0
			3	61.7
			4	121
			5	200
	3c. Uniform load w per unit length plus an end load W	$f_1 = \frac{1.732}{2\pi}\sqrt{\frac{EIg}{Wl^3 + 0.236wl^4}}$		

NOTATION: f = natural frequency; k_n = constant where n refers to the mode of vibration; g = gravitational acceleration; E = modulus of elasticity; I = area moment of inertia

2.2.1. Deflection due to Flexure: Continuity

Continuous Joists, Beams or Girders

For continuous beam over equal spans, the natural frequency can be calculated from one simply supported beam with the same span length. When the spans are not equal, the natural frequency is calculated from both Equation 2.7 and the deflection equations below. δ_{ss} is the deflection of the simply supported model for the main (larger) beam. For two continuous spans:

$$\delta = \left(\frac{0.4 + \frac{k_m}{k_s}(1 + 0.6\frac{L_s^2}{L_m^2}}{1 + \frac{k_m}{k_s}} \right) \delta_{ss} \tag{2.8}$$

For three continuous spans:

$$\delta = \left(\frac{0.6 + 2\frac{k_m}{k_s}(1 + 1.2\frac{L_s^2}{L_m^2}}{3 + 2\frac{k_m}{k_s}} \right) \delta_{ss} \tag{2.9}$$

where

L_m, I_m, L_S, I_S are defined in Figure 2.2.

$k_m = \frac{I_m}{L_m}, \; k_s = \frac{I_s}{L_s}$

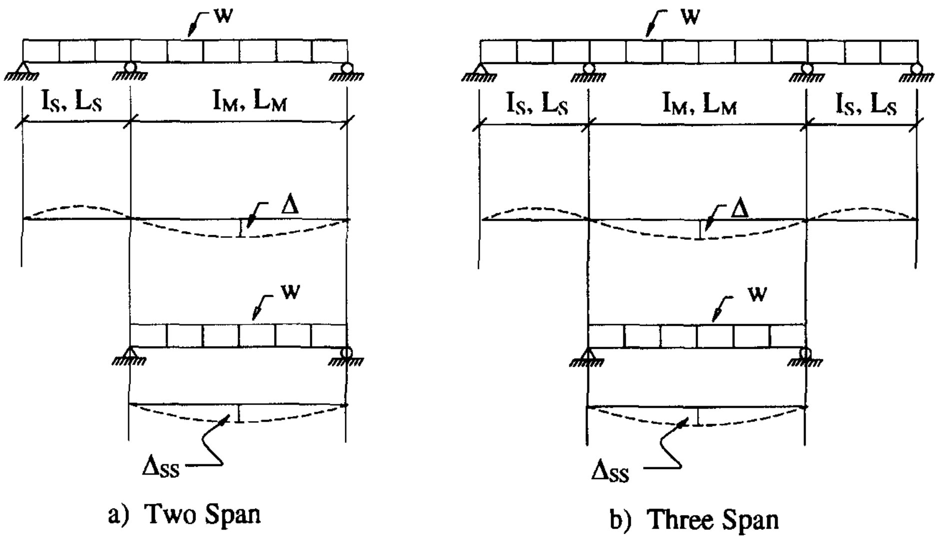


Fig. 2.2. Modal flexural deflections [5]

Members Continuous with Columns

The deflection of the slabs is decreased when they are moment-connected to columns. As a result, the natural frequency increases. This is important when the columns are large especially in tall buildings. The following relation can be used for moment-connected columns to girders as shown in Figure 2.3.

$$\delta = \left(\frac{0.6 + 2\frac{k_m}{k_c}(1 + 1.2\lambda) + 1.2n_c\frac{k_c}{k_s}}{3 + 2\frac{k_m}{k_s} + 6n_c\frac{k_c}{k_s}} \right)\delta_{ss} \tag{2.10}$$

where:

$L_m, I_m, L_S, I_S, L_c, I_c$ are defined in Figure 2.3.

$k_m = \frac{I_m}{L_m}, \quad k_s = \frac{I_s}{L_s}, \quad k_c = \frac{I_c}{Lc}$

$\lambda = \left(\frac{L_s}{L_m}\right)^2$

$n_c = 2$ if columns occur above and below, 1 if only above or below.

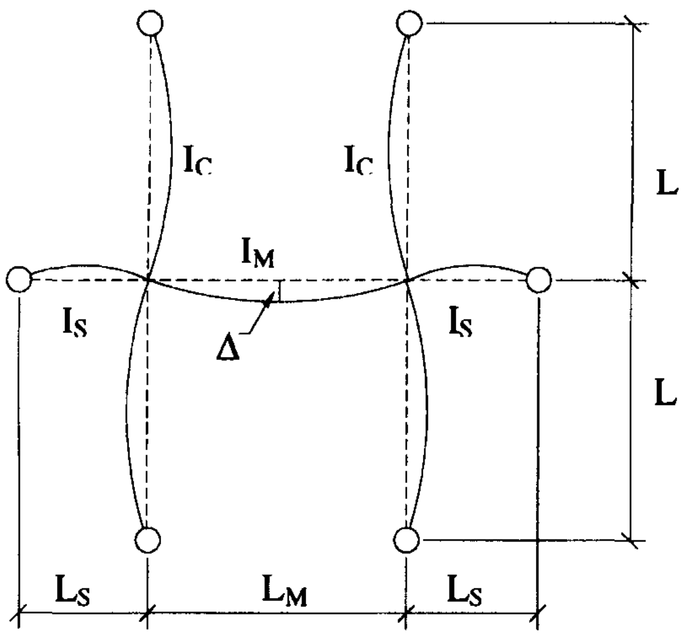


Fig. 2.3. Modal flexural deflections, for beams or girders continuous with columns [5]

Cantilevers

The deflection of the simply supported cantilever under uniformly distributed mass can be calculated from:

$$\delta_f = \frac{wL^4}{8EI} \tag{2.11}$$

and for a concentrated mass at the tip:

$$\delta_f = \frac{wL^3}{3EI} \tag{2.12}$$

However, cantilevers are rarely fully fixed. The deflection of the back-span has an influence on the deflection of the cantilever. If the cantilever deflection δ_T, exceeds the deflection of the back-span δ_B, then

$$\delta = \delta_T = C_m \left(1 + \frac{4}{3}\frac{L_B}{L_T} * \frac{1 + 0.25\frac{L_B^2}{L_T^2}}{1 + \frac{n_c k_c}{k_b}} \right) \delta_F \tag{2.13}$$

If the opposite is true, then

$$\delta = \delta_B = \left[1 + 2.4 \left(\frac{\frac{L_T^2}{L_B^2} - \frac{0.5k_c}{k_b}}{1 + \frac{n_c k_c}{k_b}} \right) \right] \delta_{ss} \tag{2.14}$$

where
C_m = 0.81 for distributed mass and 1.06 for mass concentrated at the tip

δ_F = Flexural deflection of a fixed cantilever, due to the weight supported

δ_{ss} = Flexural deflection of back-span, assumed simply supported

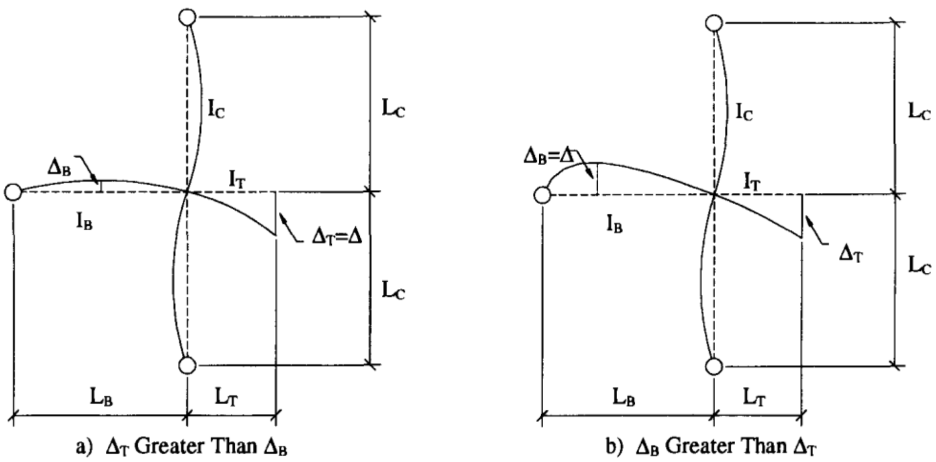


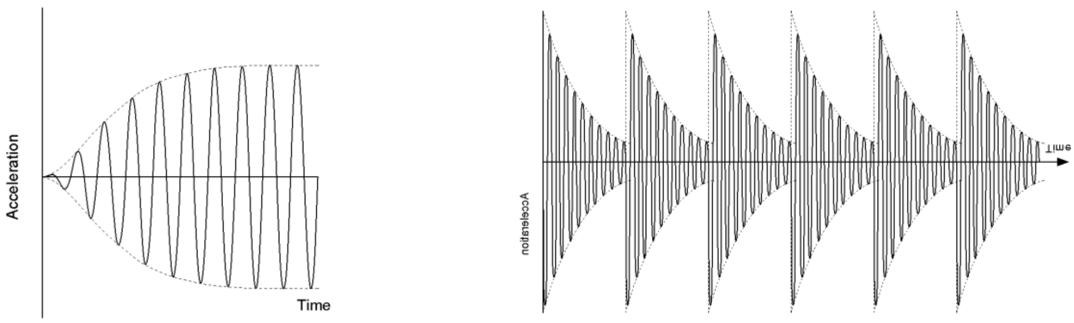
Fig. 2.4. Modal flexural deflections, for cantilever/back-span/columns [5]

2.2.2. Floor Natural Frequency Limitation

Stiffness has a major influence on the natural frequency of a floor. The floors should be stiff enough to resist the design loads and have limited deflection and at the same time, its natural frequency should be within an allowable limit. The steel construction institute suggested that the floors should have frequencies higher than 3 Hz [12]. In the case of composite floors, composite action between slab units, beams and girders should be evaluated correctly. More stiffness can be achieved by full-height walls.

2.3. Response to Walking

Walking is the most common human activity in buildings which causes the floors to vibrate. The floors can be divided into low-frequency and high-frequency floors. Low-frequency floors have a low natural frequency which can match with walking frequency. In this case, each pace contributes energy to every acceleration interval of the floor and resonant response takes place (Figure 2.5a). For high-frequency floors, there is a low probability for resonance to take place due to the significant difference between the natural frequency of floor and pace frequency. In other words, paces only contribute energy to not every but acceleration intervals at pacing time. In this case, the transient response occurs (Figure 2.5b). The floors having a natural frequency lower than 10 Hz are categorized as low-frequency floors and higher than 10 Hz as high-frequency floors [12].



(a) Resonant response (b) Transient response

Fig. 2.5. Envelopes of dynamic response [10]

2.3.1. Damping and Mass

Dynamic properties of a floor are determined by mass, stiffness, and damping. Estimating their values are particularly important to calculate the dynamic behaviour of the floor. Both mass and stiffness of a floor can be accurately estimated from its physical and mechanical properties. However, the damping, which is the key parameter influencing the floor response at resonance, cannot be evaluated correctly from physical principles.

It is more practical to combine all types of damping from different sources to an equivalent viscous damping ratio which is a fraction of critical damping [13]. The damping ratio ζ can be written as a function of c, m, k (see Equation 2.2) as in Equation 2.15 [11]:

$$\zeta = \frac{1}{2}\left(\frac{c}{\sqrt{km}}\right) \tag{2.15}$$

Increasing damping can increase the decay of free vibration and reduce vibration magnitude during resonance. Both structural and non-structural parts of a building have an influence on damping. For instance, damping of a bare structure increased by completing the floor system with partitions, furnishings, suspended ceilings, etc...

In general, as per European technical report [10], damping sources can be structural damping, damping due to furniture and damping due to finishes as shown in Table 2.3.

For floors composed of more than one material, current design guides suggest different damping ratios depending on levels of furnishing. Table 2.4 summarizes the estimated damping for composite floors suggested by the North American design guide AISC/CISC DG11 [5], UK guidelines [10] and the steel construction institute guideline [12].

Table 2.3. Estimation of damping ratio [10]

Type	Damping (% of critical damping)
Structural Damping D1	
Wood	6%
Concrete	2%
Steel	1%
Composite (steel-concrete)	1%
Damping due to Furniture D2	
Traditional office for 1 to 3 persons with separation walls	2%
Paperless office	0%
Open plan office	1%
Library	1%
Houses	1%
Schools	0%
Gymnastic	0%
Damping due to Finishes D3	
Ceiling under the floor	1%
Free-floating floor	0%
Swimming screed	1%
Total Damping $D = D1 + D2 + D3$	

Table 2.4. Damping of composite floors

Floor finishes	AISC/CISC DG11	CCIP-016	SCI P354
Completed composite floors with low fit-out, few non-structural components	2.0%	1.5% - 2.5%	1.1%
Completed composite floors, fully furnished with typical fit-out in normal use	3.0%	2.0% - 3.0%	3.0%
Completed composite floors with extensive fit-out and full height partitions between floors	5.0%	3.0% - 4.5%	4.5%

2.3.2. Resonance

Resonance is a phenomenon that occurs when the natural frequency of a system is the same or similar to activity frequency. The activity causes the system to vibrate and at every acceleration and displacement peak, further energy from the activity source is fed into the system causing the structure to reach maximum acceleration and displacement (Figure 2.6). The resonance can occur not only at activity frequency but also at multiples of this frequency known as harmonics [14].

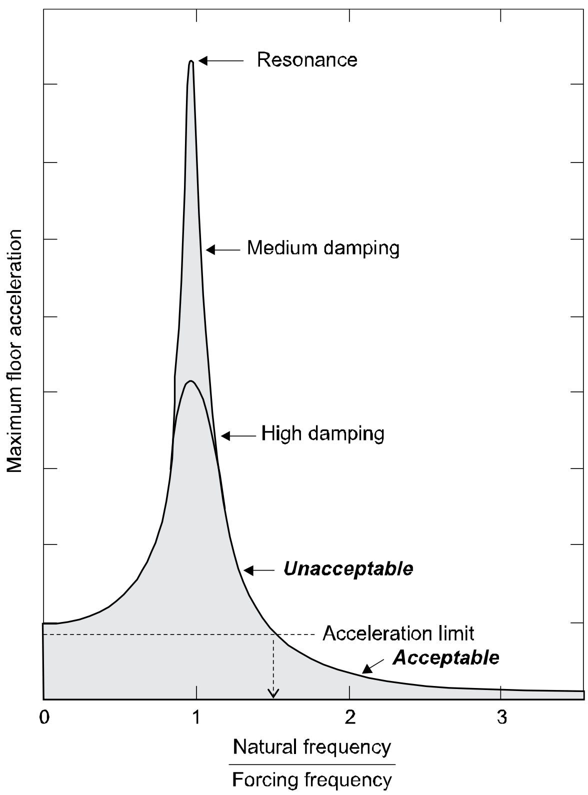


Fig. 2.6. Floor acceleration due to a cyclic force for a range of natural frequencies [15]

2.3.3. Vibration Response and Acceptance Criteria

Human response to vibrations may vary from a person to another due to psychological and physiological factors. Hence, determining unpleasant vibration is not easy. Still, lots of researches have been made to mathematically define this phenomenon. A well-known criterion for acceleration limits for walking is the Reiher-Meister scale which dates to the 1930s. Their research involved a group of people standing on a vibrating surface with a steady-state vibration of 3 - 10 Hz. The people reported vibrations as:

Curve 1: Imperceptible;

Curve 2: Slightly perceptible;

Curve 3: Distinctly perceptible;

Curve 4: Strongly perceptible;

Curve 5: Disturbing;

Curve 6: Intolerable.

From the feedback of the people, researchers could draw human comfort limits as in Figure 2.7. The direction of motion to the human body also has a major influence on vibration perception. ISO 2631-2 has defined human body positions via three axes of X, Y, and Z shown in Figure 2.8.

The vibration in the Z-axis (i.e. foot-to-head direction) could be a critical concern in the frequency range of 4-8 Hz. However, lateral vibrations along X and Y axes with 1-2 Hz vibrations may result in the greatest perceptibly by occupants. Based on these perception differences, offices can be designed for only z-axis while the vibration in residences and hotels should be checked for all three directions [16].

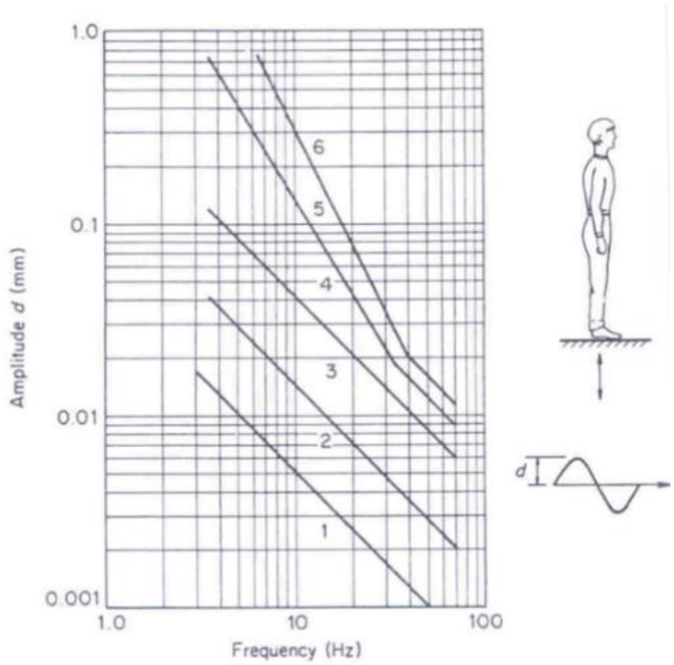


Fig. 2.7. The Reiher-Meister scale [17]

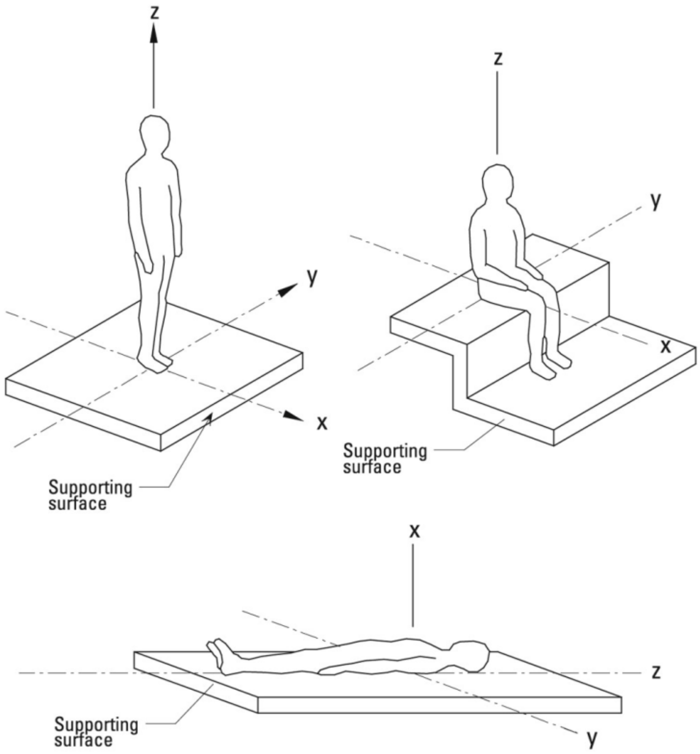


Fig. 2.8. Body postures and basicentric co-ordinate systems [18]

There are two main design guidelines to limit floor acceleration levels. Root-mean-square (RMS) acceleration and peak acceleration. Both acceleration limits are derived from the baseline curve of the International Standard Organization ISO 10137. Figure 2.9 shows a baseline curve for RMS in a vertical direction.

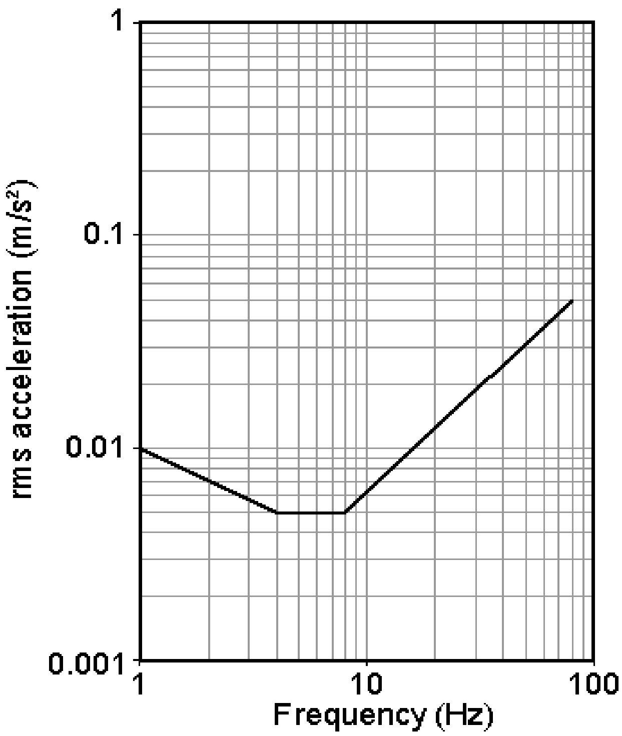


Fig. 2.9. RMS acceleration limit [8]

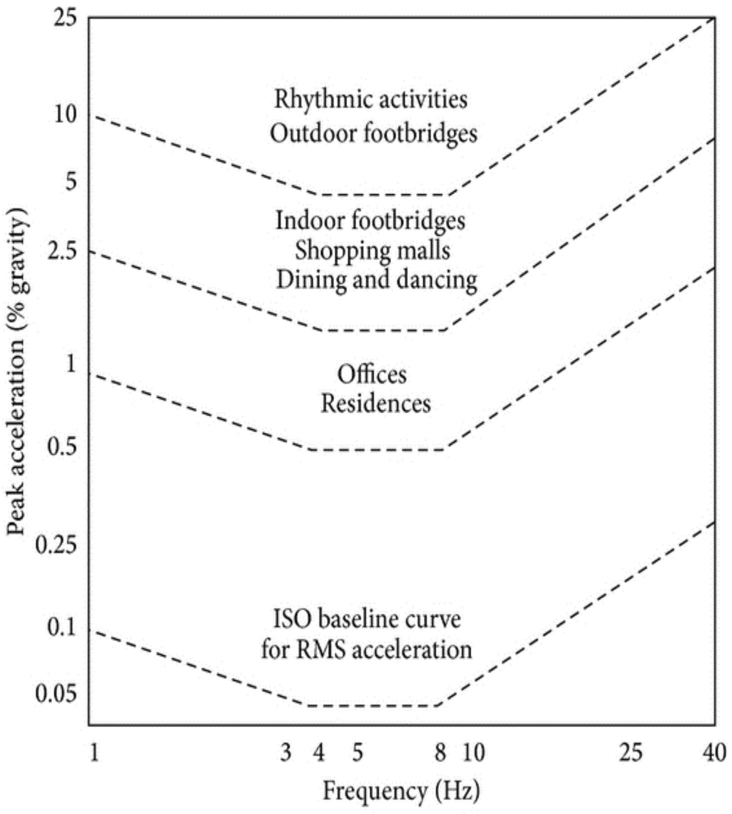


Fig. 2.10. Peak acceleration limit [5]

Peak Acceleration

Peak acceleration baseline curves are recommended by AISC/CISC DG11. It is usually used in North America design codes. This curve correlates the peak acceleration of floors to a percentage of gravitational acceleration. For instance, the peak acceleration for offices is limited to 0.5% g (i.e. 0.05 m/s^2) of gravity in the frequency ranges of 4-8 Hz. The acceleration limit can be higher for less sensitive structures like footbridges.

RMS Acceleration

Root mean square acceleration method cancels out unrepresentative peak acceleration in the response history [19]. The limits of RMS acceleration are defined from the multiplying factors of the frequency-base curve. This is known as the response factor. For example, the recommended multiplying factor for offices is 4. This value corresponds to an RMS acceleration of 0.02 m/s^2. The multiplying factor is the same as the response factor.

ISO 10137 recommends response factors shown in Table 2.5. The UK steel Construction Institute also provided a table of response factors limits in SCI P354 as shown in Table 2.6.

Table 2.5. Multiplying factors for low probability of adverse comment [9]

Place	Time	Continuous vibration and intermittent vibration	Impulsive vibration excitation with several occurrences per day
Critical working areas (e.g. hospital, operating theatre, precision laboratories, etc.)	Day	1	1
	Night	1	1
Residential (e.g. flats, homes, hospital)	Day	2 to 4	30 to 90
	Night	1.4	1.4 to 20
Quite office, open plan	Day	2	60 to 128
	Night	2	60 to 128
General office (e.g. schools, offices)	Day	4	60 to 128
	Night	4	60 to 128
Workshops	Day	8	90 to 128
	Night	8	90 to 128

Table 2.6. Multiplying factors recommended by SCI P354 [12]

Place	The multiplying factor for exposure to continuous vibration
Office	8
Shopping mall	4
Dealing floor	4
Stair-Light use	32
Stair-Heavy use	24

3. Calculation Method of Dynamic Properties and Walking Response of Floors

Structural modelling of a building is the first step of designing. Floor plans are converted to appropriate structural models based on the complexity of the plan. This is the most crucial step before making any structural analysis. For simple models, natural frequency and response factor can be estimated with hand calculation. However, complex models of floor plans may need a computer program for analysis. This chapter describes modelling techniques and calculation method of the floor models. A brief explanation of the SCIA Engineer and Autodesk Robot Structural Analysis software is also included at the end of this chapter.

3.1. Modelling Techniques

Composite members in a floor system have more than one structural material. The structures analysed in this research consist of composite steel beams and hollow-core slab units. Composite beams include a steel beam with a welded base plate, core concrete, a part of hollow-core slab units at sides and topping concrete if available as in Figure 3.1.

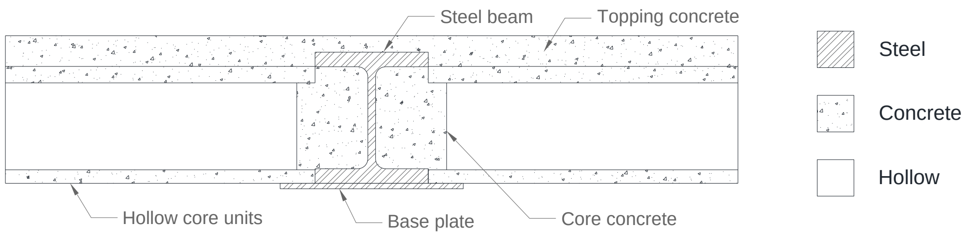


Fig. 3.1. Typical composite HCS unit, topping and steel profile section

The first step of the calculation is to determine the stiffness of the floor members. It is important to calculate the second moment of area of every member and sum them up to achieve total stiffness of the composite floor. The geometry of the composite elements of the section shown in Figure 3.1, can be re-defined as shown in Figure 3.2 to simplify calculations.

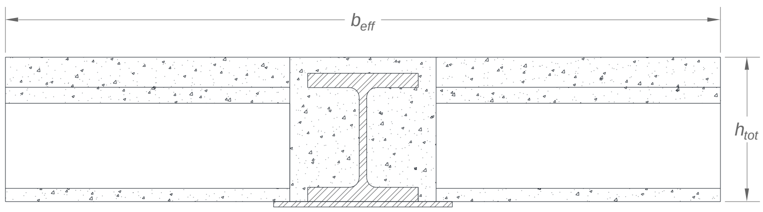


Fig. 3.2. Transformed section of composite floor

3.2. Natural Frequency of a System

The natural frequency of each element is calculated from Equation 2.6 separately. For systems, having more than one vibrating members (i.e. beams with girders and/or slabs), Dunkerley's formula can be used to obtain the natural frequency of the system

$$\frac{1}{f_n^2} = \frac{1}{f_b^2} + \frac{1}{f_g^2} + \frac{1}{f_x^2} \tag{3.1}$$

where f_b, f_g and f_x are natural frequencies of beams, girders or any other structural member which acts as a system with the rest of other members of the system.

The simplest approach to calculate natural frequency of the system, is to convert members to wire-frames (1 dimensional members) with known structural properties (Figure 3.3). This way, main beams, secondary beams, and hollow-core units are converted to lines standing for the real members. Choosing veridical member supports and load distribution scheme has a major influence on dynamic analysis results.

In the case of the combination of point load and uniform load on a member, Equations from Table 2.2 can be used to convert the uniform load to a concentric load acting in the middle of the span by coefficients. Another approach is to calculate the frequency for uniform and point loads separately, then by using Dunkerley's formula, a single natural frequency for the member can be achieved. This approach is easier and gives exact values as Table 2.2.

In both SCI P354 and AISC/CISC DG11 design guides, Equation 3.1 is used to determine the natural frequency of beams and slabs. SCI P354 suggests considering only 10% of the variable loads for floor dynamic properties calculations. However, AISC/CISC DG11 does not clearly determine the percentage of the live load that should be used.

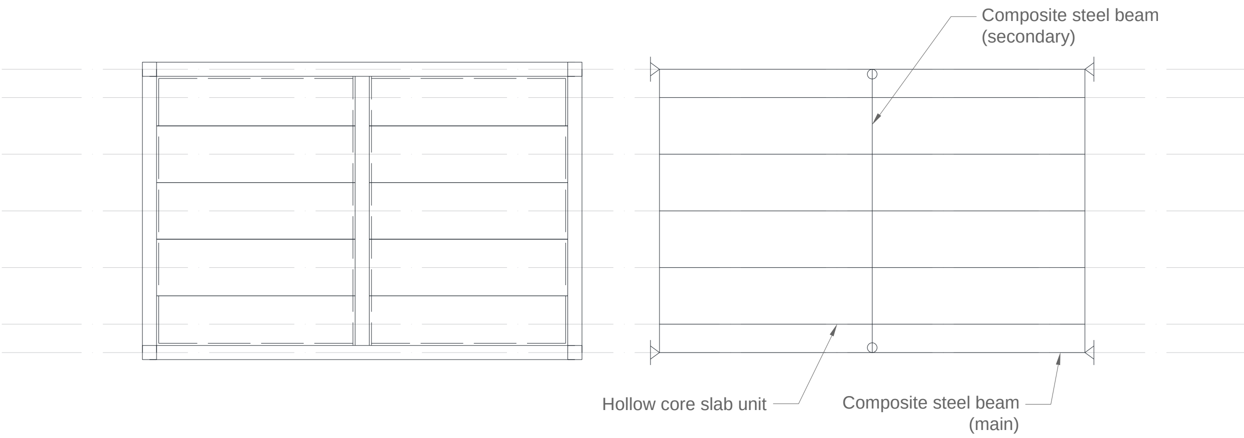


Fig. 3.3. Wireframe model of a typical composite slim floor

3.3. Walking Force Model and the Response of Floors

Dynamic analysis of the floors is a complex task. Guidelines suggest different methods for predicting dynamic properties of floors. Among several guidelines, the most used ones are AISC/CISC DG11 and SCI P354. In this section, the design methods with both guidelines are explained. In Chapter 4, SCI P354 method is used for analysis.

3.3.1. AISC/CISC DG11

Walking Force Model

Fourier series is used to calculate walking force $F(t)$

$$F(t) = P\left[1 + \sum \alpha_i \cos(2\pi i f_p t + \phi_i)\right] \tag{3.2}$$

where

P is weight of walking person.

α_i and ϕ_i are dynamic coefficient and phase lag.

f_p is pacing frequency

The guide assumes only one harmonic force component matches with the fundamental frequency of the floor. Therefore, Equation 3.2 can be re-written as

$$F(t) = \alpha_i P cos(2\pi i f_p t) \tag{3.3}$$

The guide also provides the following table for if_p and α_i values:

Table 3.1. Forcing frequencies and dynamic coefficients

Harmonic, i	if_p (Hz)	α_i
1	1.6 – 2.2	0.5
2	3.2 – 4.4	0.2
3	4.8 – 6.6	0.1
4	6.4 – 8.8	0.05

Resonant Response

The provided equation to calculate the resonant response of floors is:

$$\alpha_{peak} = \frac{\alpha_i R P}{2\zeta m} \tag{3.4}$$

where

ζ is modal damping ratio

R is a reduction value (0.5 for floors)

The guide also provides an approximate relation between dynamic coefficient and frequency as shown in Equation 3.5.

$$\alpha_i = 0.83 \exp(-0.35 i f_p) \tag{3.5}$$

Then Equation 3.6 can be obtained which is mainly used in the design

$$\frac{a_{peak}}{g} = \frac{P_0 \exp(-0.35 f_n)}{\zeta W} \leq \frac{\alpha_0}{g} \tag{3.6}$$

where P_0 is taken as 0.29 kN for floors. The obtained value of a_{peak}/g is then compared with α_0/g in Figure 2.10.

3.3.2. SCI P354

Modal Mass

Modal mass is the portion of the floor mass which takes part in floor motion as follows:

$$M = mL_{eff}S \tag{3.7}$$

where

m is the mass per unit area

L_{eff} is the effective length of the floor

S is the effective width of the floor

For slim floor beams, the effective length and width can be calculated as:

$$L_{eff} = 1.09\left(\frac{EI_b}{mL_x f_0^2}\right)^{\frac{1}{4}} \quad L_{eff} < n_y L_y \tag{3.8}$$

$$S = 2.25\left(\frac{EI_s}{mf_0^2}\right)^{\frac{1}{4}} \quad S \leq n_x L_x \tag{3.9}$$

where

n_y and n_x are the number of bays in direction of beams and hollow-core slabs, respectively.

L_y and L_x are the length of the beams and hollow-core slabs, respectively.

EI_b and EI_s are the dynamic stiffness of floor beams and hollow-core slabs, respectively.

Resonance Built-up Factor

When the walking paths are short, the steady-state condition may not be reached. This condition can be considered in rms acceleration from the following equation:

$$\rho = 1 - e^{\frac{-2\pi\zeta L_p f_p}{v}} \tag{3.10}$$

where

f_p is the pacing frequency

ζ is the damping ratio

L_p is the walking path length

v is the walking velocity

Walking Force Model

This guide suggests the following Fourier series (Equation 3.11) for walking force model $F(t)$. The values of the dynamic coefficient α_i and phase angle ϕ_i are shown in Table 3.2 for pacing frequencies of 1.8-2.2 Hz.

$$F(t) = P\left[1 + \sum \alpha_i \sin\left(2\pi i f_p t + \phi_i\right)\right] \tag{3.11}$$

Table 3.2. Design Fourier coefficients for walking activities

Harmonic i	α_i	ϕ_i
1	$0.436(if_p - 0.95)$	0
2	$0.006(if_p + 12.3)$	$-\pi/2$
3	$0.007(if_p + 5.2)$	π
4	$0.007(if_p + 2.0)$	$\pi/2$

Steady-state Response (Low-frequency Floors)

The main difference between the two guidelines is AISC/CISC DG11 uses the response for one harmonic. While SCI P354 estimates response for several harmonics. After calculating response for each vibration mode, the total response a_{RMS} can be calculated by Eq. (3.12).

$$a_{RMS} = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{H} \left(\sum_{n=1}^{N} \left(\phi_{e,n} \phi_{r,n} \frac{\alpha_i P}{m_n} D_{i,n} \lambda_i \right) \right)^2} \tag{3.12}$$

where

a_{RMS} is frequency-weighted root-mean-square acceleration

H, N are the number of harmonics and modes

$\phi_{e,n}$ and $\phi_{r,n}$ are mode shape values at the point of response and excitation

f_n is natural frequency

m_n is modal mass

ζ is the damping ratio

λ_i is the frequency weighting factor

Equation 3.12 can be further simplified and finally, Equation 3.13 can be used to determine the steady-state response of low-frequency floors.

$$a_{RMS} = \frac{1}{\sqrt{2}} \phi_e \phi_r \frac{0.1P}{2\zeta m} \lambda \rho \tag{3.13}$$

Transient Response (High-frequency Floors)

In case of having fundamental frequency higher than the activity one, the applied force acts like a series of impulses. In this state, it is recommended to check the transient response of the floor. The weighed peak acceleration at point r due to walking excitation at point e is given by the following:

$$a_{peak,n} = 2\pi f_n \sqrt{1-\zeta^2}\phi_{e,n}\phi_{r,n}\frac{F_I}{m_n}\lambda_n \tag{3.14}$$

where F_I is an impulsive force, and it can be shown as

$$F_I = 60\frac{f_p^{1.43}}{f_n^{1.3}}\frac{Q}{700} \tag{3.15}$$

where

f_p is step frequency

f_n is natural frequency

Q is static weight

The sum of all responses at each node from each impulse is obtained from time history $a(t)$ equation.

$$a(t) = \sum_{n=1}^{N} 2\pi f_n \sqrt{1-\zeta^2}\phi_{e,n}\phi_{r,n}\frac{F_I}{m_n}\lambda_n \sin\left(2\pi f_n\sqrt{1-\zeta^2}t\right)\exp(-\zeta 2\pi f_n t) \tag{3.16}$$

Equation 3.17 can be used to obtain RMS acceleration where T is taken as $1/f_n$:

$$a_{RMS} = \sqrt{\frac{1}{T}\int_0^T a(t)^2 dt} \tag{3.17}$$

From both Equation 3.17 and Equation 3.18, the following simplified formula can be obtained to calculate RMS acceleration for high-frequency floors:

$$a_{RMS} = 2\pi\phi_e\phi_r\frac{185}{mf_n^{0.3}}\frac{Q}{700}\frac{1}{\sqrt{2}}\lambda_n \tag{3.18}$$

The response factor is obtained from

$$R = \frac{a_{RMS}}{0.005} \tag{3.19}$$

The response factor is compared with multiplying factors shown in Table 2.5. According to the type of the building the maximum response factor is chosen.

3.4. Computer Analysis Software

When the floor structures are regular rectangular shapes, with similar span distances and exposed loads, the manual approach can be used to predict natural frequency and response factor. In the case of irregular slab shapes and different spans, it is recommended to use the Finite Element Method (FEM) software to analyse the structure.

Nowadays FEM computer programs are extremely popular to design structural elements of buildings. To show how to use FEM programs for dynamic analysis purposes, two programs are used in this thesis namely, SCIA Engineer and Autodesk Robot Structural Analysis software.

Computer results are reliable when all parameters entered correctly. Usually, to start analysis on FEM software, the steps mentioned below are followed:

1. Define materials needed for the structures;
2. Specify mechanical properties of the materials including stiffness, strength…etc;
3. Decide section properties of the materials;
4. Define nodes and draw structural members;
5. Establish the supports of the structural members;
6. Define load cases and their values;
7. Determine the dynamic analysis setting;
8. Perform analysis and check results;
9. Spot the mistakes and analyse again.

In SCIA Engineer, it is possible to change section properties with a multiplying factor called k-factor. To edit the stiffness of the beams, their second moment of area can be multiplied by k-factor to achieve new stiffness. This way, the composite section can be defined. It is also important to choose the correct dynamic modulus of elasticity of concrete in material properties.

In Autodesk Robot Structural Analysis, the modulus of elasticity of the structural materials can be multiplied by the same k-factor value obtained for SCIA Engineer. This way, the effect of stiffness boost due to composite action will be included.

4. Worked Examples

Here in this chapter, natural frequency and dynamic response of two floor samples are analysed by using manual calculation, nomographs, and Finite Element Method software.

4.1. Example 1

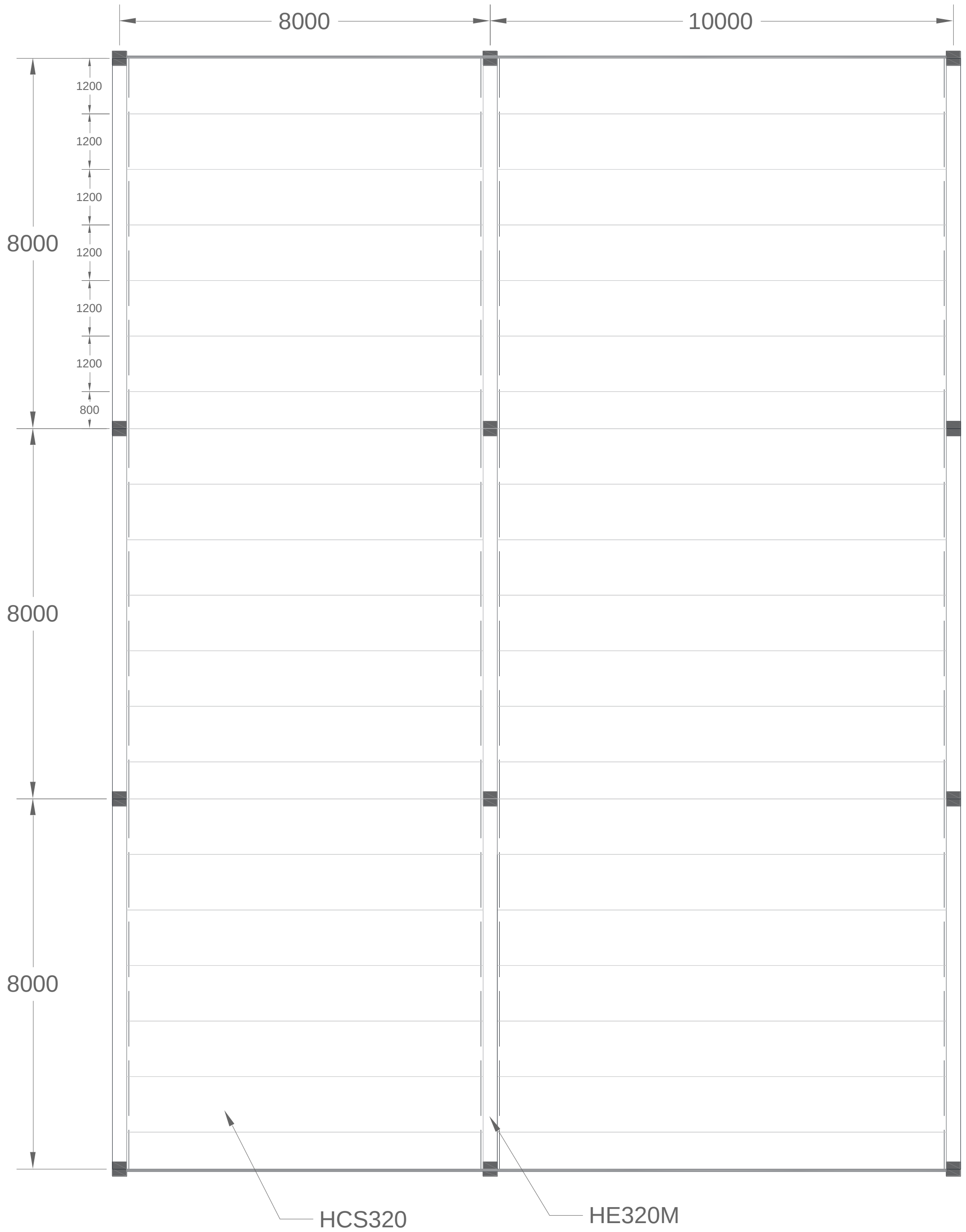


Fig. 4.1. Typical floor with slim floor beams and hollow-core slabs (Example 1)

For calculation of dynamic properties of hollow-core slab units, the cross-sectional properties of the HCS units should be used in calculations in contrast to the dynamic calculation of the beams, in which longitudinal section properties of the hollow-core slab units are needed.

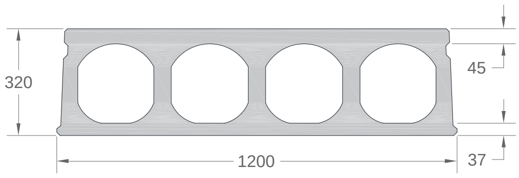


Fig. 4.2. Typical cross-section of HCS with 320 mm thickness

The joints which are assumed to be pinned in ULS design, they can be counted as rigid in the dynamic analysis because the strains in vibration are not that large to conquer the available friction in the joints [12]. Hollow-core slabs are assumed to have rigid joins with steel beams, and they work as composite. While steel beams are still assumed to be pinned to obtain the lowest possible natural frequency of the system.

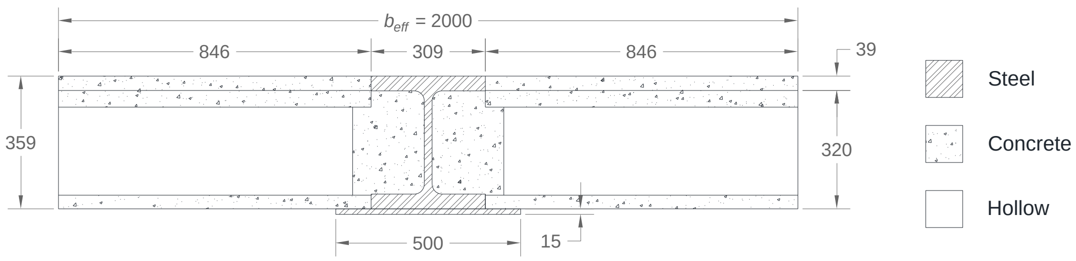


Fig. 4.3. Slim floor beam cross-section with effective part of HCS and concrete (Example 1)

In order to ease the second moment of area calculations, The boundaries of concrete parts can be changed to have more regular shapes like rectangles for the calculations as shown in Figure 4.4.

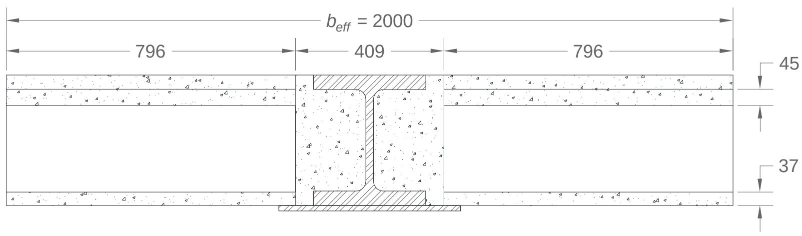


Fig. 4.4. Composite slab with modified concrete boundaries (Example 1)

It is assumed the system satisfies all required Ultimate Limit State design checks and there is enough bond between hollow-core slab units and steel sections. Rebars through the web of the steel beam are provided to assure the integrity of the composite floor.

For dynamic behaviour analysis, uncracked inertia is used. The dynamic modulus of normal weight concrete ($\gamma_{conc} = 2500\ kg/m^3$) is assumed to be $E_c = 38\ GPa$. For steel beams, $\gamma_{steel} = 7850\ kg/m^3$ and $E_{steel} = 210\ GPa$.

4.1.1. Manual Calculation

Natural Frequency of Hollow-core Slab Units

Hollow-core Slab Units (HCS320)

Length of slab at left side: $L_{hcs.l} = 8\ m$

Length of slab at right side: $L_{hcs.r} = 10\ m$

Height: $h_{hcs} = 32\ cm$

Width: $b_{hcs} = 120\ cm$

Cross-sectional area: $A_{hcs} = 1834.4\ cm^2$

The cross-sectional second moment of area: $I_{hcs} = 244155\ cm^4$

Neutral axis: $y_{hcs} = 16.56\ cm$

Topping Concrete (Non-structural Topping)

Height: $h_{top} = 3.9\ cm$

Width: $b_{top} = 120\ cm$

Cross-sectional area:

$$A_{top} = h_{topping}\ b_{topping}$$

$$A_{top} = 3.9\ cm \times 120\ cm = 468\ cm^2$$

Loads on the Floor

Floor finishing: $w_{finishing} = 1.0\ kPa$

Partition walls: $w_{partition} = 1.2\ kPa$

Services and ceiling: $w_{mechanical} = 0.5\ kPa$

Variable load: $w_{variable} = 3.6\ kPa$

Total Distributed Load

Self-weight of slab units:

$$w_{hcs} = \frac{A_{hcs}\ \gamma_{conc}\ g}{b_{hcs}}$$

$$w_{hcs} = \frac{1834.4\ \times\ 10^{-4}\ \times\ 2500\ \times\ 9.81}{1.2\ \times 1000} = 3.749\ kPa$$

Self-weight of topping concrete:

$$w_{top} = \frac{\mathrm{A_{top}}\,\gamma_{\mathrm{conc}}\,\mathrm{g}}{\mathrm{b_{top}}}$$

$$\mathrm{w_{top}} = \frac{468 \times 10^{-4} \times 2500 \times 9.81}{1.2 \times 1000} = 0.956\ kPa$$

Total Weight

It is assumed only 10% of the variable load will be present during service.

$$w_{total} = w_{finishing} + w_{partition} + w_{mechanical} + 0.1w_{variable} + w_{hcs} + w_{top}$$

$$w_{total} = 1 + 1.2 + 0.5 + 0.1 \times 3.6 + 3.749 + 0.956 = 7.765\ kPa$$

Second Moment of Area

$$I_{total} = \frac{I_{hcs}}{b_{hcs}}$$

$$I_{total} = \frac{244155}{1.2}$$

$$I_{total} = 203463\ cm^4/m$$

Deflection and Natural Frequency

$$\delta = \frac{w\,L^4}{384\,E\,I}$$

$$f_n = \frac{22.4}{2\pi}\sqrt{\frac{E\,I\,g}{w\,L^4}}$$

For left slab

$$\delta = \frac{7.765 \times 8^4}{384 \times 38 \times 203463} \times 10^5 = 1.07\ mm$$

$$f_{hcs} = \frac{22.4}{2\pi}\sqrt{\frac{38 \times 203463 \times 9.81}{7.765 \times 8^4 \times 100}} = 17.41\ Hz$$

For right slab

$$\delta = \frac{7.765 \times 10^4}{384 \times 38 \times 203463} \times 10^5 = 2.615\ mm$$

$$f_{hcs} = \frac{22.4}{2\pi}\sqrt{\frac{38 \times 203463 \times 9.81}{7.765 \times 10^4 \times 100}} = 11.14\ Hz$$

Natural Frequency of Composite Beams

Total height of slab:

$$h_{tot} = 35.9\ cm$$

Tributary width of slabs:

$$b_{trib} = 0.5(L_{hcs.l} + L_{hcs.r})$$
$$b_{trib} = 0.5 \times (8 + 10) = 9\ m$$

Steel Beam (HE320M)

Height:

$$h_{steel} = 35.9\ cm$$

Width:

$$b_{steel} = 30.9\ cm$$

Cross-sectional area:

$$A_{steel} = 312.05\ cm^2$$

Second moment of area:

$$I_{steel} = 68130\ cm^4$$

Neutral axis:

$$y_{steel} = 0.5 \times h_{steel} = 17.95\ cm$$

Length:

$$L_{steel} = 8\ m$$

Base Plate

Height:

$$h_{plate} = 1.5\ cm$$

Width:

$$b_{plate} = 50\ cm$$

Cross-sectional area:

$$A_{plate} = 75\ cm^2$$

Second moment of area:

$$I_{plate} = 14.06\ cm^4$$

Neutral axis:

$$y_{plate} = 0.75\ cm$$

Hollow-core Slab Units

Height: $$h_{hcu} = 32\ cm$$

Effective breadth:

$$b_{eff} = 0.125\ L_{steel} - 0.5\ b_{steel} - 0.05$$

$$b_{eff} = 0.125 \times 8 - 0.5 \times 0.309 - 0.05 = 79.55\ cm$$

Area:

$$A_{hcs} = \frac{820\ cm^2}{100\ cm} \times b_{eff}$$

$$A_{hcs} = \frac{820\ cm^2}{100\ cm} \times 79.55 = 652.31\ cm^2$$

Second moment of area:

$$I_{hcs} = \frac{159232\ cm^4}{100\ cm} \times b_{eff}$$

$$I_{hcs} = \frac{159232\ cm^4}{100\ cm} \times 79.55 = 126669.06\ cm^4$$

Neutral axis:

$$y_{hcu} = 17.2\ cm$$

Topping Concrete (Non-structural Topping)

Height:

$$h_{top} = 3.9\ cm$$

Area:

$$A_{top} = h_{top} \times b_{eff}$$

$$A_{top} = 3.9 \times 79.55 = 310.245\ cm^2$$

Core Concrete

Height:

$$h_{core} = 35.9\ cm$$

Width:

$$b_{core} = b_{steel} + 10$$
$$b_{core} = 30.9 + 10 = 40.9\ cm$$

Area:

$$A_{core} = b_{core} h_{core} - A_{steel}$$
$$A_{core} = 40.9 \times 35.9 - 312.05 = 1156.26\ cm^2$$

Second moment of area:

$$I_{core} = \frac{b_{core}\ (h_{core})^3}{12} - I_{steel}$$
$$I_{core} = \frac{40.9 \times 35.9^3}{12} - 68130 = 89568\ cm^4$$

Neutral axis:

$$y_{core} = 0.5\ h_{core}$$
$$y_{core} = 0.5 \times 35.9 = 17.95\ cm$$

Loads on the Composite Beam

Self-weight:

$$w_{steel} = A_{steel} \gamma_{steel}$$
$$w_{steel} = 312.05 \times 10^{-4} \times 7850 \times 9.81 \times 10^{-3} = 2.402\ kN/m$$

Plate:

$$w_{plate} = A_{plate} \gamma_{plate}$$
$$w_{plate} = 75 \times 10^{-4} \times 7850 \times 9.81 \times 10^{-3} = 0.577\ kN/m$$

Hollow-core units:

$$w_{hcs} = w_{hcs}(b_{trib} - b_{steel} - 10\ cm)$$
$$w_{hcs} = 3.748 \times (9 - 0.309 - 0.1) = 32.2\ kN/m$$

Topping concrete:

$$w_{top} = h_{top}(b_{trib} - b_{steel} - 10\ cm)\gamma_{conc} g$$
$$w_{top} = 0.039 \times (9 - 0.309 - 0.1) \times 2500 \times 9.81 \times 10^{-3} = 8.217\ kN/m$$

Core concrete:

$$w_{core} = A_{core} \times \gamma_{conc} \times g$$
$$w_{core} = 1156.26 \times 10^{-4} \times 2500 \times 9.81 \times 10^{-3} = 2.835\ kN/m$$

Floor finishing:

$$w_{fin} = 1 \times b_{trib}$$
$$w_{fin} = 1 \times 9 = 9\ kN/m$$

Partition walls:

$$w_{part} = 1.2 \times b_{trib}$$
$$w_{part} = 1.2 \times 9 = 10.8\ kN/m$$

Services and ceiling:

$$w_{mech} = 0.5 \times b_{trib}$$
$$w_{mech} = 0.5 \times 9 = 4.5\ kN/m$$

Variable load (only 10% assumed to be present in service):

$$w_{var} = 0.1 \times 3.6 \times b_{trib}$$
$$w_{var} = 0.1 \times 3.6 \times 9 = 3.24\ kN/m$$

Total Weight on the Composite Beam

$$w_{total} = w_{steel} + w_{plate} + w_{hcs} + w_{top} + w_{core} + w_{fin} + w_{part} + w_{mech} + w_{var}$$
$$w_{total} = 2.402 + 0.577 + 32.2 + 8.217 + 2.835 + 9 + 10.8 + 4.5 + 3.24 = 73.771\ kN/m$$

Position of Elastic Neutral Axis

$$n = \frac{E_{conc}}{E_{steel}}$$
$$n = \frac{38}{210} = 0.181$$

Base plate Area:

$$A_{plate} = 75\ cm^2$$

Distance from bottom:

$$y_{plate} = 0.5h_{plate}$$
$$y_{plate} = 0.5 \times 1.5 = 0.75\ cm$$

Steel beam area:

$$A_{steel} = 312.05\ cm^2$$

Distance from bottom:

$$y_{steel} = h_{plate} + 0.5h_{steel}$$
$$y_{steel} = 1.5 + 0.5 \times 35.9 = 19.45\ cm$$

Core concrete area:

$$A_{core} = n\ A$$
$$A_{core} = 0.181 \times 1156.26 = 209.228\ cm^2$$

Distance from bottom:

$$y_{core} = y_{steel} = 19.45\ cm$$

Hollow-core units' area:

$$A_{hcs} = n\ A$$
$$A_{hcs} = 0.181 \times 652.31 = 118.068\ cm^2$$

Distance from bottom:

$$y_{hcs} = 17.2 + h_{plate}$$
$$y_{hcs} = 17.2 + 1.5 = 18.7\ cm$$

$$y = \frac{A_{plate}y_{plate} + A_{steel}y_{steel} + A_{core}y_{core} + 2A_{hcs}y_{hcs}}{A_{plate} + A_{steel} + A_{core} + 2A_{hcs}}$$
$$y = \frac{75 \times 0.75 + 312.05 \times 19.45 + 209.228 \times 19.45 + 2 \times 118.068 \times 18.7}{75 + 312.05 + 209.228 + 2 \times 118.068} = 17.55\ cm$$

Second Moment of Area

Base plate:

$$I_{Plate} = I_{plate} + A_{plate}\left(y_{plate} - y\right)^2$$
$$I_{Plate} = 14.06 + 75 \times (0.75 - 17.55)^2 = 21182\ cm^4$$

Steel beam:

$$I_{Steel} = I_{steel} + A_{steel}(y_{steel} - y)^2$$
$$I_{Steel} = 68130 + 312.05 \times (19.45 - 17.55)^2 = 69257\ cm^4$$

Core concrete:

$$I_{Core} = n\, I_{core} + A_{core}(y_{core} - y)^2$$
$$I_{Core} = 0.181 \times 89568 + 209.228 \times (19.45 - 17.55)^2 = 16967\ cm^4$$

Hollow-core units:

$$I_{Hcs} = 2(n\, I_{hcs} + A_{hcs}(y_{hcs} - y)^2)$$
$$I_{Hcs} = 2(0.181 \times 126669 + 118.068 \times (18.7 - 17.55)^2) = 46166\ cm^4$$

The total second moment of area:

$$I_{tot} = I_{Plate} + I_{Steel} + I_{Core} + I_{Hcs} + I_{Top} = 153572\ cm^4$$

Deflection

$$\delta = \frac{5\, w\, L^4}{384\, E\, I}$$
$$\delta = \frac{5 \times 73.771 \times 8^4}{384 \times 210 \times 153572} \times 10^5 = 12.2\ mm$$

Natural Frequency

$$f_b = \frac{\pi^2}{2\pi}\sqrt{\frac{E\, I\, g}{w\, L^4}}$$
$$f_b = \frac{\pi^2}{2\pi}\sqrt{\frac{210 \times 153572 \times 9.81}{73.771 \times 8^4 \times 100}} = 5.08\ Hz$$

Natural Frequency of the Floor System

$$\frac{1}{f_n^2} = \frac{1}{f_{hcs}^2} + \frac{1}{f_b^2}$$
$$f_n = \frac{1}{\sqrt{\frac{1}{f_s^2} + \frac{1}{f_b^2}}}$$

Case 1: Left hollow-core slab units with composite beam

$$f_n = \frac{1}{\sqrt{\frac{1}{17.41^2} + \frac{1}{5.08^2}}} = 4.88\ Hz$$

Case 2: Right hollow-core slab units with composite beam

$$f_n = \frac{1}{\sqrt{\frac{1}{11.14^2} + \frac{1}{5.08^2}}} = \mathbf{4.62\ Hz}$$

Case 2 governs the design. Another approach is to use Equation 2.7 with a total deflection from both right hollow-core slab units and composite beam

$$f_n = \frac{18}{\sqrt{2.615 + 12.2}} = 4.68\ Hz$$

Dynamic Response of the Floor System

Modal Mass

$$m = \frac{73.771 \times 10^3}{9 \times 9.81} = 835.553 \frac{kg}{m^2}$$

The number of spans of composite beams is $n_y = 3$ and their span length is $L_y = 8\ m$, The number of spans of hollow-core slabs are $n_x = 2$ and their shortest length is $L_x = 8\ m$

Effective length of the composite beam:

$$L_{eff} = 1.09 \left(\frac{EI_b}{mL_x f_0^2}\right)^{\frac{1}{4}}$$

$$L_{eff} = 1.09 \left(\frac{210 \times 153572}{835.553 \times 9 \times 4.62^2} \times 10^1\right)^{\frac{1}{4}} = 7.3\ m < n_y L_y = 24\ m$$

$$L_{eff} = 7.3\ m$$

Effective length of the composite beam:

$$S = 2.25 \left(\frac{EI_S}{mf_0^2}\right)^{\frac{1}{4}}$$

$$S = 2.25 \left(\frac{38 \times 203463}{835.553 \times 4.62^2} \times 10^1\right)^{\frac{1}{4}} = 18.26\ m > \ n_x L_x = 16\ m$$

$$S = 16\ m$$

$$M = mL_{eff}S$$

$$M = 835.553 \times 7.3 \times 16 = 97592.6\ kg$$

Floor Response

$f_0 = 4.68\ Hz < 10\ Hz$, the floor is categorised as "Low-frequency floor"

$$a_{w,rms} = \mu_e \mu_r \frac{0.1Q}{2\sqrt{2}M\zeta} W\rho$$

The average weight of human, $Q = 76\ kg \times g = 745\ kN$

$\zeta = 2.5\%$

$\mu_e = 1$

$\mu_r = 1$

$$W = \begin{cases} 0.5f, & 1\ Hz < f < 4\ Hz \\ 1.00, & 4\ Hz < f < 8\ Hz \\ \dfrac{8}{f}, & f > 8\ Hz \end{cases}$$

$W = 1.00$

Assuming the largest corridor length $L_p = 24\ m$ and pace frequency $f_p = 2\ Hz$,

$$v = 1.67f_p^2 - 4.83f_p + 4.5$$

$$v = 1.67 \times 2^2 - 4.83 \times 2 + 4.5 = 1.52\frac{m}{sec}$$

$$\rho = 1 - e^{\frac{-2\pi\zeta L_p f_p}{v}}$$

$$\rho = 1 - e^{\left(\frac{-2\pi \times 0.025 \times 24 \times 2}{1.52}\right)} = 1$$

$$a_{w,rms} = 1 \times 1 \times \frac{0.1 \times 745 \times 1 \times 1}{2\sqrt{2} \times 97592.6 \times 0.025} = 0.0179\frac{m}{sec^2}$$

Response Factor

$$R = \frac{a_{w,rms}}{0.005}$$

$$R = \frac{0.0179}{0.005} = 2.16$$

Comparing the response factor to Table 2.5, the floor is suitable for residential buildings and offices.

4.1.2. Nomographs

Natural Frequency of Hollow-core Slab Units

1. $L = 10\ m$;
2. The supports are rigid at both sides;
3. The permanent effective load on slabs $w_{total} = 7.765\ kPa$;
4. From Figure 4.5, $f_{hcs} \approx 11.1\ Hz$.

Natural Frequency of Composite Beams

1. $L = 8\ m$;
2. The supports are pinned at both sides;
3. Stiffness to load $\zeta = 4.378$ from calculations below;

$$E = 210 \times 10^3\ MPa$$
$$I = 1.538 \times 10^{-3}\ m^4$$
$$w = 73.771\ \frac{kN}{m}$$
$$\zeta = \frac{EI}{w} = 4.378$$

4. From Figure 4.6, $f_b \approx 5.1\ Hz$.

Natural Frequency of Floor System

$$f_n = \frac{1}{\sqrt{\frac{1}{f_s^2} + \frac{1}{f_b^2}}}$$

$$f_n = \frac{1}{\sqrt{\frac{1}{11.1^2} + \frac{1}{5.1^2}}} = 4.63\ Hz$$

Section: HCS320

E= 38000 Mpa

I= 2.03E+09 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

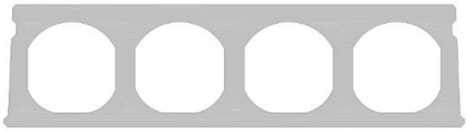
f_b: Frequency of beam (uniform load)

f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.



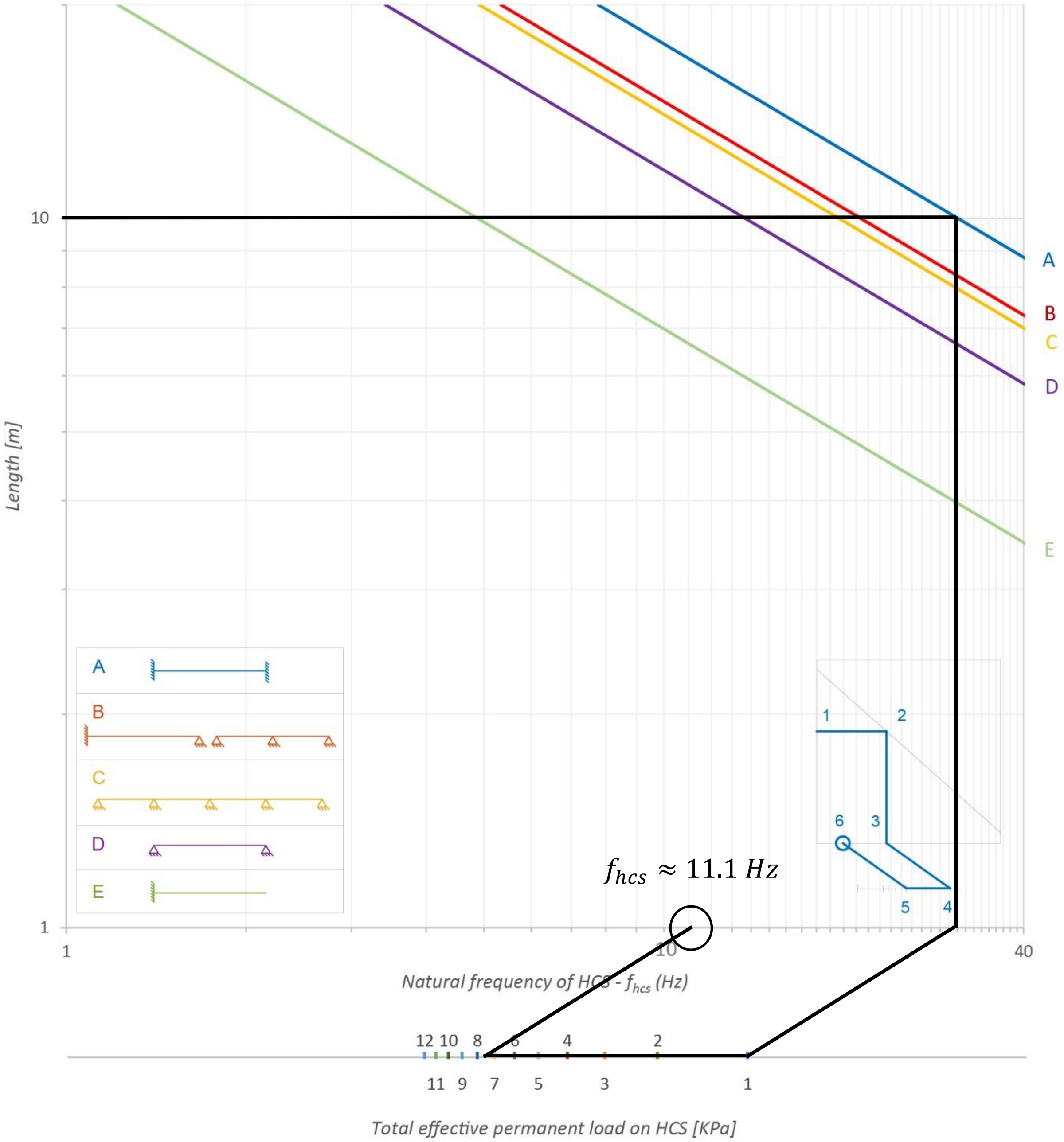


Fig. 4.5. HCS natural frequency calculation using nomograph

Section: Flexural members (uniform load)

Notes:
a) This diagram can be used to define natural frequency of a member due to uniformly distributed load only.
b) Stiffness per load ratio (ζ) should be calculated with E, I and w in given units
c) Chosen member length should satisfy ULS design requirements.

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

$$\zeta = \frac{EI}{w}$$

f_0: Frequency of system
f_{hcs}: Frequency of HCS (uniform load)
f_b: Frequency of beam (uniform load)
f_p: Frequency of beam (point load)

E: Young's modulus in Mpa
I: Second moment of area in m^4
w: Effective permanent load in in kN/m

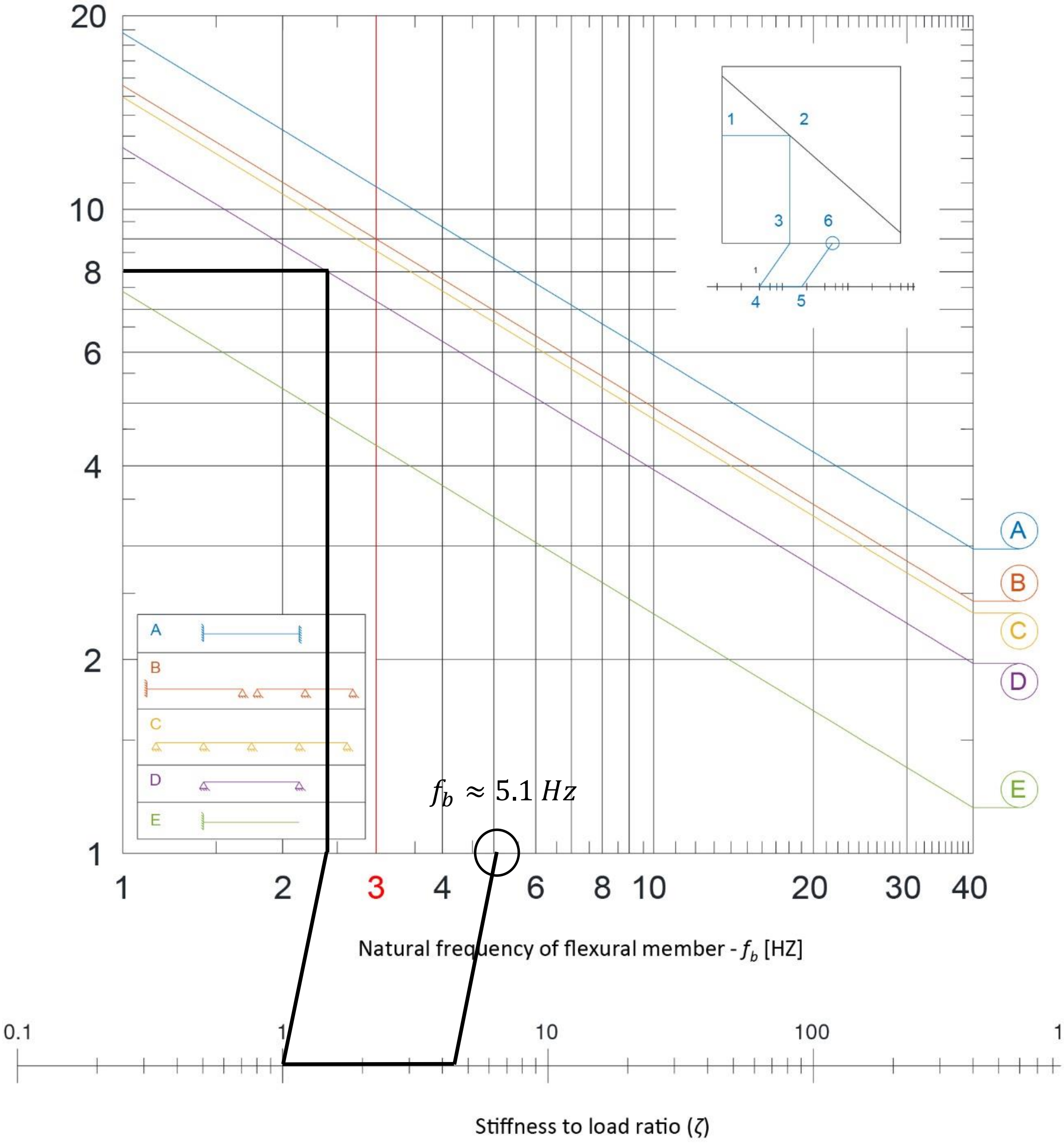


Fig. 4.6. flexural members natural frequency calculation using nomograph

4.1.3. SCIA Engineer

SCIA engineer can calculate the natural frequency of composite floors with decks only. In case of hollow-core slab units with composite beams, All materials and members can be defined as non-composite members then it is recommended to increase the second moment of area of the members which work as a composite to represent their real behaviour in calculations.

$$I_{steel} = 68130\ cm^4$$

$$I_{composite} = 153572\ cm^4$$

Multiplication factor:

$$k = \frac{I_{composite}}{I_{steel}} = 2.254$$

It is also important to choose correct member supports. The nodes in continuous members are assumed to be rigid even over the supports. Only 10% of the variable load should be entered. SCIA Engineer does not reduce it automatically. Young modulus of concrete should be increased to its dynamic modulus. Again, SCIA does not count dynamic modulus factor in dynamic calculations. The software does not have any hollow-core sections. It should be imported to the software. It is also not possible to define stiffness of transverse bending of the hollow-core slab units.

Appendix 2 shows the report by the software. The obtained natural frequency of the system with FEM analysis in SCIA Engineer is $f_0 = 4.79\ Hz$. SCIA Engineer is not capable of calculating the dynamic response.

4.1.4. Autodesk Robot Structural Analysis

Autodesk Robot Structural Analysis (ARSA) is programmed to analyse the floors according to SCI P354. Unlike SCIA Engineer, in ARSA the hollow-core slab sections can be defined by entering the slab parameters. In ARSA the beams are not calculated as composite beams. For composite steel beams, there is a possibility to increase the young modulus of the steel to have the same stiffness of the composite one. Analysis can be performed for the slabs with either including or neglecting stiffness of transverse bending.

Appendix 3 includes software calculation report. For the natural frequency, it is assumed the floor does not have stiffness of transverse bending to compare it with manual calculation. On the other hand, it is assumed the floor has transverse bending stiffness for response factor calculation as assumed in manual calculation. The values for natural frequency and response factor are $f_0 = 5.78\ Hz$ and $R = 2.33$, respectively.

4.2. Example 2

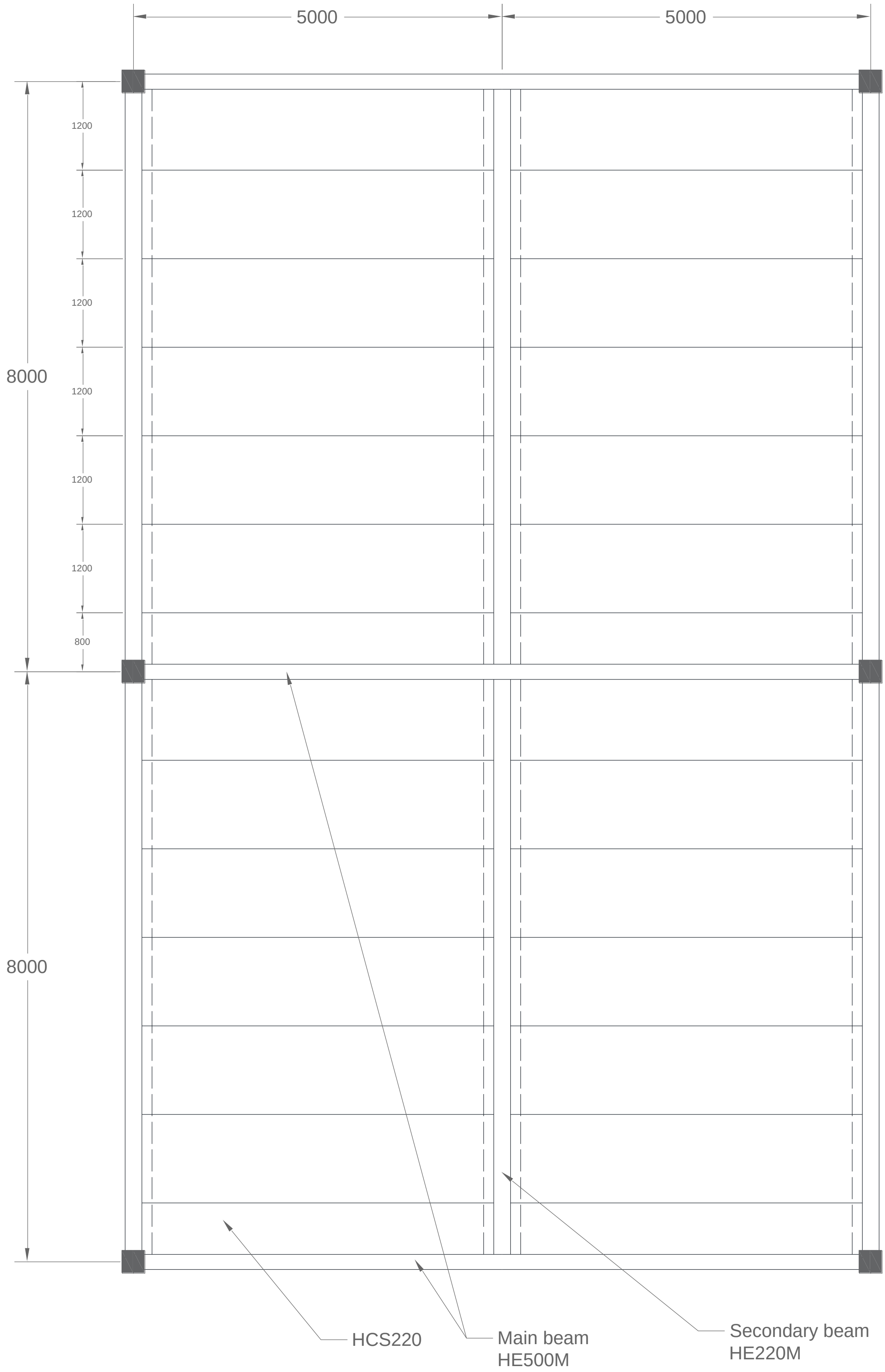


Fig. 4.7. Typical floor with slim floor beams and hollow-core slabs (example 2)

Materials and Floor Properties

Hollow-core slab units

Length of slabs: $L_{hcs} = 5\ m$

Height: $h_{hcs} = 22\ cm$

Width: $b_{hcs} = 120\ cm$

Cross-sectional area: $A_{hcs} = 1434.3\ cm^2$

Cross-sectional second moment of area: $I_{hcs} = 86763.4\ cm^4$

Neutral axis: $y_{hcs} = 10.9\ cm$

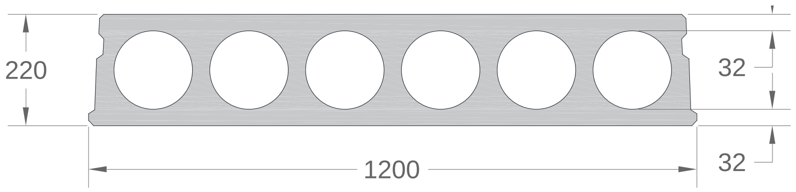


Fig. 4.8. Used hollow-core slab section (Example 2)

Main Beam (HE500M)

Length: $L_{main} = 10\ m$

Height: $h_{main} = 52.4\ cm$

Width: $b_{main} = 30.6\ cm$

Cross-sectional area: $A_{main} = 344.3\ cm^2$

Second moment of area: $I_{main} = 161900\ cm^4$

Secondary Beam (HE220M)

Length: $L_{sec.} = 8\ m$

Height: $h_{sec.} = 24\ cm$

Width: $b_{sec.} = 22.6\ cm$

Cross-sectional area: $A_{sec.} = 149.4\ cm^2$

Second moment of area: $I_{sec.} = 14600\ cm^4$

Base Plate

Height: $h_{plate} = 1.5\ cm$

Width: $b_{plate} = 50\ cm$

Cross-sectional area: $A_{plate} = 75\ cm^2$

Second moment of area: $I_{plate} = 14.06\ cm^4$

Topping Concrete (Non-structural Topping)

Height: $h_{top} = 2\ cm$

Width: $b_{top} = 120\ cm$

Cross-sectional area: $A_{top} = 2\ cm \times 120\ cm = 240\ cm^2$

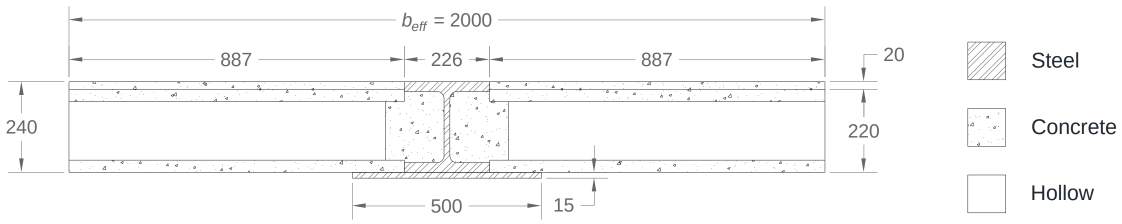


Fig. 4.9. Slim floor beam cross-section with effective part of HCS and concrete (Example 2)

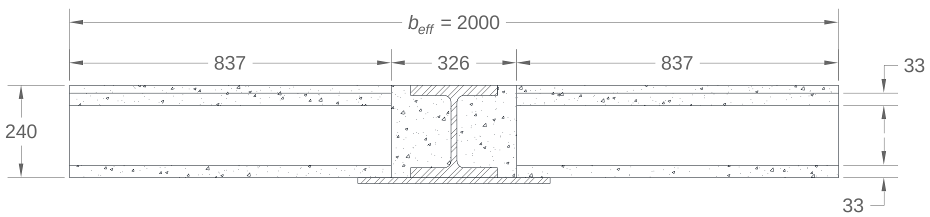


Fig. 4.10. Composite slab with modified concrete boundaries (Example 2)

Loads on the Floor

Permanent loads: $w_{permanent} = 2\ kPa$

Variable loads: $w_{variable} = 2.5\ kPa$

4.2.1. Manual calculation

Natural Frequency for Hollow-core slab units: $f_{hcs} = 31.09\ Hz$

Natural Frequency for secondary beams: $f_{b.sec} = 4.33\ Hz$

Natural Frequency for Main Beams (Due to Self-weight): $f_{b.main} = 17.62\ Hz$

Natural Frequency for Main Beams (Due to Point-load): $f_{p.main} = 4.1\ Hz$

System Natural Frequency:

$$f_s = \frac{1}{\sqrt{\frac{1}{31.09^2} + \frac{1}{4.33^2} + \frac{1}{17.62^2} + \frac{1}{4.1^2}}}$$

$$f_s = 2.92\ Hz$$

Response factor ($\zeta = 5\%$): $R = 2.42$

4.2.2. Nomographs

Natural Frequency for Hollow-core slab units: $f_{hcs} = 30\ Hz$

Natural Frequency for secondary beams: $f_{b.sec} = 4.3\ Hz$

Natural Frequency for Main Beams (Due to Self-weight): $f_{b.main} = 17.5\ Hz$

Natural Frequency for Main Beams (Due to Point-load): $f_{p.main} = 4.2\ Hz$

System Natural Frequency:

$$f_s = \frac{1}{\sqrt{\frac{1}{30^2} + \frac{1}{4.3^2} + \frac{1}{17.5^2} + \frac{1}{4.2^2}}}$$

$$f_s = 2.95\ Hz$$

4.2.3. SCIA Engineer

System Natural Frequency: $f_s = 3.92\ Hz$

4.2.4. Autodesk Robot Structural Analysis

System Natural Frequency without Stiffness of Transverse Bending: $f_s = 4.62\ Hz$

Response Factor without Stiffness of Transverse Bending ($\zeta = 5\%$): $R = 6.62\ Hz$

System Natural Frequency with Stiffness of Transverse Bending: $f_s = 5.12\ Hz$

Response Factor with Stiffness of Transverse Bending ($\zeta = 5\%$): $R = 3.98$

5. Results

In this chapter, the results of dynamic analysis by manual calculation, nomographs, SCIA Engineer, and Autodesk Robot Structural Analysis are shown. Though it is not possible to calculate response factor with nomographs and SCIA Engineer

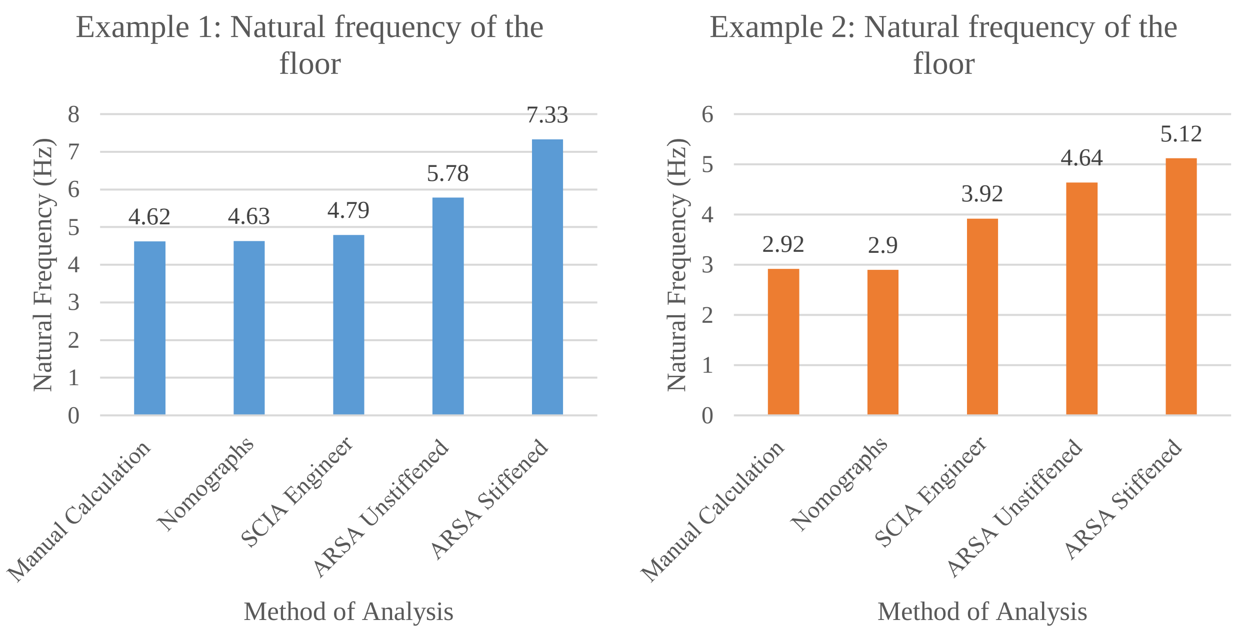


Fig. 5.1. Comparison of natural frequency results from various analysis methods

In both examples, the results of natural frequency from manual calculation and nomographs are remarkably similar. Comparing SCIA Engineer with manual calculation results; in example 1, SCIA Engineer has slightly higher natural frequency. But in example 2, SCIA Engineer result is 35% higher. Talking about Autodesk Robot Structural Analysis (ARSA) software, the natural frequency of the floor is higher when stiffness of transverse bending is not assumed. The value of the frequency is the highest when the stiffness of transverse bending is assumed.

Table 5.1. Response factor results

Examples	Manual Calculation	Nomographs	SCIA Engineer	ARSA Unstiffened	ARSA Stiffened
Example 1	2.16	-	-	29.06	2.33
Example 2	2.42	-	-	6.62	3.98

The response factor value from manual calculation and ARSA with transverse bending stiffness is similar in example 1. In example 2, the value from the software has increased by 64%. The values of response factor without encountering transverse bending stiffness are the highest but they can be ignored.

Conclusions

1. Using nomographs is an easy way to estimate natural frequency of the composite floors. These graphs can be used for natural frequency of uniform and point loads on hollow-core slabs as well as flexural members. The nomographs have been created with the same equations used in manual calculations, that is why the results are very similar. Nomographs can be used for various support types.

2. Finite Element Method (FEM) programs like SCIA Engineer and Autodesk Robot Structural Analysis can analyse dynamic properties of composite steel deck floors. For the composite hollow-core units with steel beams, the composite stiffness of the members should be increased manually for more precise calculation of dynamic properties.

3. Stiffness of transverse bending has remarkable influence on dynamic analysis results. In manual calculations of natural frequency, the effect of transverse bending stiffness is not included. It is assumed each member of the system oscillates separately within the system. In SCIA Engineer, the calculations are made based on the same assumption. In Autodesk Robot Structural Analysis, it is possible to consider transverse bending stiffness in calculations. Therefore, the natural frequency of the floor improves, making a more realistic case.

4. Response factor equations are derived for the cases where the slabs are continuous in both directions. That way the effect of transverse bending stiffness is included. High values of response factor can be achieved without considering lateral stiffness. These high values are not comparable with the limit values of the guidelines.

5. The dynamic analysis results of manual calculations and FEM programs are similar in the case of having main beams only (Uniformly distributed load only). When having secondary beams (Point loads on main beams), the obtained values from manual and FEM analysis can be different due to complexity of the structure. In any case, FEM analysis is more accurate and reliable when it is possible to use.

6. It is very important to assume the most realistic case of supports. In SLS design for vibration, the supports should not be assumed as in ULS. A member with joints assumed as pins can be analysed as having fixed joints because the available friction in the joint can be enough to overcome the strains that can happen due to vibration. Choosing the right type of the joints has considerable influence on dynamic analysis results. This consideration is debatable and can be a subject for future works.

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Appendices

Appendix 1. Nomographs for Calculating Natural Frequency of Slabs

Procedure:

1. Draw a horizontal line from vertical axis to the correct line of support type (line 1-2);
2. Draw a vertical line to the upper horizontal axis (line 2-3);
3. Connect the upper horizontal axis to the lower horizontal axis to the value labelled 1 (line 3-4);
4. Slide to the correct value at the lower horizontal axis (line 4-5);
5. Connect the lower horizontal axis to the upper horizontal axis with a line with the same angle as the one in step 3 (line 5-6);
6. The value on the upper horizontal axis is the natural frequency of the member.

1. 200 mm Hollow-core Slabs

Section: HCS200

E= 38000 Mpa

I= 5.09E+08 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

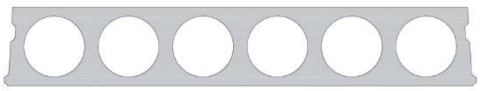
f_b: Frequency of beam (uniform load)

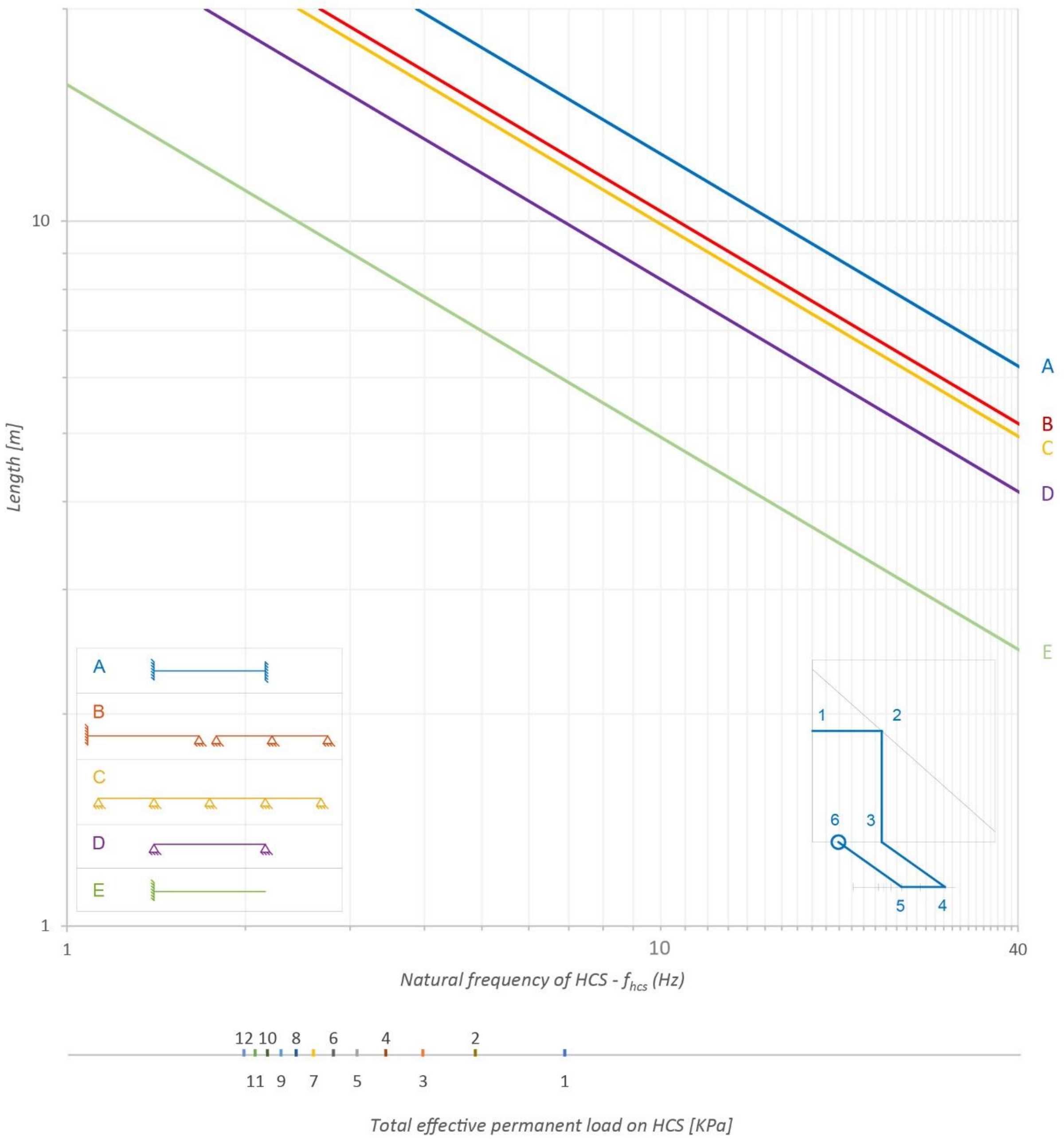
f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.





2. 220 mm Hollow-core Slabs

Section: HCS220

E= 38000 Mpa

I= 7.23E+08 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

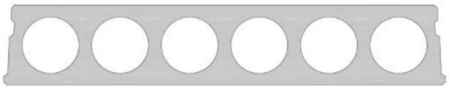
f_b: Frequency of beam (uniform load)

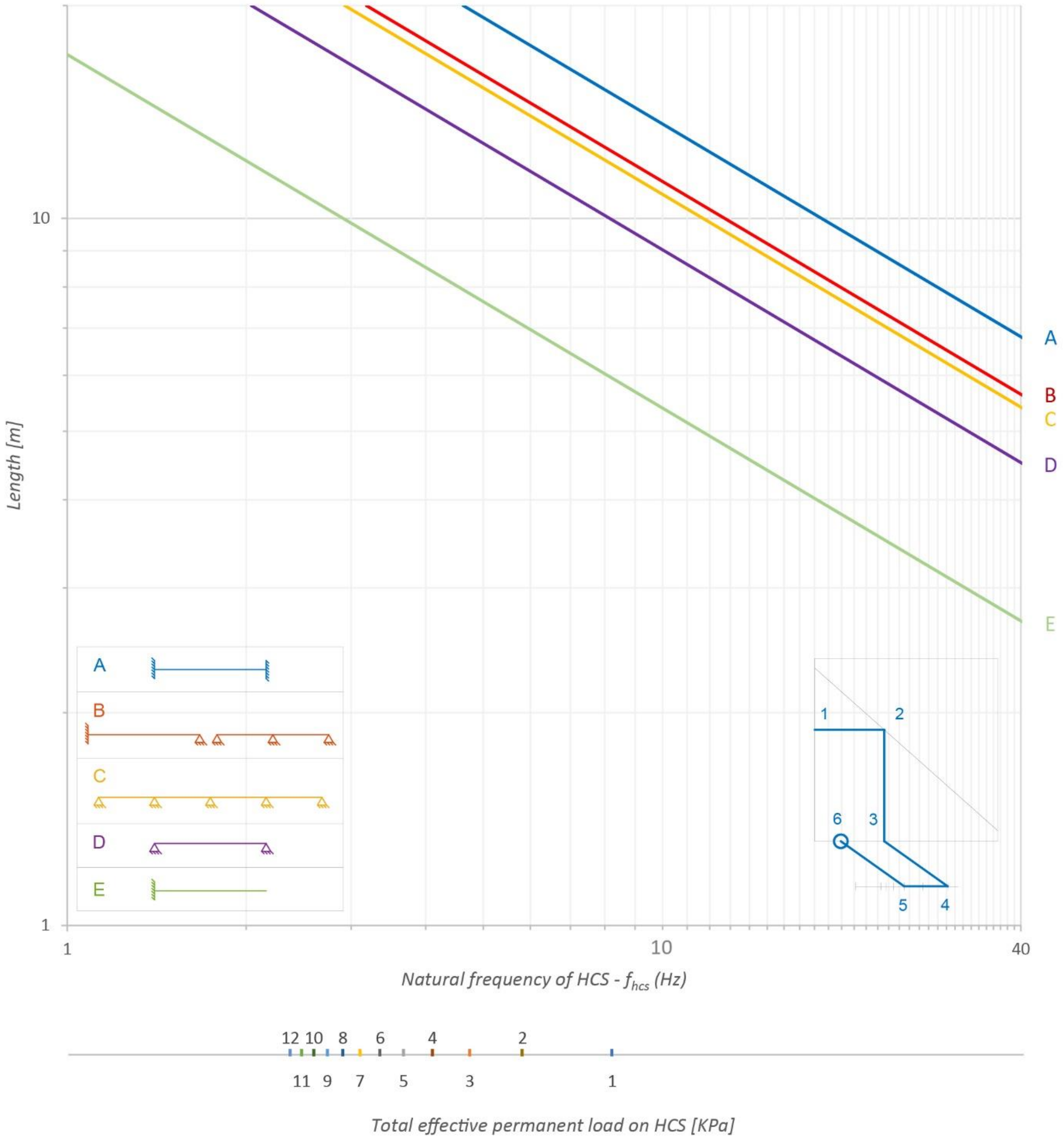
f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.





3. 265 mm Hollow-core Slabs

Section: HCS265

E= 38000 Mpa

I= 1.27E+09 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

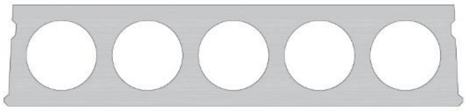
f_b: Frequency of beam (uniform load)

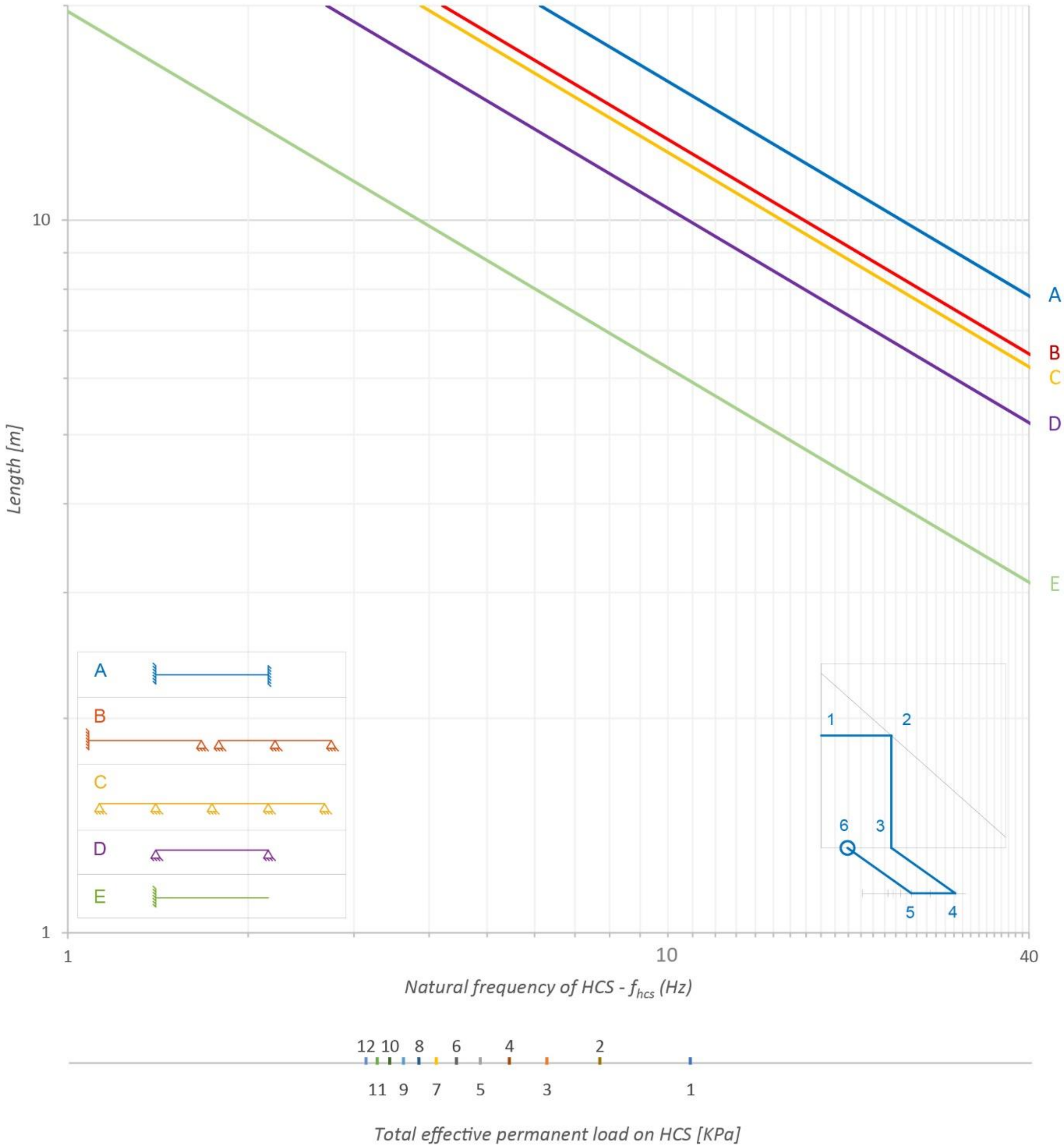
f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.





4. 300 mm Hollow-core Slabs

Section: HCS300

E= 38000 Mpa

I= 1.57E+09 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2}+\frac{1}{f_b^2}+\frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

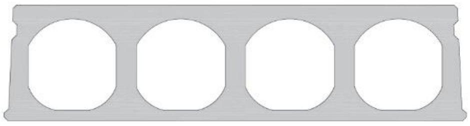
f_b: Frequency of beam (uniform load)

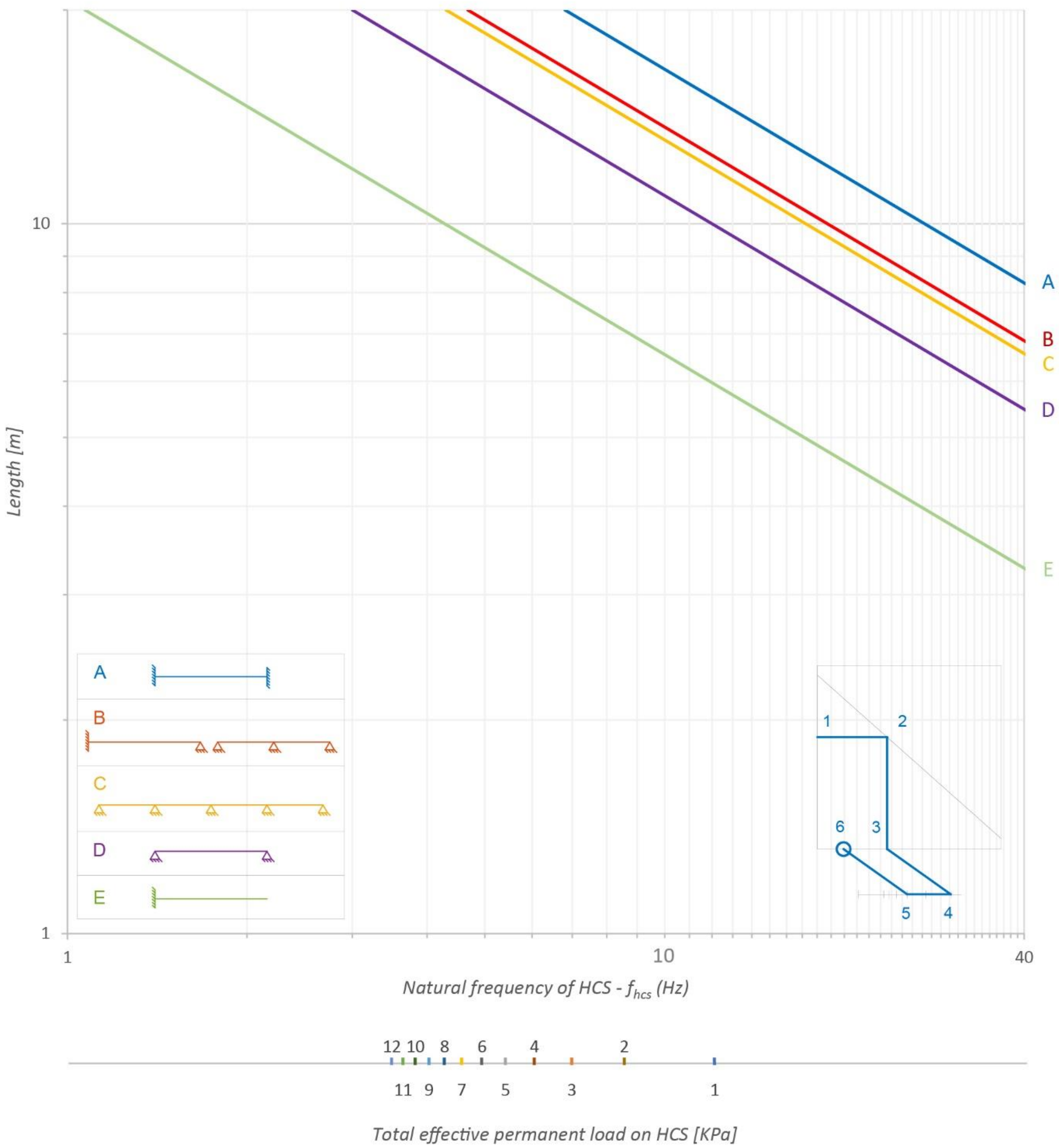
f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.





5. 320 mm Hollow-core Slabs

Section: HCS320

E= 38000 Mpa

I= 2.03E+09 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

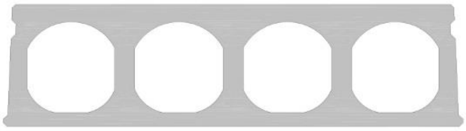
f_b: Frequency of beam (uniform load)

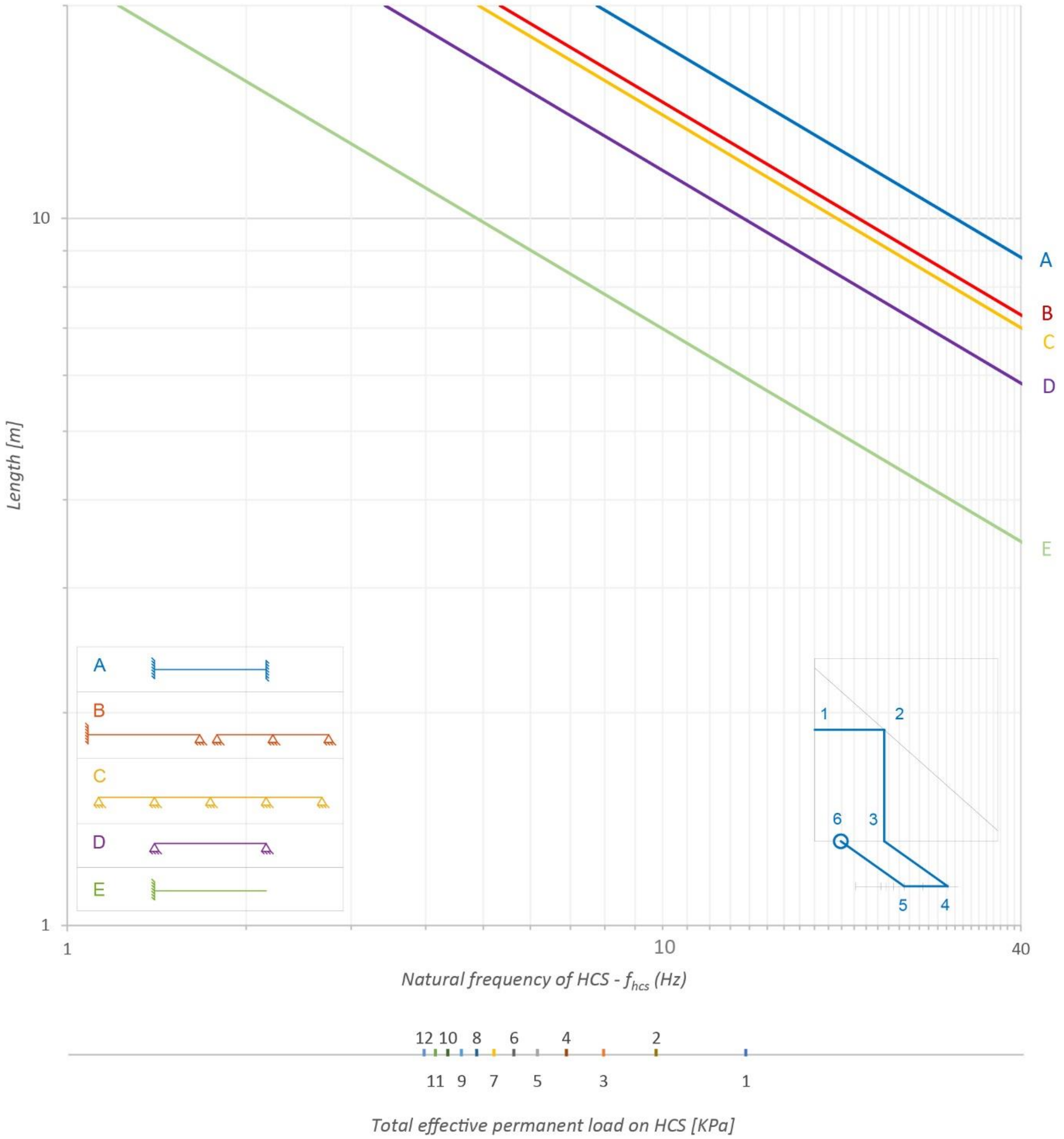
f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.





6. 400 mm Hollow-core Slabs

Section: HCS400

E= 38000 Mpa

I= 3.60E+09 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

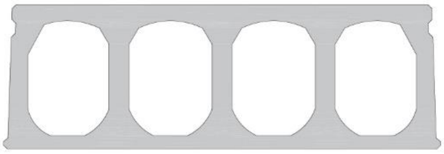
f_b: Frequency of beam (uniform load)

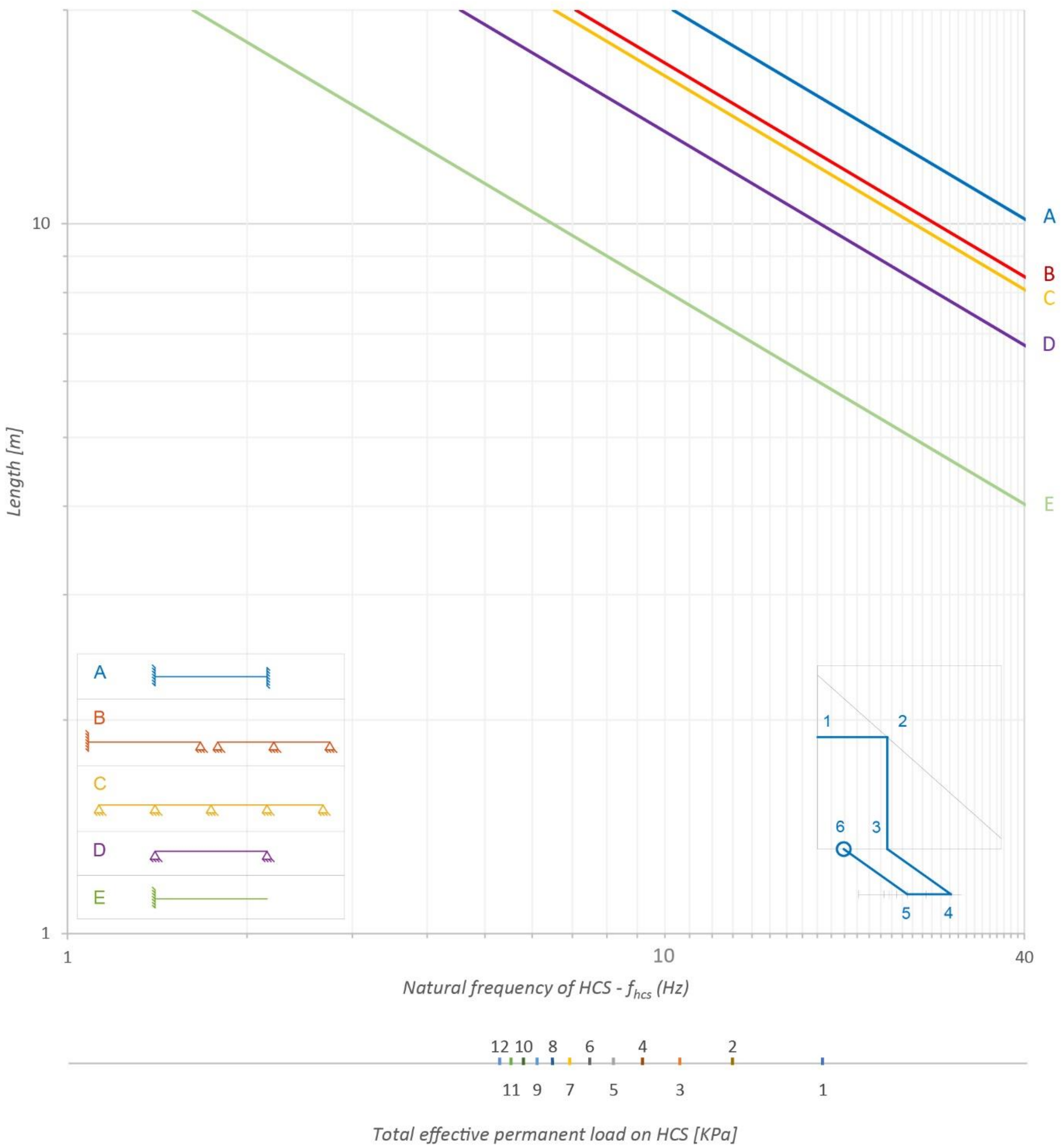
f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.





7. 500 mm Hollow-core Slabs

Section: HCS500

E= 38000 Mpa

I= 7.64E+09 mm^4/m

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

f_0: Frequency of system

f_{hcs}: Frequency of HCS (uniform load)

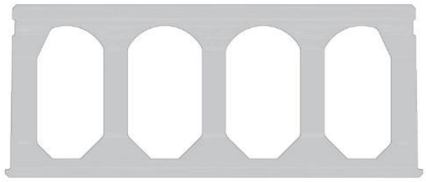
f_b: Frequency of beam (uniform load)

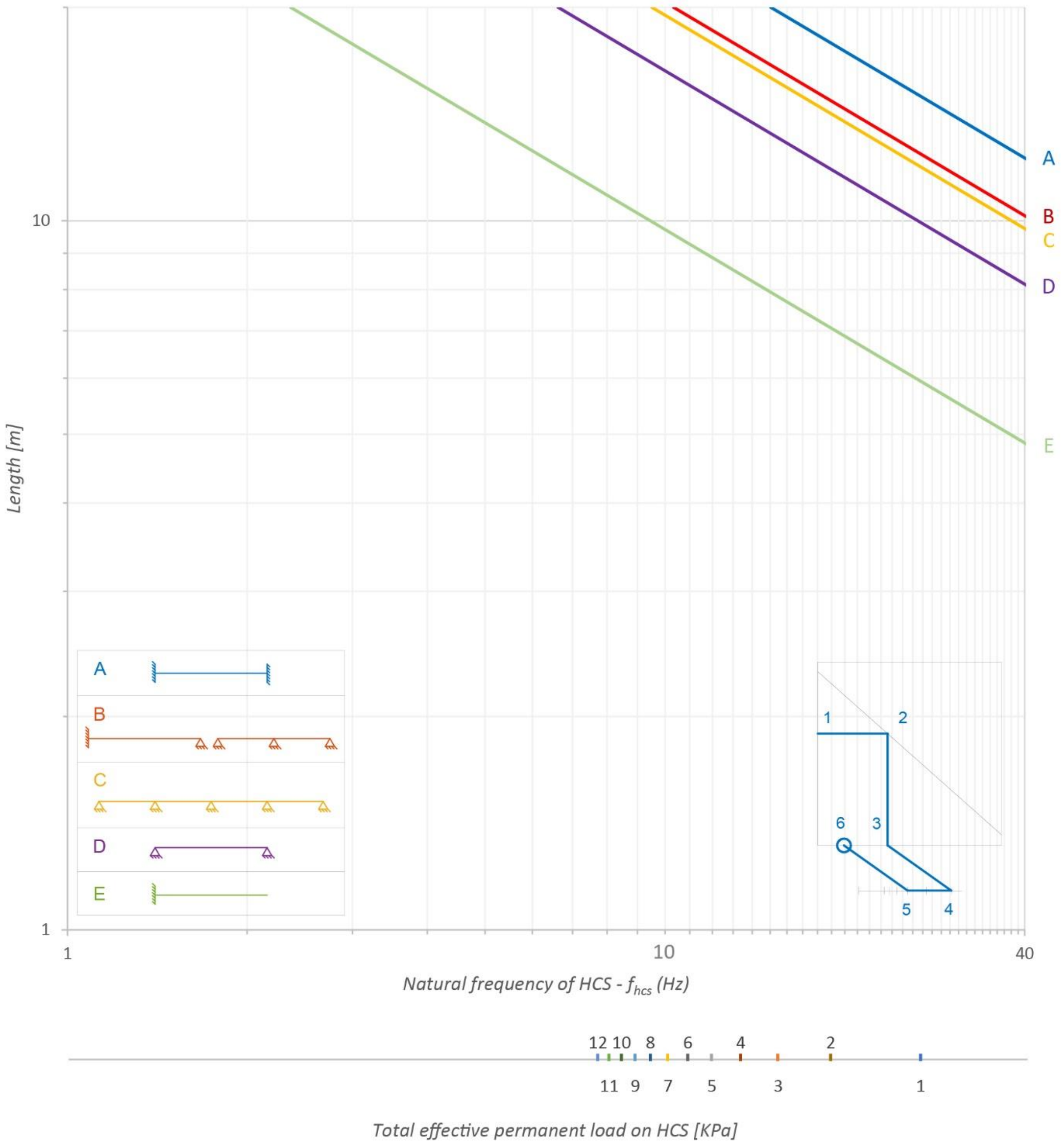
f_p: Frequency of beam (point load)

Notes:

a) Effective permanent load is sum of total permenant load and 10% of imposed load.

b) Chosen member length should satisfy ULS design requirements.





8. Flexural Members under Uniformly Distributed Load

Section: Flexural members (uniform load)

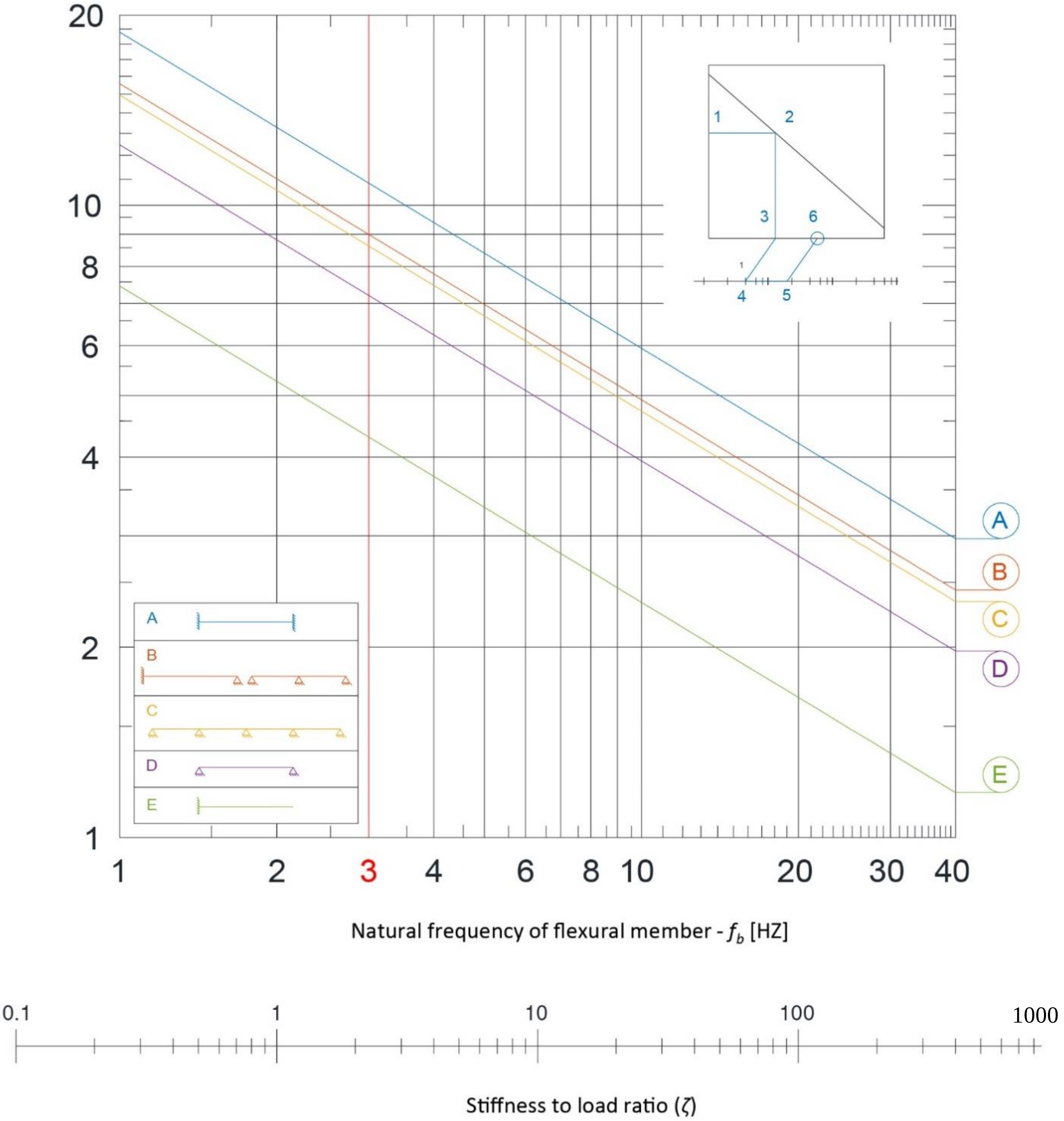
Notes:
a) This diagram can be used to define natural frequency of a member due to uniformly distributed load only.
b) Stiffness per load ratio (ζ) should be calculated with E, I and w in given units
c) Chosen member length should satisfy ULS design requirements.

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

$$\zeta = \frac{EI}{w}$$

f_0: Frequency of system
f_{hcs}: Frequency of HCS (uniform load)
f_b: Frequency of beam (uniform load)
f_p: Frequency of beam (point load)

E: Young's modulus in Mpa
I: Second moment of area in m^4
w: Effective permanent load in in kN/m



9. Flexural Members under Point Load

Section: Flexural members (point load)

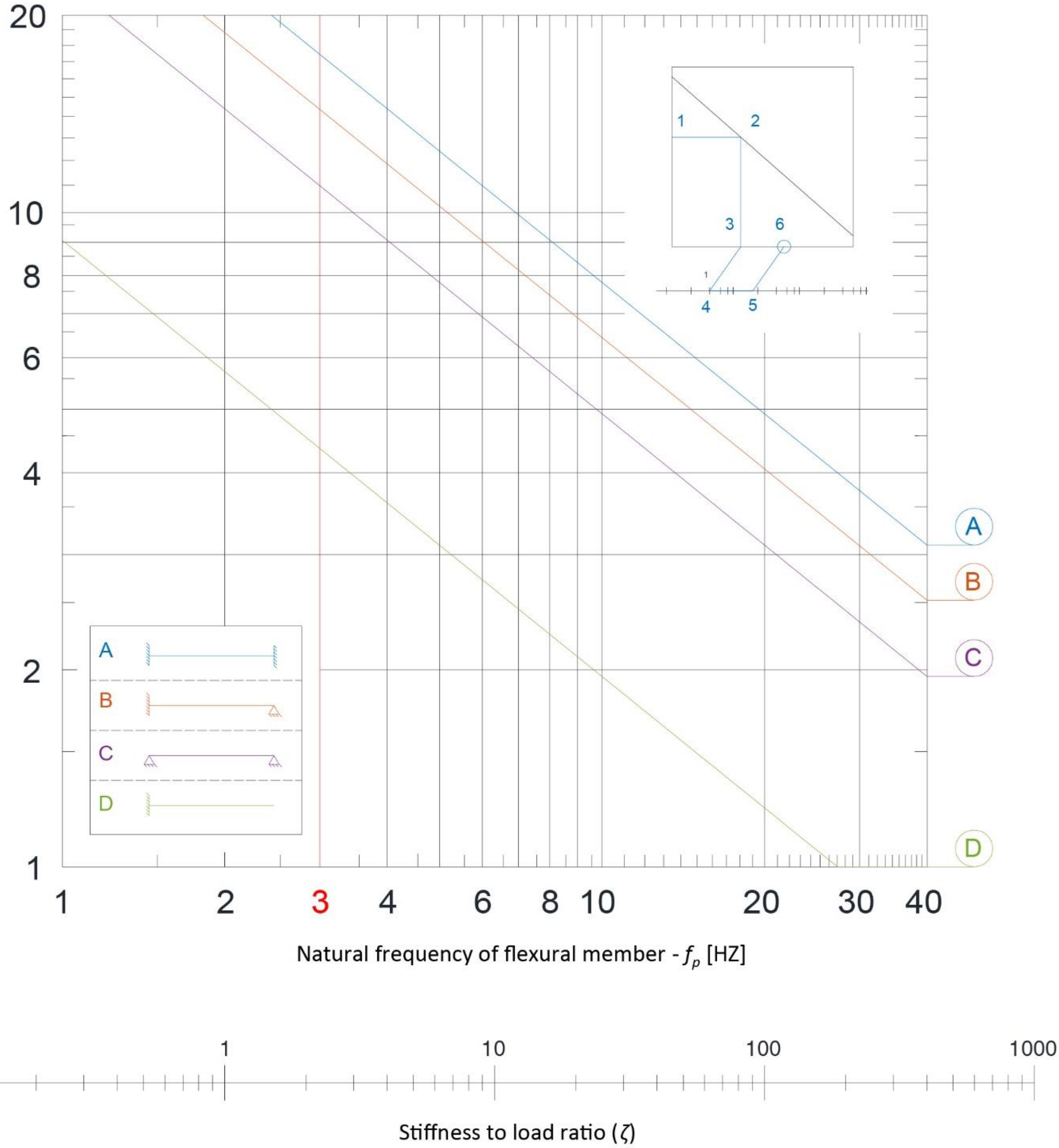
Notes:
a) This diagram can be used to define natural frequency of a member due to point load only.
b) Stiffness per load ratio (ζ) should be calculated with E, I and p in given units
c) Chosen member length should satisfy ULS design requirements.

$$f_0 = \frac{1}{\sqrt{\frac{1}{f_{hcs}^2} + \frac{1}{f_b^2} + \frac{1}{f_p^2}}}$$

$$\zeta = \frac{EI}{p}$$

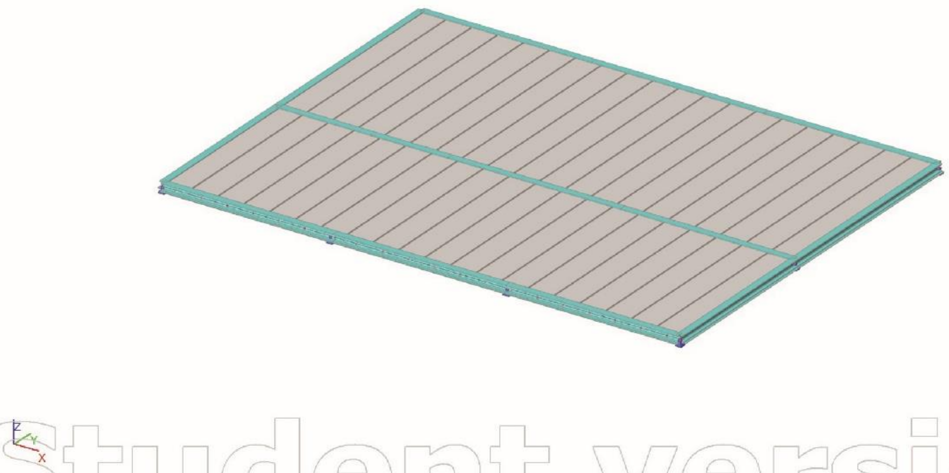
f_0: Frequency of system
f_{hcs}: Frequency of HCS (uniform load)
f_b: Frequency of beam (uniform load)
f_p: Frequency of beam (point load)

E: Young's modulus in Mpa
I: Second moment of area in m^4
p: Effective point load in in kN



Appendix 2. Dynamic Analysis Report of SCIA Engineer Software

1. Structural model



2. Materials

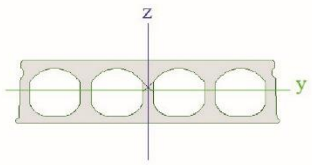
Steel EC3

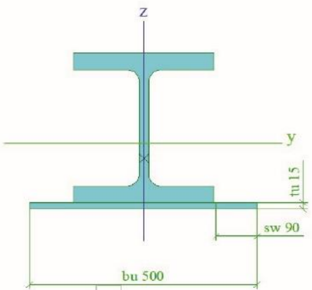
Name	ρ [kg/m³]	E_{mod} [MPa]	μ	Lower limit [mm]	Upper limit [mm]	F_y [MPa]	F_u [MPa]	Colour
		G_{mod} [MPa]	α [m/mK]					
S 235	7850.0	2.1000e+05	0.3	0	40	235.0	360.0	■
		8.0769e+04	0.00	40	80	215.0	360.0	

Name	Type	ρ [kg/m³]	Density in fresh state [kg/m³]	E_{mod} [MPa]	μ	α [m/mK]	$f_{c.k.28}$ [MPa]	Colour
C55/67	Concrete	2500.0	2500.0	3.8200e+04	0.2	0.00	55.00	■

Explanations of symbols	
Density in fresh state	The value in the density in fresh state property is used only in case a composite deck is input and its self-weight load is taken into account.

3. Cross-sections

HCS320		
Type	General cross-section	
Shape type	Thin-walled	
Item material	C55/67	
Fabrication	general	
Colour		
A [m²]	1.8346e-01	
A_y [m²], A_z [m²]	1.8346e-01	1.0882e-01
A_L [m²/m], A_D [m²/m]	3.0025e+00	6.1228e+00
α [deg]	0.00	
I_y [m⁴], I_z [m⁴]	2.4418e-03	2.2637e-02
i_y [mm], i_z [mm]	115	351
$W_{el.y}$ [m³], $W_{el.z}$ [m³]	1.4742e-02	3.7728e-02
$W_{pl.y}$ [m³], $W_{pl.z}$ [m³]	0.0000e+00	0.0000e+00
d_y [mm], d_z [mm]	0	16
I_t [m⁴], I_w [m⁶]	6.9459e-03	1.5928e-04
$β_y$ [mm], $β_z$ [mm]	-90	0
Picture		

HE320M1		
Type	SFB	
Detailed	HE320M; 500; 15; 90	
Formcode	153 - Slim Floor Beam	
Shape type	Thin-walled	
Item material	S 235	
Fabrication	welded	
Colour		
Flexural buckling y-y, Flexural buckling z-z	b	c
A [m²]	3.8716e-02	
A_y [m²], A_z [m²]	3.2553e-02	8.6016e-03
A_L [m²/m], A_D [m²/m]	2.2774e+00	2.2774e+00
α [deg]	0.00	
I_y [m⁴], I_z [m⁴]	2.0132e-03	3.5335e-04
k Iy	2.254	
i_y [mm], i_z [mm]	228	96
$W_{el.y}$ [m³], $W_{el.z}$ [m³]	9.3320e-03	1.4134e-03
$W_{pl.y}$ [m³], $W_{pl.z}$ [m³]	5.2045e-03	2.8885e-03
d_y [mm], d_z [mm]	0	-37
I_t [m⁴], I_w [m⁶]	2.4928e-05	7.3553e-06
$β_y$ [mm], $β_z$ [mm]	111	0
Picture		

4. Members

Name	Cross-section	Material	Length [m]	Beg. node	End node	Type
B1	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N1	N2	beam (80)
B2	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N2	N3	beam (80)
B3	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N3	N4	beam (80)
B4	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N4	N5	beam (80)
B5	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N1	N6	beam (80)
B6	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	10.000	N6	N7	beam (80)
B7	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	10.000	N5	N8	beam (80)
B8	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N7	N9	beam (80)
B9	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N9	N10	beam (80)
B10	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N10	N8	beam (80)
B11	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N6	N11	beam (80)
B12	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N11	N12	beam (80)
B13	HE320M1 - SFB (HE320M; 500; 15; 90)	S 235	8.000	N12	N5	beam (80)
B15	HCS320 - General cross-section	C55/67	8.000	N15	N16	beam (80)
B16	HCS320 - General cross-section	C55/67	8.000	N17	N18	beam (80)
B17	HCS320 - General cross-section	C55/67	8.000	N19	N20	beam (80)
B18	HCS320 - General cross-section	C55/67	8.000	N21	N22	beam (80)
B19	HCS320 - General cross-section	C55/67	8.000	N23	N24	beam (80)
B20	HCS320 - General cross-section	C55/67	8.000	N25	N26	beam (80)
B21	HCS320 - General cross-section	C55/67	8.000	N27	N28	beam (80)
B22	HCS320 - General cross-section	C55/67	8.000	N29	N30	beam (80)
B23	HCS320 - General cross-section	C55/67	8.000	N31	N32	beam (80)
B24	HCS320 - General cross-section	C55/67	8.000	N33	N34	beam (80)
B25	HCS320 - General cross-section	C55/67	8.000	N35	N36	beam (80)
B26	HCS320 - General cross-section	C55/67	8.000	N37	N38	beam (80)
B27	HCS320 - General cross-section	C55/67	8.000	N39	N40	beam (80)
B28	HCS320 - General cross-section	C55/67	8.000	N41	N42	beam (80)
B29	HCS320 - General cross-section	C55/67	8.000	N43	N44	beam (80)
B30	HCS320 - General cross-section	C55/67	8.000	N45	N46	beam (80)
B31	HCS320 - General cross-section	C55/67	8.000	N47	N48	beam (80)
B32	HCS320 - General cross-section	C55/67	8.000	N49	N50	beam (80)
B33	HCS320 - General cross-section	C55/67	8.000	N51	N52	beam (80)
B34	HCS320 - General cross-section	C55/67	8.000	N53	N54	beam (80)
B35	HCS320 - General cross-section	C55/67	10.000	N54	N56	beam (80)
B36	HCS320 - General cross-section	C55/67	10.000	N52	N58	beam (80)
B37	HCS320 - General cross-section	C55/67	10.000	N50	N60	beam (80)
B38	HCS320 - General cross-section	C55/67	10.000	N48	N62	beam (80)
B39	HCS320 - General cross-section	C55/67	10.000	N46	N64	beam (80)
B40	HCS320 - General cross-section	C55/67	10.000	N44	N66	beam (80)
B41	HCS320 - General cross-section	C55/67	10.000	N42	N68	beam (80)
B42	HCS320 - General cross-section	C55/67	10.000	N40	N70	beam (80)
B43	HCS320 - General cross-section	C55/67	10.000	N38	N72	beam (80)
B44	HCS320 - General cross-section	C55/67	10.000	N36	N74	beam (80)
B45	HCS320 - General cross-section	C55/67	10.000	N34	N76	beam (80)
B46	HCS320 - General cross-section	C55/67	10.000	N32	N78	beam (80)
B47	HCS320 - General cross-section	C55/67	10.000	N30	N80	beam (80)
B48	HCS320 - General cross-section	C55/67	10.000	N28	N82	beam (80)
B49	HCS320 - General cross-section	C55/67	10.000	N26	N84	beam (80)
B50	HCS320 - General cross-section	C55/67	10.000	N24	N86	beam (80)
B51	HCS320 - General cross-section	C55/67	10.000	N22	N88	beam (80)
B52	HCS320 - General cross-section	C55/67	10.000	N20	N90	beam (80)
B53	HCS320 - General cross-section	C55/67	10.000	N18	N92	beam (80)
B54	HCS320 - General cross-section	C55/67	10.000	N16	N94	beam (80)

Student version

5. Hinges

Name	Member	Position	ux	uy	uz	fix	fiy	fiz
H1	B10	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H2	B1	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H3	B2	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H4	B3	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H5	B4	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H6	B5	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H7	B6	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H8	B7	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H9	B8	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H10	B9	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H11	B11	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H12	B12	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid
H13	B13	Both	Rigid	Rigid	Rigid	Rigid	Free	Rigid

6. Nodal supports

Name	Node	System	Type	X	Y	Z	Rx	Ry	Rz
Sn1	N4	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn2	N3	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn3	N27	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn4	N1	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn5	N5	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn6	N12	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn7	N11	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn8	N6	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn9	N8	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn10	N10	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn11	N9	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn12	N7	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free

7. Load cases

Name	Description	Action type	Load group	Direction	Duration	Master load case
	Spec	Load type				
LC1	Self weight	Permanent Self weight	LG1	-Z		
LC2	Dead loads	Permanent Standard	LG1			
LC3	Topping	Permanent Standard	LG1			
LC4	Live loads Standard	Variable Static	LG2		Short	None

8. Mass groups

Name	Load case
MG1	LC1 - Self weight
MG2	LC2 - Dead loads
MG3	LC3 - Topping
MG4	LC4 - Live loads

9. Combination of mass groups

Name	Mass group	Coeff. [-]
CM1	MG1	1.00
	MG2	1.00
	MG3	1.00
	MG4	0.10

10. Line force

Name	Member	Type	Dir	Value - P_1 [kN/m]	Pos x_1	Coor	Orig	Ecc ey [m]
	Load case	System	Distribution	Value - P_2 [kN/m]	Pos x_2	Loc		Ecc ez [m]
LF14	B15	Force	Z	-3.24	0.000	Rela	From start	0.000
	LC2 - Dead loads	LCS	Uniform		1.000	Length		0.000
LF42	B15	Force	Z	-1.15	0.000	Rela	From start	0.000
	LC3 - Topping	LCS	Uniform		1.000	Length		0.000
LF85	B15	Force	Z	-4.32	0.000	Rela	From start	0.000
	LC4 - Live loads	LCS	Uniform		1.000	Length		0.000

11. 3D displacement

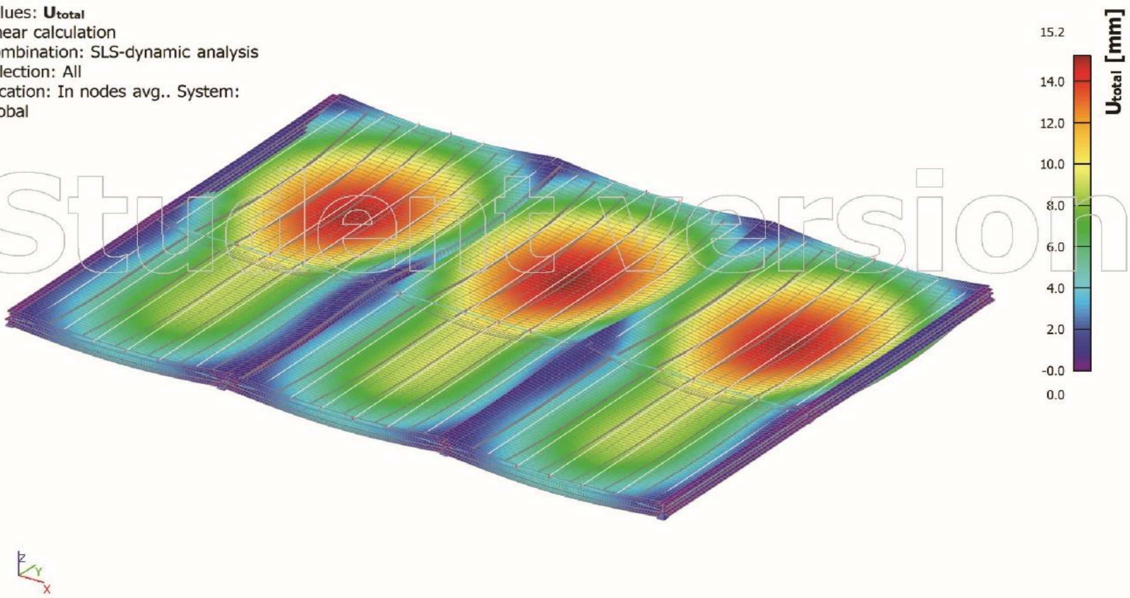
Linear calculation
Combination: SLS-dynamic analysis
Selection: All
Location: In nodes avg. on macro. System: LCS mesh element
Results on 1D member:
Extreme 1D: Global

Name	dx [m]	Fibre	Case	u_x [mm]	u_y [mm]	u_z [mm]	φ_x [mrad]	φ_y [mrad]	φ_z [mrad]	U_{total} [mm]
B4	0.000	8	SLS-dynamic analysis/1	0.0	0.0	0.0	0.0	0.2	0.0	**0.0**
B45	4.800-	14	SLS-dynamic analysis/2	0.0	0.1	-15.2	0.7	0.0	0.0	**15.2**

Name	Combination key
SLS-dynamic analysis/1	LC1 + LC2 + LC3
SLS-dynamic analysis/2	LC1 + LC2 + LC3 + 0.10*LC4

12. 3D displacement; U_total

Values: U_{total}
Linear calculation
Combination: SLS-dynamic analysis
Selection: All
Location: In nodes avg.. System: Global



13. Eigen frequencies

N	f [Hz]	ω [1/s]	ω² [1/s²]	T [s]
Mass combination : CM1				
1	4.79	30.11	906.48	0.21
2	4.82	30.30	918.13	0.21
3	4.82	30.30	918.32	0.21
4	5.68	35.68	1272.99	0.18
5	5.69	35.74	1277.28	0.18

14. Combination of mass groups

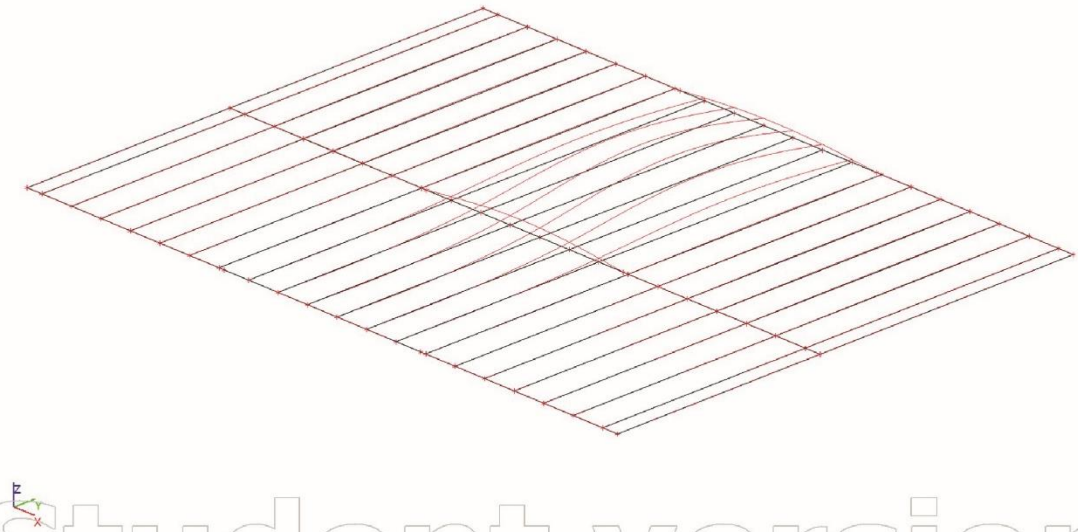
14.1. Combination of mass groups - CM1/1 - 4.79

Name
CM1/1 - 4.79

14.1.1. Deformed structure

Eigen solution, Extreme : Global, System : Principal
Selection : All
Mass combinations : CM1/1 - 4.79
Modal shapes are normalized, so that the generalized modal mass of each mode is equal to 1kg.

Case	Member	dx [m]	Ux [mm]	Uy [mm]	Uz [mm]	Fix [mrad]	Fiy [mrad]	Fiz [mrad]
CM1/1 - 4.79	B1	0.000	**0.0**	**0.0**	0.0	0.0	0.0	**0.0**
CM1/1 - 4.79	B24	2.560	0.0	0.0	**-0.2**	0.0	-0.1	0.0
CM1/1 - 4.79	B45	5.200	0.0	0.0	**7.0**	0.0	-0.2	0.0
CM1/1 - 4.79	B9	3.400	0.0	0.0	1.7	**-1.7**	-0.2	0.0
CM1/1 - 4.79	B45	1.200	0.0	0.0	3.7	**1.3**	-0.2	0.0
CM1/1 - 4.79	B12	0.000	0.0	0.0	0.0	0.1	**-0.9**	0.0
CM1/1 - 4.79	B12	8.000	0.0	0.0	0.0	0.1	**0.9**	0.0



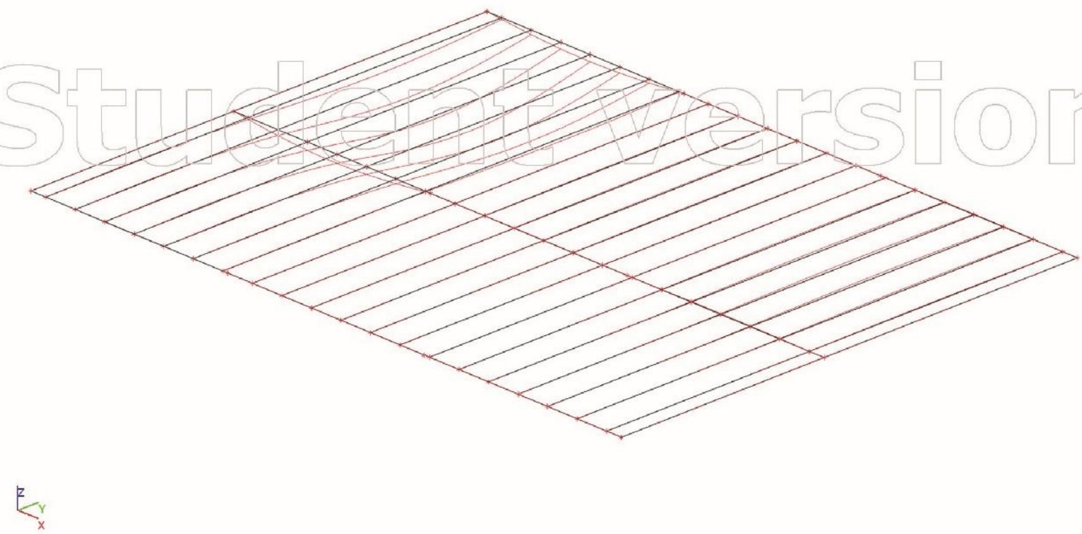
14.2. Combination of mass groups - CM1/2 - 4.82

Name
CM1/2 - 4.82

14.2.1. Deformed structure

Eigen solution, Extreme : Global, System : Principal
Selection : All
Mass combinations : CM1/2 - 4.82
Modal shapes are normalized, so that the generalized modal mass of each mode is equal to 1kg.

Case	Member	dx [m]	Ux [mm]	Uy [mm]	Uz [mm]	Fix [mrad]	Fiy [mrad]	Fiz [mrad]
CM1/2 - 4.82	B1	0.000	**0.0**	**0.0**	0.0	0.1	0.0	**0.0**
CM1/2 - 4.82	B51	5.200	0.0	0.0	**-7.2**	0.0	-0.1	0.0
CM1/2 - 4.82	B38	5.200	0.0	0.0	**0.6**	0.0	0.0	0.0
CM1/2 - 4.82	B51	1.400	0.0	0.0	-4.0	**-1.3**	-0.1	0.0
CM1/2 - 4.82	B8	4.200	0.0	0.0	-1.8	**1.8**	-0.1	0.0
CM1/2 - 4.82	B11	8.000	0.0	0.0	0.0	-0.1	**-0.9**	0.0
CM1/2 - 4.82	B11	0.000	0.0	0.0	0.0	-0.3	**0.9**	0.0



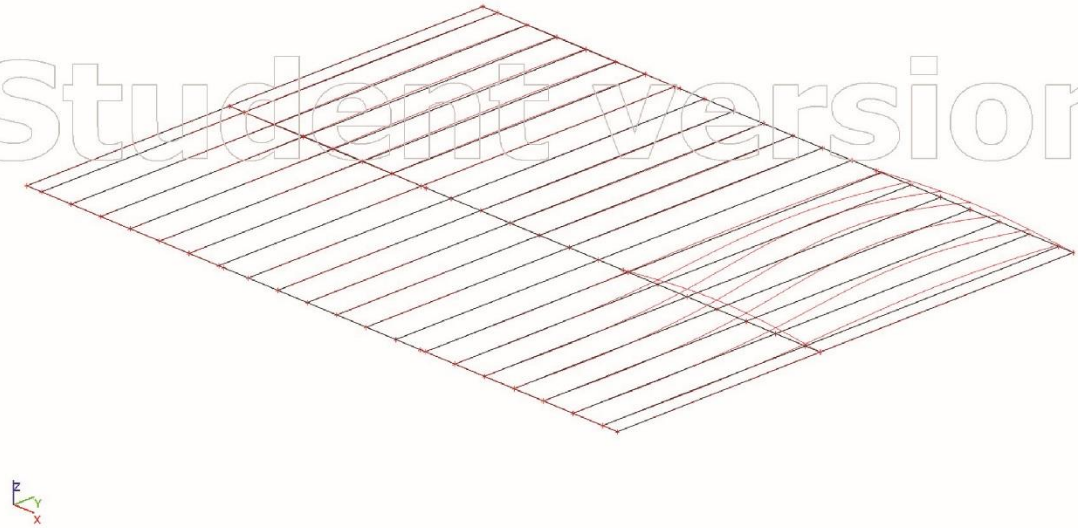
14.3. Combination of mass groups - CM1/3 - 4.82

Name
CM1/3 - 4.82

14.3.1. Deformed structure

Eigen solution, Extreme : Global, System : Principal
Selection : All
Mass combinations : CM1/3 - 4.82
Modal shapes are normalized, so that the generalized modal mass of each mode is equal to 1kg.

Case	Member	dx [m]	Ux [mm]	Uy [mm]	Uz [mm]	Fix [mrad]	Fiy [mrad]	Fiz [mrad]
CM1/3 - 4.82	B1	0.000	**0.0**	**0.0**	0.0	0.0	0.0	**0.0**
CM1/3 - 4.82	B45	5.200	0.0	0.0	**-0.3**	0.0	0.0	0.0
CM1/3 - 4.82	B38	5.200	0.0	0.0	**7.2**	0.0	-0.1	0.0
CM1/3 - 4.82	B10	3.800	0.0	0.0	1.8	**-1.8**	-0.1	0.0
CM1/3 - 4.82	B38	1.400	0.0	0.0	4.0	**1.3**	-0.1	0.0
CM1/3 - 4.82	B13	0.000	0.0	0.0	0.0	0.1	**-0.9**	0.0
CM1/3 - 4.82	B13	8.000	0.0	0.0	0.0	0.3	**0.9**	0.0



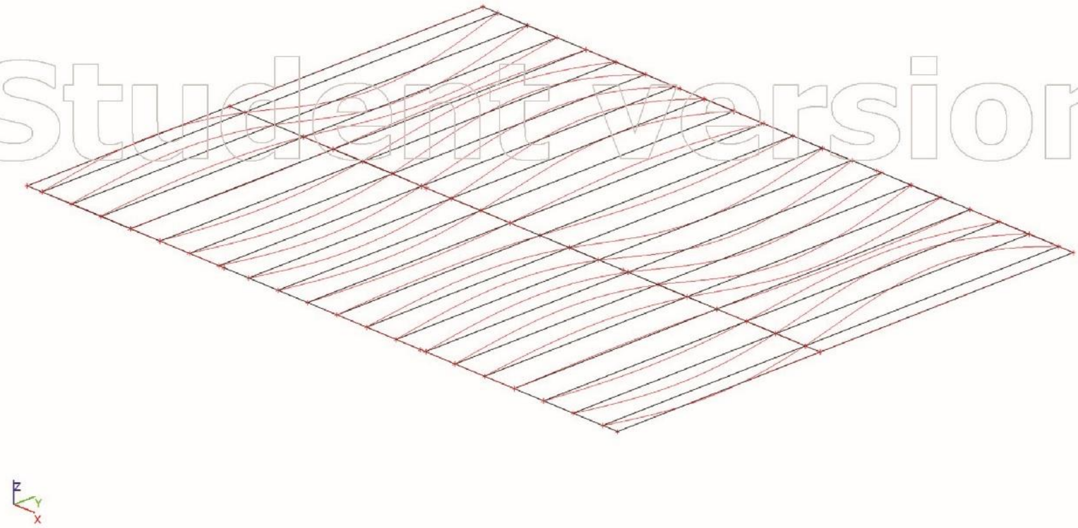
14.4. Combination of mass groups - CM1/4 - 5.68

Name
CM1/4 - 5.68

14.4.1. Deformed structure

Eigen solution, Extreme : Global, System : Principal
Selection : All
Mass combinations : CM1/4 - 5.68
Modal shapes are normalized, so that the generalized modal mass of each mode is equal to 1kg.

Case	Member	dx [m]	Ux [mm]	Uy [mm]	Uz [mm]	Fix [mrad]	Fiy [mrad]	Fiz [mrad]
CM1/4 - 5.68	B1	0.000	**0.0**	**0.0**	0.0	0.6	0.0	**0.0**
CM1/4 - 5.68	B42	5.200	0.0	0.0	**-3.8**	0.0	0.0	0.0
CM1/4 - 5.68	B35	5.200	0.0	0.0	**3.9**	0.0	0.0	0.0
CM1/4 - 5.68	B7	10.000	0.0	0.0	0.0	**-1.3**	0.0	0.0
CM1/4 - 5.68	B9	7.000	0.0	0.0	0.0	**1.2**	0.0	0.0
CM1/4 - 5.68	B1	0.000	0.0	0.0	0.0	0.6	**-0.1**	0.0
CM1/4 - 5.68	B11	8.000	0.0	0.0	0.0	0.9	**0.1**	0.0



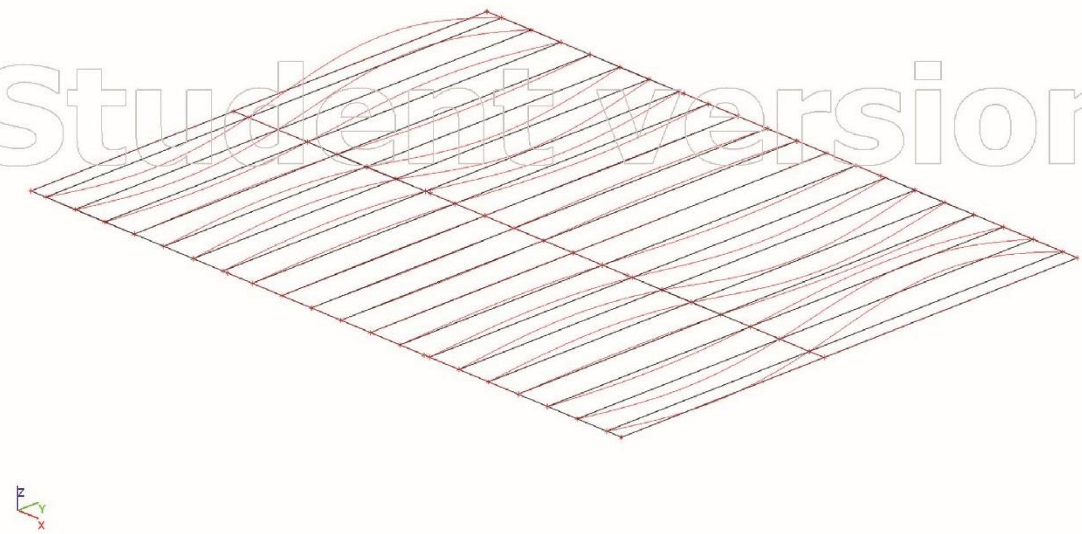
14.5. Combination of mass groups - CM1/5 - 5.69

Name
CM1/5 - 5.69

14.5.1. Deformed structure

Eigen solution, Extreme : Global, System : Principal
Selection : All
Mass combinations : CM1/5 - 5.69
Modal shapes are normalized, so that the generalized modal mass of each mode is equal to 1kg.

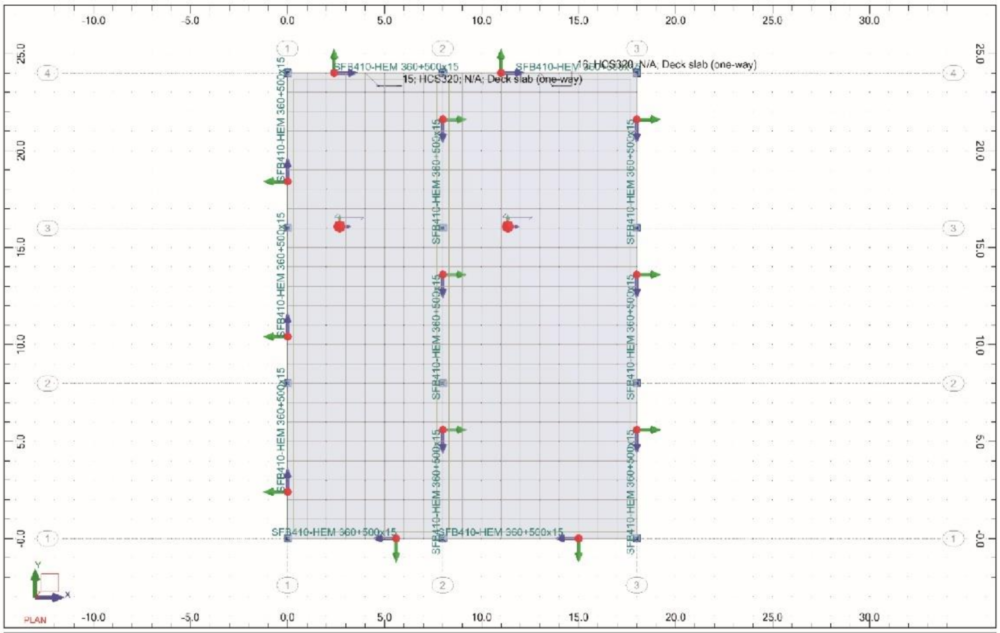
Case	Member	dx [m]	Ux [mm]	Uy [mm]	Uz [mm]	Fix [mrad]	Fiy [mrad]	Fiz [mrad]
CM1/5 - 5.69	B1	0.000	**0.0**	**0.0**	0.0	-0.9	0.0	**0.0**
CM1/5 - 5.69	B49	5.200	0.0	0.0	**-4.1**	0.0	0.0	0.0
CM1/5 - 5.69	B54	5.200	0.0	0.0	**5.2**	0.0	0.0	0.0
CM1/5 - 5.69	B6	10.000	0.0	0.0	0.0	**-1.7**	0.0	0.0
CM1/5 - 5.69	B54	1.200	0.0	0.0	1.7	**1.4**	0.0	0.0
CM1/5 - 5.69	B11	8.000	0.0	0.0	0.0	-0.6	**-0.1**	0.0
CM1/5 - 5.69	B10	0.000	0.0	0.0	0.0	0.8	**0.1**	0.0

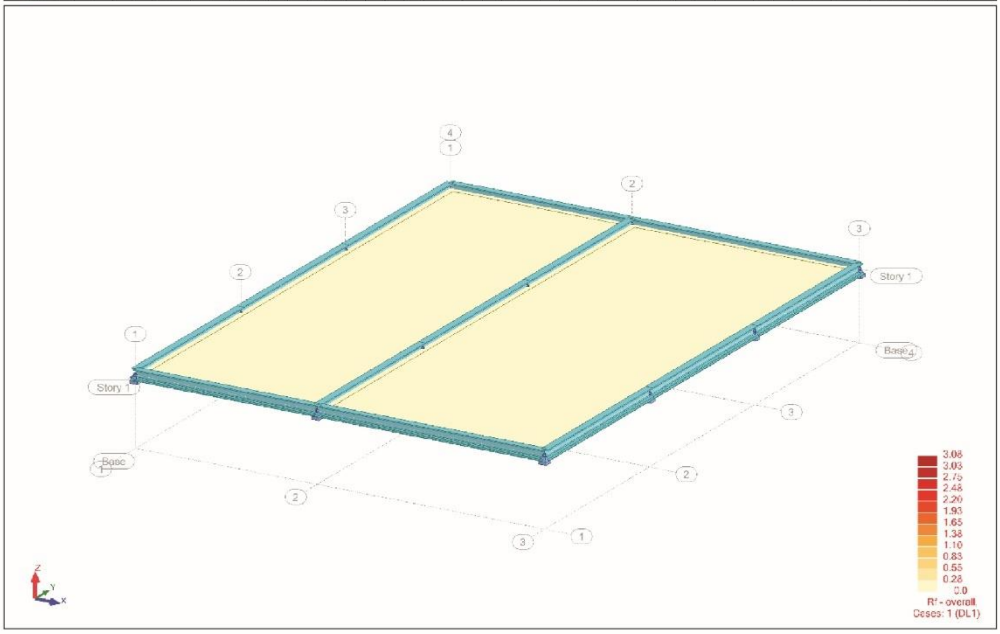


Student version

Appendix 3. Dynamic Analysis Report of Autodesk Robot Structural Analysis

Structure View





Data - Bars

Bar	Node 1	Node 2	Section	Material	Length (m)	Gamma (Deg)	Type
1	1	2	SFB410-HEM 360+500x15	S 235	8.00	0.0	Simple bar
2	2	3	SFB410-HEM 360+500x15	S 235	10.00	0.0	Simple bar
3	3	6	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
4	6	9	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
5	9	12	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
6	12	11	SFB410-HEM 360+500x15	S 235	10.00	0.0	Simple bar
7	11	10	SFB410-HEM 360+500x15	S 235	8.00	0.0	Simple bar
8	10	7	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
9	7	4	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
10	4	1	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
12	5	8	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
13	2	5	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar
14	8	11	SFB410-HEM 360+500x15	Steel	8.00	0.0	Simple bar

Data - Sections

Section name	Bar list	AX (cm2)	AY (cm2)	AZ (cm2)
SFB410-HEM 360+500x15	1to10 12to14	393.81	308.70	87.46

Section name	IX (cm4)	IY (cm4)	IZ (cm4)
SFB410-HEM 360+500x15	1820.25	110442.03	35146.80

Data - Materials

	Material	E (MPa)	G (MPa)	NI	LX (1/°C)	RO (kN/m3)	Re (MPa)
1	Steel	473340.00	81000.00	0.30	0.00	77.01	235.00
2	C20/25	30000.00	12500.00	0.20	0.00	24.53	20.00
3	S 235	210000.00	81000.00	0.30	0.00	77.01	235.00

Loads - Cases

Case	Label	Case name	Nature	Analysis type
1	DL1	DL1	Structural	Static - Linear
2	DL	Dead loads	Non-structural	Static - Linear
3	DL2	Topping	Non-Structural	Static - Linear
4	LL	Live load	Category B	Static - Linear
5	MOD5	Modal		Modal

Loads - Values

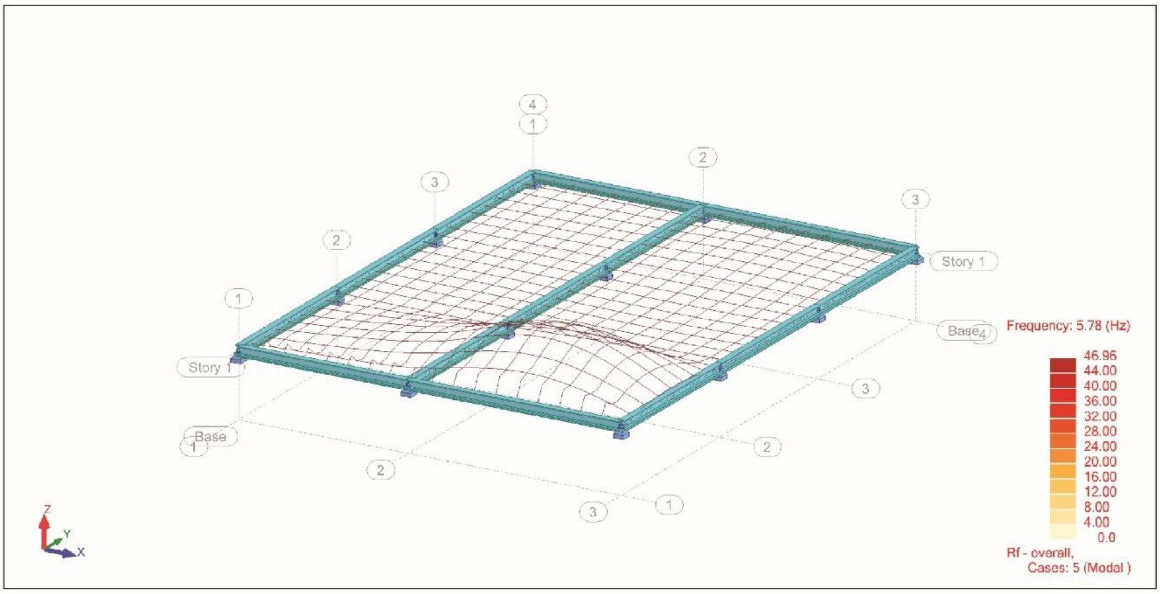
Case	Load type	List	Load values
1	self-weight	1to10 12to16	PZ Negative Factor=1.00
2	(FE) uniform	15 16	PZ=-2.70(kN/m2)
2	uniform load	3to5 8to10 12to14	PZ=-2.83(kN/m)
3	(FE) uniform	15 16	PZ=-1.00(kN/m2)
4	(FE) uniform	15 16	PZ=-3.60(kN/m2)

Eigenvalues

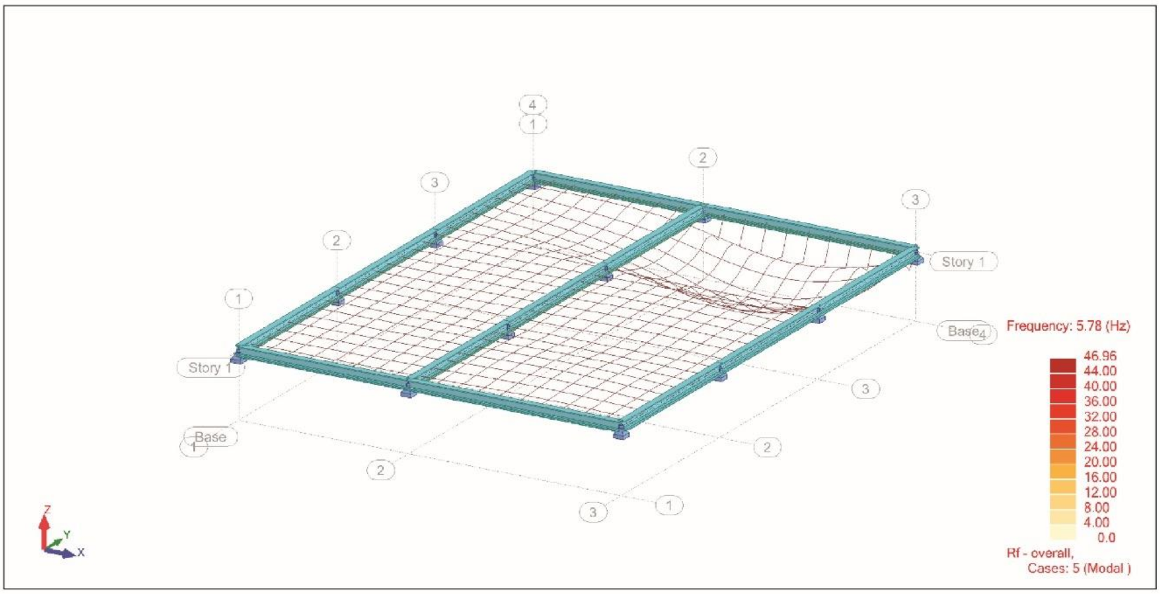
Case/Mode	Frequency (Hz)	Period (sec)	Rel.mas.UX (%)	Rel.mas.UY (%)	Rel.mas.UZ (%)	Cur.mas.UX (%)
5/ 1	5.78	0.17	0.00	0.00	17.56	0.00
5/ 2	5.78	0.17	0.00	0.00	27.98	0.00
5/ 3	5.80	0.17	0.00	0.00	48.58	0.00
5/ 4	6.13	0.16	0.00	0.00	49.21	0.00
5/ 5	6.27	0.16	0.00	0.00	49.64	0.00
5/ 6	6.49	0.15	0.00	0.00	49.67	0.00
5/ 7	7.19	0.14	0.00	0.00	55.65	0.00
5/ 8	7.20	0.14	0.00	0.00	55.66	0.00
5/ 9	7.21	0.14	0.00	0.00	58.29	0.00
5/ 10	7.35	0.14	0.00	0.00	58.29	0.00

Case/Mode	Cur.mas.UY (%)	Cur.mas.UZ (%)	Total mass UX (kg)	Total mass UY (kg)	Total mass UZ (kg)
5/ 1	0.00	17.56	412652.83	412652.83	412652.83
5/ 2	0.00	10.42	412652.83	412652.83	412652.83
5/ 3	0.00	20.61	412652.83	412652.83	412652.83
5/ 4	0.00	0.62	412652.83	412652.83	412652.83
5/ 5	0.00	0.43	412652.83	412652.83	412652.83
5/ 6	0.00	0.03	412652.83	412652.83	412652.83
5/ 7	0.00	5.98	412652.83	412652.83	412652.83
5/ 8	0.00	0.01	412652.83	412652.83	412652.83
5/ 9	0.00	2.63	412652.83	412652.83	412652.83
5/ 10	0.00	0.00	412652.83	412652.83	412652.83

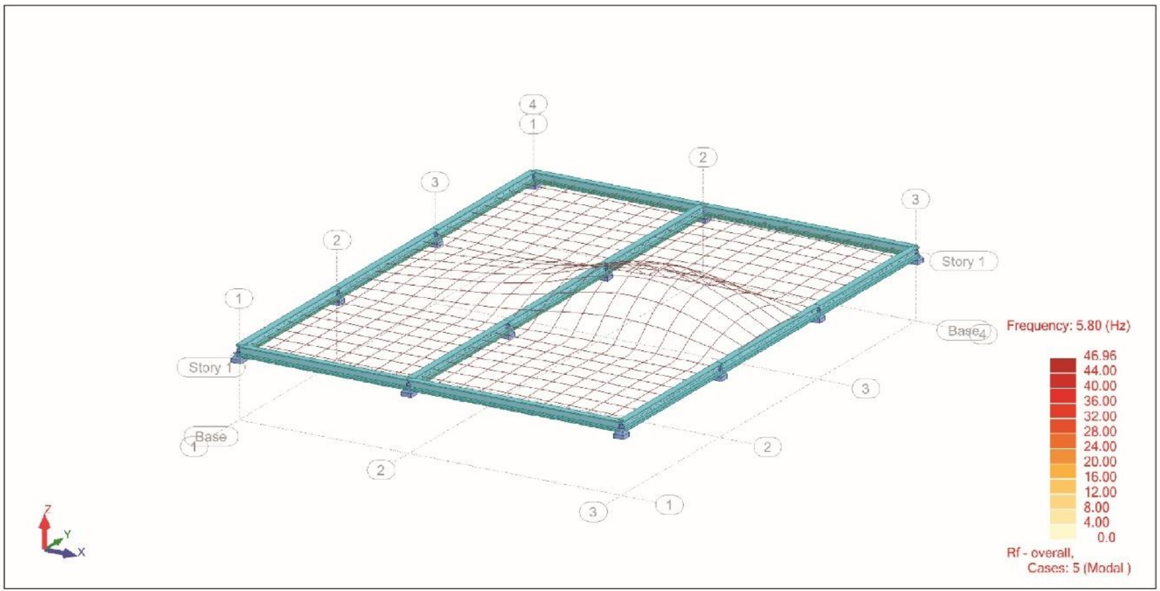
Mode 1


Frequency: 5.78 (Hz)
46.96
44.00
40.00
36.00
32.00
28.00
24.00
20.00
16.00
12.00
8.00
4.00
0.0
Rf - overall,
Cases: 5 (Modal)
Story 1
Base

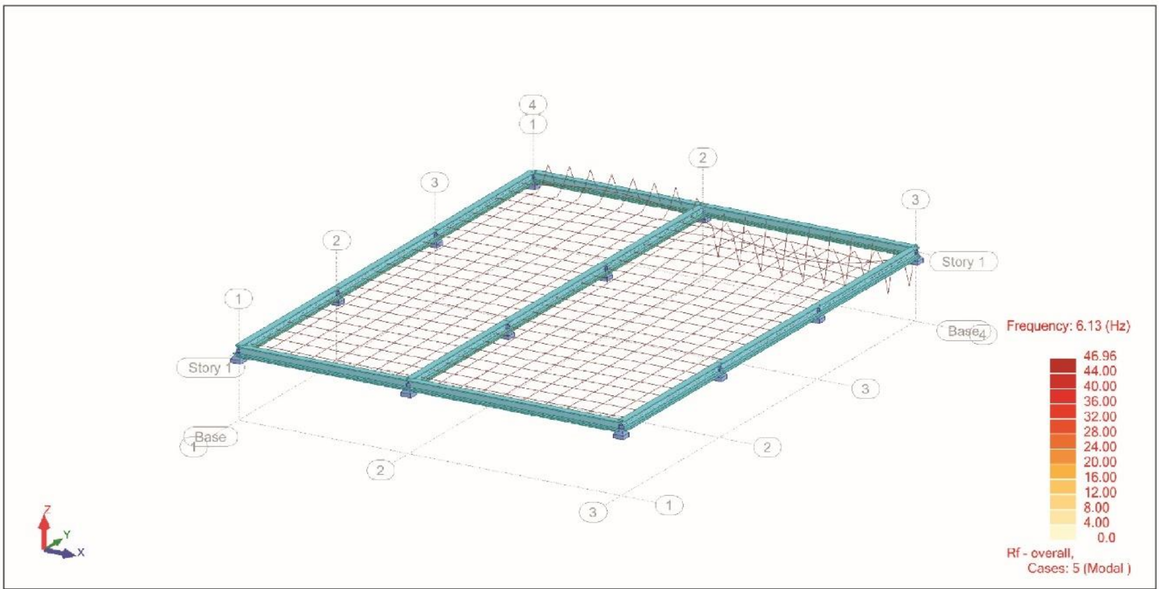
Mode 2


Frequency: 5.78 (Hz)
46.96
44.00
40.00
36.00
32.00
28.00
24.00
20.00
16.00
12.00
8.00
4.00
0.0
Rf - overall,
Cases: 5 (Modal)
Story 1
Base

Mode 3


Story 1
Base
Frequency: 5.80 (Hz)
46.96
44.00
40.00
36.00
32.00
28.00
24.00
20.00
16.00
12.00
8.00
4.00
0.0
Rf - overall,
Cases: 5 (Modal)

Mode 4


Story 1
Base
Frequency: 6.13 (Hz)
46.96
44.00
40.00
36.00
32.00
28.00
24.00
20.00
16.00
12.00
8.00
4.00
0.0
Rf - overall,
Cases: 5 (Modal)

Maximum Response Factor