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Bifurcation Characteristic-based Damage Identification for Plates undergoing Large Amplitude Vibration

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ABSTRACT

A nonlinear damage indicator based on bifurcation characteristics of cracked plates with large deflections is established in this paper. Firstly, von Kármán plate theory is applied to derive the governing equations of motions for large deflection plates. The derived model comprises quadratic and cubic terms describing the specific nonlinear dynamical behaviours within large deflection plates. Secondly, Galerkin's method is employed to discrete the PDE governing equations of large deflection plates and the simplified dimensionless equation is dealt with multiple scale perturbation method. After that, the classic hardening-type frequency responses can be observed clearly. Featured the hardening-type frequency responses, the bifurcation characteristics are discussed in detail, and crack effect on it is argued. Then, the nonlinear damage indicator is established to identify the existence of cracks in flat plates with large deflections. Next, Runge-Kutta iteration algorithm is used to achieve numerical results of the simplified dimensionless equation for plates at various damage levels. Finally, the effectiveness of established damage indicator for identifying the existence of cracks has been validated using these dynamical responses. It is demonstrated that the established damage indicator can identify the existence of cracks successfully and has a promising application in structural health monitoring of plate type structures with large deflections.

1. Introduction

Flat plate type structures are fundamental components and have a wide utilization in engineering, ranging from civil engineering to space technology, such as steel bridges, aircraft wings, and ships[1-3]. The dynamic behaviours of thin plate structures have gained great attention from the scientific and engineering communities due to their importance to a wide variety of technical applications. In general, theoretical analysis of plate structures is extensively based on the linear theory, known as Love-Kirchhoff plate theory[4]. However, this theory works well for small deformations only. Taking large deflections into account, the linear theory causes relatively big errors - what prevents the use of Love-Kirchhoff plate theory in such engineering applications where the structural response is considered not only around the state of equilibrium.

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Characterizing nonlinear dynamic behaviours of large deflection plates is one of the key problems in the engineering for its significance and complexity[5, 6]. The nonlinear plate theory was introduced by von Kármán in 1910, known as von Kármán plate theory[7]. It plays an important role in analysing the dynamic behaviours of large deflection plate structures. Back in 1961, after analysing the approximate solutions of thin plates undergoing large amplitude deflections, Yamaki[8] obtained a hard spring type frequency response, that is, the frequency of nonlinear vibration increases with increasing amplitude. These hard/soft spring type response curves were also observed experimentally by Kirchman and Greenspon[9] and Lassiter et al.[10], which began to be regarded as a representation of nonlinear vibrations. Studies on hard/soft type nonlinear dynamical behaviour springs out from then on. Eisley got the soft spring type response when investigating the influence of initial membrane stress on nonlinear vibration of beams and rectangular plates[11], and the hard spring type response when inspecting buckled beams and plates[12]. Sathyamoorthy and Pandalai[13, 14] obtained the hard type nonlinearity in analysis of nonlinear flexural vibration of skew plates. Prathap and Varadan[15] found that the nonlinearity of large amplitude vibration of stiffened plates is of the hardening-type in all cases of boundary conditions and the in-plane conditions considered. In recent decades, with the rapid development of computational science, it is allowed for researchers to consider more and more alternative situations in nonlinear vibration analysis of plates and shells, such as composite materials[16-20], complex boundary conditions[21], thermal effects[22] and fluid conditions[23-25]. It presents that hard/soft type responses are common in vibrations of plates and shells with large deflections. Additionally, Amabili and his colleagues have confirmed it from the aspect of experiment[26].

Except for the effect of large deflection (geometry nonlinearity), the existence of a crack has an important influence on nonlinear dynamics responses of plates as well. According to changes of specific dynamics features due to a crack, vast majority of vibration-based structural health monitoring methods have been developed to detect and localize damage[27], including resonant frequencies, mode shapes, and curvature/strain mode shape-related indices. Above modal-based approaches work well for linear system. However, the low sensitivity to damage in nonlinear system limits their application to such engineering problems[28].

In present work, a bifurcation behaviour based structural health monitoring method is established to detect a crack in flat plates with large deflections. Firstly, the dimensionless governing equation is obtained using von Kármán's plate theory and Galerkin's method. Secondly, with the application of multiple scale perturbation method, bifurcation behaviours of large deflection plates are achieved and the classic nonlinearity, hardening-type frequency responses, can be observed clearly. Then, after analysing the effect of a crack on hardening-type frequency responses, a combined damage index is established to identify the existence of a crack in flat plates with large deflections. At last, the method is validated using numerical results from Runge-Kutta iteration algorithm. As expected, the damage indicator has a positive relationship with damage levels and cracks can be identified successfully.

2. Equation of motion for cracked plates with large amplitude vibration

Large deformation of vibration can easily induce local cracking of a large deflected plate. When a crack occurs to a largely-deflected plate, the crack will significantly influence the vibration characteristics of the plate. To address crack effect, the modelling of the crack is implemented by using the line-spring model developed by Rice and Levy[29]. In terms of the line-spring model, the cracked section is represented as a continuous line spring, which has both stretching and bending resistance.

The governing equation of motion for a largely-deflected plate with a crack can be formed by introducing the modelled crack into the governing equation of motion for an intact largely-deflected plate. Solving the vibration of the simply supported cracked largely-deflected plate subjected to the excitation gives rise to a temporal governing equation of the cracked plate, given by

$$\psi'' + 2\xi^* \psi' + \omega^{*2}\psi + \alpha^* \psi^2 + \beta^* \psi^3 = f^* \cos \lambda \tau , \tag{1}$$

where ξ^*, ω^*, α^*, β^*, f^*, λ are dimensionless coefficients; ψ, λ and τ are dimensionless response amplitude, dimensionless excitation frequency and dimensionless time, respectively.

3. Bifurcation characteristics and damage indicator

The exact explicit solution of Eq. (1) is unavailable due to its nonlinearity and complexity. To this end, the perturbation method of multiple scales, a sophisticated tool to analyse nonlinear dynamic systems, is used to attain approximate solution of the equation. When Eq. (1) is processed by the perturbation method of multiple scales, a steady-state frequency-response function, is derived, of the form:

$$\sigma = \frac{3\beta^* b^2}{8\omega^*} \pm \sqrt{\frac{f^{*2}}{4\omega^{*2} b^2} - \xi^{*2}}\,, \tag{2}$$

where σ is the detuning parameter, measuring the deviation between the excitation frequency and the primary natural frequency; b represents the amplitude of steady-state response. The operator $\pm$ indicates a certain response amplitude corresponds two values of detuning parameter.

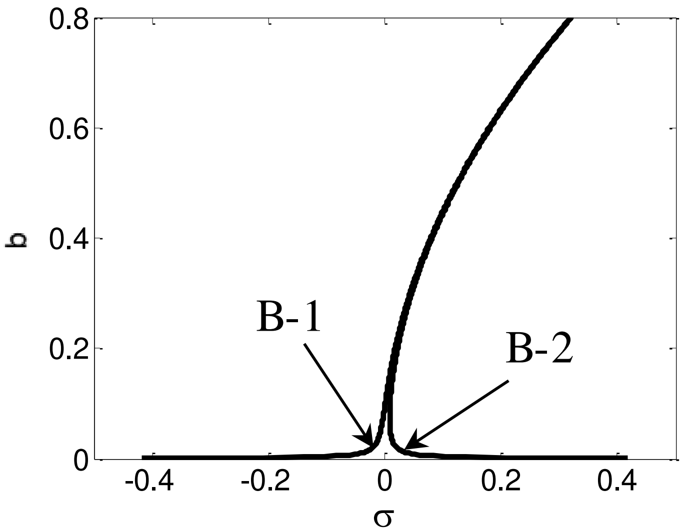

Figure 1. Frequency-response relation for largely-deflected plates.

The steady-state frequency-response function stated in Eq. (2) reveals the relation between excitation frequency and the corresponding steady-state response amplitude, namely $\sigma - b$ relation, as shown in Fig. 1. This relation characterizes the essence of the vibration of a largely-deflected plate, and the quadratic coefficient, α^*, has no effect on it. For a positive σ, there are two different dynamic responses when the excitation frequency varies from low-to-high and from high-to-low, resulting in two branches, represented by B-1 and B-2. The two branches overhang to the right-side of the frequency domain, indicating a hardening-type nonlinearity[8, 9]. This one-to-two relation is called nonlinear bifurcation in frequency domain. The excitation frequency at which the bifurcation starts to occur, is called jump point. From the jump point, the steady-state response of the vibration system jumps from B-2 to B-1 when σ varies from high to low. This nonlinear bifurcation describes the relationship between the excitation frequency and the dynamic response.

Without loss of generality, two states of largely-deflected plates are considered: the intact one represented by $(\delta,\zeta)=(0,0)$, and the cracked one represented by $(\delta,\zeta)\neq(0,0)$. By analysing the frequency response curves, two bifurcation characteristics due to damage can be observed: (1) the change of maximal dynamic response amplitudes; (2) the left-side shift of jump points of bifurcations.

A comprehensive damage indicator according to above two parts is established to quantify the change of nonlinear bifurcations of largely-deflected plates due to damage. In general, it is expressed as

$$DI = \sqrt{DI_1^2 + DI_2^2}\ , \tag{3}$$

where DI_1 measures the average increasement of dynamic response amplitude due to damage; DI_2 deals with the shift of the jump point of the bifurcations. For an intact large-deflected plate, DI ought to be zero for the stable nonlinear bifurcation. When damage occurs, the nonlinear bifurcation makes a difference, and DI should be positive. This characterization of DI indicates that it is a promising tool to distinguish the nonlinearity due to damage from the complex nonlinear dynamic responses.

4. Damage interrogation using established indicator

An aluminium largely-deflected plate by SSSS boundary condition is considered for damage interrogation. The damage interrogation still needs a baseline. With the dynamic responses obtained, the bifurcations for each case can be achieved. The obtained bifurcations are the typical hardening-type nonlinearity. When damage occurs, the response amplitude increases with the growth of crack length and the bifurcation shifts to the left-side of the frequency domain. These characteristics are consistent with the discussion in forward problem analysis. Damage indicator, DI , is used to quantify the change of bifurcations for damage interrogation. The calculated indicators for each case are presented in Fig. 2. The indicator increases with the growth of crack length. It is implied that the established damage indicator based on bifurcation characteristics is capable of distinguishing the nonlinearity due to damage, and promising for damage interrogation of nonlinear dynamic systems.

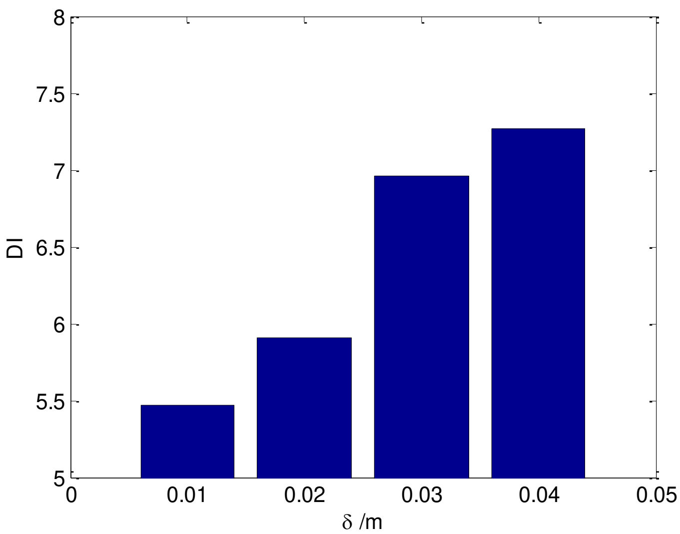

Figure 2. Damage interrogation using established indicator.

5. Conclusions

Bifurcation behaviours of cracked flat rectangular plates with large deflections have been studied using von Kármán's plate theory and multiple scale perturbation method. Firstly, the partial differential equation based on von Kármán's plate theory is discretized by Galerkin's method and then the governing equation of cracked plate with large deflections is obtained. Secondly, on the basis of multiple scale perturbation method, bifurcation behaviours of cracked plates are obtained and the hardening-type frequency responses are clearly observed. The frequency responses of small deformations and large deflections are also compared. By analysing three features of hardening-type frequency responses, a combined damage index is established to identify the existence of cracks in flat rectangular plates with large deflections. Using Runge-Kutta iteration method, the numerical results are obtained and they are used to validate the effectiveness of established damage index for identifying cracks. As expected, the damage index has a positive relationship with damage levels and cracks can be identified successfully.

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References

[1] X Zhao, H Gao, G Zhang, B Ayhan, F Yan, C Kwan, JL Rose, "Active Health Monitoring of an Aircraft Wing with Embedded Piezoelectric Sensor/Actuator Network: I. Defect Detection, Localization and Growth Monitoring", SMART MATER STRUCT, 16(4), pp 1208-1217, 2007.

[2] LC Hollaway, "The Evolution of and the Way Forward for Advanced Polymer Composites in the Civil Infrastructure", Construction and Building Materials, 17(6), pp 365-378, 2003.

[3] RA Ibrahim, "Recent Advances in Vibro-Impact Dynamics and Collision of Ocean Vessels", Journal of Sound and Vibration, 333(23), pp 5900-5916, 2014.

[4] AEH Love, "The Small Free Vibrations and Deformation of a Thin Elastic Shell", Philosophical Transactions of the Royal Society of London A, 179(pp 491-546, 1888.

[5] CW Bert, S Jang, A Striz, "Nonlinear Bending Analysis of Orthotropic Rectangular Plates by the Method of Differential Quadrature", CompM, 5(2-3), pp 217-226, 1989.

[6] WS Yoo, JH Lee, SJ Park, JH Sohn, D Pogorelov, O Dmitrochenko, "Large Deflection Analysis of a Thin Plate: Computer Simulations and Experiments", Multibody System Dynamics, 11(2), pp 185-208, 2004.

[7] TV Kármán, "Festigkeitsprobleme Im Maschinenbau", Encyklopadie der Mathematischen Wissenschaften, 4(4), pp 311-385, 1910.

[8] N Yamaki, "Influence of Large Amplitudes on Flexural Vibrations of Elastic Plates", ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik, 41(12), pp 501–510, 1961.

[9] EJ Kirchman, JE Greenspon, "Nonlinear Response of Aircraft Panels to Acoustic Noise", J Acous Soc Am, 29(7), pp 854-857, 1957.

[10] LW Lassiter, RW Hess, "Calculated and Measured Stresses in Simple Panels Subject to Intense Random Acoustic Loading Including the near Noise Field of a Turbojet Engine", Technical Report Archive & Image Library, pp, 1957.

[11] JG Eisley, "Nonlinear Vibration of Beams and Rectangular Plates", Zeitschrift für angewandte Mathematik und Physik ZAMP, 15(2), pp 167-175, 1964.

[12] JG Eisley, "Large Amplitude Vibration of Buckled Beams and Rectangular Plates", AIAAJ, 2(12), pp 2207-2209, 1964.

[13] M Sathyamoorthy, KAV Pandalai, "Non-Linear Flexural Vibration of Orthotropic Skew Plates", Journal of Sound and Vibration, 24(1), pp 115-120, 1972.

[14] M Sathyamoorthy, K Pandalai, "Large Amplitude Flexural Vibration of Simply Supported Skew Plates", AIAAJ, 11(9), pp 1279-1282, 1973.

[15] G Prathap, TK Varadan, "Large Amplitude Flexural Vibration of Stiffened Plates", Journal of Sound and Vibration, 57(4), pp 583-593, 1978.

[16] CY Chia, "Large Amplitude Vibrations of Laminated Rectangular Plates", Fibre Science and Technology, 17(2), pp 123-131, 1982.

[17] S Nair, J Woo, "Nonlinear Vibrations of Rectangular Laminated Thin Plates", AIAAJ, 30(1), pp 180-188, 1992.

[18] M Amabili, S Farhadi, "Shear Deformable Versus Classical Theories for Nonlinear Vibrations of Rectangular Isotropic and Laminated Composite Plates", Journal of Sound and Vibration, 320(3), pp 649-667, 2009.

[19] F Alijani, M Amabili, "Nonlinear Vibrations of Laminated and Sandwich Rectangular Plates with Free Edges. Part 1: Theory and Numerical Simulations", CmpSt, 105(pp 422-436, 2013.
[20] F Alijani, M Amabili, G Ferrari, V D'Alessandro, "Nonlinear Vibrations of Laminated and Sandwich Rectangular Plates with Free Edges. Part 2: Experiments & Comparisons", CmpSt, 105(pp 437-445, 2013.
[21] M Amabili, "Nonlinear Vibrations of Rectangular Plates with Different Boundary Conditions: Theory and Experiments", Computers & Structures, 82(31), pp 2587-2605, 2004.
[22] M Amabili, S Carra, "Thermal Effects on Geometrically Nonlinear Vibrations of Rectangular Plates with Fixed Edges", Journal of Sound and Vibration, 321(3), pp 936-954, 2009.
[23] KN Karagiozis, M Amabili, MP Païdoussis, AK Misra, "Nonlinear Vibrations of Fluid-Filled Clamped Circular Cylindrical Shells", JFS, 21(5), pp 579-595, 2005.
[24] M Amabili, C Touzé, "Reduced-Order Models for Nonlinear Vibrations of Fluid-Filled Circular Cylindrical Shells: Comparison of Pod and Asymptotic Nonlinear Normal Modes Methods", JFS, 23(6), pp 885-903, 2007.
[25] K Karagiozis, M Amabili, MP Païdoussis, "Nonlinear Dynamics of Harmonically Excited Circular Cylindrical Shells Containing Fluid Flow", Journal of Sound and Vibration, 329(18), pp 3813-3834, 2010.
[26] F Alijani, M Amabili, "Non-Linear Vibrations of Shells: A Literature Review from 2003 to 2013", International Journal of Non-Linear Mechanics, 58(pp 233-257, 2014.
[27] W Fan, P Qiao, "Vibration-Based Damage Identification Methods: A Review and Comparative Study", Structural Health Monitoring-an International Journal, 10(1), pp 83-111, 2011.
[28] BI Epureanu, SH Yin, "Identification of Damage in an Aeroelastic System Based on Attractor Deformations", Computers &E Structures, 82(31-32), pp 2743-2751, 2004.
[29] JR Rice, N Levy, "The Part-through Surface Crack in an Elastic Plate", Journal of Applied Mechanics, 39(1), pp 185-194, 1971.