Gap Distributions for Analysing Buyer Behaviour in Agent-based Simulation

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Abstract: Simulation models allow predicting the development of real situations in various technical, business and social systems. However, many real situations in business environment are of bursty nature. Buyers often appear concentrated or, in other words, bursty. Different approaches for analysing buyers' behaviour have been developed. One of these approaches focuses on analysis of gaps between buyers, and the buyers' scenario is completely described by the sequence of gaps. The present research is interdisciplinary, namely telecommunications and business management. The methodology of the present contribution is built on adaptation of gap distribution functions from data transmission theory in telecommunications to bursty business process in business management. The aim of the paper is to demonstrate inter-connections between different gap distribution functions such as Weibull, Exponential and Wilhelm as well as to compare different gap distribution functions for their suitability when analysing bursty processes. Furthermore, this contribution provides the mathematical description of gap processes. The comparison results of different gap distribution functions are presented. The theoretical results are confirmed by practical implementation in agent-based simulation environment.

1 INTRODUCTION

Phenomenon's simulation or, in other words, imitation of a situation or process, allows predicting the development of real situations in various technical, business and social systems. Many real situations in business environment are of bursty nature as shown in telecommunication systems by Gilbert and Elliot in the 1960s (Gilbert, 1960; Elliott, 1963). Contemporary computers and information and communications technology (ICT) facilitate phenomenon simulation. When the mathematics is intractable, agent-based simulation provides an efficient solution to simulate the bursty process of buying by taking decisions of individual buyers, so called agents, into account (Axelrod, 2006; Albanese, 2006; Tesfatsion, 2006). Thus, in business process, agent-based simulation assists in analysing buyers' behaviour.

Different approaches for analysing buyers' behaviour have been developed. One of these approaches focuses on analysis of gaps between buyers, and the buyers' scenario is completely described by the sequence of gaps. However, in many situations, buyers appear concentrated or, in other words, bursty. Bursty processes are described by gap distributions in data transmission theory in such a research field as telecommunications. In such scenarios the classical Bernoulli model, also known as memoryless scenario in data transmission theory, cannot be applied. Mostly the Weibull gap distribution is used to describe bursty processes. Unfortunately, the parameters of the Weibull gap distribution are not directly connected with the process of buying.

A promising approach was formulated by Wilhelm. Wilhelm described the distribution of bit-errors (i. e. gaps between bit-errors) in data transmission by defining a bit-error probability as well as a bit-error concentration (Wilhelm, 1976). The Wilhelm approach was adapted by Ahrens to the bursty process of buying by defining a buyer probability as well as a buyer concentration (Ahrens et al., 2015; Ahrens and Zaščerinska, 2016). Both gap distribution functions, namely Weibull as well as Wilhelm, can be applied to describe bursty and non-bursty business processes. Whereas the Bernoulli approach is well established in statistical theory, in this paper we are going to show that the Wilhelm gap distribution function is an extension of the Bernoulli model (the same as for the Weibull gap distribution).

The present research is interdisciplinary, namely telecommunications and business management. The methodology of the present contribution is built on adaptation of gap distribution functions from data transmission theory in telecommunications to bursty business process in business management.

The novelty of this paper is the demonstration of inter-connections between different distribution functions as well as a comparison of different distribution functions for their suitability when analysing bursty processes. The paper provides the mathematical description of gap process and presents the comparison results of different gap distribution functions. Practical implementation in agent-based simulation is used to confirm the theoretical results.

The remaining part of this paper is structured as follows: In section 2 the mathematical description of gap processes is presented. In section 3 the probability of arbitrarily buyer's patterns is demonstrated. The comparison results of different gap distribution functions are shown in section 4. Finally, the practical implementation in agent-based simulation is shown in section 5. Some concluding remarks are given in section 6.

2 MATHEMATICAL DESCRIPTION OF GAP PROCESSES

Bursty buyer processes can be defined by gaps between consecutive buyers as highlighted in Fig. 1 and 2.

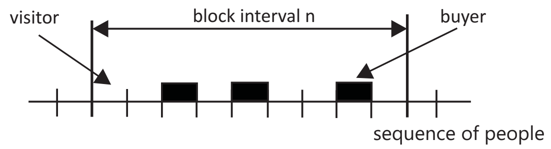

Figure 1: Buyer processes defined by gaps between consecutive buyers.

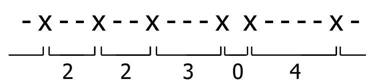

Figure 2: Definition of gaps between consecutive buyers (a buyer (represented by "x") within a sequence of non-buying visitors (represented by "-")).

Frequently used and well-suited practical approximations are provided, if the model is based on the independence of gap intervals. The gap distribution indicates the probability that a gap X between two buyers is greater than or at least equal to a given number k, i. e.

$$u(k) = P(X \geq k) \tag{1}$$

as well as the gap density function $v(k)$ defining the probability that a gap X between two buyers is equal to a given number k, i. e.

$$v(k) = P(X = k) \ . \tag{2}$$

Models that are based on the independence of gap intervals are completely described by the gap density or the gap distribution, respectively. The assumption that successive gaps are statistically independent is regarded as a good practical approximation. Models with these requirements are described as regenerative models in the literature (Wilhelm, 1976; Ahrens, 2000). Several gap distribution functions $u(k)$ are shown in Table 1. Whereas the Exponential distribution function is described by one parameter, Wilhelm distribution as well as Weibull distribution are defined by two parameters.

Table 1: Several gap distribution functions.

Type	Distribution $u(k)$
Exponential	$\mathrm{e}^{-\beta_e k}$
Weibull	$\mathrm{e}^{-(\beta_w k)^{\alpha_w}}$
Wilhelm	$((k+1)^{\alpha} - k^{\alpha}) \cdot \mathrm{e}^{-\beta \cdot k}$

When taking the gap distribution defined by Wilhelm into account the following expression was identified

$$u(k) = ((k+1)^{\alpha} - k^{\alpha}) \cdot e^{-\beta \cdot k} \quad 0 \leq k < \infty \tag{3}$$

with

$$\lim_{k \to \infty} e^{-\beta \cdot k} = 0 \qquad \beta > 0 \tag{4}$$

and

$$\beta \approx {p_e}^{1/\alpha} \ . \tag{5}$$

Here, the business process, i. e. the buyers' characteristics, is modelled by two parameters, namely visitor probability to buy (also referred as the buyers' probability) p_e and the buyers' concentration $(1-\alpha)$. Typical values for the buyer concentration are $(1-\alpha) = 0$ for the memoryless buyer scenario (also known as the Bernoulli scenario), i. e. the buyers appear independently distributed and $0 < (1-\alpha) \leq 0.5$ for a bursty buyer scenario. Assuming that the buyers appear independently form each other, i. e. $(1-\alpha) = 0$, the buyers' gap distribution function $u(k)$ defined by Wilhelm simplifies to

$$u(k) = e^{-p_e \cdot k} = (e^{-p_e})^k \ . \tag{6}$$

Taking the Taylor series of the exponential function e^{-x} for small x into account, the function e^{-x} can be re-written as

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \cdots \tag{7}$$

and approximated by

$$e^{-x} \approx 1 - x \tag{8}$$

for small x. Finally, the buyers' gap distribution function $u(k)$, defined in (3), results for small p_e in

$$u(k) = P(X \geq k) = (1 - p_\mathrm{e})^k = \overline{p}_\mathrm{e}^k \ . \tag{9}$$

The parameter $\overline{p}_\mathrm{e}$ described the probability of non-buying and can be defined as

$$\overline{p}_\mathrm{e} = \frac{\text{Number of Visitors - Number of Buyers}}{\text{Number of Visitors}} \ . \tag{10}$$

It should be noted that equation (9) is well-known in probability theory for the product of independent events and is valid for any p_e. Obtaining equation (9) testifies the correctness of the equality (3) for the memoryless buyer scenario.

The probability of a non-buying visitor is given by $\overline{p}_\mathrm{e} = (1 - p_\mathrm{e})$. Finally, the probability $u(k) = P(X \geq k)$ that $X \geq k$ consecutive visitors are non-buying visitors results in

$$u(k) = \overline{p}_\mathrm{e}^k = (1 - p_\mathrm{e})^k \ . \tag{11}$$

Re-writing of $u(k)$ leads to the buyers' gap density function $v(k)$, i. e.

$$v(k) = P(X = k) \ , \tag{12}$$

which describes the probability of a gap X equal to k. The buyers' gap density function $v(k)$ can be calculated as follows

$$\begin{aligned} u(k) &= v(k) + v(k+1) + v(k+2) + \cdots \\ u(k+1) &= v(k+1) + v(k+2) + \cdots \ . \end{aligned}$$

By calculating the difference between $u(k)$ and $u(k+1)$ the buyers' gap density function $v(k) = P(X = k)$ can be obtained

$$v(k) = u(k) - u(k+1) \tag{13}$$

and results for the memoryless buying process with (9) in

$$v(k) = \overline{p}_\mathrm{e}^k - \overline{p}_\mathrm{e}^{k+1} = \overline{p}_\mathrm{e}^k \cdot (1 - \overline{p}_\mathrm{e}) \tag{14}$$

and can be simplified as

$$v(k) = (1 - p_\mathrm{e})^k \cdot (1 - (1 - p_\mathrm{e})) = (1 - p_\mathrm{e})^k \cdot p_\mathrm{e} \ . \tag{15}$$

The probability that after a buyer in the distance of $k = 0$ another buyer appears results in

$$v(0) = p_\mathrm{e} \tag{16}$$

and is solely defined by the buyer probability p_e as expected for the memoryless buyer scenario. In situations with bursty buyers, the probability $v(0)$ increased as highlighted in Fig. 3

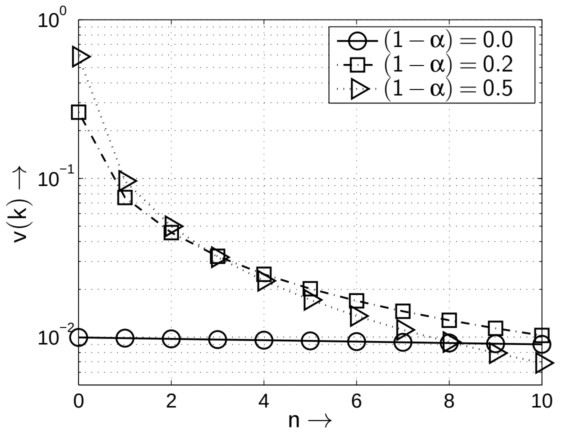

Figure 3: Buyers' gap density function $v(k)$ for different parameters of the $(1 - \alpha)$ at a buyer's probability of $p_\mathrm{e} = 10^{-2}$.

3 STATISTICAL ANALYSIS OF ARBITRARILY BUYERS' PATTERNS

In this section, the proof is given that the beforehand defined functions $u(k)$ and $v(k)$, introduced by Wilhelm, can be used to calculate the probability of arbitrarily buyer pattern with e buyers within an interval of n visitors. Let us denote the number of pattern with $K_{n,e}$ and start with the memoryless buyer scenario with $(1 - \alpha) = 0$. Analysing a pattern E of n visitors with e buyers, total number of pattern within an interval n is given by

$$K_{n,e} = \binom{n}{e} = \frac{n!}{e!\,(n-e)!} \ . \tag{17}$$

Here it is worth noting that when analysing the memoryless buyer scenario all pattern E within an interval of n visitors with e buyers appear with the same probability.

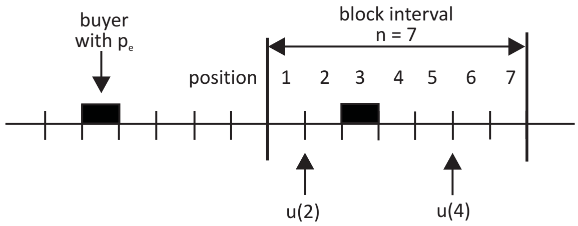

Figure 4: Calculation of the probability $\mathbf{P}_\mathrm{E}(7,1)$ of a pattern E within an interval of $n = 7$ visitors with $e = 1$ buyer at the position $n_1 = 3$.

According to Fig. 4, the probability $\mathbf{P}_\mathrm{E}(7,1)$ of such a pattern E within an interval of $n = 7$ visitors with

$e = 1$ buyer results in

$$\mathbf{P}_\mathrm{E}(7,1) = p_\mathrm{e} \cdot u(2) \cdot u(4) \ . \tag{18}$$

The term $p_\mathrm{e} \cdot u(2)$ defines the probability that there will be a buyer before the considered interval followed by a gap of at least two visitors. With

$$u(k) = (1 - p_\mathrm{e})^k = \overline{p}_\mathrm{e}^k \ . \tag{19}$$

the probability $\mathbf{P}_\mathrm{E}(7,1)$ can be expressed for the considered pattern and the memoryless buyer scenario as

$$\mathbf{P}_\mathrm{E}(7,1) = p_\mathrm{e} \cdot \overline{p}_\mathrm{e}^2 \cdot \overline{p}_\mathrm{e}^4 = p_\mathrm{e} \cdot \overline{p}_\mathrm{e}^6 \ , \tag{20}$$

indicating that six non-buying visitors within an interval of $n = 7$ visitors appear. Here the position of the buyer is not of any interest, as the pattern depicted in Fig. 5 results in the same probability for $\mathbf{P}_\mathrm{E}(7,1)$.

Taking two buyers within an interval of n visitors into consideration as exemplary depicted in Fig. 6, the probability $\mathbf{P}_\mathrm{E}(7,2)$ of such a pattern E within an interval of $n = 7$ visitors results in

$$\mathbf{P}_\mathrm{E}(7,2) = p_\mathrm{e} \cdot u(2) \cdot v(1) \cdot u(2) \ . \tag{21}$$

With

$$u(k) = (1 - p_\mathrm{e})^k = \overline{p}_\mathrm{e}^k \tag{22}$$

and

$$v(k) = (1 - p_\mathrm{e})^k \cdot p_\mathrm{e} = \overline{p}_\mathrm{e}^k \cdot p_\mathrm{e} \tag{23}$$

the probability $\mathbf{P}_\mathrm{E}(7,2)$ can be expressed for the memoryless buyer scenario as

$$\mathbf{P}_\mathrm{E}(7,2) = p_\mathrm{e} \cdot \overline{p}_\mathrm{e}^2 \cdot \overline{p}_\mathrm{e} \cdot p_\mathrm{e} \cdot \overline{p}_\mathrm{e}^2 = p_\mathrm{e}^2 \cdot \overline{p}_\mathrm{e}^5 \tag{24}$$

indicating that five non-buying visitors within an interval of $n = 7$ visitors with 2 buyers appear.

On the other hand, 2 buyers between 7 visitors may appear in another sequence.

It yields from (17) that the total number of such combinations (pattern) equals to

$$K_{7,2} = \binom{7}{2} = \frac{7!}{2!\,(7-2)!} = 21 \ . \tag{25}$$

Therefore, the probability of 2 buyers between 7 visitors of all patterns is given by the Bernoulli formula

$$\mathbf{P}(7,2) = K_{7,2} \cdot \mathbf{P}_\mathrm{E}(7,2) = \frac{7!}{2!\,(7-2)!} \cdot p_\mathrm{e}^2 \cdot \overline{p}_\mathrm{e}^5 \ . \tag{26}$$

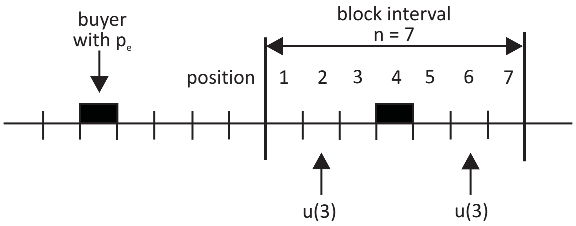

Figure 5: Calculation of the probability $\mathbf{P}_\mathrm{E}(7,1))$ of a pattern E within an interval of $n = 7$ visitors with $e = 1$ buyer at the position $n_1 = 4$.

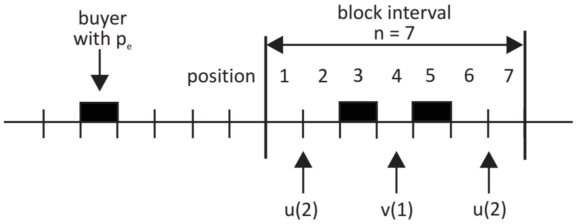

Figure 6: Calculation of the probability $\mathbf{P}_\mathrm{E}(7,1)$ of a pattern E within an interval of $n = 7$ visitors with $e = 2$ buyers at the position $n_1 = 3$ and $n_2 = 5$.

Therefore, for the memoryless channel it can be shown that the obtained results coincide with the Bernoulli model.

Having more than 2 buyers within an interval of n visitors, the probability $\mathbf{P}_\mathrm{E}(n,e)$ of a pattern E in an interval of n visitors with e buyers at the positions $n_1, n_2, \cdots, n_e$ can be obtained as

$$\mathbf{P}_\mathrm{E}(n,e) = p_\mathrm{e} \cdot u(n_1 - 1) \cdot u(n - n_\mathrm{e}) \cdot \prod_{\nu=2}^{e} v(n_\nu - n_{\nu-1} - 1) \ . \tag{27}$$

The ith pattern (with $1 \leq i \leq K_{n,e}$) is determined by the buyers' position $n_1, n_2, \cdots, n_e$.

Fig. 7 illustrates the calculation of the probability $\mathbf{P}_\mathrm{E}(7,3)$ of a pattern E within an interval of $n = 7$ visitors. Here, the $e = 3$ buyers are at the positions $n_1 = 3$, $n_2 = 4$ and $n_3 = 6$. The probability $\mathbf{P}_\mathrm{E}(7,3)$ of such a pattern E within an interval of $n = 7$ visitors is given by

$$\mathbf{P}_\mathrm{E}(7,3) = p_\mathrm{e} \cdot u(2) \cdot v(0) \cdot v(1) \cdot u(1) \ . \tag{28}$$

With

$$u(k) = \overline{p}_\mathrm{e}^k \tag{29}$$

and

$$v(k) = \overline{p}_\mathrm{e}^k \cdot p_\mathrm{e} \tag{30}$$

the probability $\mathbf{P}_\mathrm{E}(7,3)$ can be expressed for the memoryless buyer scenario as

$$\mathbf{P}_\mathrm{E}(7,3) \approx p_\mathrm{e} \cdot \overline{p}_\mathrm{e}^2 \cdot p_\mathrm{e} \cdot \overline{p}_\mathrm{e} \cdot p_\mathrm{e} \cdot \overline{p}_\mathrm{e} = p_\mathrm{e}^3 \cdot \overline{p}_\mathrm{e}^4 \ . \tag{31}$$

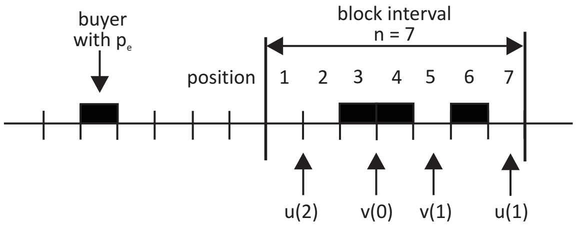

Figure 7: Calculation of the probability $\mathbf{P}_\mathrm{E}(7,3)$ of a pattern E within an interval of $n = 7$ visitors with $e = 3$ buyers at the positions $n_1 = 3$, $n_2 = 4$ and $n_3 = 6$.

Taking the total number of such combinations (pattern) such as

$$K_{7,3} = \binom{7}{3} = \frac{7!}{3!\,(7-4)!} = 35 \qquad (32)$$

into account, the probability of 3 buyers in a sequence of 7 visitors is given by the Bernoulli formula

$$\mathbf{P}(7,3) = K_{7,3} \cdot \mathbf{P}_{\mathrm{E}}(7,3) = \frac{7!}{3!\,(7-3)!} \cdot p_{\mathrm{e}}^3 \cdot \overline{p}_{\mathrm{e}}^4 \ . \qquad (33)$$

Analysing the memoryless buyer scenario, the probability $\mathbf{P}_{\mathrm{E}}(n,e)$ can be written as

$$\mathbf{P}_{\mathrm{E}}(n,e) \approx = p_{\mathrm{e}}^e \cdot \overline{p}_{\mathrm{e}}^{n-e} \qquad (34)$$

and is independent of the individual pattern when analysing the memoryless buyer scenario where each pattern E appears with the same probability.

4 COMPARISON OF GAP DISTRIBUTIONS

In this section the interconnections of Exponential, Weibull and Wilhelm distribution are to be shown. It is assumed that the given buyers' characteristic undergoes the Wilhelm distribution with given parameters p_{e} and $(1-\alpha)$.

Table 2 and 3 show the resulting estimation errors for $p_{\mathrm{e}} = 10^{-2}$. As quality parameter for the approximation between the given Wilhelm gap interval distribution $u_{\mathrm{Wilhelm}}(k)$ and the investigated distribution function $u(k)$ (Exponential, Weibull) the mean square error

$$\mathrm{MSE}_{\min} = \sum_{k=0}^{k_{\max}-1} |u(k) - u_{\mathrm{Wilhelm}}(k)|^2 \qquad (35)$$

is used and minimized when using least-square optimization. The parameter $k_{\max}$ specifies the maximum gap length to be considered.

Tab. 2 shows the obtained results when using an Exponential gap distribution instead of the Wilhelm distribution. As the Exponential gap distribution equals the Wilhelm distribution for the memoryless (non-bursty) buyer scenario, a perfect mapping can be achieved. With increasing buyers' concentration, the gap between the Wilhelm and Exponential gap distribution becomes larger as the Exponential gap distribution function is not able to take the buyers' concentration into account.

Tab. 3 highlights the obtained results when using Weibull gap distribution instead of the Wilhelm one. Here, a better adaptation can be reached, as gap distribution functions with two parameters lead to a better adaptation. Finally, Fig. 8 shows the approximated gap distributions as a function of the interval length k at a buyer concentration of $(1-\alpha) = 0,2$ and a buyer probability of $p_{\mathrm{e}} = 10^{-2}$.

Table 2: Estimation errors when using the Exponential distribution instead of the Wilhelm distribution.

$(1-\alpha)$	β_{e}	MSE
0,0	0,010	0,000
0,1	0,016	0,989
0.2	0,026	2,377
0,3	0,055	2,613

Table 3: Estimation errors when using the Weibull distribution instead of the Wilhelm distribution.

$(1-\alpha)$	β_{w}	α_{w}	Error
0,0	0,010	1,000	0,000
0,1	0,013	0,460	0,007
0.2	0,033	0,310	0.002
0,3	0,119	0,264	0,007

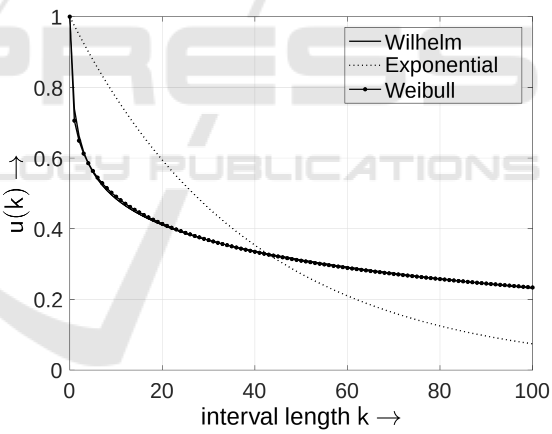

Figure 8: Approximated gap distributions as a function of the interval length n at a buyer concentration of $(1-\alpha) = 0,2$ and a buyer probability of $p_{\mathrm{e}} = 10^{-2}$.

5 AGENT-BASED SIMULATION

In this section a model validation is carried out by demonstrating that the model is a reasonable representation of the investigated system (Martis, 2006). Therefore, the objective of this section is compare the probability $\mathbf{P}(n,e)$ with simulation outcomes obtained by agent-based approach (Sajjad et al., 2016). By agents relatively autonomous computational objects are understood. Agents of the same environment

Table 4: Comparison of the frequency γ_e with the probability $\mathbf{P}(n,e)$ for memoryless buying scenarios.

block length n	number m_e of buying events	probability p_e	mumber m of blocks in simulated sample	probability (theory) $\mathbf{P}(n,e)$	relative frequency (simulation) γ_e
5	2	0.01	200	0.001	0.000
5	1	0.01	200	0.048	0.040
5	2	0.1	200	0.073	0.051
5	1	0.1	200	0.328	0.328
7	4	0.1	142	0.003	0.007
7	2	0.1	142	0.128	0.078
5	2	0.5	200	0.031	0.030
5	1	0.5	200	0.156	0.197
7	4	0.5	142	0.273	0.234
7	2	0.5	142	0.164	0.170

may slightly differ in values of their properties, called also attributes. Agents from different environments may differ essentially. They exchange messages and carry out activities influencing other agents and their environment. Finally, agent activities are defined by their own rules. Results of agent activities may be message sending to other agents or the change of its own state. The state of the agent may be changed by other agents as well. Therefore according to various authors, agents may have the following properties (Ahrens et al., 2019):

- Intelligence: this property is implemented with simple If then rules, fuzzy logic methods, built-in neural networks, genetic algorithms, etc.
- Autonomy: agents are able to make decisions independently.
- Reactivity: agents have an ability to respond to the activities of other agents and environment.
- Pro-activity: the agent may have a goal and are programmed to reach it; the agent also may be able to foresee possible negative events and to try to avoid them.
- Adaptivity: agents may change their own rules of behaviour responding to activities of other agents and changes of environment as well as evaluating accumulated statistics.
- Robustness: the ability to carry out activities and survive in different environments.
- Goal-orientation: agents act according to their goals and do nothing more.
- Mobility: agents may change their virtual place in 2D and 3D environments and they may be placed on GIS map.

The process of making decisions when buyers enter the shop was modelled using agent-based approach. Each visitor was simulated as an autonomous agent (Fig. 9). The visitors' decisions were implemented as rules. Each agent after entering the shop generated a random decision to buy a product or service with a given probability p_e. Various statistics and logs of the whole process were collected as well as various properties of the burstiness were calculated (Ahrens et al., 2019).

For the purposes of this work, the simulation was supplemented for the memoryless buyer scenario by the export of the sequence of one thousand agents' decisions to the Excel file. Fig. 9 shows the structure of an individual agent simulating the decision of an individual buyer.

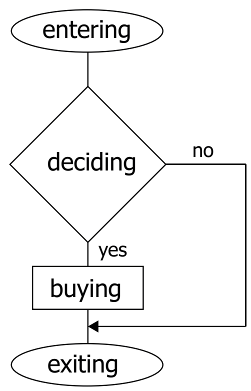

Figure 9: Individual Agent simulating the decision of an individual buyer.

These decisions known as binary customer behaviour were coded by series of 1 (decision to buy) and 0 (not to buy). The series were splitted into m blocks each of the length n (number of visitors, pattern length). Therefore the total number of visitors is given by $n \cdot m$. For instance, when a pattern length of $n = 10$ is considered, the series contained $m = 100$ blocks and therefore a series of $n \cdot m = 1000$ visitors is considered. Then the number m_e of blocks containing e buying events were counted. According to the statistical definition of the probability, the relative frequency

$$\gamma_e = \frac{m_e}{m} \tag{36}$$

is an estimation of the theoretical probability $\mathbf{P}(n,e)$. Tab. 4 presents the comparison of the simulated relative frequency γ_e with the probability $\mathbf{P}(n,e)$. As Tab. 4 shows, the difference between the probability $\mathbf{P}(n,e)$ and the relative frequency γ_e is not large, and can be be explained by the randomness of the agents' behaviour. The match testifies the consistency of analytical and simulation models.

6 CONCLUSIONS

The present research has successfully demonstrated the adaptation of gap distribution functions from data transmission theory in telecommunications to business processes. The similar nature, namely bursty nature, of bit-errors in telecommunications and buyers in business management has been outlined. Consequently, the present paper has emphasized the bursty nature of business processes such as buying and selling, too. The complex process of buying by analysing such properties of buyers' behaviour as buyer probability and buyer concentration has been highlighted. The research has resulted in proposing the use of gaps for the description of the buying process. The mathematical description of gap processes built on the independence of gap intervals has been revealed in the present paper. As shown by our research results, distribution functions with two parameters such as Weibull or Wilhelm have been found to be an adequate tool for the analysis of both, namely the buyers' behaviour and the process of buying.

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