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Kaunas University of Technology

Faculty of Mechanical Engineering and Design

Glass Fiber Composites Characterisation for Engineering Structures Design by Finite Element Method

Master's Final Degree Project

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Kaunas, 2019

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Mechanical Engineering (code 6211EX009)

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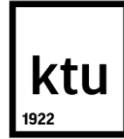
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Kaunas University of Technology

Faculty of Mechanical Engineering and Design

Study programme: MECHANICAL ENGINERING 6211EX009

Task Assignment for Final Degree Project of Masters Studies

Student – Antanas Valinčius

1. Title of the Project –

Glass Fiber Composites Characterisation For Engineering Structures Design By Finite Element Method

(In English)

Stiklo Pluošto Kompozitų Charakterizavimas Inžinerinių Konstrukcijų Projektavimui Baigtinių Elementų Metodu

(In Lithuaniain)

2. Aim and Tasks of the Project –

This project is designed to determine and analyze the mechanical characteristics of glass fiber lamina, and investigate the possibility to use this properties in laminate analysis by fine element method.

3. Initial Data –

4. Main Requirements and Conditions –

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Summary

This project is designed to determine and analyze the mechanical characteristics of glass fiber lamina, and investigate the possibility to use this properties in laminate analysis by fine element method. In order to achieve this goal, the experimental and analytical methods of determining composite properties were analyzed and applied. Obtained values were compared with each other. Using lamina mechanical properties by the finite element method, a multilayer multidirectional glass fiber laminate sample was modeled. The finite elements were used to calculate composite elasticity and strength, then the resulting values were compared with experimental values.

Santrauka

Šiame projekte siekiama pateikti stiklo pluošto laminos (vienos perdengimo-sluoksnio) mechaninių charakteristikų nustatymo metodika ir išnagrinėti galimybe panaudoti nustatytas savybes projektuojant kompozito laminatą (daugiasluoksnį įvairiakryptį kompozitą) baigtinių elementų metodu. Minėtam tikslui pasiekti buvo išanalizuotos ir pritaikytos eksperimentinės ir analitinės kompozito savybių nustatymo metodikos. Gauti rezultatai buvo palyginti tarpusavyje. Panaudojus laminos mechaninęs charakteristikas, baigtiniu elementu metodu buvo sumodeliuotas daugiasluoksnis įvairiakryptis stiklo pluošto bandinys. Naudojant baigtinius elementus buvo apskaičiuotas jo elastingumas ir stiprumas, gautos reikšmės buvo palyginti su eksperimentiškai gautomis reikšmėmis.

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Introduction

Composite materials have become an important part of today's industry due to its wide range of use and many advantages compared with traditional materials. In recent years, a lot of research has been carried out in the field of engineering design of composites material and development to improve mechanical and thermal properties such as rigidity, high strength to weight ratio, high stiffness to weight ratio, thermal resistance, and so on. [1].

The fiber-reinforced polymer (composite) is mainly made of continuous or short (chopped) fiber supported by polymer matrix[2]. Polymer matrix usually are made up of polyester thermosetting plastic or epoxy vinylester [3] the reason of this is very good properties and relatively low material prices. Glass, aramid, carbon, cellulose and silicon carbide are commonly used in composite as reinforce fibers in composites. Fiber added to the matrix helps to improve such cuolitis as strength and elasticity modulus and impact resistance[4 There are many factors that affect final properties of fiber-reinforced polymer. The three most important of these are interface adhesion, matrix properties, geometry, and fiber orientation. Composites are mostly used in construction and transport industry and in many old applications [1].

A key factor in increasing the properties of fiber-reinforced material is to validate various options, such as the fiber volume fraction, different thickness, differtnt laminated martial and sifferent fiber orientation[3-5].

To obtain mechanical properties of composite materials there are several ways such as experimental, analytical and computational methods.Computational methods need specialized software which can create additional costs. But in nowadays there are appearing computer-aided design (CAD) software which without traditional package (creation, modification, analysis, or optimization of a design) allow and simulation whit composite. One of such software is SolidWorks. Unfortunate this software need additional composite lamina properties. The material properties needed for a SolidWorks simulation solution vary by the solution type i.e. static, thermal, fatigue, frequency and buckling[6]. However, they can generally be put into tow categories: 1. Properties required to calculate a response; Input Properties. 2. Properties required to interpret a response; Failure properties.[6]. This project is designed to determine and analyze the mechanical characteristics of glass fiber lamina, and investigate the possibility to use this properties in laminate analysis by fine element method.

Tasks:

- Determinate analytical calculation methods for composite mechanical properties;
- Calculate mechanical properties of composite laminate using defined approaches;
- Expermentaly determinate mechanical characteristics of composite samples manufactured by UAB "Traidenis" using glass fiber filament winding process;
- Create SolidWork FE model of simple shape composite structure and experimentally validate it;
- Analyze experimentally and theoretically obtained mechanical properties of composite;

1. Brief introduction to the composite:

A composite material (also known as composition material, fiber reinforce plastic) a material made of two or more ingredients (Fig. 1) having substantially different physical and chemical properties, when mixed together create material with properties different from bout of initial components [7]. Separate constituents retain their structure, which is why the composites are separated from mixtures and solid bodies.
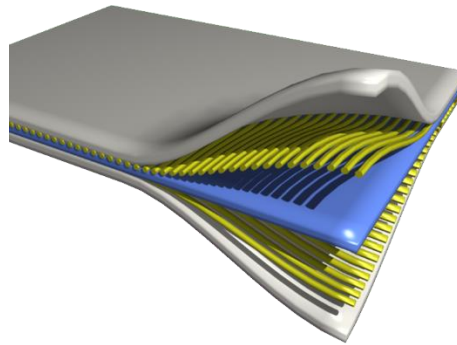
Fig. 1. Composites made by combining materials and forming a common structure with properties that are unlike sum of the properties of the individual ingredient. [8]

Reinforced fiber is increasingly being used for reasons such as higher strength, low weight and in some case being cheaper compared to traditional materials.

The composites can be sorted according to the nature and arrangement of the reinforcing material such breakdown is provided in (Fig.2) [9]:
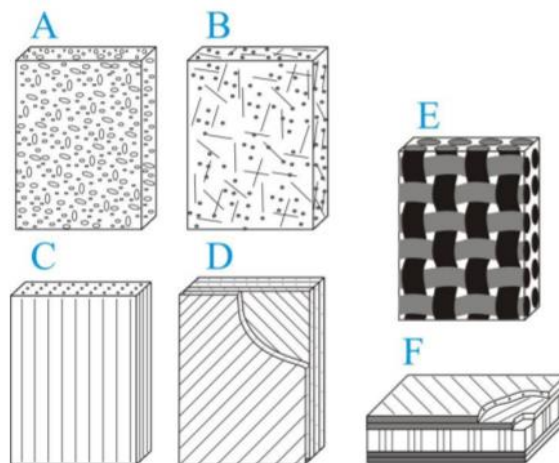
Fig. 2. Different types of composites**:** A - composites reinforced by particles; B - composites reinforced by chopped strands;C - unidirectional composites; D - laminates; E - fabric reinforced plastics; F - honeycomb composite structure[9].

As mentioned above, the composites are composed of two or more independent materials. Therefore it is acceptable to divide composite components in two categories: matrix and reinforcement. The material is called composite only if it has at least one element from the abovementioned categories. The matrix material provides the solidity of reinforcement material to form the shape and keep it. Reinforcement provides its special physical and mechanical properties to improve composite

properties. This combination allows you to get the material with features that are not available from individual materials, and the choice of matrix and reinforcement materials allows the product or structure designer to choose the most appropriate combination.

Created composite materials are formed according to a particular shape. The matrix material may be incorporated into the reinforcement under or prior placing fiber on the mould surface or into the mould cavity. The matrix material goes through the process of concoction, after which the composite gets a rigid form. Depending on the nature of the matrix material, the yield of the material may vary for example, chemical polymerization of a thermosetting polymer matrix or curing of a thermoplastic polymer matrix composite from a melted state [10].

Depending on the final product design needs, various composite manufacturing techniques may be used. One of the most important factors influencing the methodology is the nature of the selected components. Also, when choosing a production method, attention should be paid to the overall volume of production. Having large production in might you can invest in automated manufacturing technologies. Low production volumes are associated with lower investment, but higher labor, tool and equipment costs which have led to lower pace and higher unit cost of production.

Commercially produced composites often use a polymeric matrix which may be referred to as the resin solution. We have many different polymers, which are dependent on raw materials. Among which several types dominate with lots of variations. The most favored are well-known as, vinyl ester, epoxy, phenolic, polyimide, polyamide, polypropylene, PEEK, and others [8]. Most often reinforcement materials are fiber, but as reinforcement materials also commonly fine mineral substance can be used. Many different techniques have been developed to reduce the amount of resin or increase the amount of fiber it is becose fiber fraction is one of the most important factors influencing strenght of composite. A few examples of developed technology is rule of thumb, wich lay up results in a product containing 60% resin and 40% fibre, and vacuum infusion gives where final product givs 40% resin and 60% fibre [11].

1.1. Matrices:

In order to use the high strength and stiffness of the fiber in practice, the fiber is combined with a matrix, which is a material with much lower strength and stiffness than the reinforcing material. The Composite matrix not only provides the final form of the composite structure but also determines the production process. A correct chousen combination of fiber and matrix results in the desired production and operational results.

Recently, the most popular material in the composition used as a matrix is made of polymers. In most sources, polymer matrices are divided into two main types: thermoplastic and thermoset. Thermoset Polymers are the most widely used matrix materials for the production of progressive composites. Most popular thermoset are polyester, polyimide, epoxy and other resins vulcanize under elevated or room temperature[12]. Cured epoxy resin has typical stress-strain diagram shown in (Fig.3) which are provide from source (12). The polymerized thermoset matrix cannot be re-formed or used. Repeated heating of this material reduces its strength and stiffness and causes premature failure.
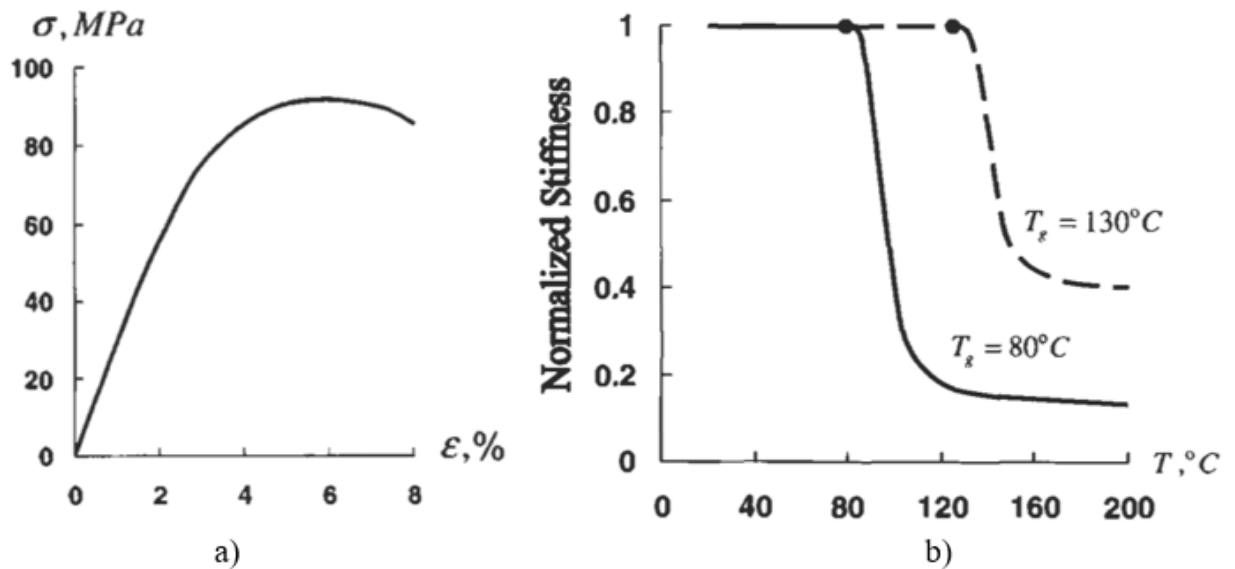
Fig. 3. a – stress (σ) and strain (ε) diagram for a typical cured epoxy matrix; b – typical thermo-mechanical diagrams for cured epoxy resins [12].

In contrast to thermostatic resins, thermoplastic resins do not need curing reactions. When heated it melts and hardens after cooling. Thermoplastic matrices can be repeatedly melted and hardened. This feature gives a big plus for thermoplastic in comparison to thermoset, which allows to change or return the appropriate form for composite parts also simplifies material recycling.

Polymeric matrices combined with glass fibres can provide composites with impresiv stiffness and strength, good fatigue resistance, smoll density, and great chemical resistance. But polymers have drawbacks, they are sensitive to ultraviolet rays, and has a low temperature resistance compared with metals. Thermomechanical curves are used to determine the effect of temperature on the matrix. some of these curves are provided in (Fig.3-b), from the source (12). Said curve shows the stiffness characteristics dependence from temperature, also the curve enables to determine glass transition temperature (T_g). Glass transition temperature which determines the temperature at which the material stiffness decreases significantly [12]. In the light of the previous observation, wanting to keep best set of composite properties it is recommended not to exceed the glass transition temperature. But it is worth noting that the performed studies [12] have shown that the material response to the temperature is influenced by the type of loading (Fig.4). From the graph we can see that overheating has huge effect on bending and compression. Composite longitudinal moduli and strength under longitudinal tension is less affected heating over the glass transition temperatureIt is known that the glass transition temperature depends on the processing temperature at which the material is made, the higher the processing temperatures, the higher the glass transition temperature [12].

Further composite temperature resistance improvement is related to the use of metal matrix. Fiber reinforcement substantially improves the mechanical properties of metals.
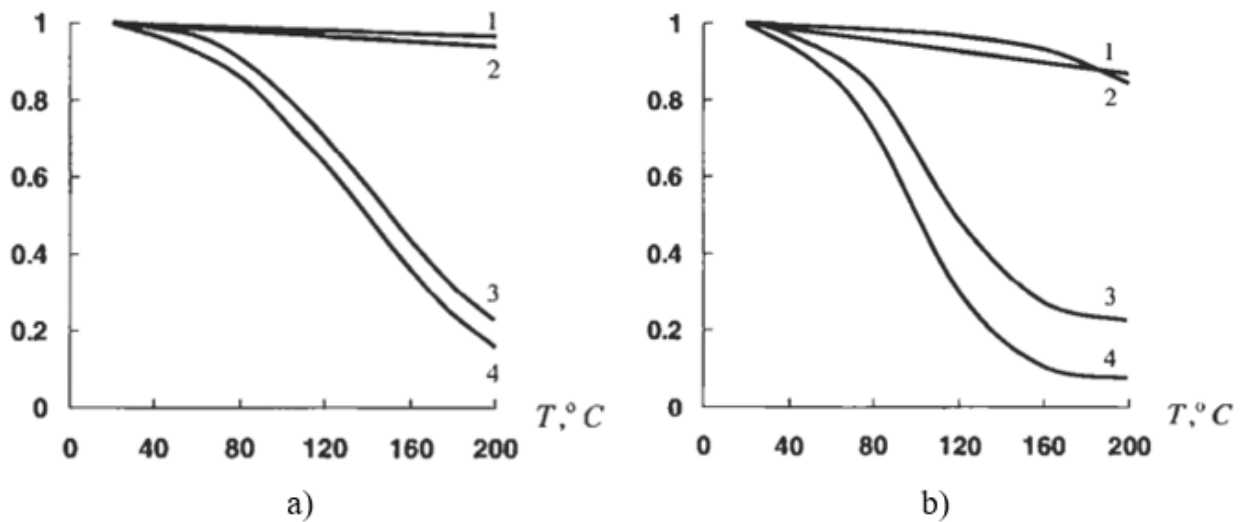
Fig. 4. Dependence of normalized longitudinal moduli (1), strength under longitudinal tension (2), bending (3), and compression (4) on temperature for unidirectional carbon composites with epoxy matrices having T_g= 130°C (a) and T_g, =80°C (b) [13].

1.2. Fibre:

Reinforcement usually allows to achieve higher stiffness and reduces the possibility of cracking [12]. Different fibers have different effects on the final properties of the composite, some of most popular fiber properties are shown (Fig.5).
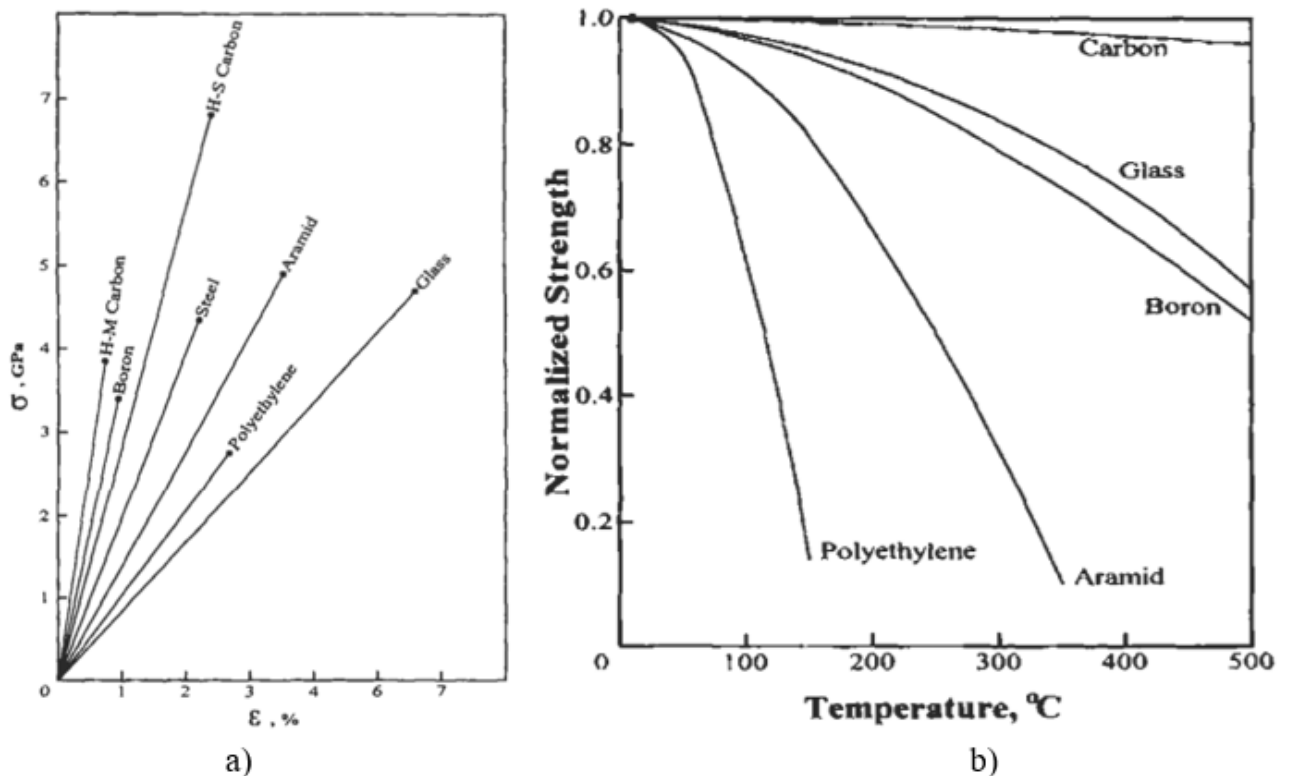
Fig. 5. a - Stress-strain diagrams for typical fibres of advanced composites; b – Temperature degradation of fibre strength normalized by the strength at 20°C.[12]

The fiber material according to its shape can be divided into three categories: 1 - particles (Fig.2 - A); 2 - chopped or short (Fig.2 - B) and 3 - continuous fiber (Fig.2 - C). From continuous fiber layers

(lamina) laminate are made. Such material are used where high strength and stiffness are needed. The particles and short fibres are usually are used in compression moulding and sheet moulding operations.

According to the analyzed sources, the most popular fibers used for reinforcement of composite is glass fibers, carbon fibers, and aramid.

1.3. Processing:

Matrix and reinforcement are combined while the structure are fabricating. Until now there was developed a wide range of composite fabrication technologies for composite production. The list of most popular/know fabrication technology are represented below [10]:

- Advanced fibre placement;
- Tailored fibre placement;
- Fibreglass spray lay-up process;
- Filament winding;
- Lanxide process;
- Tufting;
- Z-pinning.

The process of composite production consists this steps: 1. fiber wetting; 2. matrix induction (from chemical reaction or heat exposure matrix will gain solid structure) [10]: The aforementioned operation is usually carried out in forming mould which can be open or closed. However, the procedures and methods for introducing the ingredients may vary depending on the method. A list of popular methods is given below [10]:

- Mold overview;
- Vacuum bag moulding;
- Pressure bag moulding;
- Autoclave moulding;
- Resin transfer moulding.

1.4. Physical properties of composite material:

The physical properties of composite materials are commonly not isotropic it means that material properties are independent of direction of applied force in nature, but they are typically anisotropic it means that material properties have different depending on the direction of the applied force or load. For instance the stiffness of a composite panel will mainly rely upon the orientation of the applied forces and moments. The strength of a composite is bounded by two loading conditions as shown in (Fig.6)[13].

The physical properties of composite materials are usually orthotropic, which means that the properties of the materials depend on the direction of the load. Kompozito stiprumą riboja dvi apkrovos sąlygos išilginė (Iso-stress) ir skersine (Iso-strane) kryptimis(6 - 7 pav.) [13]. In the case of longitudinal loading it is determined that both phases of the composite will experience the same stress but the deformations between the phases will be different. A similar situation is obtained with the

load in the longitudinal direction only in this case the phases experience the same load but different stresses.

Elastic modulus of a composite
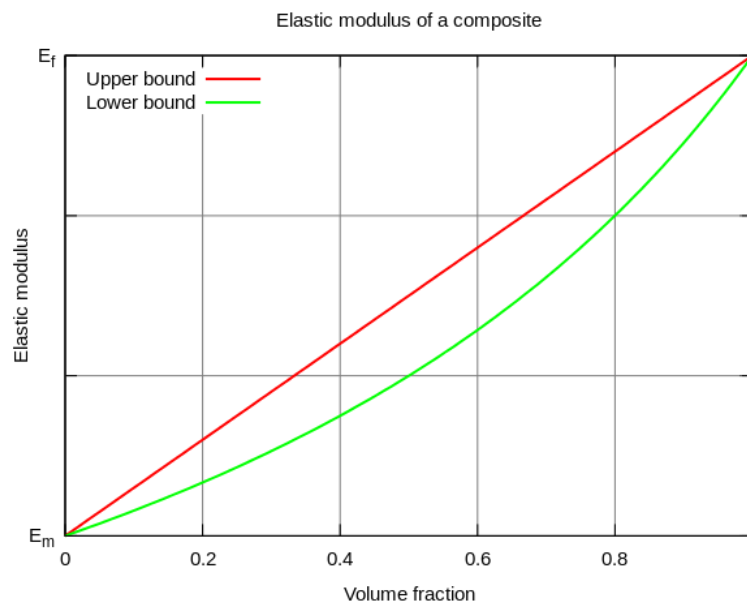
Fig. 6. Plot of the overall strength of a composite material as a function of fiber volume fraction limited by the upper bound "isostrain" and lower bound "isostress" conditions.[13].

(a) Iso-stress   **(b) Iso-strain**
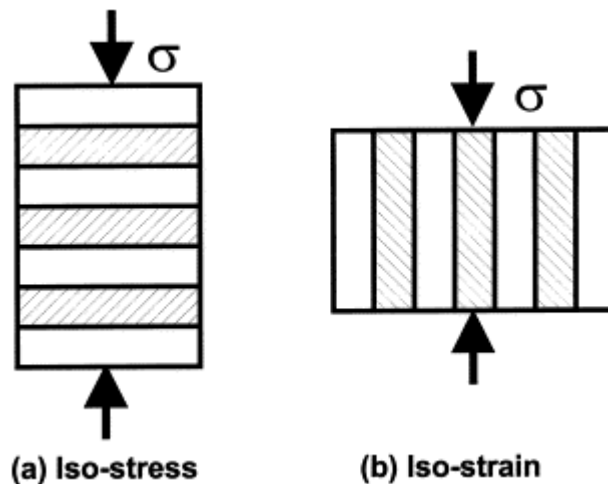
Fig. 7. a - shows the isostress condition where the composite materials are perpendicular to the applied force and b - is the isostrain condition that has the layers parallel to the force.[14]

Maximum composite stiffness and strength is achieved when the load is oriented along the fiber. Wen fiber has some angle misorientation there appears different failure causes. When the fiber orientation angle is close to 45 deg. the matrix shear failure occurs. When the misorientation angle is about 180 deg. the transverse matrix failure occurs, it is because the fiber are no longer obtain more loads the matrix.

Unlike composites, isotropic material, made from traditional materials have the same material properties in different directions. One of the most delicate differences between the mechanical characteristics of isotopic and anisotropic material properties (Poisson ratio, shear modulus and

Young module) is that anisotropic material instead of 3 parameter have 9, it is becose characteristic has different meanings in different directions (x, y, z). But since the composites are usually analyzed as the surface parameters number decreases to 6.

1.5. Design of composite products:

Design is defined as a process by which the problems of a customer are solved and transformed into product in such a way as to provide a profit to the producer [16]. Main goal of engineer is to succeed in these two tasks. In nowadays industry, it is not enough for the product to have sufficient strength and stiffness it is necessary to take into account factors such as cost, ecology, number of service cycles, mass and attractiveness for consumer. Therefore, the solution for any problem have to be well thought out and evaluate. So, it is recommended to evaluate each solution effect on all factors of the product and maintaining feedback with customers to ensure appropriate balance between profit and product cost.

Designer when designing a product from composite must understand that this material is different from traditional. One of the differences is that in manufacturing composite product we are producing material and in the same time shaping it to the final form of product. This can make the composite more expensive than other alternatives. Another difference is that composite is anisotropic material.

It is known that the matrix and reinforcing components have a major influence on composite characteristics. From this we have one of the major challenges which we face by designing a product from composite: it is to choose the best combination of matrix and fabric according given conditions. This task requires quick mathematical tasks solution for recurring equations, because there is a large choice of fibers and matrices, and it is also necessary to evaluate the arrangement of the fiber or lamina (one layer of composite).

2. Analytical approach of composite material mechanical properties.

2.1. Macromechanical analysis of lamina:

In this chapter we describe mathematical approach for composite lamina (layer) mechanical properties calculation. A lamina is a slim ply of a composite material [19] . A laminate of composite is made by stacking a few or more of such layers by increasing thickness (Fig.8). In order to analyze the properties of laminate, we first need to understand the properties and behavior of the single laminate [19].

Fiber cross-section   Matrix material
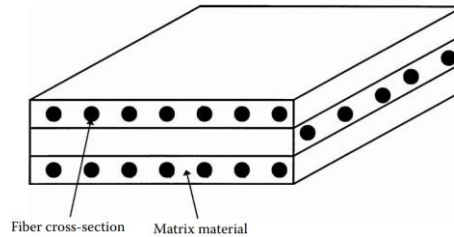
Fig. 8. Typical laminate made of three laminas.[18]

A lamina is not an isotropic homogeneous material it means that properties of this material depends from the place and direction [19]. For this reason, Lamin mechanical modeling is very complex process. For this reason the macromechanical analysis of a lamina is ground on average properties and taking into consideration the lamina to be homogeneous [18]. This methodology is based on the mechanical properties of individual fibers and matrices in terms of fiber content, packing geometry and shape.

2.2. Unidirectional composites fiber tensile properties calculation using rule-of-mixture

The rule-of-mixture is one of straightforward way for the calculation of mechanical properties in composite material[18-20]. Witch and was used to estimated some of mechanical behavior of glass fiber composite material. Calculation method presented further.

Volume fraction calculate according equations (1-5) [18-20]. Composit stiffnes can be calculated according equation (6) . In this research the density of composite was measured and then volume fraction of fiber was calculated according equation (2). The fiber and matrix weight fractions can be found using equation (1). In our case the densities of the fiber,matrix and composite was known so volume fraction of fiber was calculated according equation (3).

$$v_m = \frac{V_m}{V_c}; v_f = \frac{V_f}{V_c} \tag{1}$$

$$v_f + v_m = 1 \tag{2}$$

$$v_f = \frac{\rho_c - \rho_m}{\rho_f - \rho_m} \tag{3}$$

$$\rho_c = \rho_f \cdot v_f + \rho_m v_m \tag{4}$$

$$W_f = \frac{\rho_f}{\rho_c} v_f \qquad (5)$$

Where v – volume fraction; V – volume; ρ – density; W – weight; m – matrix index; f – reinforcement material index; c – composite index;

Elasticity modulus along fiber E_1 is calculated using equation [19]:

$$E_1 = E_f v_f + E_m v_m \qquad (6)$$

Elasticity modulus perpendicular to fiber direction E_2 is calculated using equation [19]:

$$E_2 = E_m \left(\frac{1 + 2\beta \cdot v_p}{1 - \beta \cdot v_p} \right) \qquad (7)$$

Where $\beta = \frac{(E_p/E_m)-1}{(E_p/E_m)+2}$; E_p, E_m – reinforcement fiber and matrix elasticity modulus.

Shear modulus G_{12} calculated using equation[19]:

$$G_{12} = G_m \frac{(G_p + G_m) + v_p(G_P - G_m)}{(G_p + G_m) - v_p(G_P - G_m)} \qquad (8)$$

Where G_p, G_m – reinforcement fiber and matrix shear modulus calculated according equations[19]:

$$G_p = \frac{E_p}{2(1 + v_p)}; G_m = \frac{E_m}{2(1 + v_m)} \qquad (9)$$

Where v_p, v_m – reinforcement fiber and matrix Poisson ratio.

Poisson ratio v_{12} can be calculated using equation[19]:

$$v_{12} = v_p v_p + v_m v_m \qquad (10)$$

2.3. Hooke's Law for a Two-Dimensional Angle Lamina

In laminate made just from unidirectional laminae has small stiffness and strength properties in transverse direction for this reson laminate contane plays arranged in different directions. Therefore, in order to calculate compozite, it is necessary to know the connections between lamina stacked at different angles

The global and local stresses in an angle lamina are related to each other through the angle of the lamina, θ [19]:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \qquad (11)$$

where [T] is called the transformation matrix and is defined as [19]

$$[T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}, \tag{12}$$

and

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}, \tag{13}$$

where $c = \cos\theta$ and $s = \sin\theta$.

Using the stress–strain equation in the local axes, equation can be written as [19]

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1}[Q] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{14}$$

were $[Q]$ stiffness elements matrix.

The global and local strains are also related through the transformation matrix[19]:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} - [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{bmatrix} \tag{15}$$

which can be rewritten as [19]

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{16}$$

where [R] is the Reuter matrix and is defined as [19]

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \tag{17}$$

Then, substituting equation (16) in equation (14) gives [19]

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1}[Q][R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{18}$$

On carrying the multiplication of the first five matrices on the right-hand side of equation (18), [19]

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{19}$$

where $\overline{Q_{ij}}$ are the elements of the transformed reduced stiffness matrix $[\overline{Q}\,]$ and are given below[19]

$$\overline{Q_{11}} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2, \tag{20}$$

$$\overline{Q_{12}} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4),$$ (21)

$$\overline{Q_{22}} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2,$$ (22)

$$\overline{Q_{16}} = (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})s^3c,$$ (23)

$$\overline{Q_{26}} = (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s,$$ (24)

$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4).$$ (25)

The transformed reduced stiffness matrix is just functions of the four stiffness elements, and the angle of lamina.

Inverting equation (19) gives us equation below [19]

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \overline{S_{11}} & \overline{S_{12}} & \overline{S_{16}} \\ \overline{S_{12}} & \overline{S_{22}} & \overline{S_{26}} \\ \overline{S_{16}} & \overline{S_{26}} & \overline{S_{66}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$ (26)

where S_{ij} are the elements of transformed reduced compliance matrix and is given by equation below [19]

$$\overline{S_{11}} = S_{11}c^4 + 2(S_{12} + 2S_{66})s^2c^2 + S_{22}s^4,$$ (27)

$$\overline{S_{12}} = S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66})s^2c^2,$$ (28)

$$\overline{S_{22}} = S_{11}s^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}c^4,$$ (29)

$$\overline{S_{16}} = (2S_{11} - S2_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c,$$ (30)

$$\overline{S_{26}} = (2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3,$$ (31)

$$\overline{S_{66}} = 2(2S_{11} + 2S_{22} - 4S_{12} - 2S_{66})s^2c^2 + S_{66}(s^4 + c^4).$$ (32)

Equation (14) and equation (19) is stress–strain equations for what are called a generally orthotropic lamina[19].

2.4. Engineering Constants of an Angle Lamina

In this section, techniques shown below are applied according references to relate the engineering constants of an angle ply to its transformed stiffness and compliance matrices[19].

a)   b)   c)
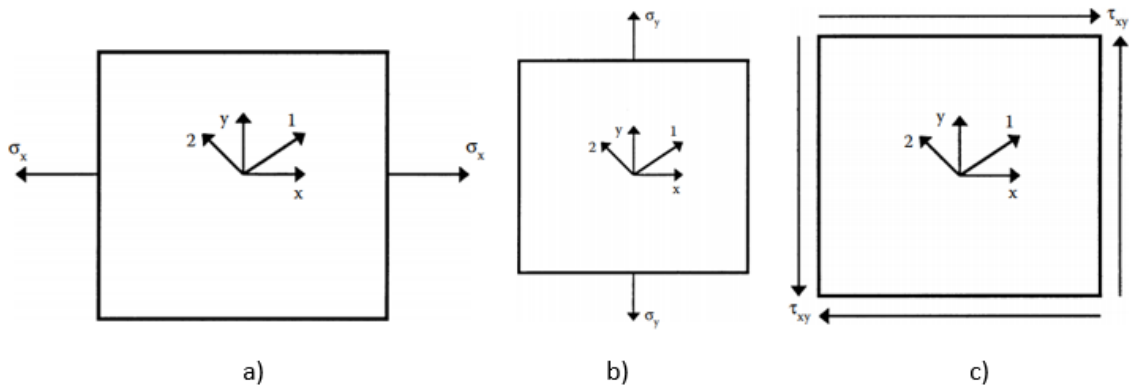
Fig. 9. Application of stresses to find engineering constants of an angle lamina[19].

Application of stresses to find engineering constants of an angle lamina[19].

1. For finding the engineering elastic moduli in direction x (Fig.9 - a), we are applying such conditions[19]:

$$\sigma_x \neq 0, \sigma_y = 0, \tau_{xy} = 0. \tag{33}$$

Next, from equation (33),we get :[19]

$$\varepsilon_x = \overline{S_{11}}\sigma_x \tag{34}$$

$$\varepsilon_y = \overline{S_{12}}\sigma_x \tag{35}$$

$$\gamma_{xy} = \overline{S_{16}}\sigma_x \tag{36}$$

The elastic moduli in direction x are defined as shown below[19]:

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{\overline{S_{11}}} \tag{37}$$

Also, the Poisson's ratio, v_{xy}, are defined as shown below[19]:

$$v_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\overline{S_{12}}}{\overline{S_{11}}} \tag{38}$$

In an angle lamina, dissimilar as in a unidirectional lamina, intercourse also occurs within the shear strain and the normal stresses[19]. Is called shear coupling. The shear coupling term relates normal stress in the x-direction to the shear strain is indicate as m_x and is defined by equation below [19]:

$$\frac{1}{m_x} = -\frac{\sigma_x}{\gamma_{xy}E_1} = -\frac{1}{\overline{S_{16}}E_1} \tag{39}$$

It can be noticed that m_x is a not dimensional parameter as a Poisson's ratio. Later was noticed that the same parameter m_x relates shearing stress in the x–y plane to the normal strain to direction -x [20].

1. For finding the engineering elastic moduli in direction y (Fig.9 - b), we are applying such conditions[19]:

$$\sigma_x = 0, \sigma_y \neq 0, \tau_{xy} = 0. \tag{40}$$

as shown in (Fig.9 - b), it can be found that elastic moduli (E_y), Poisson's ratio (v_{yx}) and shear coupling (m_y) can be calculated according equations (41-43) [19]:

$$E_y = \frac{1}{\overline{S_{22}}} \tag{41}$$

$$v_{yx} = -\frac{\overline{S_{12}}}{\overline{S_{22}}} \tag{42}$$

$$\frac{1}{m_y} = -\frac{1}{S_{26}E_1} \tag{43}$$

From equations (37), (38), (41), and (42), the reciprocal relationship can be given by equation (44) [19]:

$$\frac{v_{yx}}{E_y} = \frac{v_{xy}}{E_x} \tag{44}$$

2. For finding the engineering share moduli (Fig.9 - c), we are applying such conditions[19]:

$$\sigma_x = 0, \sigma_y = 0, \tau_{xy} \neq 0. \tag{45}$$

as shown in (Fig.9 - c), it can be found that shear coupling in x direction (m_x), coupling in y direction (m_y) and shear modulus (m_y) can be calculated according equations (46-48) [19]:

$$\frac{1}{m_x} = -\frac{1}{\overline{S_{16}}E_1}, \tag{46}$$

$$\frac{1}{m_y} = -\frac{1}{\overline{S_{26}}E_1}, \tag{47}$$

$$G_{xy} = \frac{1}{\overline{S_{66}}} \tag{48}$$

The strain–stress equation (26) of an angle lamina also can be written in terms of the engineering constants of an angle lamina in matrix form as shown in equation (49) [19]:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E_x} & -\dfrac{v_{xy}}{E_x} & -\dfrac{m_x}{E_1} \\ -\dfrac{v_{xy}}{E_x} & \dfrac{1}{E_y} & -\dfrac{m_y}{E_1} \\ -\dfrac{m_x}{E_1} & -\dfrac{m_y}{E_1} & \dfrac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{49}$$

The previous six engineering constants also be written in terms of the engineering constants of a unidirectional ply using compliance matrix [S] expression from reference [19] to equation (27-32) in equation (32) through equations (38), (41), (43), and (48):

$$\frac{1}{E_x} = \overline{S_{11}} = S_{11}c^4 + 2(S_{12} + 2S_{66})s^2c^2 + S_{22}s^4 = \frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - \frac{2v_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}s^4 \tag{50}$$

$$v_{xy} = -E_x\overline{S_{12}} = -E_x(S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66})s^2c^2) =$$

$$= E_x\left(\frac{v_{12}}{E_1}(s^4 + c^4) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)s^2c^2\right) \tag{51}$$

$$\frac{1}{E_y} = \overline{S_{22}} = S_{11}s^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}c^4 = \frac{1}{E_1}s^4 + \left(-\frac{2v_{12}}{E_1} + \frac{1}{G_{12}}\right)s^2c^2 + \frac{1}{E_2}c^4 \tag{52}$$

$$\frac{1}{G_{xy}} = \overline{S_{66}} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4) =$$

$$= 2(2\frac{1}{E_1} + 2\frac{1}{E_2} - \frac{1}{G_{12}})s^2c^2 + \frac{1}{G_{12}}(s^4 + c^4) \tag{53}$$

$$m_x = -\overline{S_{16}}E_1 = -E_1((2S_{11} - S2_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c) =$$

$$= E_1\left(\left(-\frac{2}{E_1} - \frac{2v_{12}}{E_1} + \frac{1}{G_{12}}\right)sc^3 + \left(\frac{2}{E_2} + \frac{2v_{12}}{E_1} - \frac{1}{G_{12}}\right)s^3c\right) \tag{54}$$

$$m_y = -\overline{S_{26}}E_1 = -E_1((2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3) =$$

$$= -E_1\left(\left(-\frac{2}{E_1} - \frac{2v_{12}}{E_1} + \frac{1}{G_{12}}\right)s^3c + \left(\frac{2}{E_2} + \frac{2v_{12}}{E_1} - \frac{1}{G_{12}}\right)sc^3\right) \tag{55}$$

2.5. In-Plane and Flexural Modulus of a Laminate

Laminate engineering constants are another way of defining laminate stiffnesses. Showing Equation (56) [19]:

$$\begin{bmatrix} N \\ \hline M \end{bmatrix} = \begin{bmatrix} A & B \\ \hline B & D \end{bmatrix}\begin{bmatrix} \varepsilon^0 \\ \hline \kappa \end{bmatrix} \tag{56}$$

where

$$[N] = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} \tag{57}$$

$$[M] = \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} \tag{58}$$

$$[\varepsilon^0] = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} \tag{59}$$

$$[\kappa] = \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \tag{60}$$

Inverting equation (56) gives (61) expression[19]:

$$\begin{bmatrix} \varepsilon^0 \\ \hline \kappa \end{bmatrix} = \begin{bmatrix} A^* & B^* \\ \hline C^* & D^* \end{bmatrix}\begin{bmatrix} N \\ \hline M \end{bmatrix} \tag{61}$$

where

$$\begin{bmatrix} A^* & B^* \\ \hline C^* & D^* \end{bmatrix} = \begin{bmatrix} A & B \\ \hline B & D \end{bmatrix}^{-1} \tag{62}$$

and

$$[C^*] = [B^*]^T \tag{63}$$

The [A*] - extensional compliance matrix; [B*] - coupling compliance matrix, and [D*] - bending compliance matrix[19].

2.6. In-Plane Engineering Constants of a Laminate

For a symmetric laminate, [B] = 0 and it can be shown that $[A^*] = [A]^{-1}$ and $[D^*] = [D]^{-1}$. Then, from equation (62) we get[19]:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} \tag{64}$$

The previous equations allow us to effectively define in-plane moduli in terms of the extensional compliance matrix [A*] as follows[19]:

Effective in-plane longitudinal modulus, E_x: can be find as described below

Applying the load $N_x \neq 0$, $N_y = 0$, $N_{xy} = 0$ and then modifying equation (64) as shown in equation (65) [19]:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} N_x \\ 0 \\ 0 \end{bmatrix} \tag{65}$$

which gives as equation (66)[19].

$$\varepsilon_x^0 = A_{11}^* N_x \tag{66}$$

Now the effective in-plane longitudinal modulus E_x can be calculated according equation (67) [19].

$$E_x = \frac{\sigma_x}{\varepsilon_x^0} = \frac{N_x/h}{A_{11}^* N_x} = \frac{1}{h A_{11}^*} \tag{67}$$

Effective in-plane transverse modulus, E_y: can be find as described below

Applying the loads $N_x = 0$, $N_y \neq 0$, $N_{xy} = 0$ and then modifying equation (64) we get expression below[19]:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} 0 \\ N_y \\ 0 \end{bmatrix} \tag{68}$$

which gives as]

$$\varepsilon_y^0 = A_{22}^* N_y \tag{69}$$

Then the effective in-plane transverse modulus E_y can be calculated by equation below[19]:

$$E_y = \frac{\sigma_y}{\varepsilon_y^0} = \frac{N_y/h}{A_{22}^* N_y} = \frac{1}{h A_{22}^*} \tag{70}$$

Effective in-plane shear modulus G_{xy} can be find as described below:

Appling loads $N_x = 0$, $N_y = 0$, $N_{xy} \neq 0$ and then modifying equation (64) as shown below:[19]

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A_{26}^* \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ N_{xy} \end{bmatrix} \tag{71}$$

which gives as:

$$\gamma_{xy}^0 = A_{66}^* N_{xy} \tag{72}$$

Then the effective in-plane shear modulus G_{xy} can be calculated as shown in equation (73) below [19]:

$$G_{xy} = \frac{\tau_{xy}}{\gamma_{xy}^0} = \frac{N_{xy}/h}{A_{66}^* N_{xy}} = \frac{1}{h A_{66}^*} \tag{73}$$

Effective in-plane Poisson's ratio, v_{xy} can be find as described below:

From the implication for the effective longitudinal Young's modulus, E_x, where the loads applied is $N_x \neq 0$, $N_y = 0$, $N_{xy} = 0$, from Equation (65), we have [19]

$$\varepsilon_y^0 = A_{12}^* N_x \tag{74}$$

$$\varepsilon_x^0 = A_{11}^* N_y \tag{75}$$

The effective Poisson's ratio, v_{xy}, are defined as shown below[19]:

$$v_{xy} = -\frac{\varepsilon_y^0}{\varepsilon_x^0} = -\frac{A_{12}^* N_x}{A_{11}^* N_x} = -\frac{A_{12}^*}{A_{11}^*} \tag{76}$$

Effective in-plane Poisson's ratio v_{yx} can be find as described below:

From the derivation for the effective transverse Young's modulus, E_y , where the load applied is $N_x = 0$, $N_y \neq 0$, $N_{xy} = 0$, from Equation (75), we can get this expression[19]:

$$\varepsilon_y^0 = A_{12}^* N_x \tag{77}$$

$$\varepsilon_x^0 = A_{11}^* N_y \tag{78}$$

The effective Poisson's ratio, v_{yx}, is then defined as shown below[19]:

$$\nu_{yx} = -\frac{\varepsilon_x^0}{\varepsilon_y^0} = -\frac{A_{12}^* N_y}{A_{22}^* N_y} = -\frac{A_{12}^*}{A_{22}^*} \qquad (79)$$

It is worth to notice that here a reciprocal relationship exists between the two effective Poisson's ratios ν_{xy} and ν_{yx}. from equations (70) and (76),we get expresion below [19]:

$$\frac{\nu_{xy}}{E_x} = \left(-\frac{A_{12}^*}{A_{11}^*}\right) h A_{11}^* = -A_{12}^* h \qquad (80)$$

From equation (70) and equation (79),we get expresion below[19]:

$$\frac{\nu_{yx}}{E_y} = \left(-\frac{A_{12}^*}{A_{11}^*}\right) h A_{22}^* = -A_{22}^* h \qquad (81)$$

From equation (80) and equation (71), we can get expression shown below[19]:

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad (82)$$

2.7. Flexural Engineering Constants of a Laminate

For a symmetric laminate, the coupling matrix [B] = 0; then, from equation (61),we can get expression below [19]:

$$\begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} D_{11}^* & D_{12}^* & D_{16}^* \\ D_{12}^* & D_{22}^* & D_{26}^* \\ D_{16}^* & D_{26}^* & D_{66}^* \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} \qquad (83)$$

Equation (83) allows us to define effective flexural moduli in terms of the bending compliance matrix [D*] as described above following steps shown below:

Apply $M_x \neq 0$, $M_y = 0$, $M_{xy} = 0$ and then substitute in equation (83) we can get expression below [19]:

$$\begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} D_{11}^* & D_{12}^* & D_{16}^* \\ D_{12}^* & D_{22}^* & D_{26}^* \\ D_{16}^* & D_{26}^* & D_{66}^* \end{bmatrix} \begin{bmatrix} M_x \\ 0 \\ 0 \end{bmatrix} \qquad (84)$$

which gives us [19]

$$\kappa_x = D_{11}^* M_x \qquad (85)$$

Then the effective flexural longitudinal modulus, E_x^f, can be calculated as shown below[19]:

$$E_x^f = \frac{12 M_x}{\kappa_x h^3} = \frac{12}{h^3 D_{11}^*}. \qquad (86)$$

Similarly, one can show that the other flexural elastic moduli are given below[19]:

$$E_y^f = \frac{12}{h^3 D_{22}^*}, \qquad (87)$$

$$G_{xy}^f = \frac{12}{h^3 D_{66}^*}, \tag{88}$$

$$v_{xy}^f = -\frac{D_{12}^*}{D_{11}^*}, \tag{89}$$

$$v_{yx}^f = -\frac{D_{12}^*}{D_{22}^*}. \tag{90}$$

Flexural Poisson's ratios v_{xy}^f and v_{yx}^f also have a reciprocal relationship, as given below[19]:

$$\frac{v_{xy}^f}{E_x} = \frac{v_{yx}^f}{E_y} \tag{91}$$

2.8. Ultimate Strengths of a Unidirectional Lamina

To carry out an analysis of the strength of the composite in final elements, its requires five lamin strength characteristics: longitudinal tensile and compressive strength, transverse tensile and compressive strength and in-plane shear strength.

This section summarizes the material mechanics methodology for calculating strength properties. It follows a similar principle as the rule of the mixture, the properties are obtained by evaluating individual components of composite properties taking into account their fraction in composite. Unfortunately the calculations result and deeper analysis of methodology showed that this method is inapplicable in our case. Therefore, the following methodologies and comparisons are only for the purpose of the observation.

2.8.1. Longitudinal Tensile Strength

The approach of longitudinal tensile strength calculation are represented below, which calculation model are shown in (Fig.10). For this methodic we need to apply this assumptions[19]:

- Reinforces and matrix are assumed to be isotropic material with liner elasticity until destruction:
- Strain of the matrix are bigger than fibers.
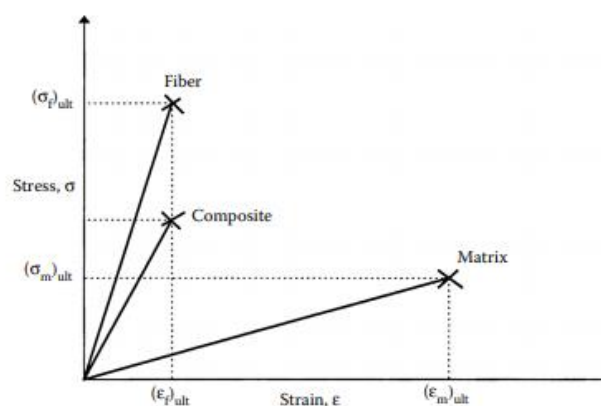
Fig. 10. Stress–strain curve for a unidirectional composite under uniaxial tensile load along fibers[19].

Now, if $(\sigma_f)_{ult}$ is ultimate tensile strength of fiber; $(\sigma_m)_{ult}$ is ultimate tensile strength of matrix; E_f is Young's modulus of fiber; E_m is Young's modulus of matrix then according (Fig.10) the ultimate failure strain of the fiber can be calculated by equation below[19]:

$$(\varepsilon_f)_{ult} = \frac{(\sigma_f)_{ult}}{E_f} \tag{92}$$

and the ultimate failure strain of the matrix can be calculated by equation below[19]:

$$(\varepsilon_m)_{ult} = \frac{(\sigma_m)_{ult}}{E_m} \tag{93}$$

as a known fiber material withstands most of the composite loads, so it can be concluded that when fibers fail at strain $(\varepsilon_f)_{ult}$, the entire laminate breaks. This assumption makes it possible to express composite tensile strength by equation below:[19]

$$(\sigma_1^T)_{ult} = (\sigma_f)_{ult} V_f + (\varepsilon_f)_{ult} E_m (1 - V_f) \tag{94}$$

Once the fibers broke, sometimes composite can take more load. It can occur when stress that the matrix can take alone $(\sigma_{mult})(1 - V_f)$ is greater than $(\sigma_1^T)_{ult}$, it make composite lamina capable to take larger force. The volume fraction when this possible is named as minimum fraction, $(V_f)_{minimum}$, and can be calculated by equation below[19]:

$$(\sigma_m)_{ult}\left[1 - (V_f)_{minimum}\right] > (\sigma_f)_{ult}(V_f)_{minimum} + (\varepsilon_f)_{ult} E_m \left[1 - (V_f)_{minimum}\right]$$

$$(V_f)_{minimum} < \frac{(\sigma_m)_{ult} - (\varepsilon_f)_{ult} E_m}{(\sigma_f)_{ult} - (\varepsilon_f)_{ult} E_m + (\sigma_m)_{ult}} \tag{95}$$

It is also possible that, by increasing fiber volume fraction in composite, the composite will have it will respond lower lamina tensile strength than the matrix. In this case the fiber volume fraction is named as critical fraction, $(V_f)_{critical}$, and can be calculated by equation below[19]:

$$(\sigma_m)_{ult} > (\sigma_f)_{ult}(V_f)_{critical} + (\varepsilon_f)_{ult} E_m \left[1 - (V_f)_{critical}\right]$$

$$(V_f)_{critical} < \frac{(\sigma_m)_{ult} - (\varepsilon_f)_{ult} E_m}{(\sigma_f)_{ult} - (\varepsilon_f)_{ult} E_m} \tag{96}$$

2.8.2. Longitudinal Compressive Strength

The methodology for longitudinal tensile strength calculation cannot be used to calculate the longitudinal compressive strength. It is mainly because typical of failure modes are different (Fig.14):
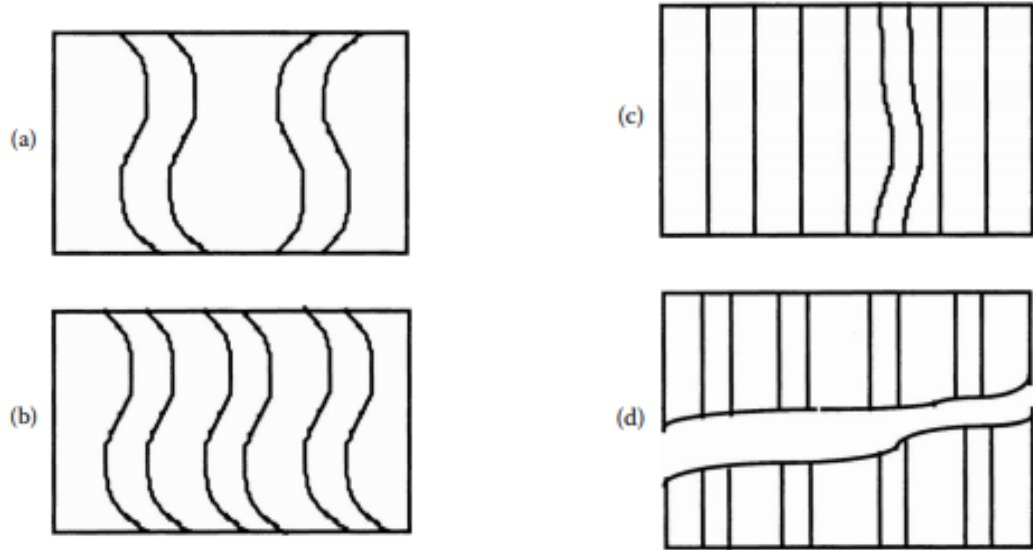
Fig. 11. Modes of failure of a unidirectional lamina under a longitudinal compressive load: a - fiber micro buckling extensional mode; b - fiber microbuckling shear mode; c - transverse tensile failure of matrix; d - shear failure [19].

Wen longitudinal compressive stress σ_1, are applayed the longitudinal compressive strain can be calculated by equation below[44]:

$$|\varepsilon_1| = \frac{|\sigma_1|}{E_1} \tag{97}$$

Because the major Poisson's ratio is ν_{12}, the transverse strain is tensile and can be given by equation below[44]:

$$|\varepsilon_2| = \nu_{12}\frac{|\sigma_1|}{E_1} \tag{98}$$

After applying the maximum deformation failure theory when the transverse tension exceeds the maximum transverse tensile strength composite will fail in transverse direction. Longitudinal compressive strength can be expressed thru equation given below[44]:

$$(\sigma_1^C)_{ult} = \frac{E_1(\varepsilon_2^T)_{ult}}{\nu_{12}} \tag{99}$$

The value of the longitudinal modulus, E_1, and the major Poisson's ratio, ν_{12}, can be calculated using rule of mixture. The ultimate tensile strain $(\varepsilon_2^T)_{ult}$ can be calculated by equation presented below,[44]

$$(\varepsilon_2^T)_{ult} = (\varepsilon_m^T)_{ult}\left(1 - V_f^{1/3}\right) \tag{100}$$

or the mechanics of materials formula presented below[44]:

$$(\varepsilon_2^T)_{ult} = (\varepsilon_m^T)_{ult}\left[\frac{d}{s}\left(\frac{E_m}{E_f} - 1\right) + 1\right], \tag{101}$$

Where $(\varepsilon_m^T)_{ult}$ − is ultimate tensile strain of the matrix; d − is diameter of the fibers; s − is center-to-center spacing between the fibers.

In fiber microbuckling shear mode (Fig.11 – b) compressive strength can be calculated according equation presented below [19]:

$$(\sigma_1^c)_{ult} = min[S_1^c, S_2^c] \tag{102}$$

Where

$$S_1^c = 2\left[V_f + \frac{(1 - V_f)E_m}{E_f}\right]\sqrt{\frac{V_f E_m E_f}{3(1 - V_f)}} \tag{103}$$

and

$$S_2^c = \frac{G_m}{1 - V_f} \tag{104}$$

Most often extensional mode buckling stress will exceed shear mode buckling stress except in composites wight small fiber volume fraction.

In shear failure (Fig.11 – d) the compressive strength can be calculated according to the following methodologies. In such occasion shear strength of the unidirectional composite $(\tau_{12})_{ult}$ can be calculated applying rule of mixture[44]:

$$(\tau_{12})_{ult} = \left(\tau_f\right)_{ult}V_f + (\tau_m)_{ult}V_m \tag{105}$$

where $\left(\tau_f\right)_{ult}$ is ultimate shear strength of the fiber and (τ_m) is ultimate shear strength of the matrix.

In the section at 45° to the loading axis we have maximum shear stress. Then longitudinal compresive strength are expressed according equation below[44]:

$$(\sigma_1^c)_{ult} = 2\left[\left(\tau_f\right)_{ult}V_f + (\tau_m)_{ult}V_m\right] \tag{106}$$

2.8.3. Transverse Tensile Strength:

Next we determinate methodology for finding the transverse tensile strength of a unidirectional lamina. For this model these assumptions must be made[19]:

- We have ideal connection between fiber and matrix;
- Spacing between fibers thread are even.;
- We can apply Hooke's law for fiber and matrix.;
- No residual stresses in modelate system..

The methodology are desined acording (Fig.12) where transverse tensile presented.
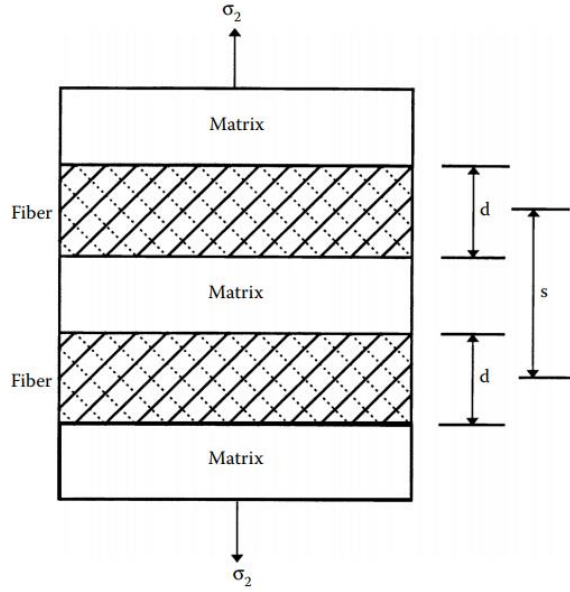
Fig. 12. Representative volume element to calculate transverse tensile strength of a unidirectional lamina, s is distance between center of fibers,d is diameter of fibers and σ_2 is tensile stress.[19].

The transverse deformations of the fiber,(δ_f), the matrix,(δ_m), and the composite, (δ_c) , dependency are expressed by equation below[19]:

$$\delta_c = \delta_f + \delta_m \tag{107}$$

The deformation dependence from strains, as shown in equations below[19]:

$$\delta_c = s\varepsilon_c \tag{108}$$

$$\delta_f = d\varepsilon_f \tag{109}$$

$$\delta_m = (s - d)\varepsilon_m \tag{110}$$

where $\varepsilon_c, \varepsilon_f, \varepsilon_m$ is ttransverse strain in the composite, fiber, and matrix, respectively.

Replacing expressions from equation (108-110) in to equation (107), we get:[19]

$$\varepsilon_c = \frac{d}{s}\varepsilon_f + \left(1 - \frac{d}{s}\right)\varepsilon_m \tag{111}$$

If we consider that stresses in the fiber and matrix being same at transverse loading, then for strains in fiber and matrix we can adapt Hooke's law as shown in equation below[19]:

$$E_f\varepsilon_f = E_m\varepsilon_m \tag{112}$$

By replacing the expression for the transverse strain in the fiber, ε_f , in equation (111), with expression from equation (112) the transverse strain in the composite can be expressed by equation below[19]:

$$\varepsilon_c = \left[\frac{dE_m}{sE_f} + \left(1 - \frac{d}{s}\right)\right]\varepsilon_m \tag{113}$$

If we consider that transverse failure of the lamina occurs because of matrix failure then the transverse failure strain can be calculated by equation below[19]:

$$(\varepsilon_2^T)_{ult} = \left[\frac{dE_m}{sE_f} + \left(1 - \frac{d}{s}\right)\right](\varepsilon_m^T)_{ult} \tag{114}$$

where $(\varepsilon_m^T)_{ult}$ – ultimate tensile failure strain of the matrix.

Now knowing ultimate transverse failure we can find ultimate transverse tensile strength can be expressed thru equation below[19]:

$$(\sigma_2^T)_{ult} = E_2(\varepsilon_2^T)_{ult} \tag{115}$$

In preceding equation of ultimate transverse failure strain $(\varepsilon_2^T)_{ult}$ are made assumption that the fiber and matrix are combined perfectly. But in real life bond isn't perfect what responds in strength reduce.

2.8.4. Transverse Compressive Strength

Transverse compressive strengths of a lamina can be calculated with the same equation as evaluating transverse tensile strength (122). The real compressive strength is smoller, mostly because imperfect fiber and matrix bond and longitudinal splitting. By modifying equation (115) for compressive strength, we get equation below[45]:

$$(\sigma_2^C)_{ult} = E_2(\varepsilon_2^C)_{ult} \tag{116}$$

where ultimate compressive failure strain of matrix value can be obtained by equation below [45]:

$$(\varepsilon_2^C)_{ult} = \left[\frac{dE_m}{sE_f} + \left(1 - \frac{d}{s}\right)\right](\varepsilon_m^C)_{ult} \tag{117}$$

where $(\varepsilon_m^C)_{ult} - is$ matrix compressive failure strain

2.8.5. In-Plane Shear Strength

For shear strength calculation metodic we use similar approach as in section 2.6.3. We make these assumptions: first we are use a magnitude shear stress τ_{12} and the second is that the shear deformation Δ_c are calculated by adding fiber deformation Δ_f to matrix deformation Δ_m as shown in equation below [19]:

$$\Delta_c = \Delta_f + \Delta_m \tag{118}$$

By using definition of shearing strain will be as shown in equation below[19]:

$$\Delta_m = s(\gamma_{12})_c \tag{119}$$

$$\Delta_m = d(\gamma_{12})_f \tag{120}$$

and

$$\Delta_m = (s - d)(\gamma_{12})_m \tag{121}$$

where $(\gamma_{12})_{c,f,m}$ – is the in-plane shearing strains in the composite, fiber, and matrix.

By using equation (126-128) in to equation (125), we get equation below[19]:

$$(\gamma_{12})_c = \frac{d}{s}(\gamma_{12})_f + \left(1 - \frac{d}{s}\right)(\gamma_{12})_m \tag{122}$$

Further we under shearing stress loading are making these assumptions: shear stress in the fiber and matrix are equal. Then, the shearing strains in the fiber and matrix expressed according given equation below[19]:

$$(\gamma_{12})_m G_m = (\gamma_{12})_f G_f \tag{123}$$

By using expression for $(\gamma_{12})_f$ from equation (130) in to equation (129), we get equation below[19]:

$$(\gamma_{12})_c = \left[\frac{dG_m}{sG_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_{12})_m \tag{124}$$

If make conclusion that shear failure occurs because failure of the matrix, then strain of shear $(\gamma_{12})_{ult}$ can be calculated by equation below[19]:

$$(\gamma_{12})_{ult} = \left[\frac{dG_m}{sG_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_{12})_{mult} \tag{125}$$

where $(\gamma_{12})_{mult}$ − ultimate shearing strain of the matrix.

The ultimate shear strength can be expressed according equation below[19]:

$$(\tau_{12})_{ult} = G_{12}(\gamma_{12})_{ult} = G_{12}\left[\frac{dG_m}{sG_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_{12})_{mult} \tag{126}$$

3. Composit material simulation in SolidWorks simulation:

The SolidWorks Simulation Premium has the ability to perform a linear static, buckling, frequency analysis with composite materials and even optimize material usage under know conditions within Design Study[22]. SolidWorks Simulation uses finite element method (FEA) for calculation [23]. In SolidWorks Design composites are defined using shell elements. Composite properties are defined thru Shell Definition interface shown in (Fig.13). It allow define up to 50 layers (but not less than 2) of composite.

a)   b)
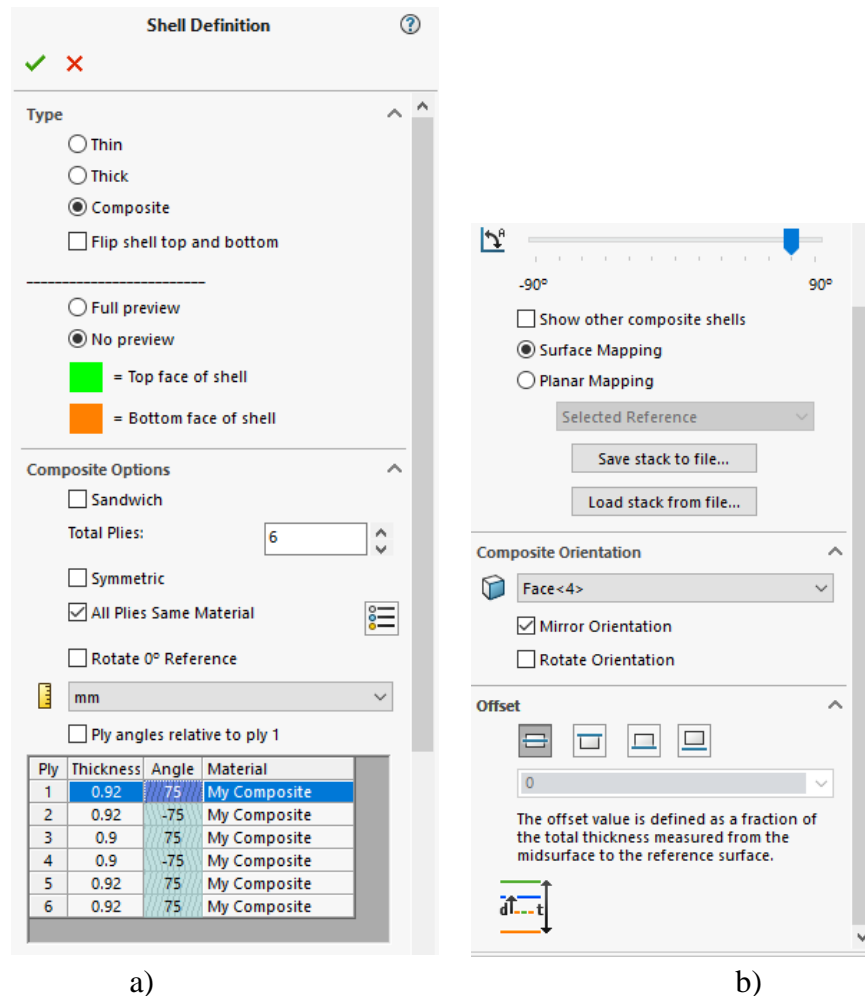
Fig. 13. Composite shell definition interface in SolidWork Premium: a – top of the interface; b – bottom of interface.

3.1. Composites shells arrangement:

SolidWorks Simulation in shell definition interface (Fig.13) provide three options for arranging lamina to laminate (Fig.14), [24].

- An unsymmetric laminate (Fig.17 - a) This laminate is made from different thickness lamina that can also be made of different materials. Their layout setup is independent.
- A symmetric laminate (Fig.17 - b) is symmetric about the midsurface it mens that plies materials, ply orientations, and plies thicknesses are mirrored . (Fig.17 - a)
- The sandwich composite (Fig.17 - c) is a separate case of symmetric composite with three layers. Such composite has higher resistance to bending loads.

38
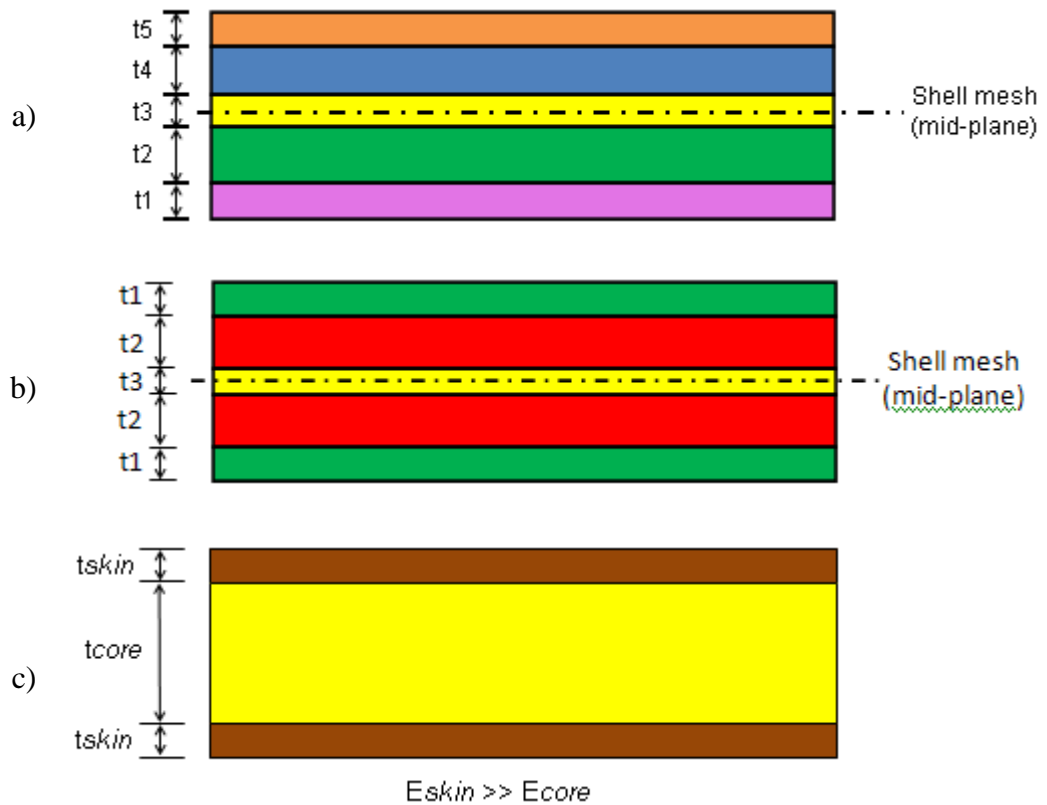
Fig. 14. Types of composite arrangement in SolidWork Simulation: a - unsymmetric laminate; b - symmetric laminate; c - sandwich composite arrangement (the same color represents the same material)[24].

3.2. Ply angle

SolidWorks Simulation allows to adjust the ply angle direction across different shells and faces using the Composite Orientation options in shell definition interface (Fig.16 - b). It is used to remove inconsistency reference ply angle for accurate simulation implementation according realistic model. There are three ways to orientate composite material:

1. The Rotate 0 degree reference this option rotates the 0 degree reference orientation by 90 degrees for all faces of a shell.
2. Mirror Orientation this option flips normal direction of the triad on an individual face.
3. Rotate Orientation this option rotates the 0 degree reference orientation by 90 degrees for a face of shell.

Also when you are stocking composite material in layers Solid Work Simulation are option to define ply angles for a stack relative to the first play.

Rotate Orientation this option rotates the 0 degree reference orientation by 90 degrees for a face of

Ply angle: 0 degrees   Ply angle: 60 degrees

shell.
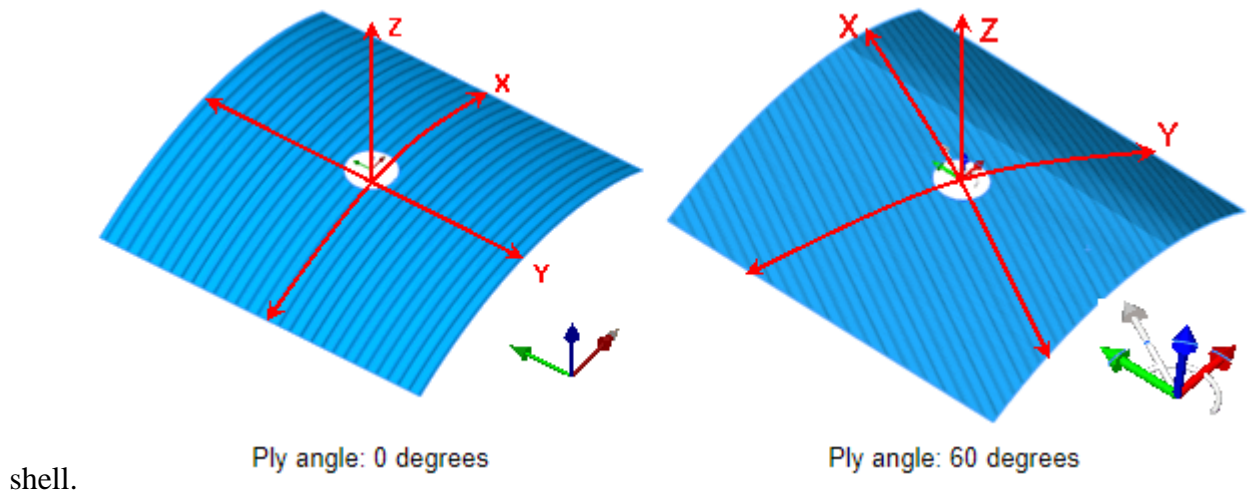
Fig. 15. The images show the X, Y and Z directions for ply angles 0 degrees and 60 degrees[24].

A composite triad is shown in (Fig.15) at the center of face. The triad has four arrows which meaning explained below:

→	Shows the 0 degree ply orientation. The red arrow is aligned with the surface stripes.
→	Shows the 90 degree ply orientation. The green arrow is aligned normal to the surface stripes.
→	The blue arrow is normal to the surface. Mirror orientation flips the blue arrow direction.
→	Shows the ply orientation direction.

3.3. Offsets for Composites

In SolidWorks Simulation shell definition interface allow to manipulate the position of a composite layer stoking according surface. The Shell Definition PropertyManager (Fig.13) for composites has these following options[24]: middle surface, top surface, bottom surface and specify ratio

3.4. Stacking Sequence of Plies

Alazysing model from unsymmetric laminates it is necessary to make sure that the stacking direction is correct across all faces before running analysis (Fig.16).

SolidWork Simulation allow to flip composite stocking direction for chosen faces to make the stacking direction consistent across each face. It gives greater reliability of simulation results.
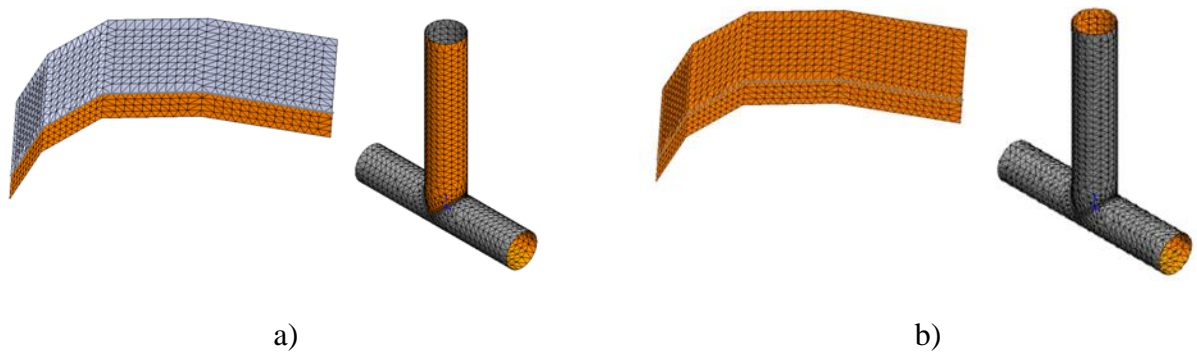
Fig. 16. Stacking direction for models with multiple faces and/or shells, orange color represents bottom face of shell: a - inconsistent stacking direction across faces; b - consistent stacking direction across every face[24].

3.5. Stress Analysis for Composite Shells:

Composite laminate have continued displacement and strain fields through thickness. But stress fields altogether are not continuous due to different material properties and orientations in to different lamina. So we get a stepped stress diagram (Fig.17).

Fig. 17. Longitudinal strain and longitudinal stress under pure bending[24].

SolidWorks Simulation allows to plot the maximum stress values of the elaborate component across all plies. It also capable to plot maximum stress across all plies for each node taking in account top and bottom faces. Apart from stress components available plot the interlaminar shear stress in the between two nerr by plies.

SolidWorks Simulation optionaly allows to analyze loading effects separately in each layer at top and bottom of faces and over all laminate. It also have available three different criteria for laminate failure [24]:

- Tsai-Hill Failure;
- Tsai-Wu Failure;
- Maximum Stress;

4. Expermental part:

To tests and make analyze of manufactured composite samples for mechanical characteristics which will further be used in to compare data obtained analytically.

The study presents experimental studies on a various composite sample series (in total test was performed for 10 series and each series consist at least 5 samples) for determining manufactured material mechanical properties in all direction. For this matter the tensile, shear tensile, interlaminar shear and compressive strength tests was performed. Each test was performed for several series witch every set was with different fiber orientation. Samples layer was made of Epoxy vinyl ester resin reinforced with a ECR 469L glass fiber. Samples was manufactured by UAB "Traidenis" using glass fiber filament winding process. For better collected data representation the statistical analysis was dan.

4.1. Material used in composite samples manufacturing and its properties:

Main samples structure manufactured by glass fiber filament winding process by UAB Traidenis" company. Materials physical characteristics according data sheets and literature are presentet in table 1.:

Table. 1. Properties of composite matrices and reinforcement materials, which was used to make samples [26,27]

Parameter	Units	ECR 469L glass		Epoxy vinyl ester resin	
			Assumed		Assumed
Density, ρ	kg/m^3	$2660 \div 2680$	2670	1140	1140
Young's modulus, E	GPa	$80 \div 81$	80.5	3.2	3.2
Poisson ratio, v	-	$0.21 \div 0.23$	0.22	$0.35 \div 0.4$	0.38
Tensile strength, σ_U	MPa	$3100 \div 3800$	3400	86	86
Elongation, ε_U	%			$5 \div 6$	
CTE, α_T	$10^{-6}\ 1/C°$	5.9	5.9	1.1	1.1
Shear strength, τ	MPa	35	35	34	34

4.2. Equipment used in testing:

Experiment was dan with 2 testing machines (Fig.18) because first testing machine was unable to reach ultimate strength of all specimens. For this case with first machine was measured all specimens elasticity , Young's modulus and Poisson's ratio. In some case the first machine was able to reach ultimate strength of same specimen in such case all experiment was carried out with first machine.
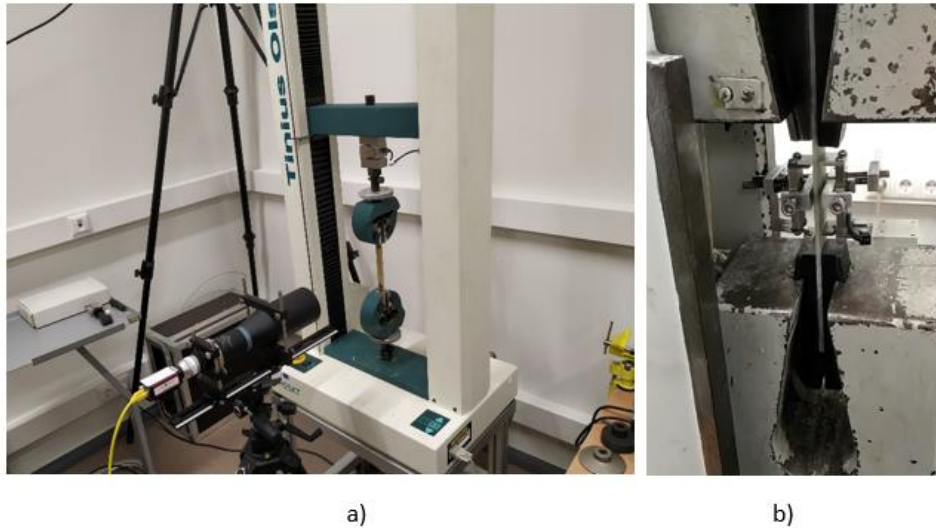
Fig. 18. Testing machines setups: a - testing machine I; b - testing machine II.

4.2.1. Testing machine I:

First machine is Tinius Olsen H25KT with video extensometer PoE 1 for strain measurement (Fig.18 – a). This test machine can develop 25 kN force, the distance between the loading part and the base can reach up to 1.1m [28-29]. The machine load speed is from 0.001 to 1000 mm / min [28-29]. This testing device can be controlled by a JOG or computer. Machines meet the accuracy of this measurement: load accuracy +/- 0.5%; distance accuracy +/- 0.01%.

4.2.2. Testing machine II specification:

Is fife tons tensile-compression machine: "ГЗИП" "ГЛАВМЕРВЕСПРОМ" Type – 5 , No – 314, years of 1948 (Fig.18 – b) with strength, position end deformations sensors connected whit computer thro Spider 8 signal convertor (Fig.19).
Spider 8 (Fig.19 - a) receives electrical signals from various sensors and converts it to digital signal then sends them to computer where over specializuota program data is visible in real time and can be analyzed.

Miniature extensometer 3475-050M-ST (Fig.19 - b) sensor allows to measure Poisson's ratio. Sensor can measure up to 25 mm with sample, it capable to measure deformation in range of ±0.5 mm. Collected data characterized by ≤0.20% linearity of full scale measuring.

WA inductive standard displacement transducer (Fig.19 - c) allows tu mesure displacement whith linearity deviation of ±0.20%.

DD1 strain transducer (Fig.19 - d) has full range of accessories for strane which allows to mesur stranes in various geometry samples. Sensor has nominal displacement is in range of ±2.5 mm; sensity tolerance of ±0.10% and linearity error including hysteresis of <±0.05%.

Used U5 force transducer (Fig.19 - f) has nominal forces of 100kN. Linearity deviation of force is less than 0.10%.
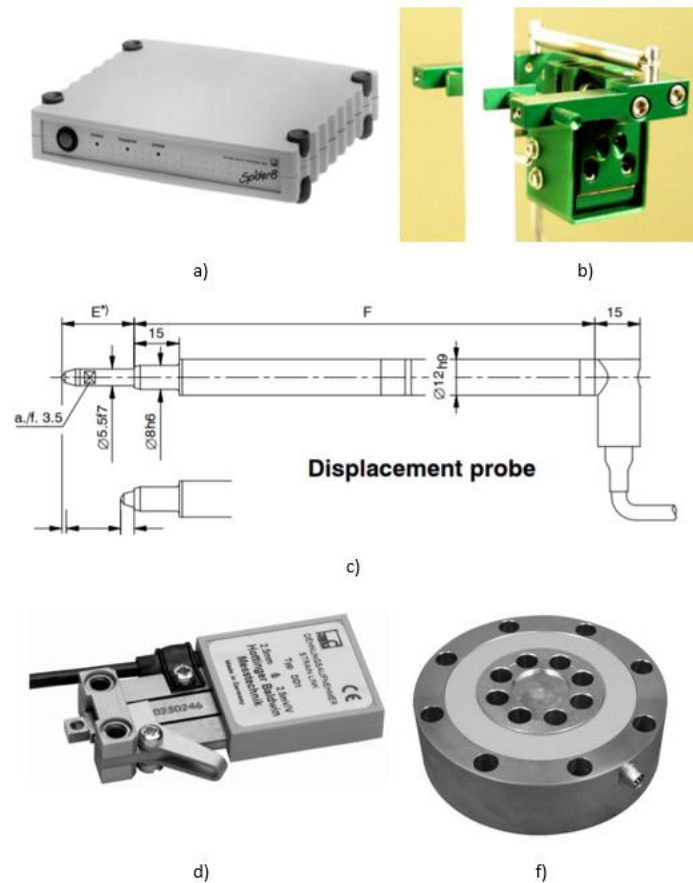
Fig. 19. a – Spider8 an electronic measuring system ;b - model 3475-050M-ST miniature extensometer designed for general purpose transverse or diametral strain measurements on small or thin specimens; c – WA inductive standard displacement transducer; d – DD1 strain transducer; f – U5 force transducer[31-35].

4.2.3. Other:

Caliper with +-0.05mm error was used for measuring width and thinness of the test specimen.

4.3. Measurement accuracy check:

To ensure better collected data accuracy in beginning of experiment there was done quick check on displacement and strain measurement accuracy. Check setup are shown in (Fig.20).

The test was done by applying displacement with micrometer for "ГЗИП" "ГЛАВМЕРВЕСПРОМ" Type – 5 test machine and with caliper for Tinius Olsen H25KT test machine. Applied displacement was measured with test machines sensors and occurred error was determinate. The measurements and error are present in table 1. Taking in account that micrometer error is +-0.01 and caliper error is +-0.05, looking in results given in table 1 we made assumption that sensor measurement is good.
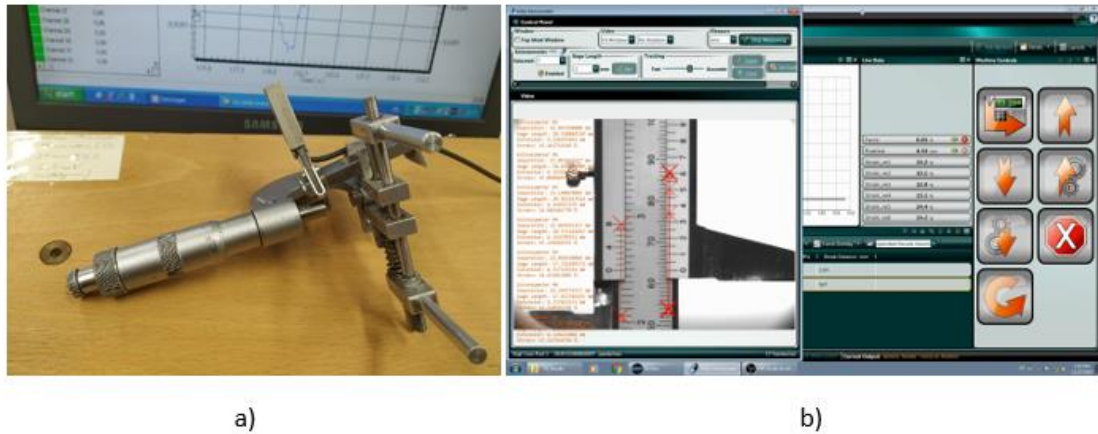
Fig. 20. Setup for a – deformation measurement check on "ГЗИП" "ГЛАВМЕРВЕСПРОМ" Type – 5 , No – 314 test machine; b – strain measurement check on Tinius Olsen H25KT video extensometer.

The test was done by applying displacement with micrometer for "ГЗИП" "ГЛАВМЕРВЕСПРОМ" Type – 5 test machine and with caliper for Tinius Olsen H25KT test machine. Applied displacement was measured with test machines sensors and occurred error was determinate. The measurements and error are present in table 2. Taking in account that micrometer error is +-0.01 and caliper error is +- 0.05, looking in results given in table 2 we made assumption that sensor measurement is good.

Table. 2. Result from check measurements:

No.	For "ГЗИП" "ГЛАВМЕРВЕСПРОМ" Type – 5			For Tinius Olsen H25KT		
	Displacement applied with micrometer	Displacement measured with sensor	Error	Displacement applied with caliper	Displacement measured with sensor	Error
1		0.0505	-0.0005		0.5255	-0.0255
2		0.0500	0.0000		0.4707	0.0293
3		0.0505	-0.0005		0.5277	-0.0277
4	0.0500	0.0495	0.0005	0.5000	0.4883	0.0117
5		0.0500	0.0000		0.5005	-0.0005
6		0.0455	0.0045		0.4720	0.0280

4.4. Tensile testing methodology

4.4.1. Principles

In tensile test the specimen is stretched at a constant speed along the main longitudinal axis until sample decomposes or reaches the investigated values. During experimentation, the applied load, sample elongation and strains are measured. Testing in different machines are shown in (Fig.21 – a,b).
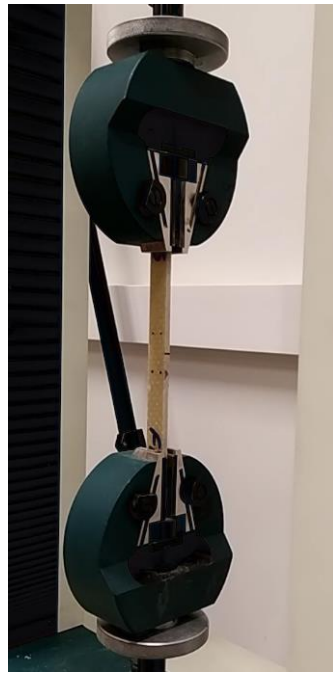
<div align="center">a)</div>

<div align="center">b)</div>
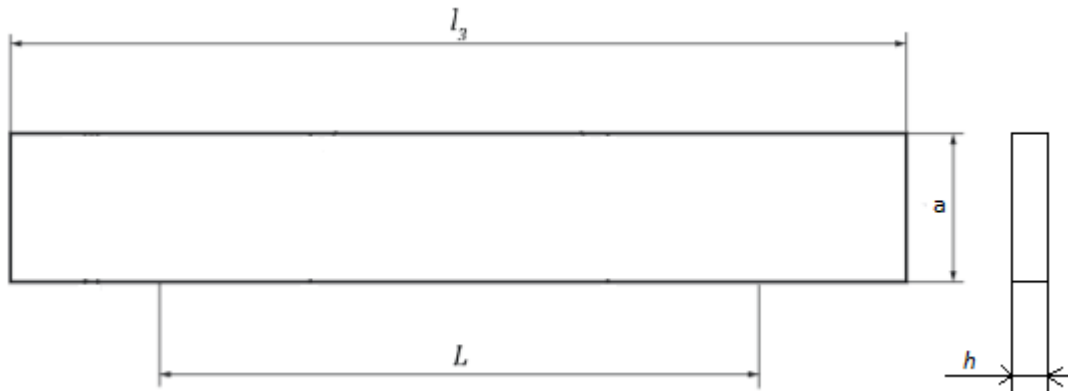
<div align="center">c)</div>

Fig. 21. Tensile tesiting: a – sample in test machine I; b - sample in test machine II; c – sample geometry[37].

The methods are applied using specimens which was manufactured by "Traidenis" company and then was to machined and finished to the chosen dimensions according ISO 527 standard. The used form of test specimen for the determination of tensile properties is shown in (Fig.21 c) it is a strip that is about 25mm wide (*a*), approximately 5mm thick (*h*) and about 225mm long (*l₃*), recomendet distance between grips (*L*) is about 100mm.

4.4.2. Expression of calculation for tensile test:

Stress value are calculated according equation[37]:

$$\sigma = \frac{F}{A} \tag{127}$$

where: σ is the stress value in *(MPa); F* is the measured force in *(N); A* is the initial cross-section area of the specimen in (mm^2)

For materials or test conditions for which a homogeneous strain distribution is prevalent in the parallel section of the test specimen, i.e. for strains prior and up to yield point, calculate all strain values, using the following equation.[37]

$$\varepsilon = \frac{\Delta L_0}{L_0} \tag{128}$$

where: ε is the strain value who are dimensionless; L_0 is the measured length of the tested specimen, expressed in millimeters *(mm)*; ΔL_0 is the increased length of the tested specimen, expressed in millimeters *(mm)*;

The determination of strain is increasing in length per unit original length of the gauge. In experiments whith testing machine I the strain in the specimen was measured by video extensometer PoE 1. This devise has opportunity to measure 9 strains at the same time. In sample strains was measured between four points, the strain measurement method shown in (22.Fig.).
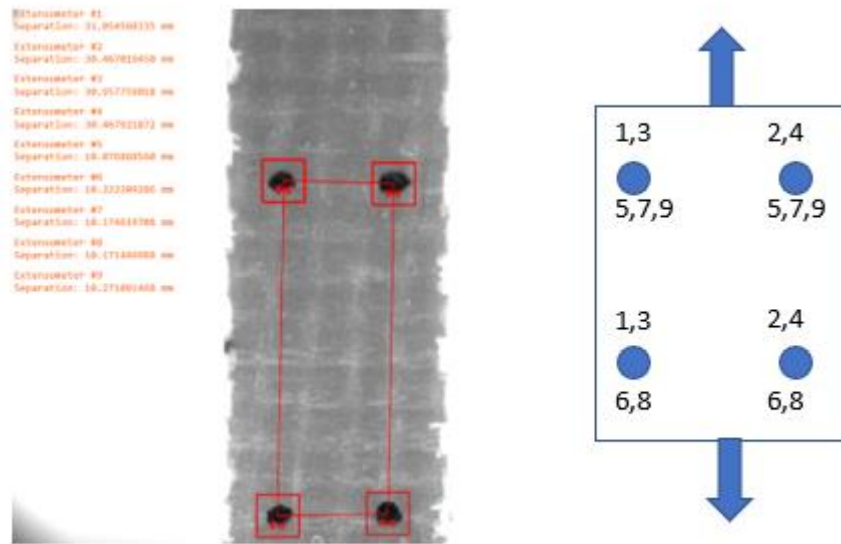
Fig. 22. Strain measurement schematics.

In this experiment for tensile modulus calculation we use chord slope equation[37]:

$$E_t = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} \tag{129}$$

where: E_t is the tensile modulus, expressed in megapascals (MPa); σ_1 is the stress, in (MPa), measured at the strain ε_1; σ_2 is the stress, in (MPa), measured at the strain ε_2.

Poisson's ratio was calculated as described below:

Plot the width or thickness of the specimen as a function of the length of the gauge section for the part of the stress/strain curve before a yield point, if present, and excluding those sections that may be influenced by changes in test speed[37].

Determine the slope $\mu/\Delta L_0$ of the change-in-width versus the change-in-length curve. This slope was calculated by using a linear least-squares regression, analysis between two limits, preferably after the

modulus region and an ensuing speed change, if applicable, that are in a linear portion of this curve. Poisson's ratio is determined from the following equation[37]:

$$\mu = -\frac{\Delta\varepsilon_n}{\Delta\varepsilon_l} = -\frac{L_0 \cdot \Delta n}{n_0 \cdot \Delta L_0}$$
(130)

where: μ -is the Poisson's ratio; it is dimensionless; $\Delta\varepsilon_n -$ is the strain decrease in the selected transverse direction, while the longitudinal strain increases by $\Delta\varepsilon_l -$, expressed as a dimensionless ratio or percentage; $\Delta\varepsilon_l -$ is the strain increase in the longitudinal direction, a dimensionless ratio or percentage; $L_0, n_0 -$ are the initial gauge (in our case distance between points) lengths in the longitudinal and transverse direction, respectively, expressed in millimeters (mm); $\Delta n -$ is the decreased of the specimen gauge length in the transverse direction: n = width or n = thickness, expressed in millimeters (mm); $\Delta L_0 -$ is the corresponding increase of the gauge length in the longitudinal direction, expressed in millimeters (mm).

4.4.3. Expression of calculation for share tensile test:

In-plane shear stress τ_{12} expressed in megapascals, are calculate using following equation [38]:

$$\tau_{12} = \frac{F}{2ab}$$
(131)

where: F – is the instantaneous force, in (N); b – is the width, in (mm), of the test specimen; h – is the thickness, in (mm), of test specimen.

Shear strain γ_{12} are calculated using following equation [38]:

$$\gamma_{12} = \varepsilon_x - \varepsilon_y$$
(132)

where: ε_x – is the strain in the direction parallel to the specimen axis; ε_y – is the strain in the direction perpendicular to the specimen axis;

In-plane shear modulus G_{12}, expressed in megapascals are calculated using following equation[38]:

$$G_{12} = \frac{\tau_{12}'' - \tau_{12}'}{\gamma_{12}'' - \gamma_{12}'}$$
(133)

where: τ_{12}' – is the shear stress at a shear strain γ_{12}'; τ_{12}''– is the shear stress at a shear strain γ_{12}'';

4.4.4. Tested specimens

According ISO 527 standard it is recommended to test at lest five specimens.

The tensile experiment was done for 6 different series to obtain mechanical characteristic in different direction (series: 1.set; 2.set; 3.set; 4.set and 5.set) and obtain mechanical characteristic of laminate which consists of different layers, each having a different orientation of glass fiber (8.set). Each batch contains at leest fife samples. The samples of (1.set) are reprezented in (Fig.26). More detailed information about each sets experimentation are represented in appendices.
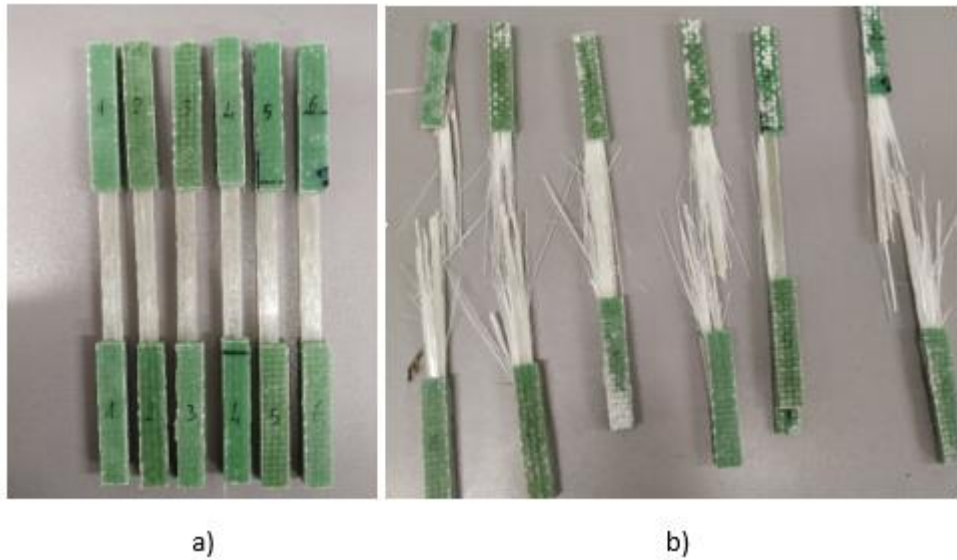
Fig. 23. Image of Unid_0 samples: a – before, b – after tensile test.

4.5. Testing methodology for flexural properties determination:

In this section we make short introduction to methodology for expermental determination of flexural properties. This section is based on ISO 14125. The material used in sample manufacture are described in this chapter first section. Geometry was chosen taking in account recommendation in ISO 14125 standard and are determinate below.

4.5.1. Principle:

For laminar flexural properties determination three point bending test was chosen. In this experiment, the test sample are supported from the bottom between the two supports and then the load from top are initiated. During this apllayed force and deflection in the speciment are measured. [42] Three-point bending test setup are shown in (Fig.27).

a)
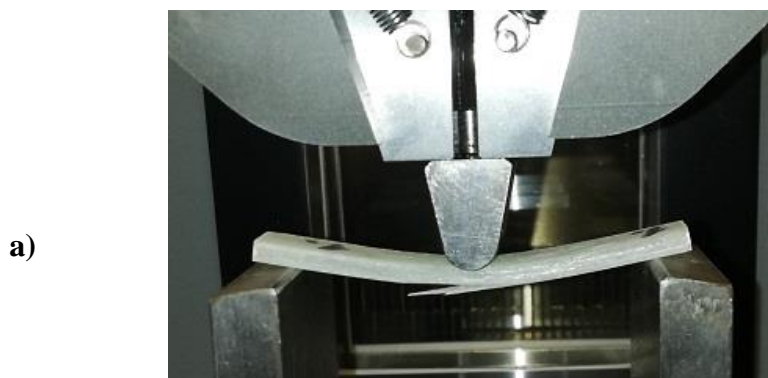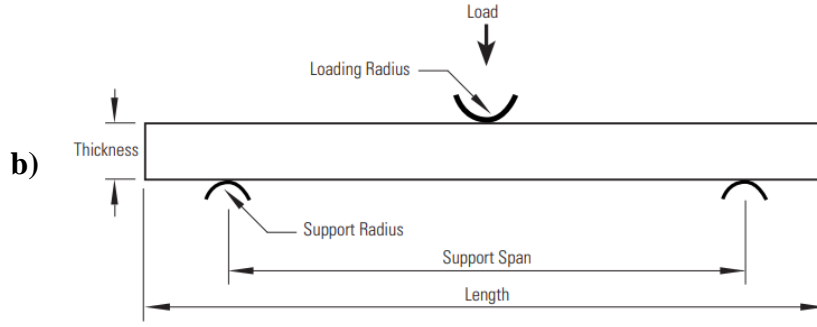
Fig. 24. Three-point bending test setup: a – in machine I; b – schematic representation of try points bending [43].

This method allow to mesure the determinate flexural behaviour of the test specimens under the defined conditions [42].

4.5.2. Calculation and expression of result:

The flexural stress σ_f can be calculated by following equation [42]:

$$\sigma_f = \frac{3FL}{2bh^2}$$

(134)

where σ_f is the flexural stress, in (MPa); F is the applied force in (N); L is the support span, in (mm); b is the thickness of the specimen, in (mm); h is the width of the test sample, in (mm).

For the measurement of the flexural modulus, we have to calculate the deflections s' and s'', which on flexural strain $\varepsilon_f' = 0,0005$ and $\varepsilon_f'' = 0,0025$, according to expression below [42]:

$$s' = \frac{\varepsilon_f'L^2}{6h} \ and \ s'' = \frac{\varepsilon_f''L^2}{6h}$$

(135)

where s' and s'' are the beam mid-point deflections, in (mm); ε_f' and ε_f'' are the flexural strains, whose values are given above.

The flexural modulus can be calculated according (136) or (137) equations:[42]

$$E_f = \frac{L^3}{4bh^3}\left(\frac{\Delta F}{\Delta s}\right)$$

(136)

where E_f is the flexural modul of elasticity, in (MPa); Δs is the difference in deflection between s' and s''; ΔF is the difference between force F' and F'' at s'' and s' respectively.

$$E_f = 500(\sigma_f'' - \sigma_f')$$

(137)

where σ_f' is the stress measured at the deflection s', in (MPa); σ_f'' is measured stress at the deflection s'', in (MPa).

The strain in the outer surface of the specimen can be calculate as follows [42]:

$$\varepsilon = \frac{6sh}{L^2} \qquad\qquad (138)$$

The try point bending experiment was done for 3 different series to obtain mechanical characteristic in different direction (series: 6.set and 7.set). Each batch contains at leest fife samples. More detailed information about each sets experimentation are represented in appendices.

4.6. Determination of stress-strain characteristics in compression:

Compression test was preformed according ISO 3386, with test machine II which represented in section 4.2. Test was done for two series. The different between series was fiber direction: 9.set – parallel; 10.set – perpendicular to loading axis. Samples were rectangular.

Compression stress and strain are calculated as same as tensile, strain according equation (128), stress according equation (127). Detailed information about each sets experimentation are represented in appendices. Testing inter laminar shear strength of laminates:

4.7. Testing inter laminar shear strength of laminates:

4.7.1. Form of test specimen:

Test specimens form and dimensions was made by UAB "Traidenis" according recommendation in LST EN 13121-3:2016. Specimen schematic shown in in (28.Fig.). They shall have a minimum thickness *t* of 3 mm and the length *L* of the specimens may be varied to accommodate the requirements of the available testing equipment. The edges of the specimen shall be smooth but not round or bevelled. Two parallel cuts, one in each opposite face and between 5 to 12 mm apart *a*, shall be sawn across the entire width *b* of the specimen[40]. They shall be as narrow as practicable and shall be parallel within 0,8 mm[40]. The incision shall be half the laminate thickness plus the thickness of one layer or half the laminate thickness + *0,1, - 0* mm if the number of layers or thickness per layer is unknown [40].
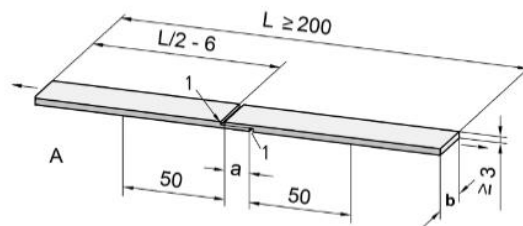
Fig. 25. Test specimen for the determination of inter laminar shear strength of a laminate for tensile test [40].

4.7.2. Tested specimens:

There was tested 3 series of composite samples made from DERAKANE MOMENTUM 411-350 epoxy vinyl ester resin and ECR 469L 2400 fiber. In one sample series fiber is orientated along tensile force (further name 11.set), in another sample series fiber is orientated disorderly (further name as 12.set), and in the last serie the fiber orientation and tensile forces axis is perpendicular (further name as 13.set). Each batch contains at leest fife samples. Tested samples are shown in (27.Fig), geometry and detailed information about each sets experimentation are represented in appendices.
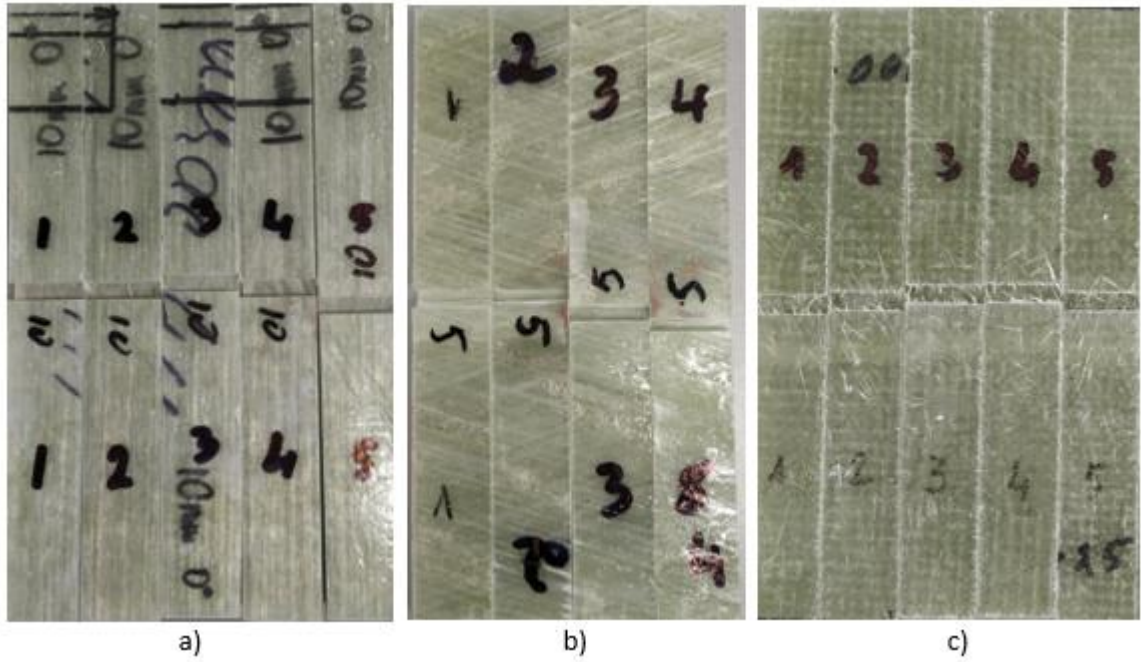
Fig. 26. Test samples for the determination of inter laminar shear strength of a laminate for tensile test, a - 11.set; b – 12.set; c – 13.set.

4.7.3. Procedure:

Condition of the specimens was according to EN ISO 291. The test was performed with test machine II which was described above.

The specimen was insert in the testing machine and load it until rupture occurs in the form of a shear-type failure with some peeling at the interlaminate bond.

4.7.4. Results:

The inter-laminar shear strength, τ (in *N/mm²*), shall be determined from the following formula [40]:

$$\tau = \frac{W}{a \cdot b} \tag{139}$$

Where: *W* - is the determined load, in *N*; a - is the distance between saw cuts; *b* - is the width of the specimen.

4.8. Statistical analysis:

Statistical analysis allows to make adequate assessment of the collected results .

4.8.1. Expression of calculation:

In this study, we will evaluate the following statistical parameters according to ISO 2602:

Arithmetic mean $\overline{x}$ of the results:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{140}$$

Standard deviation [41]:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2} \tag{141}$$

Two-sided 95% confidence interval for the mean value[41]:

$$\overline{x} - \frac{t_{0.975}}{\sqrt{n}}s < m < \overline{x} + \frac{t_{0.975}}{\sqrt{n}}s \tag{142}$$

where: $\overline{x}$ - arithmetic mean of n measurements; n - number of all measurements; x_i - value of the i-measurement; s - standard deviation; m - mean value for the population;

The value of $t_{0.975}$, which was used to obtane the two-sided confidence interval with a probability of 95% , is set from t-student distribution of $\vartheta = n - 1$ degrees of freedom. This property meaning can be obtained from standard [39].

5. Overall result:

5.1. Statistically evaluated of experimentally obtained results:

In this section are presented statistically analyzed data obtained from experiment (table 1). Samples numeration and description are represented below:

- 1.set – is samples from tensile test earlier refer as vyn_0 sample. In this sample fiber are oriented along tensile force axis.
- 2.set – samples from laminar shear stress earlier refer as vyn_45. In this sample fiber are oriented 45 deg. angle along tensile force axis.
- 3.set – samples from tensile test earlier refuse as vyn_90. In this sample fiber are oriented perpendicular to tensile force axis.
- 4.set – samplses from tensile test, made of helical wound laminate in to axial direction of cylinder (Diameter 5.5 m, ±75 deg), layup: (75/-75/75/-75/75/75).
- 5.set – samples from tensile test, made of helical wound laminate in to circumferential direction of cylinder (Diameter 5.5 m, ±75deg), layup: (75/-75/75/-75/75/75).
- 6.set– samples from three point bending test, made of helical wound laminate in to axial direction of cylinder (Diameter 5.5 m, ±75deg), layup: (75/-75/75/-75/75/75).
- 7.set – samples from three point bending test, made of helical wound laminate in to circumferential direction of cylinder (Diameter 5.5 m, ±75deg), layup: (75/-75/75/-75/75/75).
- 8.set - samples from tensile test, made of helical wound laminate in to axial direction of cylinder (Diameter 4.0 m), Layup: (70/-70/90/90/-70/70)

Table. 3. Statistical analysis result for all series:

Sierias	1.set		2.set	3.set		4.set	5.set	6.set	7.set	8.set
Characteristics	Tensile modulus GPa	Poisson's ratio	In plane shear modulus, GPa	Tensile modulus, GPa	Poisson's ratio	Tensile modulus, GPa	Tensile modulus, GPa	Flexural modulus, GPa	Flexural modulus, GPa	Tensile modulus, GPa
Arithmetic mean	43.28	0.48	7.17	12.65	0.15	10.66	29.34	10.00	24.84	11.73
Standard deviation	2.00	0.10	1.49	2.21	0.12	2.22	11.12	1.65	8.76	5.28
Confidence Level(95.0%)	2.10	0.10	1.56	2.32	0.13	2.75	13.80	2.05	10.87	4.88
Lower confidence limit	41.18	0.38	5.61	10.32	0.02	7.91	15.53	7.95	13.97	6.84
Upper confidence limit	45.38	0.58	8.73	14.97	0.28	13.41	43.14	12.05	35.71	16.61

Statistic analysis of sets 9-13 are not reprezented in the table 3.

- 9.set – samples from compression test fiber direction was parallel to loding direction;
- 10.set - samples from compression test fiber direction was perpendicular to loding direction.
- 11.set – samples from laminar shear strength tests, fiber was orientated along tensile force;
- 12.set – samples from laminar shear strength tests, fiber is orientated disorderly;

- 13.set - samples from laminar shear strength tests, fiber orientation and tensile forces axis is perpendicular

5.2. Properties calculation using rule-of-mixture:

In this section the sheare modulus, Poisson's ratio and elastic modulus dependence from volume fraction was calculated and plotted in (27.Fig. – 29.Fig.). The experimentally determined volume fraction is 0.51. The parameters calculated analytically and obtained experimentally with volume fraction of 0.51 are represented in Table.4:

Table. 4. Initial lamina properties.

	Analiticaly obtaned value	Expermentaly obtaned veliu
Longitudinal elastic modulus, GPa	42.62	43.28
Transverse elastic modulus, GPa	11.17	11.25
Poisson's ratio	0.29	0.48
In-plane shear modulus, GPa	3.26	-

Calculation was dan according second chapter taking in account samples material properties described in chapter four.
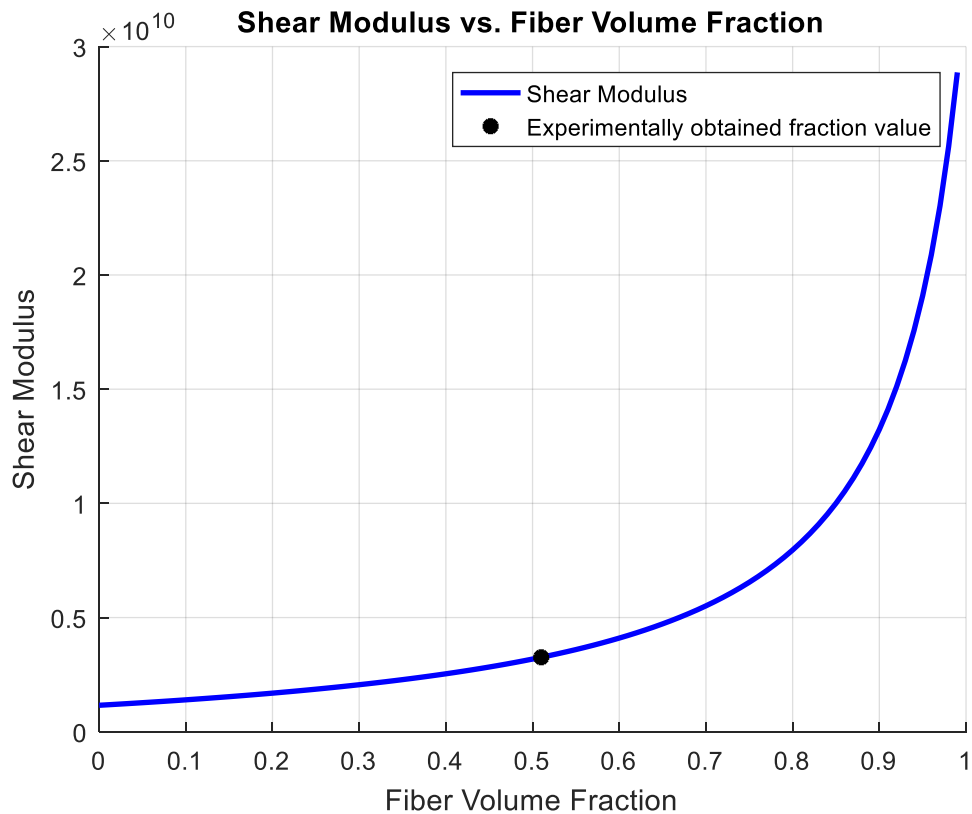
Fig. 27. Shere modulus dependence from volume fraction.

In graph of shear modulus dependence from volume fraction calculation analytical (27.Fig.) we can see non-linear behavior shear modulus dependence from fiber volume. We cansy that the effective shear modulus progressively rise when fiber volume fraction increases. We can make

assumption that the biger volume fraction of fiber the better performance of composites under shear loading.
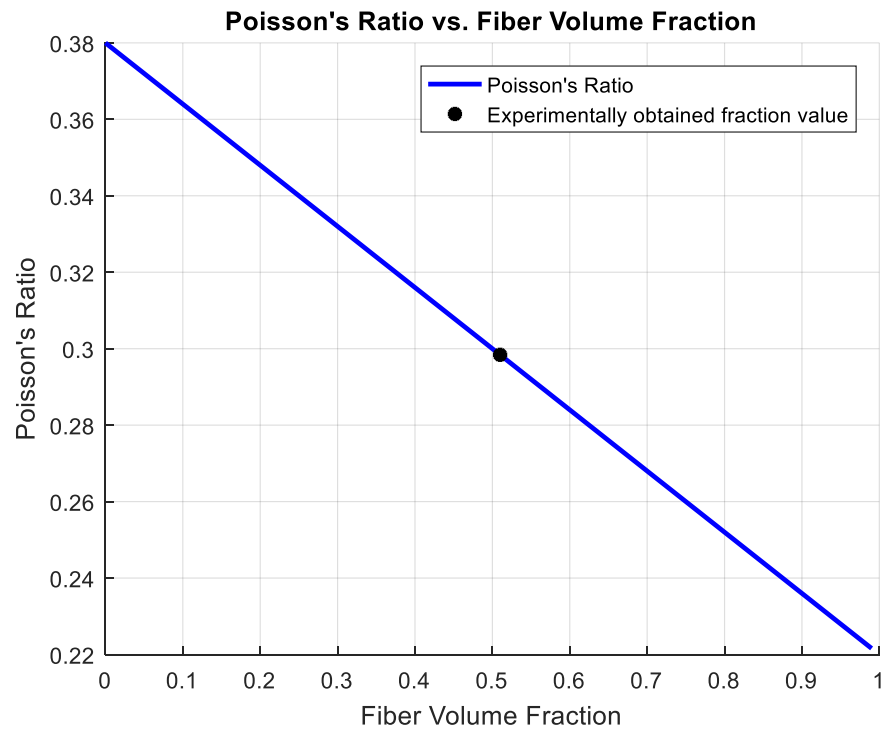
Fig. 28. Poisson's ratio dependence from volume fraction.

The values of Poisson's ratio results for varying fiber volume fractions are plotted in a graph (28.Fig). As we can see effective Poisson's ratio reducing linearly when volume fraction of fiber are increasing.
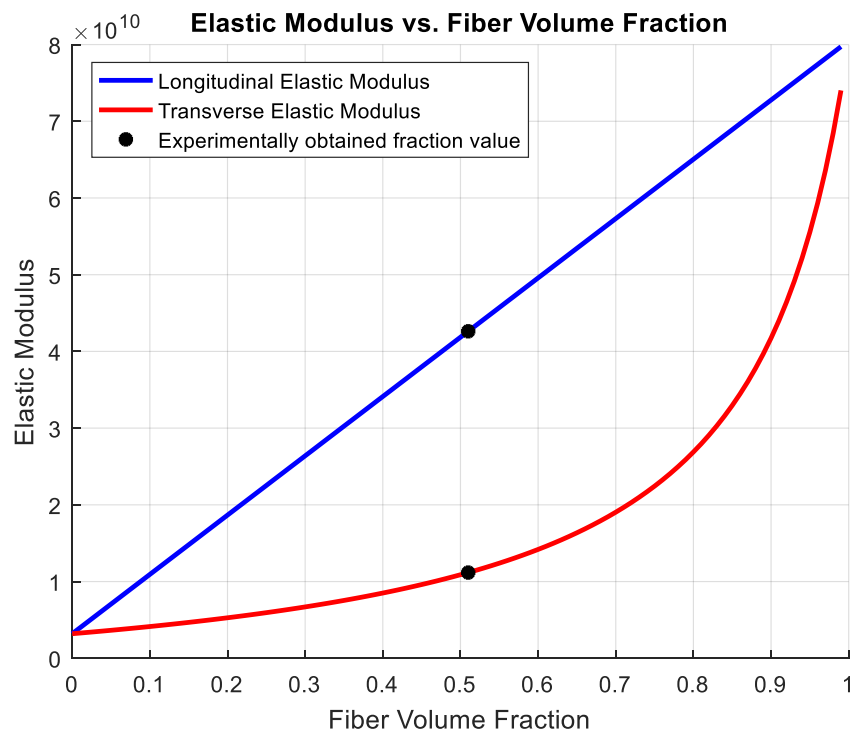
Fig. 29. Elastic modulus dependence from volume fraction.

The values of Young's modulus variation from volume fractions calculated analytically are plotted in a graph (Fig.29). From the graph we can clearly see that the longitudinal elastic modulus have linearly dependenc from fiber volume fraction. And the trasnsveres elastic modulus has non-linear dependency whit fiber volume fraction. The value of elastic modulus increases as fiber volume fraction is increasing.

5.3. Properties calculated using macromechanical approach:

In this section the elastic, shear, flexural modulus and Poisson's ratio dependence from fiber angle was calculated and plotted in (30.Fig. – 33.Fig.). The experimentally determined volume fraction is 0.51 besides of that for comparison and further evaluation of composite mechanical properties behaviorthe under 0.60, 0.51 and 0.4 volume fraction was also calculated and ploted. Furthermore for compieroso the the experimentally obtained values from 1-7 sets was presented into grafs.

Calculation was done according second chapter taking in account samples material properties described in chapter four and calculated according rule-of-mixture. The experimental data was obtained using description in chapter four there represented walues are obtane from 1-7 sets.
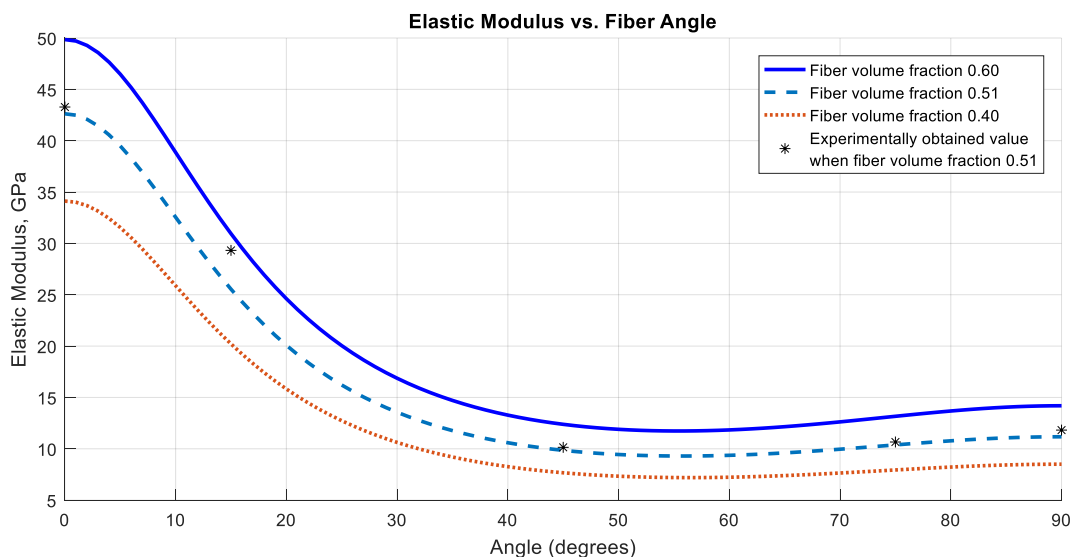
Fig. 30. Elastic modulus dependence from fiber angle.

The values of Elastic modulus calculated analytically varying from fiber angle are plotted in a graph (33.Fig.) with the results obtained experimentally. From graph we can see nonlinear behavior between Elastic modulus and fiber angle. Also from graph we can see that experimentally obtained values have good match with values calculated analytically in the determined fiber volume fraction for sample. The maxim mismach between theoretical and experimental results is from 4.set. The analytically calculated value is 2.6 % lower than the experimentally received value.
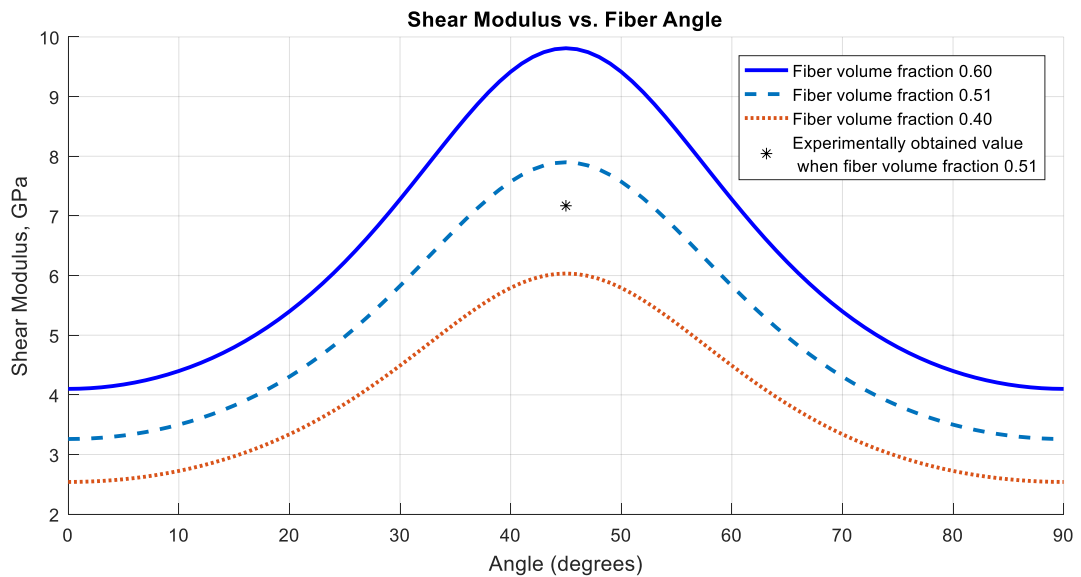
Fig. 31. Shear modulus dependence from fiber angle.

The values of shear modulus calculated analytically varying from fiber angle are plotted in a graph (34.Fig.) with the results obtained experimentally. From graph we can see nonlinear behavior between shear modulus and fiber angle. Also from graph we can see that experimentally obtained values not perfectly match with calculated analytically values in the determined fiber volume fraction for sample. The mismach between theoretical and experimental results is from 2.set. The analytically calculated value is than the experimentally obtained value..
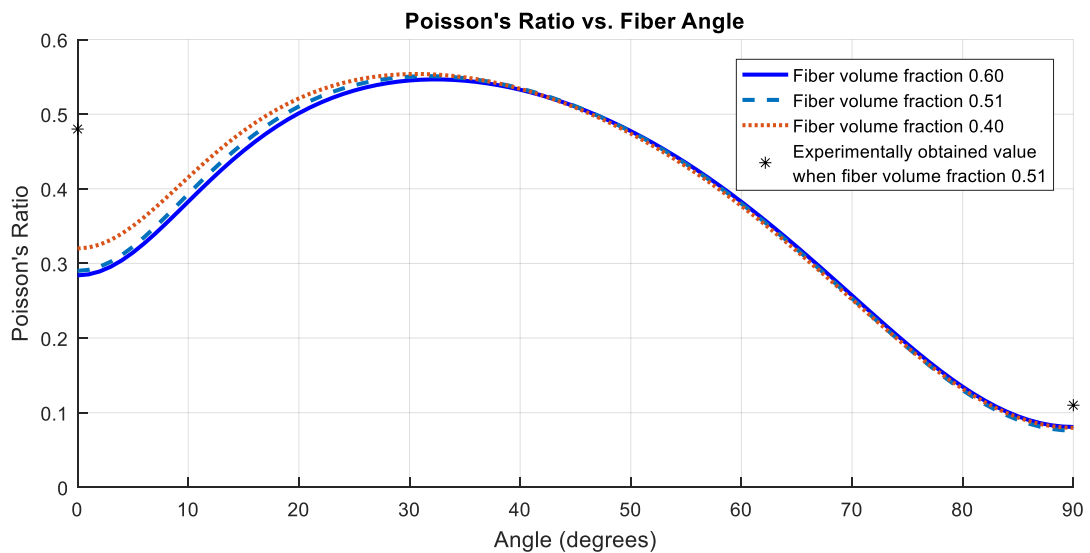
Fig. 32. Poisson's ratio dependence from fiber angle.

The values of Pousson's ratio calculated analytically varying from fiber angle are plotted in a graph (35.Fig.) with the results obtained experimentally from 1.set and 3.set. From graph we can see nonlinear behavior between Pousson's ratio and fiber angle. Also from graph we can see that experimentally obtained values are not close with calculated analytically values in the determined fiber volume fraction. The analytically calculated value is 47 percents for 1.set and 40 percents for 3.set lower than the experimentally received value.
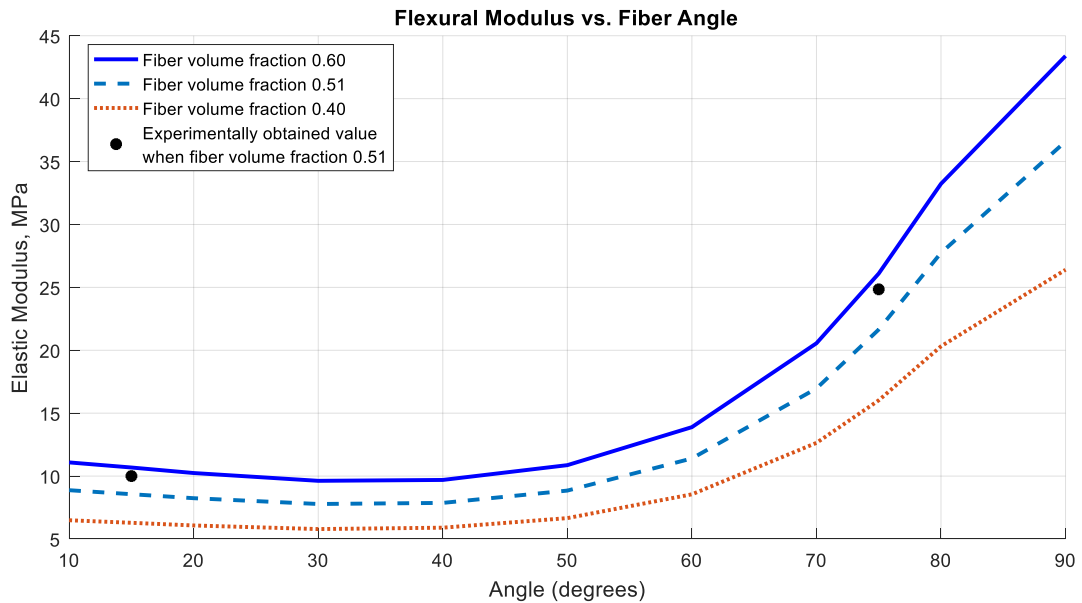
Fig. 33. Flexural modulus dependence from fiber angle.

The values of flexural modulus calculated analytically varying from fiber angle are plotted in a graph (36.Fig.) with the results obtained experimentally. From graph we can see nonlinear behavior between flexural modulus and fiber angle. Also from graph we can see that experimentally obtained values not perfectly match with calculated analytically values in the determined fiber volume fraction for sample. The analytically calculated value is 14 percents for 4.set and 13 percents for 5.set higher than the experimentally received value.

By observing the results of the elasticity properties from the Fig. 32, Fig. 31 and Fig.34. It can be seen that the results are distributed between the composite with the 0.6 and 0.4 fiber fraction. On this basis, it can be assumed that the fluctuations of the glass fiber fraction could have affected the accuracy of the results.

Overall loking in standard deviation of experimentaly obtained values we can make assumption that deviations experienced during experiments could have impact to the mismatch between experimental and analytical obtained values. So analytically calculated values can be as accurate as experimental obtained values or maybe even more accurate.

5.4. Ultimate strengths of a unidirectional lamina

In this section the longitudinal tensile strength, longitudinal compressive strength, transverse tensile strength, transverse compressive strength and in-plane shear strength calculated by adapting the mechanics of materials approach and obtaned expermentaly are represented (Table 3.). The section „Difference between experimentally obtained and calculated values“ shows how much calculated value are bigger (if value are positive) or smaller (if value are negative) comparing with experimentally obtained value.

Calculation was dan according second chapter taking in account samples material properties described in chapter four and calculated according rule-of-mixture. The experimental data was obtained using description in chapter four .

Table. 5. Calculated and experimentally obtained ultimate strength parameters of a unidirectional glass fiber lamina:

	Calculated value, MPa	Experimentally obtained value, MPa	Difference between experimentally obtained and calculated values, %
Longitudinal Tensile Strength	1800	988.58	82%
Longitudinal Compressive Strength	69.02	102.79	-33%
Transverse Tensile Strength:	60.35	19.2	214%
Transverse Compressive Strength	80.54	302.06	-73%
In-Plane Shear Strength	21.28	9.7	119%

From table.3 we can see that glass fiber composite strenght paramyters are more inconsistent with expermental data comparing with input properties discrabet in section 5.3. the differences between experimental and calculated values fluctuates from -72 % till 214%.

Determining the strength properties of composites is much more complicated because strength is more sensitive to matrix and fiber bonding imperfection errors. [44].

Overall the strength predicting is quite complicated and inaccurate. Also taking in account furde analysis from source (19;44;45;46) we can say that used models for strength parameters calculation are not found to have match with experimental data. The reason of this inconvenience can be such factors as [19;44;45;46]:

- Irregular spacing of fibers causing premature failure in matrix-rich areas;
- Less than perfect bonding between the fibers and the matrix;
- Poor alignment of fibers;
- Not accounting for Poisson's ratio mismatch between the fiber and the matrix;
- Not accounting for the transversely isotropic nature of fibers such as aramids and graphite.

For this reason we can see that used approach are not usable, and we just use values obtained experimentally, assuming that these test values are accurate.

5.5. Comparison between properties obtained experimentally with calculated by FEM approach:

In this section the elastic and strenght properties for more complex composite (in different layers of the composite, the glass fiber is oriented at different angles) are represented (37.Fig.). this values are obtained experimentally from (8.set) of samples and using FEM in SolidWorks simulation.

a)   b)
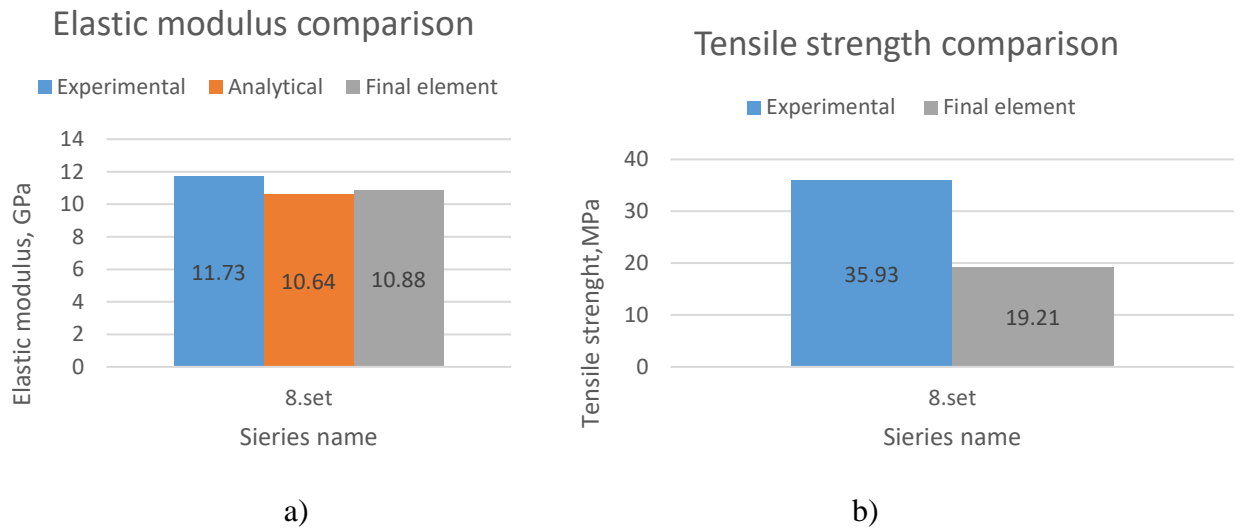
Fig. 34. Comparison: a – between the values of the elastic modulus obtained by experimental, analytical and finite elements methods; b – between the values of the ultimate strength obtained by experimental and finite elements methods.
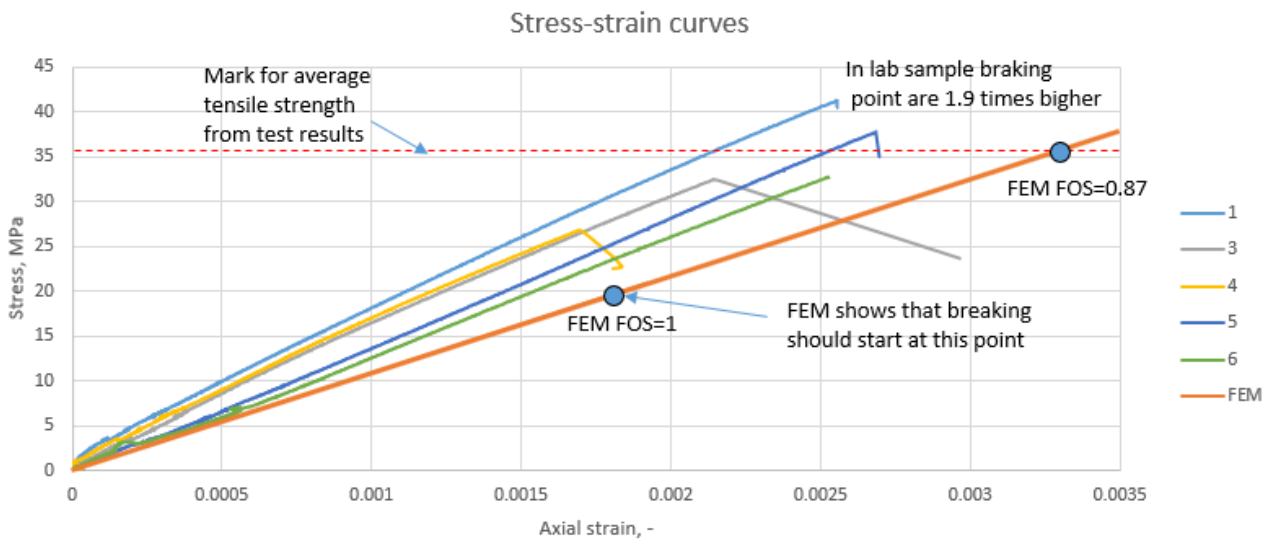
Fig. 35. Results of FE model validation.

As can be seen from the elastic modulus comparison (Fig. 34 a), the analytically obtained value is 9.3% lower and by the finite element method calculated value is 7.2% lower than the experimentally obtained value. From the strength comparison (Fig. 34 b ; Fig.35) we can see that the value calculated by the finite element method is 46.5% lower than the experimental obtained value. It is possible that Solidworks Simulation in its computational algorithm has an incremental some sort of safety coefficient, because in our case from lab obtained braking point is 1.9 times bigger.

It is worth to noting that using Tsai-Wu Criterion we get lowest strength and using Maximum Stress Criterion we get the highest strength value. In our observation we use values obtained using Maximum Stress Criterion, because result obtained by this method was closest to result for experimentation.

Calculation with SolidWorks was done using data described in section 5.1;5.2 and 5.3.

Conclusions

The present work includes details of extensive study on epoxy/glass lamina, to determine the elastic properties using expermental and analytical methodology, and the ability to use them in the laminate design by finite element method. Following are the important observations from the study:

1. From result obtained by micro mechanics methodology comparison to results obtained experimentally we can see that mechanical properties: Elastic modulus is up to 2.6% lower; Shear modulus 10% higher; Poisson's ratio up to 47%; Flexural modulus up to 14% higher. Looking to this and taking in account experimental data deviation and literature analysis we can assume that micromechanics approach are valid way to calculate mechanical properties.

2. After deep analysis of mechanics of materials approach for composite lamina strength properties calculation. We can say that this approach in current stage are not usable. The lamina strength properties should be obtained experimentally;

3. From comparison between properties obtained experimentally with calculated by fine element method (FEM) we can see that FEM provides 7.2% lower elastic modulus and 46.5% lower tensile strength. From this we can assume that engineering structures design can be safely done with finite element method.

4. The composit laminate designa by finale lement methoad can be done whit SolidWorks Simulation. But it is recommended to evaluate that SolidWorks evaluates the material as stiffer than it is in really and the calculated strenght by the finite element method is lower than the experimental obtained value. But it is also worth to notice that lab sample braking point are 1,9 times bigger than FEM. It mens that calculacion done with SolidWorks has some sort of safety.

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Appendices

Appendix 1. Data from tensile test experimentation.

1.set – is samples from tensile test. In this sample fiber are oriented along tensile force axis. Samples geometry and experment data are present in table 4-5. In (Fig.38) are shown each sample test Stress/strain function.

Table. 6. Series 1.set samples geometry and result of tensile experiments gain from test machine I:

Samples No. From 1.set	Section thicness t , mm	Section width h , mm	Sample length L , mm	Tensile modulus E, GPa	Poisson's ratio
1	6.2	25		44.00	0.49
2	5.5	25		40.79	0.40
3	6.2	25		43.63	0.50
4	5.7	25	250	46.59	0.57
5	7	25		41.90	0.33
6	5.45	25		42.79	0.59
Average				43.28	0.48

Table. 7. Series 1.set samples geometry and result of tensile experiments gain from test machine II:

Samples No. From 1.set	Section thicness t , mm	Section width h , mm	Sample length L , mm	Ultimate tensile strength F , N	Ultimate tensile stress σ_u , MPa	Average ultimate stress, MPa
1	3.63	10.9		35718	902.722	
2	3.50	11.00		39030	1013.766	
3	3.70	11.00	250	40770	1001.720	988.579
4	3.60	11.00		43302	1093.485	
5	3.70	10.70		37776	954.180	
6	3.65	10.80		38064	965.601	
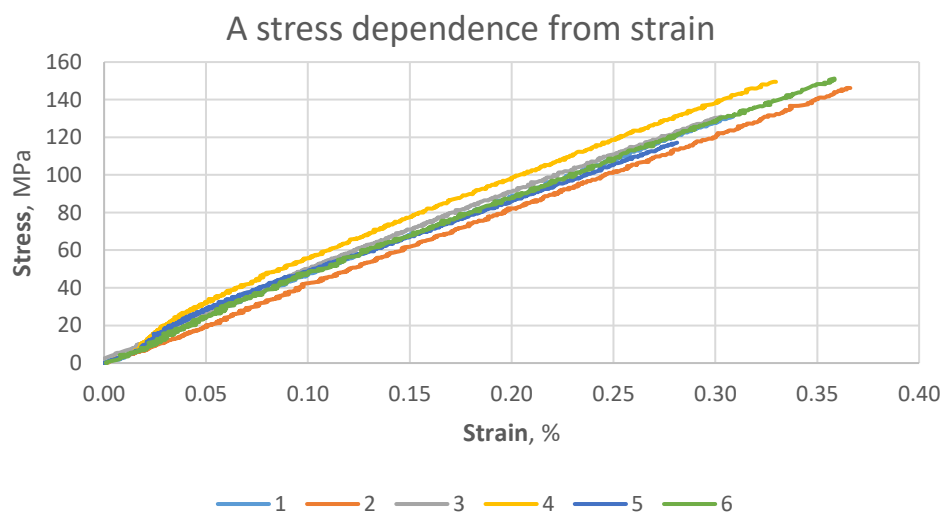
Fig. 36 Stress/strain dependence from tensile test with 1.set. Result was obtaned maschine I.

2.set – samples from laminar shear stress earlier refer as vyn_45. In this sample fiber are oriented 45 deg. angle along tensile force axis. From this sample we optane in plane shear modulus and in plane shear strane. Geometry of sample and test results are shown in table 6. In (Fig.39) is presented elongation dependents from loading force.

Table. 8. Samples from 2.set geometry and result from tensile experiments:

2.set samples No.	Section thicness t , mm	Section width h , mm	Sample length L , mm	Ultimate tensile strength, N	In plane shear strength MPa	Shear strain , %	In plane shear modulus, GPa
1	4.7	25		2040	8.68	0.12	7.20
2	5	25		4070	16.28	0.12	6.56
3	4.8	25		3750	15.63	0.15	5.89
4	5.6	25	250	2480	8.86	0.08	8.20
5	4.8	25		3200	13.33	0.18	5.64
6	4.5	25		1570	6.98	0.05	9.55
Average					11.62	0.12	7.17
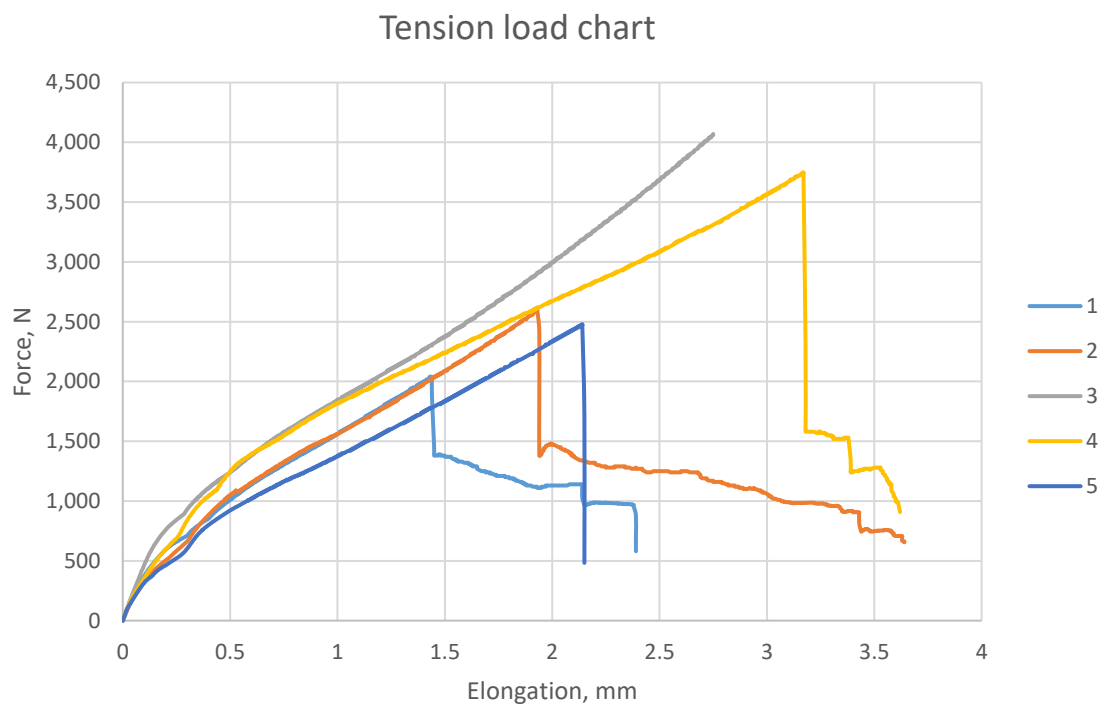
Fig. 37. Stress/strain curve approximated from series vyn_45 tensile test results.

3.set – samples from tensile test earlier refuse as vyn_90. In this sample fiber are oriented perpendicular to tensile force axis. The samples geometry parameters and experimental data are shown in table 6 also for visualization the experimental data are present in (Fig.40):

Table. 9. Samples geometry and result from tensile experiments whit 3.set:

3.set samples No.	Section thicness t, mm	Section width h, mm	Sample length L, mm	Ultimate tensile strength, N	Ultimate tensile stress, MPa	Strain under ultimate stress, %	Tensile modulus E, GPa	Poisson's ratio
1	4.8	25		2130	17.75	0.14	12.22	0.03
2	5	25		2730	21.84	0.19	11.10	0.07
3	5.2	25	250	1970	15.15385	0.12	13.39	0.13
4	5.1	25		2720	21.33333	0.18	11.58	0.13
5	5.1	25		2540	19.92157	0.18	10.83	0.17
6	4.2	25		1970	18.7619	0.11	16.76	0.38
Average					19.20	0.16	11.82	0.11
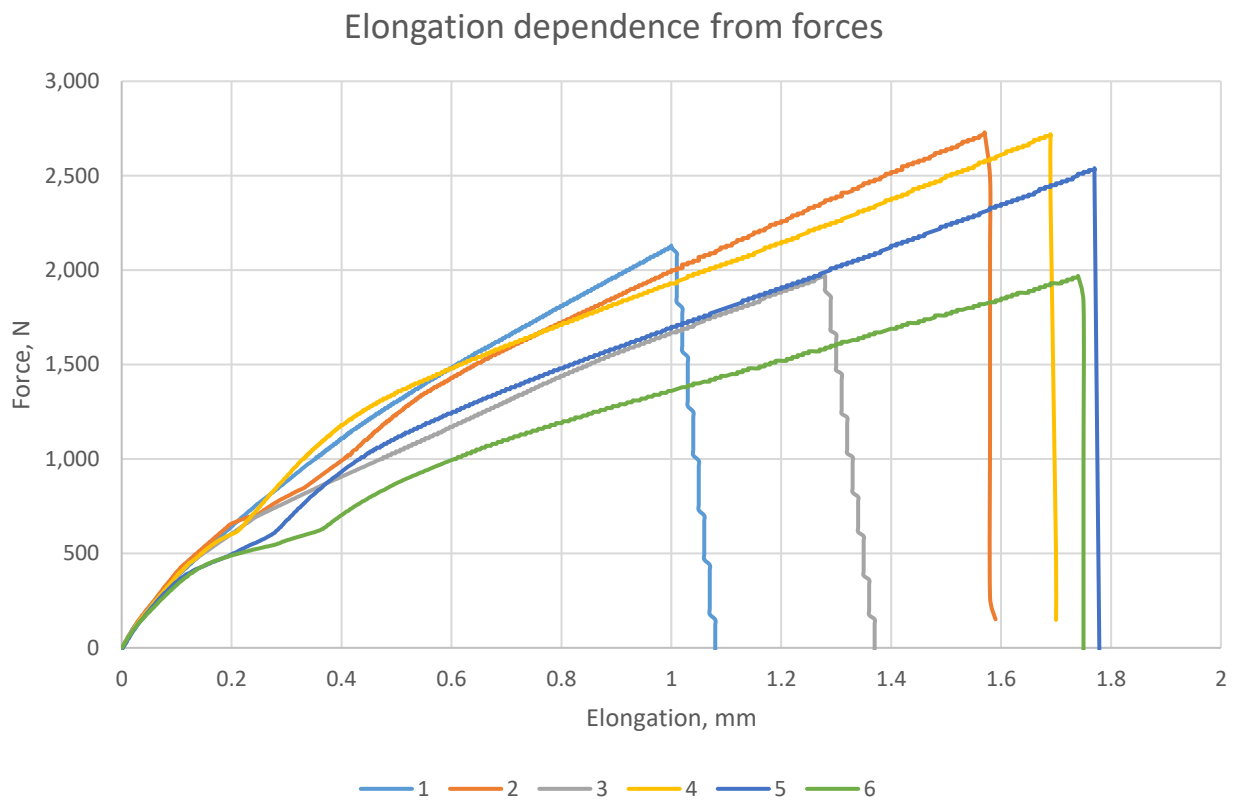
Fig. 38. Force/elongation curve approximated from series 3.set tensile test results.

4.set – samplses from tensile test, made of helical wound laminate in to axial direction of cylinder, fiber oriented 15 deg to louding axis. The samples geometry parameters and experimental data are shown in table 8 also for visualization the experimental data are present in (Fig.41):

Table. 10. Samples geometry and result from tensile experiments whit 4.set:

4.set samples No.	Section width b, mm	Section thicness t, mm	Ultimate tensile strength Fmax, kN	Ultimate tensile stress, MPa	Tensile modulus E, GPa	Poisson's ratio mu_21	Notes
1	20	5.1	2.6	25.4	12.5	0.070	Breaking in fixtures

							Breaking at extensometer
2	19.7	5.2	2.9	27.8	8.0	0.054	edge
3	19.9	5	3.6	36.0	13.3	-	
4	20	5.1	2.6	25.2	10.2	0.070	Breaking in fixtures
5	20.1	5.25	3.2	30.5	9.3	0.077	
avg	19.94	5.13	3.0	31.5	10.7	0.068	
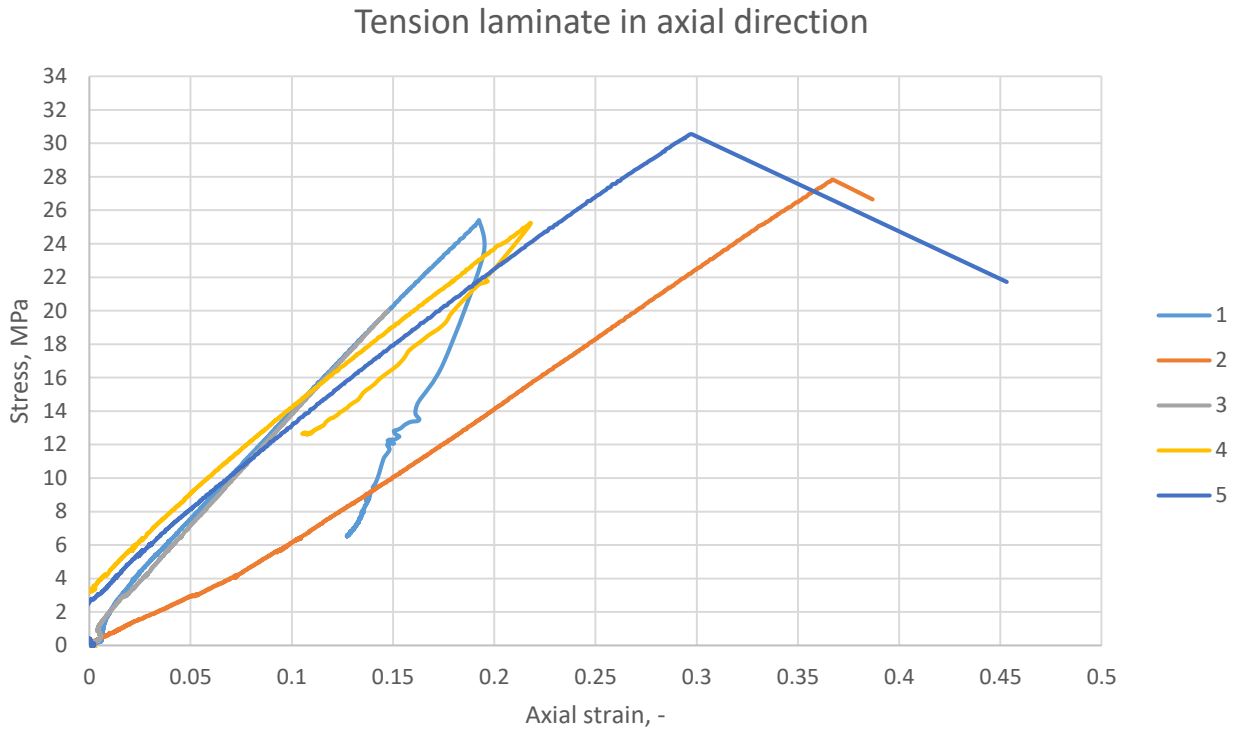
Fig. 39. Stress/strain curve approximated from 4.set tensile test results.

5.set – samples from tensile test, made of helical wound laminate in to circumferential direction of cylinder,fiber oriented by 75 deg to louding axis. The samples geometry parameters and experimental data are shown in table 9 also for visualization the experimental data are present in (Fig.42):

Table. 11. Samples geometry and result from tensile experiments with 5.set:

5.set samples No.	Section width b, mm	Section thicness t, mm	Ultimate tensile strength Fmax, kN	Ultimate tensile stress, MPa	Tensile modulus E, GPa
1	9.9	4.8	13.50	284.09	42.9
2	9.5	4.9	14.55	312.57	38.9
1	21	4.1	41.18	478.33	36.7
2	20.4	3.8	23.90	308.28	28.6
3	21	3.8	28.25	354.06	28.2
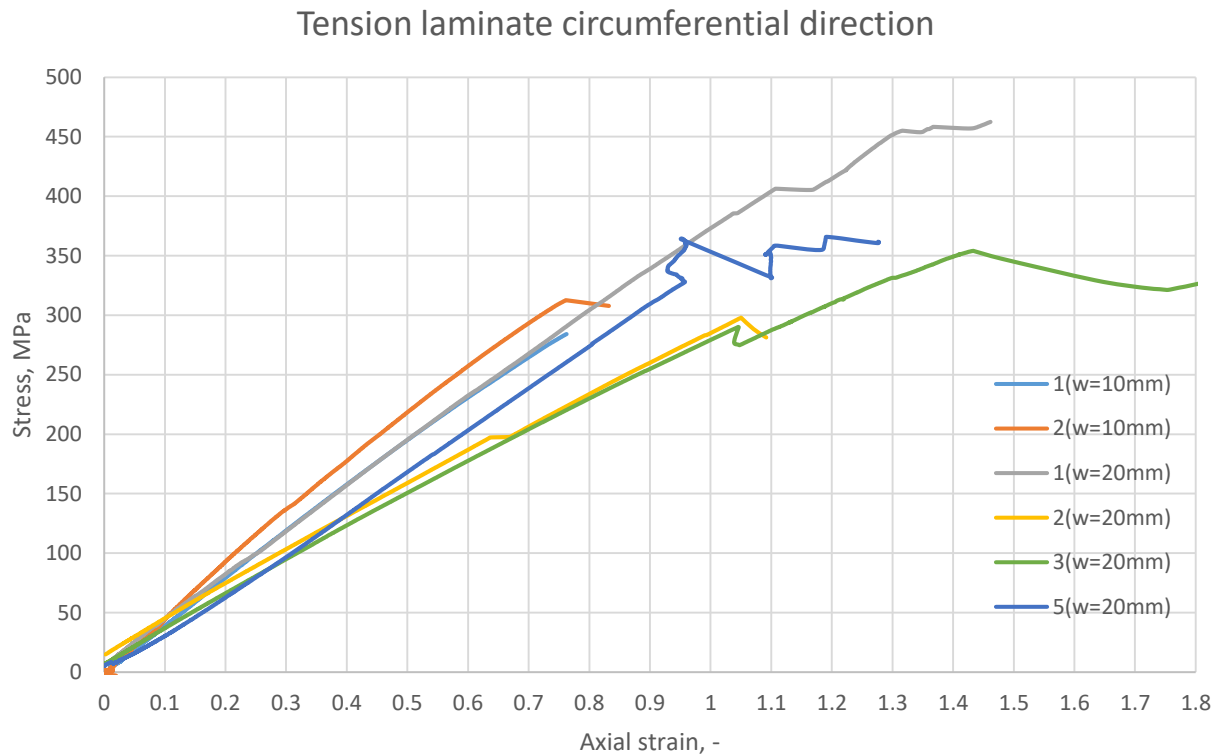
Fig. 40. Stress/strain curve approximated from series 5.set tensile test results.

8.set - samples from tensile test, made of helical wound laminate in to axial direction of cylinder (Diameter 4.0 m), Layup: (70/-70/90/90/-70/70) deg. The samples geometry parameters and experimental data are shown in table 10, also for visualization the experimental data are present in (Fig.43):

Table. 12. Samples geometry and result from tensile experiments whit 8.set:

8.set samples No.	Section width b, mm	Section thicness t, mm	Ultimate tensile strength Fmax, kN	Ultimate tensile stress, MPa	Tensile modulus E, GPa
1	19.7	5	4.06	41.18	15.00
2	20.1	5.05	2.64	26.01	12.00
3	20.1	4.9	3.19	32.41	14.40
4	19.8	4.9	2.60	26.78	14.20
4	19.8	4.9	3.32	34.20	13.50
5	20.05	5	0.00	0.00	14.00
6	20	5	0.00	0.00	12.50
avg	19.9	4.95	3.16	35.93	13.68
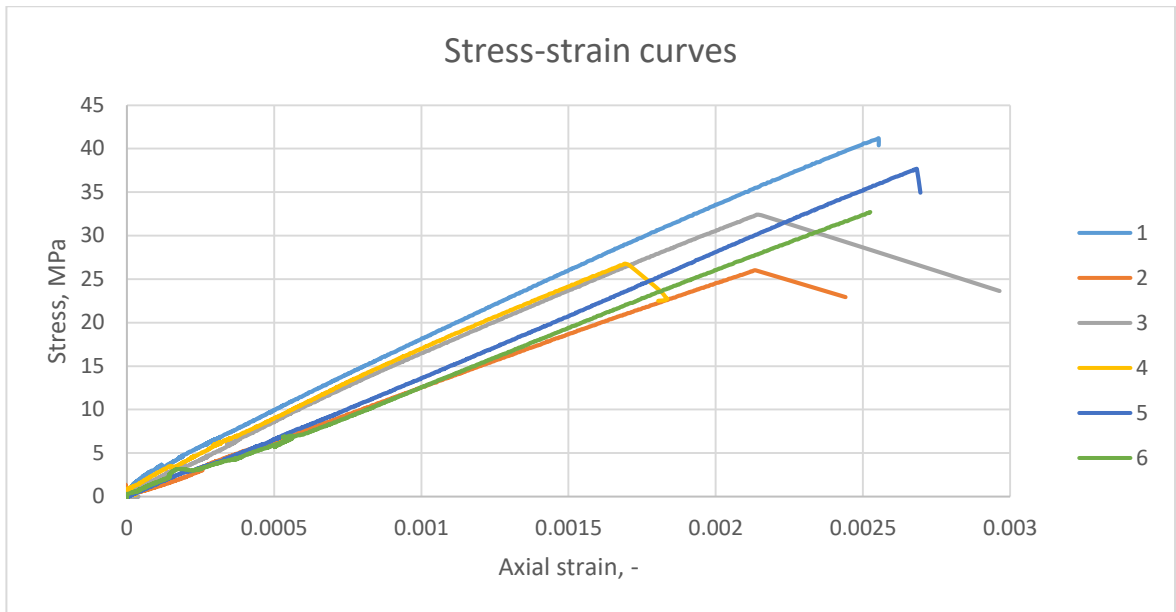
Fig. 41. Stress/strain curve approximated from series 8.set tensile test results.

Appendix 2. Data from three-point bending test experimentation.

6.set– samples from three point bending test, made of helical wound laminate in to axial direction of cylinder, fiber are oriented by 15 deg to sample longitude axis. The samples geometry parameters and experimental data are shown in table 11, also for visualization the experimental data are present in (Fig.44):

Table. 13. Samples geometry and result from flexure experiments with 6.set samples:

| 6.set samples No.	Section width b, mm	Section tihnes t, mm	Ultimate flexure strength Fmax, kN	Ultimate flexure stress, MPa	Flexure modulus E, GPa
1	20	5.1	0.216	62.3	8.6
2	19.7	5.2	0.192	55.4	11.3
3	19.9	5	0.186	53.6	12.0
4	20	5.1	0.186	53.6	8.2
5	20.1	5.25	0.216	62.3	9.9
avg	19.94	5.13	0.1992	58.8	10.0
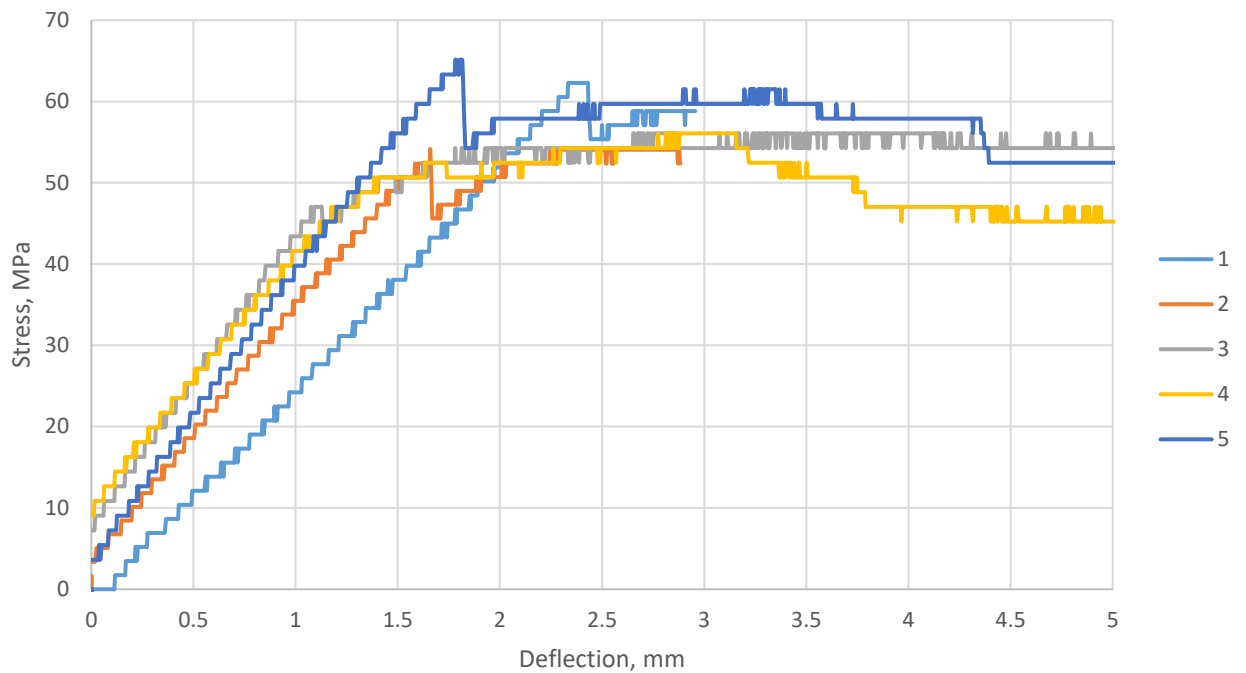
Fig. 42. Stress/strain curve approximated from 6.set bending test results.

7.set – samples from three point bending test, made of helical wound laminate in to axial direction of cylinder, fiber are oriented by 75 deg to sample longitude axis. The samples geometry parameters and experimental data are shown in table 12, also for visualization the experimental data are present in (Fig.45):

Table. 14. Samples geometry and result from flexure experiments with 7.set:

7.set samples No.	Section width b, mm	Section thicness t, mm	Ultimate flexure strength Fmax, kN	Ultimate flexure stress, MPa	Flexure modulus E, GPa
1	9.9	4.8	0.86	564.2	25.81403382
2	9.5	4.9	1.06	694.4	32.03637116
3	9.9	5	0.80	524.8	17.66203146
4	9.8	5.1	0.98	643.2	34.39792863
5	10	5.25	0.96	631.3	14.30068167
avg	20	5	0.93	629.34	24.84220935
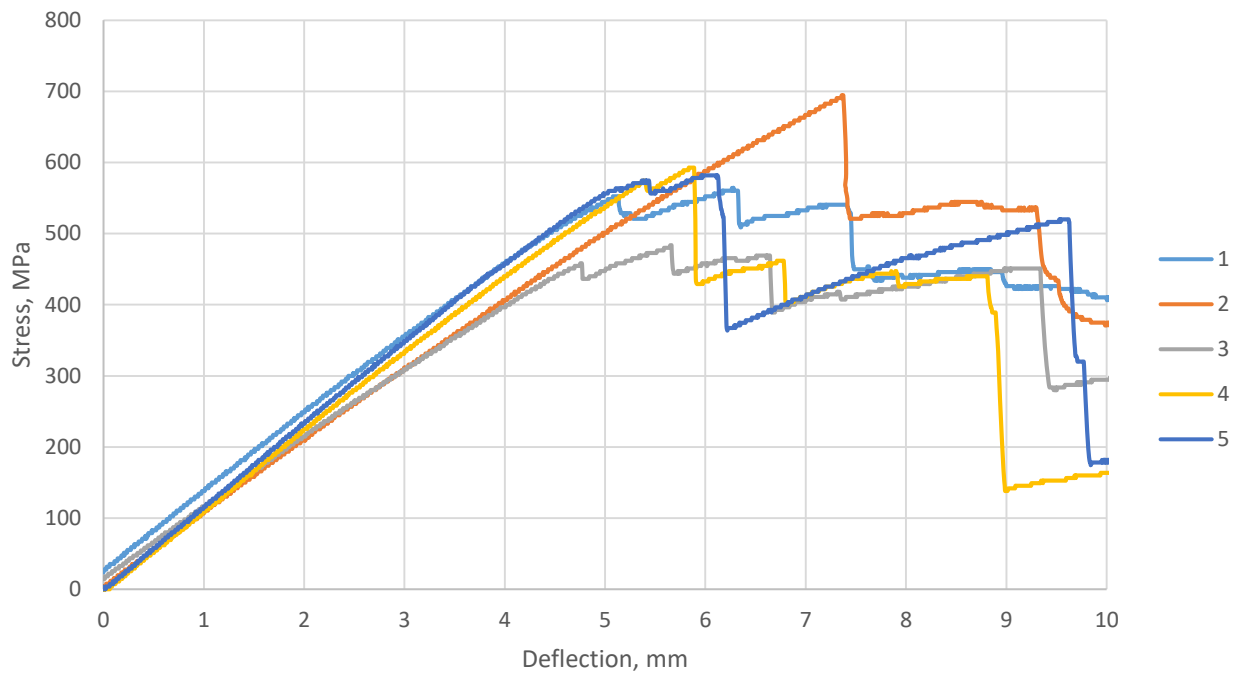
Fig. 43. Stress/strain curve approximated from 7.set tensile test results.

Appendix 3. Data from compression test experimentation.

9.set – samples from compression test fiber direction was parallel to loding direction; 10.set - samples from compression test fiber direction was perpendicular to loding direction.

Compression test was preformed with test machine II which represented abow. Test was done for two. The different between series was fiber direction: G_L_V_A (9.set) – parallel; G_S_V_A (10.set) – perpendicular. Samples were rectangular, samples geometry represented in table 6 where *t* - width, *b* - length, *h* - height.
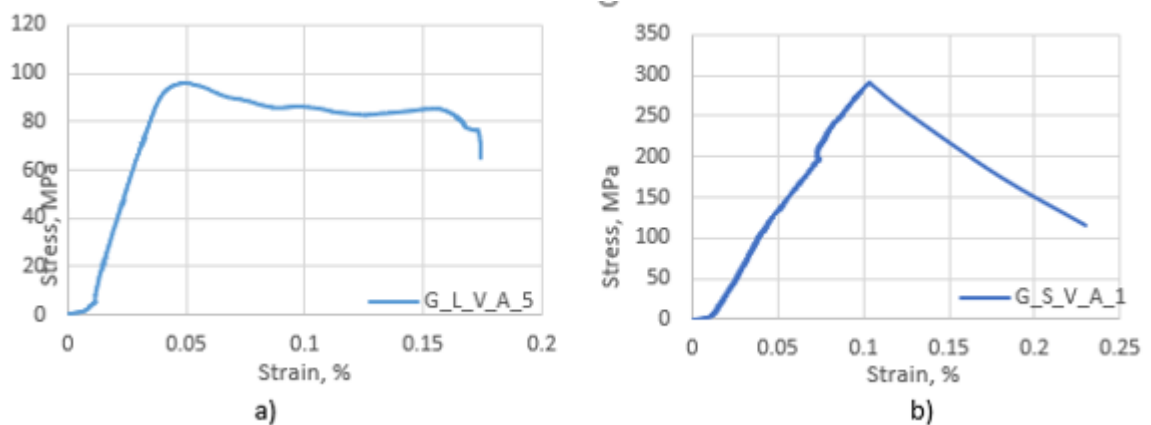
Fig. 44. Represented curves of compression stress/strain: a – for G_L_V_A serie; b - for G_S_V_A serie.

Table. 15. Compression test results:

Sample No.		Thickness t,mm	Width b, mm	Hight h, mm	F_uc, N	σ_uc, MPa	σ_uc_avg, MPa
9.set	G_L_V_A_1	9.5	10	9.6	13308	140.084	102.786
	G_L_V_A_2	9.9	10.5	9.7	9210	88.600	
	G_L_V_A_3	9.4	10.6	9.6	9192	92.252	
	G_L_V_A_4	9.5	10.5	9.6	9642	96.662	
	G_L_V_A_5	9.6	12.1	9.9	11190	96.333	
10.set	G_S_V_A_1	9.7	11.5	9.6	32544	291.744	302.055
	G_S_V_A_2	9.8	11.4	9.6	28218	252.578	
	G_S_V_A_3	9.8	10.8	9.5	29568	279.365	
	G_S_V_A_4	9.8	9.8	9.5	27888	290.379	
	G_S_V_A_5	9.8	9.8	9.5	38052	396.210	

Appendix 4. Data from laminar shear strength test experimentation.

11.set – samples from laminar shear strength tests, fiber was orientated along tensile force;12.set – samples from laminar shear strength tests, fiber is orientated disorderly;13.set - samples from laminar shear strength tests, fiber orientation and tensile forces axis is perpendicular

Sample No.		Minimum thickness a ,mm	Width b, mm	Lenght L, mm	Load W, N	Inter-laminar shear strength τ, MPa	τ average, MPa
13.set	90_5_1	5			1058	4.232	4.862
	90_5_2	5			1098	4.392	
	90_5_3	5			1284	5.136	
	90_5_4	5			1422	5.688	
11.set	A_10_1	10			4704	9.408	9.4032
	A_10_2	10			4788	9.576	
	A_10_3	10	25	250	4176	8.352	
	A_10_4	10			4704	9.408	
	A_10_5	10			5136	10.272	
12.set	C_10_1	10			3228	6.456	6.8328
	C_10_2	10			3468	6.936	
	C_10_3	10			3234	6.468	
	C_10_4	10			3720	7.44	
	C_10_5	10			3432	6.864	

Appendix 5. MatLab code for lamina properties calculation using rule of mixture.

```
%% Usage
%       function [moduli] = moduli_halphin(Ef,Em,nuf,numm,Vf,xi)
% Input Variables
%       Ef=fiber elastic modulus
%       Em=matrix elastic modulus
%       nuf=fiber Poisson's ratio
%       numm=matrix Poisson's ratio
%       Vf=fiber volume fraction
%       xi=Halphin Tsai reinforcing factor
% Output Variables
```

```
%       moduli=vector of elastic modulii and Poisson's ratio of unidirectional
lamina
%       [E1 E2 nu12 G12]

%% License Agreement
%       http://www.eng.usf.edu/~kaw/OCW/composites/license/limiteduse.pdf
%% Testing code
clc
clear all
%% Inputs
% Mateial: Glass/Epoxy
% Lamina: square array with circular fiber cross-sections
Ef=80.5E9;
Em=3.2E9;
nuf=0.22;
numm=0.38;
xi=[2 1];
fprintf('\nFiber Elastic Modulus: %g',Ef)
fprintf('\nMatrix Elastic Modulus: %g',Em)
fprintf('\nFiber Poisson''s ratio: %g',nuf)
fprintf('\nMatrix Poisson''s ratio: %g',numm)
fprintf('\nHalphin Tsai reinforcing factor for elastic moduli: %g',xi(1))
fprintf('\nHalphin Tsai reinforcing factor for shear moduli: %g',xi(2))
%% Test 1
% Testing for a particular Vf
% Input desired Vf
Vf=0.51;
% Call function: moduli_halphin
[moduli] = moduli_halphin(Ef,Em,nuf,numm,Vf,xi);
% Results for a particular Vf
fprintf('\n\nFiber Volume Fraction: %G\n\n',Vf)
disp('Lamina Modulii and Poisson''s Ratios')
disp('_____')
fprintf('\n Ex    | %G\n Ey    | %G\n nuxy  | %G\n Gxy   | %G\n\n\n\n',...
    moduli(1),moduli(2),moduli(3),moduli(4))
%% Test 2
% Table of elastic modulii and Poisson's ratios of a Unidirectional Lamina
% as a Function of Fiber Volume Fraction
for i=1:1:11
    Vf(i)=i/10-0.1;
    [moduli] = moduli_halphin(Ef,Em,nuf,numm,Vf(i),xi);
    Ex(i)=moduli(1);
    Ey(i)=moduli(2);
    nuxy(i)=moduli(3);
    Gxy(i)=moduli(4);
end
% Table of results
disp('            Lamina Modulii and Poisson''s Ratios')
disp('_____
_____')
disp('   Vf    |     Ex        |      Ey       |      nuxy     |
Gxy ')
for i=1:1:11
fprintf('  %3.2f   |    %3.4E   |    %3.4E   |    %3.4f   |
%3.4E\n',...
    Vf(i),Ex(i),Ey(i),nuxy(i),Gxy(i))
end
%% Test 3
% Graph of elastic modulii and Poisson's ratios of a Unidirectional Lamina
% as a Function of Fiber Volume Fraction
for i=1:1:100
    Vf(i)=i/100-0.01;
    [moduli] = moduli_halphin(Ef,Em,nuf,numm,Vf(i),xi);
    Ex(i)=moduli(1);
```

```matlab
    Ey(i)=moduli(2);
    nuxy(i)=moduli(3);
    Gxy(i)=moduli(4);
end
% Plot of elastic modulus
figure
hold on
plot(Vf,Ex,'b','LineWidth',2)
plot(Vf,Ey,'r','LineWidth',2)
plot(Vf(52),Ex(52),'k*','LineWidth',2)
plot(Vf(52),Ey(52),'k*','LineWidth',2)
grid
legend('Longitudinal Elastic Modulus','Transverse Elastic
Modulus','Experimentally obtained fraction value')
title('Elastic Modulus vs. Fiber Volume Fraction')
xlabel('Fiber Volume Fraction')
ylabel('Elastic Modulus')
hold off
% Plot of Poisson's ratio
figure
hold on
plot(Vf,nuxy,'b','LineWidth',2)
plot(Vf(52),nuxy(52),'k*','LineWidth',2)
grid
legend('Poisson''s Ratio','Experimentally obtained fraction value')
title('Poisson''s Ratio vs. Fiber Volume Fraction')
xlabel('Fiber Volume Fraction')
ylabel('Poisson''s Ratio')
hold off
% Plot of shear modulus
figure
hold on
plot(Vf,Gxy,'b','LineWidth',2)
plot(Vf(52),Gxy(52),'k*','LineWidth',2)
grid
legend('Shear Modulus','Experimentally obtained fraction value')
title('Shear Modulus vs. Fiber Volume Fraction')
xlabel('Fiber Volume Fraction')


function [moduli] = moduli_halphin(Ef,Em,nuf,numm,Vf,xi)
% Fiber shear modulus
Gf=Ef/(2*(1+nuf));
% Matrix shear modulus
Gm=Em/(2*(1+numm));
% Stress partitioning parameter for elastic modulii
Eta1=((Ef/Em)-1)/((Ef/Em)+xi(1));
% Stress partitioning parameter for shear modulus
Eta2=((Gf/Gm)-1)/((Gf/Gm)+xi(2));
% Longitudional elastic modulus
E1=Ef*Vf+Em*(1-Vf);
% Transverse elastic modulus
E2=Em*((1+xi(1)*Eta1*Vf)/(1-Eta1*Vf));
% Shear modulus
G12=Gm*((1+xi(2)*Eta2*Vf)/(1-Eta2*Vf));
% Major Poisson Ratio
nu12=nuf*Vf+numm*(1-Vf);
% Vector of elastic modulii and Poisson's ratio of unidirectional lamina
[moduli]=[E1 E2 nu12 G12];
end
```

Appendix 6. MatLab code for lamina properties dependenc from angle calculation.

```matlab
%% Usage
```

```matlab
%       function [moduli_ang] = anglemoduli(moduli,angle)
% Input variables
%       moduli=vector with four elastic moduli of unidirectional lamina
%       [moduli]=[E1 E2 nu12 G12]
%       angle=angle of ply in degrees
%       E1=longitudinal elastic modulus
%       E2=transverse elastic modulus
%       nu12=major Poisson's ratio
%       G12=in-plane shear modulus
% Output variables
%       Ex=elastic modulus in the x-direction
%       Ey=elastic modulus in the y-direction
%       nuxy=Poisson's ratio in the x-y plane
%       Gxy=shear modulus in x-y plane
%       Mx=shear coupling with axial load in x-direction
%       My=shear coupling with axial load in y-direction
%       moduli=vector with four elastic moduli of unidirectional lamina in
%             x,y direction
%       [moduli_ang]=[nuxy Gxy Mx My]
%% License Agreement
%       http://www.eng.usf.edu/~kaw/OCW/composites/license/limiteduse.pdf
%% Testing Code
clc
clear all
%% Inputs
% Material: Graphite/Epoxy
[moduli]=[36.564 9.197 0.311 1.78];
fprintf('\nLongitudinal Elastic Modulus: %g ',moduli(1))
fprintf('\nTransverse Elastic Modulus: %g ',moduli(2))
fprintf('\nPoisson''s Ratio: %g ',moduli(3))
fprintf('\nShear Modulus: %g ',moduli(4))
%% Test 1
% Testing for individual angle
% Input desired angle in degrees
angle=75;
% Call function: anglemoduli
[moduli_ang] = anglemoduli(moduli,angle);
% Results for a particular angle
fprintf('\n\nAngle of Lamina: %G\n\n',angle)
disp('  Angle Lamina Engineering Constants')
disp('_____')
fprintf('\n Ex   | %G\n Ey   | %G\n nuxy | %G\n Gxy  | %G\n Mx   | %G\n My   | %G\n\n\n\n'...

,moduli_ang(1),moduli_ang(2),moduli_ang(3),moduli_ang(4),moduli_ang(5),moduli_an
g(6))
%% Test 2
% Table of Engineering Constants of an Angle Lamina
% as a Function of Fiber Angle
for i = 1:10:91
    angle(i)=i-1;
    [moduli_ang] = anglemoduli(moduli,angle(i));
    Ex(i)=moduli_ang(1);
    Ey(i)=moduli_ang(2);
    nuxy(i)=moduli_ang(3);
    Gxy(i)=moduli_ang(4);
    Mx(i)=moduli_ang(5);
    My(i)=moduli_ang(6);
end
disp('                          Angle Lamina Engineering Constants')
disp('_____
_____')
disp('   angle         Ex             Ey            nuxy           Gxy
Mx             My')
```

```matlab
for i=1:10:91
    fprintf('   \t%2.0f    |\t %3.4e |\t %3.4e |\t %3.4e |\t %3.4e |\t %3.4e |\t
%3.4e \n', angle(i),Ex(i),Ey(i),nuxy(i),Gxy(i),Mx(i),My(i))
end
%% Test 3
% Graphs of Engineering Constants of an Angle Lamina
% as a Function of Fiber Angle
for i=1:1:91
    angle(i)=i-1;
    [moduli_ang] = anglemoduli(moduli,angle(i));
    Ex(i)=moduli_ang(1);
    Ey(i)=moduli_ang(2);
    nuxy(i)=moduli_ang(3);
    Gxy(i)=moduli_ang(4);
    Mx(i)=moduli_ang(5);
    My(i)=moduli_ang(6);
end
% Plot of elastic modulus
figure
hold on
plot(angle,Ex,'b','LineWidth',2)
plot(angle,Ey,'r','LineWidth',2)
grid
legend('Longitudinal Elastic Modulus','Transverse Elastic Modulus')
title('Elastic Modulus vs. Fiber Angle')
xlabel('Angle (degrees)')
ylabel('Elastic Modulus')
hold off
% Plot of Poisson's ratio
figure
hold on
plot(angle,nuxy,'b','LineWidth',2)
grid
legend('Poisson''s Ratio')
title('Poisson''s Ratio vs. Fiber Angle')
xlabel('Angle (degrees)')
ylabel('Poisson''s Ratio')
hold off
% Plot of shear modulus
figure
hold on
plot(angle,Gxy,'b','LineWidth',2)
grid
legend('Shear Modulus')
title('Shear Modulus vs. Fiber Angle')
xlabel('Angle (degrees)')
ylabel('Shear Modulus')
hold off
% Plot of shear coupling
figure
hold on
plot(angle,Mx,'b','LineWidth',2)
plot(angle,My,'r','LineWidth',2)
grid
legend('Shear Coupling x-dir','Shear Coupling y-dir')
title('Shear Coupling vs. Fiber Angle')
xlabel('Angle (degrees)')
ylabel('Shear Coupling')

function [moduli_ang] = anglemoduli(moduli,angle)
% Calculating the sine of the desired angle
s=sind(angle);
% Calculating the cosine of the desired angle
c=cosd(angle);
```

```
% Creating the S matrix from the moduli vector
S=[1/moduli(1) -moduli(3)/moduli(1) 0; -moduli(3)/moduli(1) 1/moduli(2) 0;
    0 0 1/moduli(4)];
% Defining elements of the transformed reduced compliance matrix, Sbar
sbar11 = S(1,1)*c^4 + (2*S(1,2)+S(3,3))*s^2*c^2 + S(2,2)*s^4;
sbar12 = S(1,2)*(s^4+c^4) + (S(1,1)+S(2,2)-S(3,3))*s^2*c^2;
sbar22 = S(1,1)*s^4 + (2*S(1,2)+S(3,3))*s^2*c^2 + S(2,2)*c^4;
sbar16 = (2*S(1,1)-2*S(1,2)-S(3,3))*s*c^3 - (2*S(2,2)-2*S(1,2)-S(3,3))*s^3*c;
sbar26 = (2*S(1,1)-2*S(1,2)-S(3,3))*s^3*c - (2*S(2,2)-2*S(1,2)-S(3,3))*s*c^3;
sbar66 = 2*(2*S(1,1)+2*S(2,2)-4*S(1,2)-S(3,3))*s^2*c^2 + S(3,3)*(s^4+c^4);
% Constructing the Sbar matrix from the elements
Sbar =[sbar11 sbar12 sbar16; sbar12 sbar22 sbar26; sbar16 sbar26 sbar66];
% Engineering constants of the angle ply
Ex = 1/Sbar(1,1);
Ey = 1/Sbar(2,2);
nuxy = -Sbar(1,2)/Sbar(1,1);
Gxy = 1/Sbar(3,3);
Mx = -Sbar(1,3)*moduli(1);
My = -Sbar(2,3)*moduli(1);
% Elements of engineering constants create the vector moduli_ang matrix
[moduli_ang] = [Ex Ey nuxy Gxy Mx My];
end
```

Appendix 7. MatLab code for laminate properties calculation:

```
%% Usage
%       function [A,B,D,An,Bn,Dn] = ABD(nplies,Qplies,angleplies,tplies)
% Input variables
%       nplies=number of plies
%       Qplies=reduced stiffness matrix for each ply
%       angleplies= angle of each ply in degrees
%       tplies=thickness of each ply
% Output variables
%       A=extensional stiffness matrix
%       B=coupling stiffness matrix
%       D=bending stiffness matrix
%       An=normalized extensional stiffness matriix
%       Bn=normalized coupling stiffness matrix
%       Dn=normalized bending stiffness matrix


%% License Agreement
%       http://www.eng.usf.edu/~kaw/OCW/composites/license/limiteduse.pdf


%% Testing Code
clc
clear all
%% Inputs
% Lamina: 70/-70/90/90/-70/70
% Material: Graphite/Epoxy
nplies=6;
[tplies]=[0.00088 0.00088 0.00088 0.00088 0.00088 0.00088];
[angleplies]=[15 -15 15 -15 15 -15];
t=sum([tplies]);
% Properties of lamina: Ex, Ey, mu, Gxy
moduli=[42.62 11.17 0.29 3.26];
% compliance matrix
S=[1/moduli(1) -moduli(3)/moduli(1) 0; -moduli(3)/moduli(1) 1/moduli(2) 0; 0 0
1/moduli(4)];
% reduced stiffness matrix
Q=inv(S);
%Q=[1.818*10^11 2.897*10^9 0; 2.897*10^9 1.035*10^10 0; 0 0 7.17*10^9];
fprintf('Number of plies: %g\n\n',nplies)
disp('Reduced Stiffness Matrix:')
```

```matlab
disp('_____')
disp(Q)
for i=1:1:nplies
    fprintf('For Ply: %g\n',i)
    fprintf('   Ply Angle: %g\n',angleplies(i))
    fprintf('   Ply Thickness: %g\n\n',tplies(i))
    Qplies(:,:,i)=Q;
end
disp(' ')
%% Test
% Call function: ABD
[A,B,D,An,Bn,Dn] = ABD(nplies,Qplies,angleplies,tplies);
% Extensional stiffness matrix
disp('Extensional Stiffness Matrix:')
disp('_____')
disp(A)
% Coupling stiffness matrix
disp('Coupling Stiffness Matrix:')
disp('_____')
disp(B)
% Bending stiffness matrix
disp('Bending Stiffness Matrix:')
disp('_____')
disp(D)
% Normalized extensional stiffness matrix
disp('Normalized Extensional Stiffness Matrix:')
disp('_____')
disp(An)
% Normalized coupling stiffness matrix
disp('Normalized Coupling Stiffness Matrix:')
disp('_____')
disp(Bn)
% Normalized bending stiffness matrix
disp('Normalized Bending Stiffness Matrix:')
disp('_____')
disp(Dn)
Ac=inv(A);
t
Ex=1/(t*Ac(1,1))
Ey=1/(t*Ac(2,2))
Gxy=1/(t*Ac(3,3))
S_l=[988 25 60]'
theta=75;
m=cosd(theta);
n=sind(theta);
T=[m^2 n^2 2*m*n; n^2 m^2 -2*m*n; -m*n m*n m^2-n^2];
S_gl=inv(T)*S_l

function[A,B,D,An,Bn,Dn]=ABD(nplies,Qplies,angleplies,tplies)
% Three dimensional reduced stiffness matrix consisting of a reduced
% stiffness matrix for each ply
for i=1:1:nplies
    Qbarplies(1,1,i)=(Qplies(1,1,i)*((cosd(angleplies(1,i)))^4))...
        +(Qplies(2,2,i)*((sind(angleplies(1,i)))^4))+(2*(Qplies(1,2,i)...
        +(2*Qplies(3,3,i)))*((sind(angleplies(1,i)))^2)...
        *((cosd(angleplies(1,i)))^2));
    Qbarplies(1,2,i)=((Qplies(1,1,i)+Qplies(2,2,i)-4*Qplies(3,3,i))...
        *((sind(angleplies(1,i)))^2)*((cosd(angleplies(1,i)))^2))...
        +(Qplies(1,2,i)*(((cosd(angleplies(1,i)))^4)...
        +((sind(angleplies(1,i)))^4)));
    Qbarplies(2,1,i)=((Qplies(1,1,i)+Qplies(2,2,i)-4*Qplies(3,3,i))...
        *((sind(angleplies(1,i)))^2)*((cosd(angleplies(1,i)))^2))...
        +(Qplies(1,2,i)*(((cosd(angleplies(1,i)))^4)...
        +((sind(angleplies(1,i)))^4)));
```

```
            Qbarplies(2,2,i)=((Qplies(1,1,i)*((sind(angleplies(1,i)))^4))...
                +(Qplies(2,2,i)*((cosd(angleplies(1,i)))^4))+(2*(Qplies(1,2,i)...
                +2*Qplies(3,3,i))*((sind(angleplies(1,i)))^2)...
                *((cosd(angleplies(1,i)))^2)));
            Qbarplies(1,3,i)=(Qplies(1,1,i)-Qplies(1,2,i)-2*Qplies(3,3,i))...
                *cosd(angleplies(1,i))^3*sind(angleplies(1,i))...
                -(Qplies(2,2,i)-Qplies(1,2,i)-2*Qplies(3,3,i))...
                *sind(angleplies(1,i))^3*cosd(angleplies(1,i));
            Qbarplies(3,1,i)=(Qplies(1,1,i)-Qplies(1,2,i)-2*Qplies(3,3,i))...
                *cosd(angleplies(1,i))^3*sind(angleplies(1,i))-(Qplies(2,2,i)...
                -Qplies(1,2,i)-2*Qplies(3,3,i))*(sind(angleplies(1,i))^3...
                *cosd(angleplies(1,i));
            Qbarplies(2,3,i)=((Qplies(1,1,i)-Qplies(1,2,i)-2*Qplies(3,3,i))...
                *cosd(angleplies(1,i))*((sind(angleplies(1,i)))^3))...
                -((Qplies(2,2,i)-Qplies(1,2,i)-2*Qplies(3,3,i))...
                *((cosd(angleplies(1,i)))^3)*sind(angleplies(1,i)));
            Qbarplies(3,2,i)=((Qplies(1,1,i)-Qplies(1,2,i)-2*Qplies(3,3,i))...
                *cosd(angleplies(1,i))*((sind(angleplies(1,i)))^3))...
                -((Qplies(2,2,i)-Qplies(1,2,i)-2*Qplies(3,3,i))...
                *((cosd(angleplies(1,i)))^3)*sind(angleplies(1,i)));
            Qbarplies(3,3,i)=((Qplies(1,1,i)+Qplies(2,2,i)-2*Qplies(1,2,i)...
                -2*Qplies(3,3,i))*((sind(angleplies(1,i)))^2)...
                *((cosd(angleplies(1,i)))^2))+(Qplies(3,3,i)...
                *(((sind(angleplies(1,i)))^4)+((cosd(angleplies(1,i)))^4)));
end
% Total thickness of the laminate
t=sum(tplies);
% Position of top and bottom of each lamina
h(1)=-t/2;
for i=2:1:nplies+1
    h(i)=h(i-1)+tplies(i-1);
end
% Extensional stiffness matrix, coupling stiffness matrix and bending stiffness
matrix
A=0;
d=0;
b=0;
for i=1:1:nplies
    h1(1,i)=(h(i+1)-h(i));
    h2(1,i)=(h(i+1)^2-h(i)^2);
    h3(1,i)=(h(i+1)^3-h(i)^3);
    A=A+(Qbarplies(:,:,i)*h1(1,i));
    b=b+(Qbarplies(:,:,i)*h2(1,i));
    d=d+(Qbarplies(:,:,i)*h3(1,i));
end
% Multiplication due to integration
B=0.5*b;
D=(1/3)*d;
% Normalized extensional stiffness matrix
An=A*(1/t);
% Normalized coupling stiffness matrix
Bn=B*(2/t^2);
% Normalized bending stiffness matrix
Dn=D*(12/t^3);
end
```

Appendix 8. MatLab code for lamina ultimate strength calculation:

```
%% Inputs
Ex=42.62E+09;%Pa
Ey=11.17E+09;%Pa
nuxy=0.31088;
Gxy =3.26E+09;%Pa
```

```matlab
Vf=0.51;
Ef=80.5E9;
Em=3.2E9;
nuf=0.22;
numm=0.38;
Sigmafult=3400*10^6;
Sigmamult=86*10^6;
Taufult=35*10^6;
Taumult=34*10^6;
Gm=1.159*10^9;
Gf=32.992*10^9;
SigmaCmult=102*10^6;
Tau12mult=34*10^6;
%% Usage
%       Ex=Longitudinal elastic modulus
%       Ey=Transverse elastic modulus
%       nuxy=Poisson's ratio
%       Gxy =In-plane shear modulus
%       Ef=fiber elastic modulus
%       Em=matrix elastic modulus
%       nuf=fiber Poisson's ratio
%       numm=matrix Poisson's ratio
%       Vf=fiber volume fraction%
%% Longitudinal Tensile Strength
efult=Sigmafult/Ef;
emult=Sigmamult/Em;
SigmaTult=Sigmafult*Vf+efult*Em*(1-Vf)
%% Longitudinal compressive strength
DoverS=sqrt(4*Vf/3.1415);
eT2ult1=emult*(1-Vf^(1/3));
eT2ult2=emult*(DoverS*((Em/Ef)-1)+1);
sigmacult1=(Ex*min([eT2ult1 eT2ult2]))/nuxy;
Sc1=2*(Vf+(1-Vf)*(Em/Ef))*sqrt((Vf*Em*Ef)/(3*(1-Vf)));
Sc2=Gm/(1-Vf);
sigmacult2=2*(Taufult*Vf+Taumult*(1-Vf));
sigmacult=min([sigmacult1 Sc1 Sc2 sigmacult2])
%% Transverse Tensile Strength:
SigmaTTult=Ey*min([eT2ult1 eT2ult2])
%% Transverse Compressive Strength
eCm=SigmaCmult/Em;
eC=eCm*(DoverS*(Em/Ef)+(1-DoverS));
SigmaCult=Ey*eC
%% In-Plane Shear Strength
Gama12mult=Tau12mult/Gm;
Tau12ult=Gxy*(DoverS*(Gm/Gf)+(1-DoverS))*Gama12mult
```
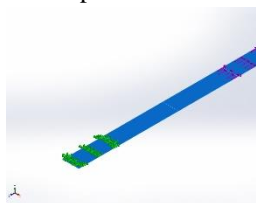
Appendix 9. SolidWorks Simulation report:

Model Information
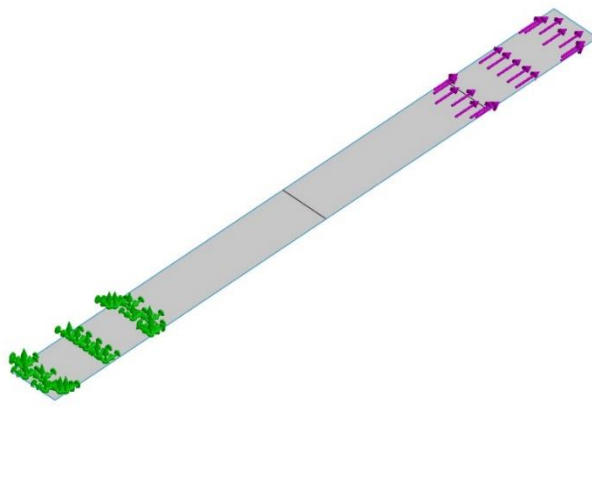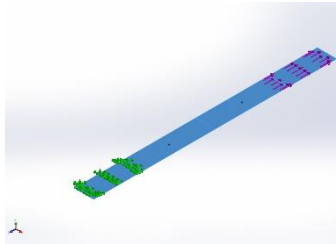
Model name: 8serie
Current Configuration: Default

Composite Bodies

Document Name and Reference	Properties
Split Line1	Total number of Plies: 6 Symmetric: Yes

Total number of Plies: 6
Symmetric: Yes

Ply	Thickness(mm)	Angle(deg)	Area(m^2)	Volume(m^3)	Density(kg/m^3)	Mass(kg)	Weight(N)
1	0.833	70	0.005	4.165e-06	1916	0.00798014	0.0782054
2	0.833	-70	0.005	4.165e-06	1916	0.00798014	0.0782054
3	0.833	90	0.005	4.165e-06	1916	0.00798014	0.0782054
4	0.833	90	0.005	4.165e-06	1916	0.00798014	0.0782054
5	0.833	-70	0.005	4.165e-06	1916	0.00798014	0.0782054
6	0.833	70	0.005	4.165e-06	1916	0.00798014	0.0782054

Document   Name:
C:\Users\antval1\Desktop\SW\Strenght\8serie.SLDPRT
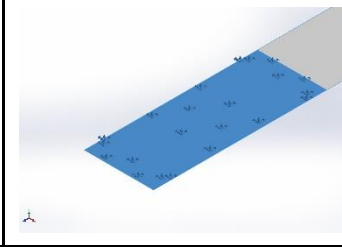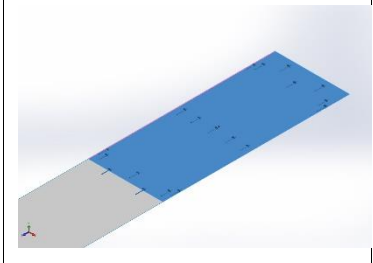**Date Modified:** May 30 01:57:41 2019

Units

Unit system:	SI (MKS)
Length/Displacement	mm
Temperature	Kelvin
Angular velocity	Rad/sec
Pressure/Stress	N/m^2

Material Properties
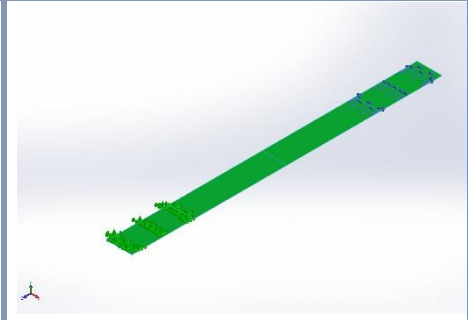
Model Reference	Properties		Components
	Name:	My Composite (3)	SurfaceBody 1(Split Line1)(8serie)
	Model type:	Linear Elastic Orthotropic	
	Default failure criterion:	Max von Mises Stress	
	Yield strength:	9.8858e+08 N/m^2	
	Tensile strength in x:	9.8858e+08 N/m^2	
	Tensile strength in y:	1.92e+07 N/m^2	
	Compressive strength in x:	1.0279e+08 N/m^2	
	Compressive strength in y:	3.0206e+08 N/m^2	
	Elastic modulus in x:	4.262e+10 N/m^2	
	Elastic modulus in y:	1.125e+10 N/m^2	
	Elastic modulus in z:	1.125e+10 N/m^2	
	Poisson's ratio in xy:	0.29	
	Poisson's ratio in yz:	0.078085	
	Poisson's ratio in xz:	0.078085	
	Mass density:	1916 kg/m^3	
	Shear modulus in xy:	3.26e+09 N/m^2	
	Shear strength:	9.7e+06 N/m^2	
Curve Data:N/A			

Loads and Fixtures

Fixture name	Fixture Image	Fixture Details
Fixed-1		**Entities:** 1 face(s) **Type:** Fixed Geometry

Resultant Forces				
Components	**X**	**Y**	**Z**	**Resultant**
Reaction force(N)	-1.33514e-05	-5.12138e-14	2000	2000
Reaction Moment(N.m)	-1.81526e-13	0	7.9036e-14	1.97986e-13

Load name	Load Image	Load Details
Force-1		**Entities:** 1 face(s) **Reference:** Edge< 1 > **Type:** Apply force **Values:** ---, ---, -2000 N **Moments:** ---, ---, --- N.m

Contact Information

Contact	Contact Image	Contact Properties
Component Contact-1		**Type:** Bonded **Options:** Compatible mesh

Mesh information - Details

Total Nodes	2111
Total Elements	968
Time to complete mesh(hh;mm;ss):	00:00:01
Computer name:	
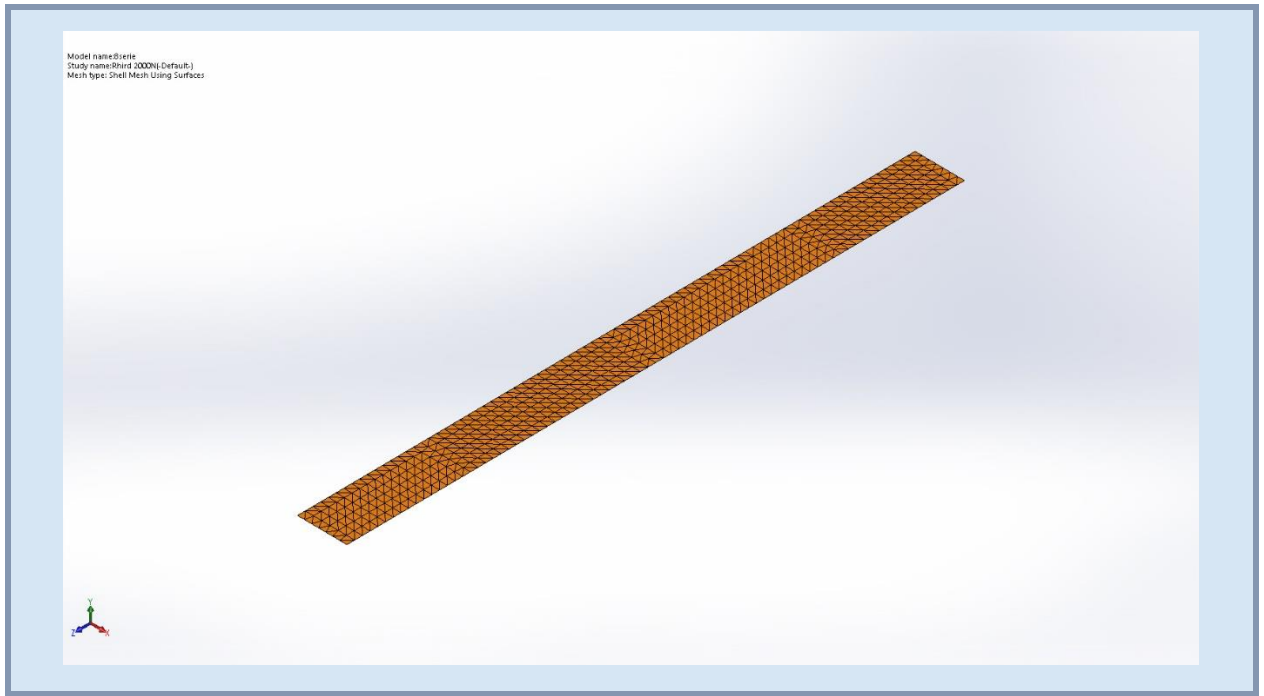
Resultant Forces

Reaction forces

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N	-1.33514e-05	-5.12138e-14	2000	2000

Reaction Moments

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N.m	-1.81526e-13	0	7.9036e-14	1.97986e-13

Study Results

Name	Type	Min	Max
Stress1	SZ: Z Normal Stress	-1.719e-08N/mm^2 (MPa) Node: 40	2.090e+01 N/mm^2 (MPa) Node: 243

8serie-Rhird 2000N-Stress-Stress1
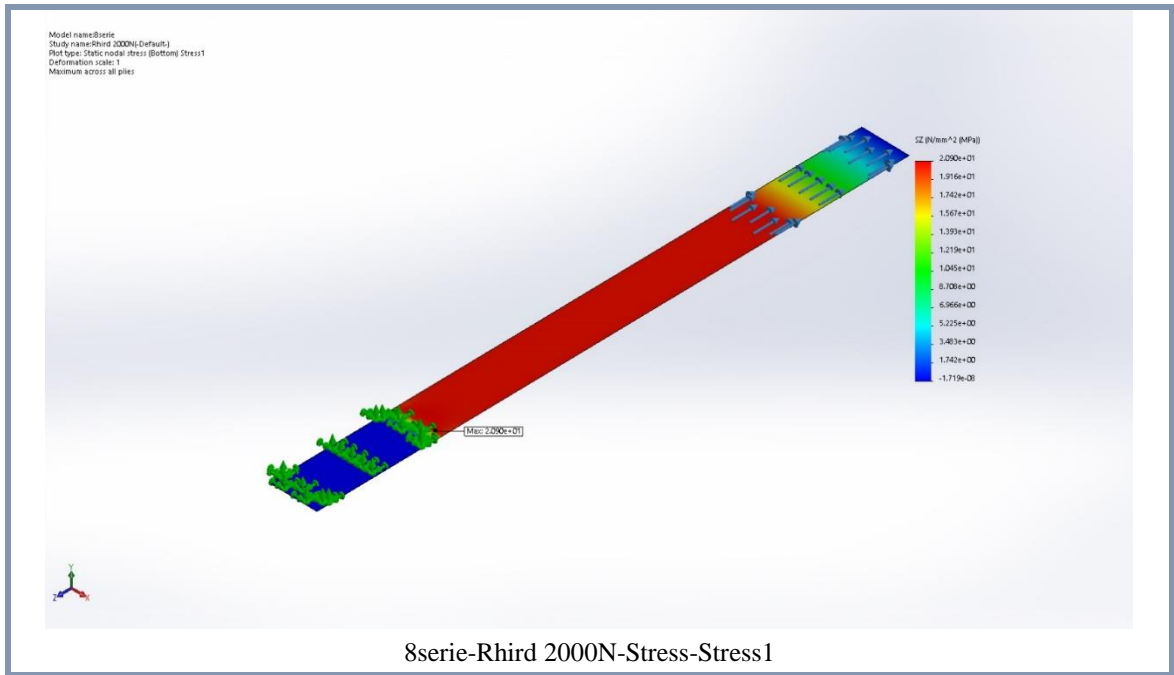
Name	Type	Min	Max
Strain1	EPSZ: Z Normal Strain	0.000e+00 Element: 775	1.921e-03 Element: 448

8serie-Rhird 2000N-Strain-Strain1
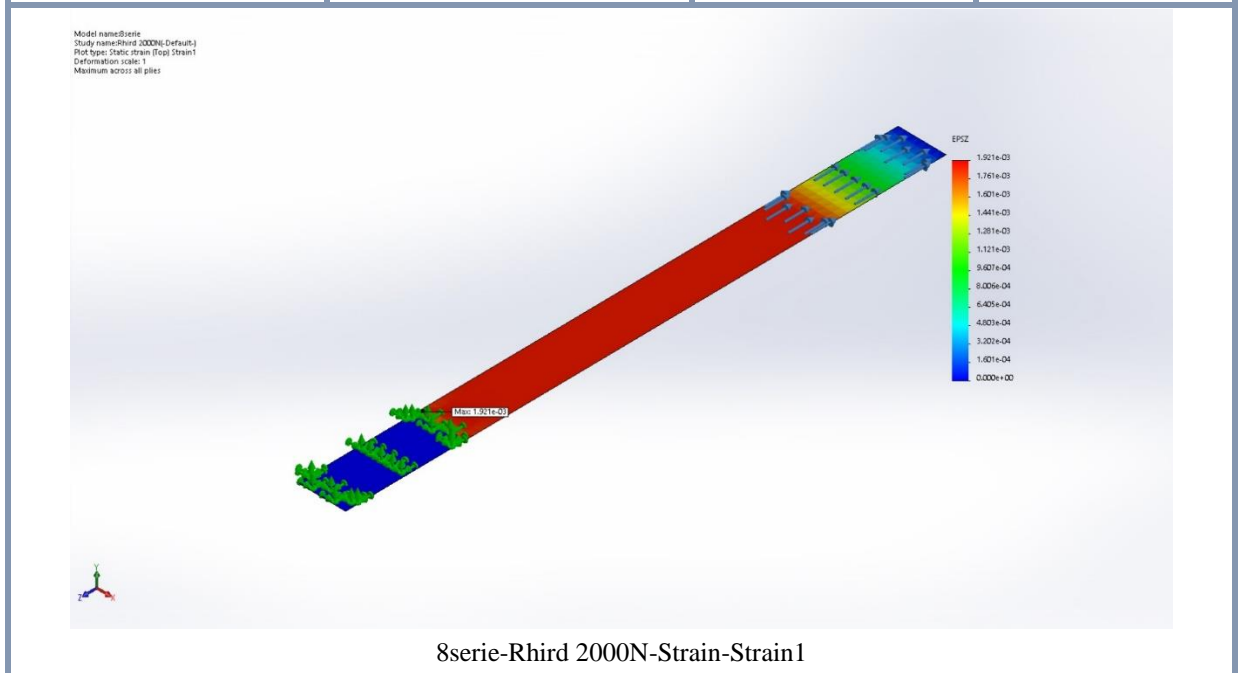
Name	Type	Min	Max
Displacement1	UZ: Z Displacement	-3.268e-01mm Node: 1979	0.000e+00 mm Node: 146

Model name:8serie
Study name:Rhird 2000N(-Default-)
Plot type: Static displacement Displacement1
Deformation scale: 1

8serie-Rhird 2000N-Displacement-Displacement1
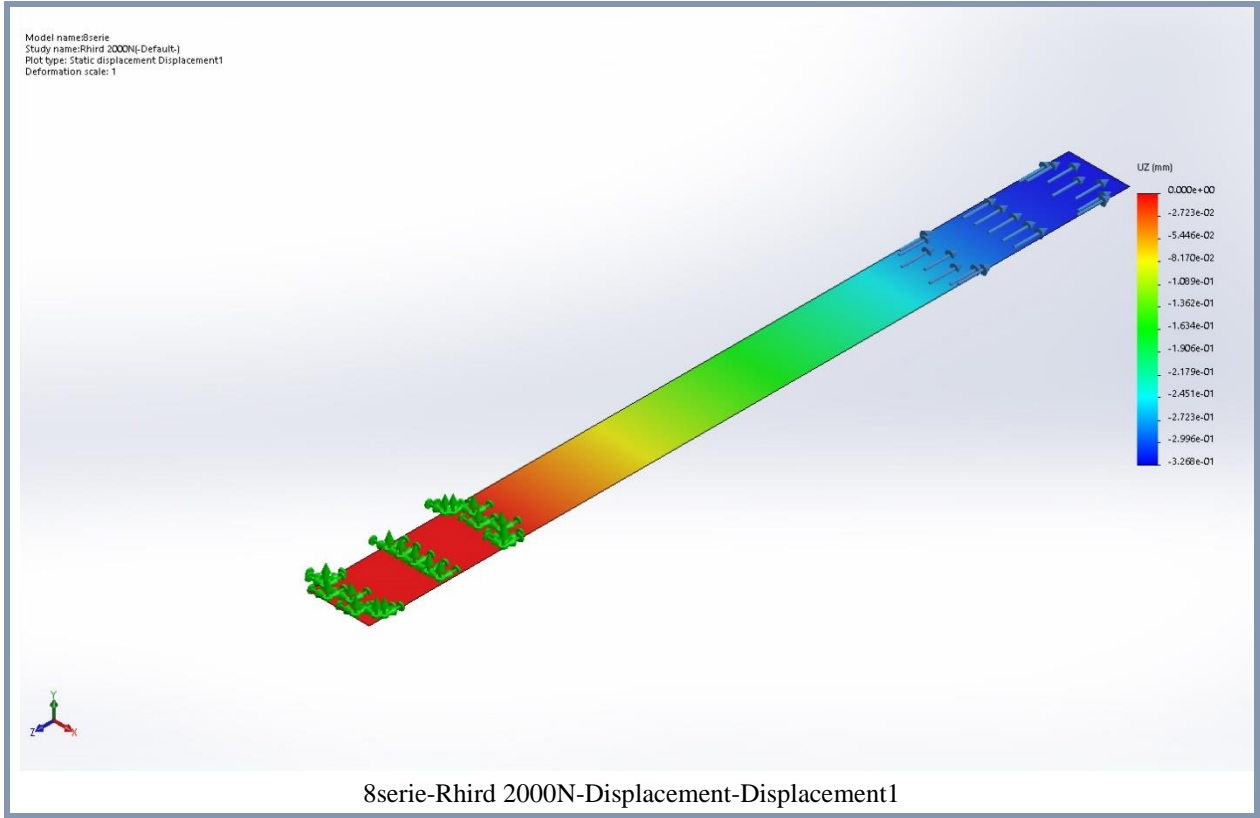
Name	Type	Min	Max
Tsai-Hill	Automatic	9.131e-01 Node: 1979	1.000e+16 Node: 146

Model name:8serie
Study name:Rhird 2000N(-Default-)
Plot type: Factor of Safety Tsai-Hill
Failure criteria for composites: Tsai-Hill Criterion
Worst-case across all plies, Min Composite FOS = 0.91309

8serie-Rhird 2000N-Factor of Safety-Tsai-Hill
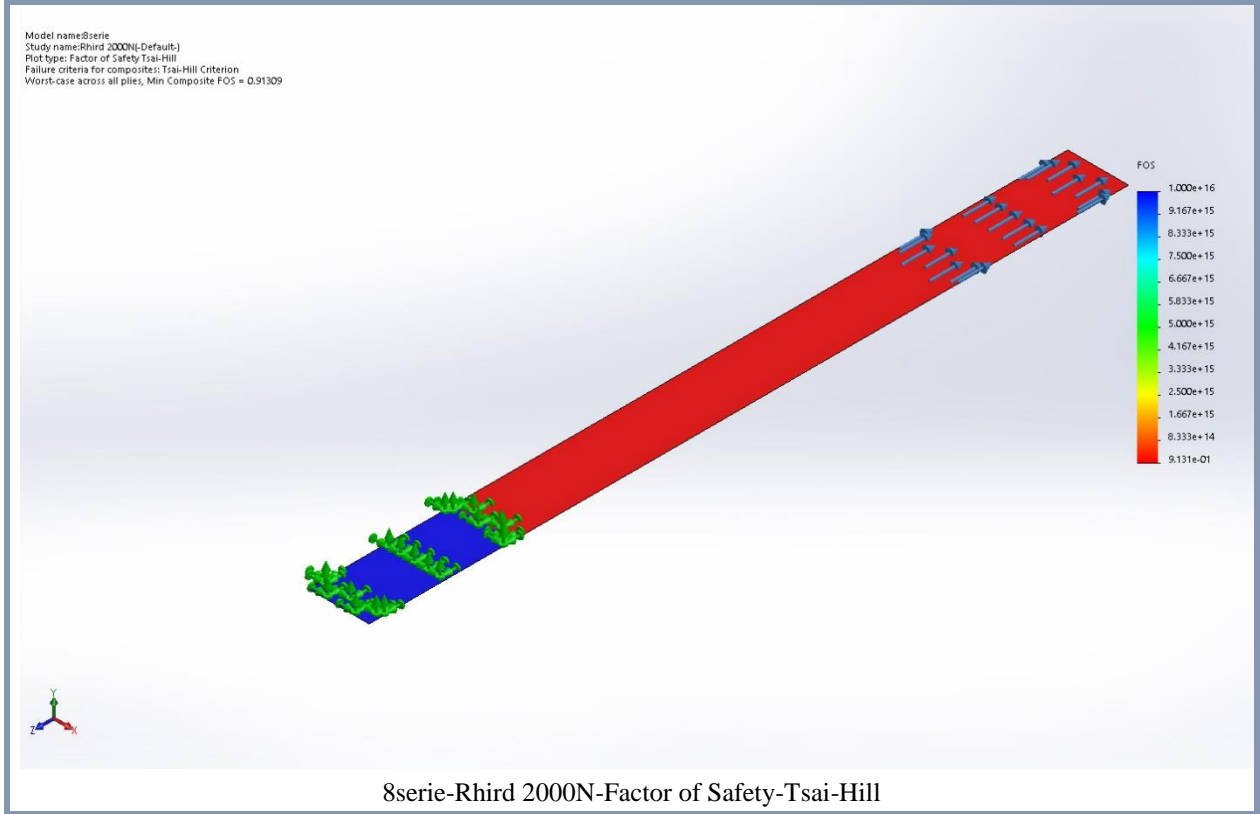
Name	Type	Min	Max
Tsai-Wu	Automatic	8.775e-01	1.000e+16

		Node: 1979	Node: 146

8serie-Rhird 2000N-Factor of Safety-Tsai-Wu

Name	Type	Min	Max
Max-stress	Automatic	9.187e-01 Node: 1979	1.000e+16 Node: 146

8serie-Rhird 2000N-Factor of Safety-Max-stress