# Measurement of Stresses in the Vibrating Elements of a Printing Device

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**Abstract.** In this paper measurement of stresses in vibrating elements of the printing device using photo-elastic techniques is investigated. Numerical procedure is applied to obtain data which is needed for the interpretation of experimental results of time averaged photo-elasticity when sinusoidal vibrations according to the eigenmode of the structure in the problem of plane stress take place. The numerical procedure is also applied to obtain data which is needed for interpretation of experimental results when the structure is slowly loaded from the status of equilibrium by linearly increasing the load till its final value. Plotting isoclinics for bending vibrations of a plate using photo-elastic coatings is analyzed. Improved smoothing procedure for two dimensional Lagrange quadratic elements is proposed.

## 1. Introduction

The investigations about time averaging in photo-elasticity presented in this paper are based on the material presented in [1]. In this previous paper the data which is necessary to interpret experimental results of time averaged photo-elasticity were obtained on the basis of analytical investigations based on manipulation of integrals and using the zero order Bessel function of the first kind. In this paper numerical procedure is applied to obtain data which is needed for the interpretation of experimental results of time averaged photo-elasticity when sinusoidal vibrations according to the eigenmode of the structure in the problem of plane stress take place.

The numerical procedure is also applied to obtain data which is needed for interpretation of experimental results of packages when the package construction is slowly loaded from the status of equilibrium by linearly increasing the load till its final value.

Plotting isoclinics for bending vibrations of a plate using photo-elastic coatings is also analyzed in this paper. The numerical procedure is based on the technique of conjugate approximation [2, 3] with smoothing [4] and the analysis of bending vibrations of a plate using photo-elastic coatings [5]. In this paper the improved smoothing procedure for two dimensional Lagrange quadratic elements is proposed, which has advantages over the conventional smoothing procedure.

## 2. Measurement of stresses in time averaged photo-elasticity

*2.1. Measurement of stresses when the structure performs vibrations according to the eigenmode*

Intensity of the static image for the circular polariscope is expressed as:

$$I = \sin^2 C, \tag{1}$$

where $C$ is a product of a constant which depends on the thickness and material of the structure with the difference of principal stresses.

Intensity of the time averaged image for the circular polariscope is expressed as:

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$$\overline{I} = \frac{1}{n}\sum_{j=1}^{n}\sin^2\left(C\sin 2\pi\frac{j-1}{n}\right), \tag{2}$$

where $C$ is a product of a constant which depends on the thickness and material of the structure with the difference of principal stresses in the state of extreme deflections according to the eigenmode, $n$ is a large integer number.

Further the roots of the equation:

$$I = \frac{1}{2} \tag{3}$$

are denoted by $C_i$.

Also the roots of the equation:

$$\overline{I} = \frac{1}{2} \tag{4}$$

are denoted by $\overline{C}_i$.

The values of $C_i$ and $\overline{C}_i$ are presented in table 1.

**Table 1.** Values of $C_i$ and $\overline{C}_i$ for the first problem of time averaging.

| $i$ | $C_i$ | $\overline{C}_i$ |
|---|---|---|
| 1 | 0.785398 | 1.20241 |
| 2 | 2.35619 | 2.76004 |
| 3 | 3.92699 | 4.32686 |
| 4 | 5.49779 | 5.89577 |
| 5 | 7.06858 | 7.46546 |
| 6 | 8.63938 | 9.03553 |

**Table 2.** Values of $C_i$ and $\overline{C}_i$ for the second problem of time averaging.

| $i$ | $C_i$ | $\overline{C}_i$ |
|---|---|---|
| 1 | 0.785398 | 1.5708 |
| 2 | 2.35619 | 3.14159 |
| 3 | 3.92699 | 4.71239 |
| 4 | 5.49779 | 6.28319 |
| 5 | 7.06858 | 7.85398 |
| 6 | 8.63938 | 9.42478 |

In the previous paper the data which is necessary to interpret experimental results of time averaged photo-elasticity were obtained on the basis of analytical investigations based on integrals and using the zero order Bessel function of the first kind. The results obtained in this paper correspond with the results obtained analytically in the previous paper and the numerical procedure does not require sophisticated mathematical techniques.

*2.2. Measurement of stresses when the structure is slowly loaded from the status of equilibrium by linearly increasing the load till its final value*

Intensity of the time averaged image for the circular polariscope is expressed as:

$$\overline{I} = \frac{1}{n}\sum_{j=1}^{n}\sin^2\left(C\frac{j-\frac{1}{2}}{n}\right), \tag{5}$$

where $C$ is a product of a constant which depends on the thickness and material of the structure with the difference of principal stresses at the final value of the load.

Variations of intensities as functions of $C$ are presented in figure 1. Variations of the values of $C_i$

and $\bar{C}_i$ as functions of $i$ are presented in figure 2. The values of $C_i$ and $\bar{C}_i$ are presented in table 2.

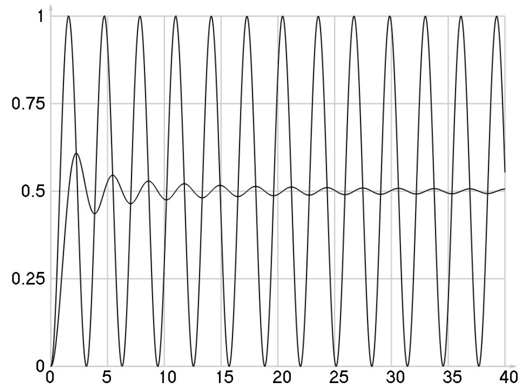


**Figure 1.** $I$ and $\bar{I}$ as functions of $C$.

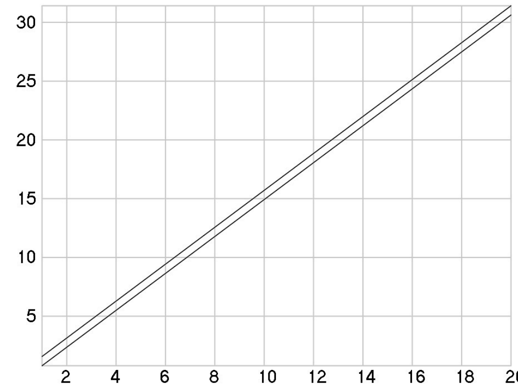


**Figure 2.** $C_i$ and $\bar{C}_i$ as functions of $i$.

Thus the numerical procedure enables to obtain the data required for the interpretation of photo-elastic measurements without the need of complicated mathematical techniques, which could be based on the use of the generalized zero order Bessel function of the first kind.

From the obtained results a constant shift $\bar{C}_i - C_i = 0.785398$ between the corresponding values of the roots of equations is observed from the data presented in the table. Of course in order to prove if this is exactly true the strict mathematical investigation based on the generalized zero order Bessel function of the first kind would be required.

## 3. Improved smoothing procedure in photo-elastic measurements

### *3.1. The improved smoothing procedure*

$x$ and $y$ denote the axes of the system of coordinates. Local coordinates ($\xi$, $\eta$) of the nodes are ($-1$, $-1$), (0, $-1$), (1, $-1$), ($-1$, 0), (0, 0), (1, 0), ($-1$, 1), (0, 1), (1, 1). $N_1$, $N_2$, …, $N_9$ are the shape functions of the two dimensional Lagrange quadratic finite element.

The nodal values of the stresses in the coating are determined from:

$$\left[\hat{K}\right]\left[\{\delta_x\} \quad \{\delta_y\} \quad \{\delta_{xy}\}\right] = \left[\hat{F}\right], \tag{6}$$

where $\{\delta_x\}$, $\{\delta_y\}$, $\{\delta_{xy}\}$ are the vectors of nodal values of the stresses $\sigma_x$, $\sigma_y$, $\tau_{xy}$ in the coating and:

$$\left[\hat{K}\right] = \int\left(\left[\hat{N}\right]^T\left[\hat{N}\right] + \left[\hat{B}\right]^T \lambda \left[\hat{B}\right]\right) dxdy, \tag{7}$$

$$\left[\hat{F}\right] = \int\left[\hat{N}\right]^T\left[\sigma_x \quad \sigma_y \quad \tau_{xy}\right] dxdy, \tag{8}$$

where $\lambda$ is the smoothing parameter and:

$$\left[\hat{N}\right] = \left[N_1 \quad N_2 \quad \dots \quad N_9\right], \tag{9}$$

$$\left[\hat{B}\right] = \left[N_1 - \bar{N}_1 \quad N_2 \quad N_3 - \bar{N}_2 \quad N_4 \quad N_5 \quad N_6 \quad N_7 - \bar{N}_3 \quad N_8 \quad N_9 - \bar{N}_4\right], \tag{10}$$

where $\bar{N}_1$, $\bar{N}_2$, $\bar{N}_3$, $\bar{N}_4$ are the shape functions of the two dimensional Lagrange linear finite element with the nodes having local coordinates $(-1,\ -1)$, $(1,\ -1)$, $(-1,\ 1)$, $(1,\ 1)$.

*3.2. Results of calculation of isoclinics in the coating of a vibrating plate*

Circular plate with fixed internal radius is investigated. Isoclinics for the second eigenmode obtained by using the procedure of conjugate approximation are presented in figure 3. The same result obtained by using the proposed procedure of conjugate smoothing is presented in figure 4.

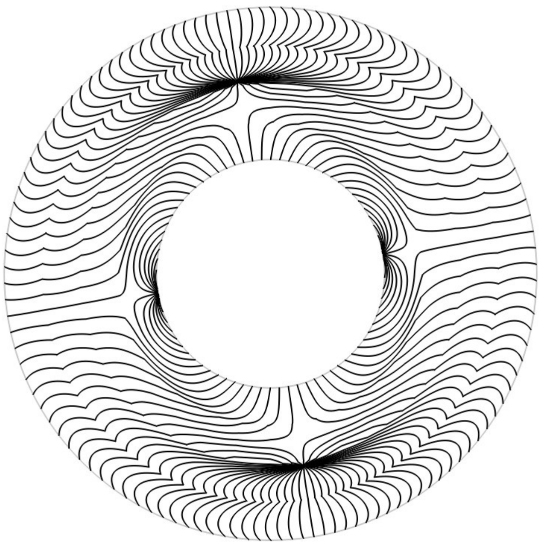

**Figure 3.** Isoclinics obtained by using the procedure of conjugate approximation.

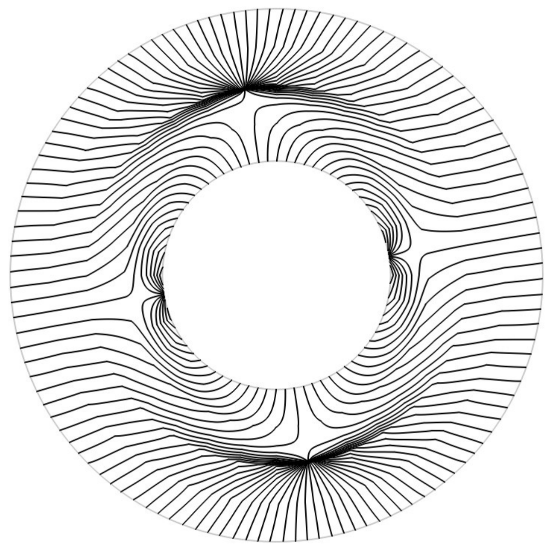

**Figure 4.** Isoclinics obtained by using the proposed procedure of conjugate smoothing.

From the presented results the applicability of the proposed procedure of conjugate smoothing is evident.

## 4. Conclusions

The obtained results are used in the process of interpretation of stress measurements in photo-elastic experiments and a numerical procedure of the proposed type might be applied to other time averaging problems in a similar way.

Improved smoothing procedure for the analysis of stresses in the coating of a plate performing bending vibrations is presented. From the obtained graphical results of isoclinics the applicability of the proposed procedure of conjugate smoothing is evident.

The results of performed investigation are used in the process of measurements of stresses in the elements of a printing device.

## 5. References

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