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The Optimization and Evaluation of Investment Portfolio

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Investment to assets is getting more and more importance today. Shares of stock options have become one of the region's economic indicators illustrating current economic situation. The main subject in researches of investment portfolios forecasting became a discussion about what values are best to measure portfolio risk and expected returns. Also there is a wide range of researches on macroeconomic and social events influence on evolution of stock returns. The main problem of today's professional investment is to achieve the master level of complex investment decision adoption, included macroeconomic and historical data return analysis.
The main aim of this work is to overview the portfolio optimization and evaluation models and to apply them to form the different portfolios of securities on the Vilnius Stock Market and compare them in the factor of conservativeness.
There are used mean-risk models, such as Markowitz classic model, Mean Absolute Deviation model and MiniMax model, to optimize the investment portfolio in this research. Also there is performed the approbation of the multifactor model to evaluate the formed portfolios and to see the influence of the macroeconomic indexes to stock returns.
Studies of this research showed 8 indicators, which have strongest linear dependence on shares return: VILIBOR interbank interest rate, consumer prices index, producer price index, construction expenses price index, trade balance, foreign direct investments, gross domestic product and inflation. Multifactorial analysis results showed that approved model to shares return data is not statistically reliable. Also, after further analysis there were found that Markowitz and Mean Absolute Deviation models create more conservative and optimal portfolio compared to MiniMax model, which is more suitable for aggressive and speculative investor.

Keywords: *mean-risk models, portfolio optimization, portfolio evaluation, Mean Absolute Deviation, Markowitz, MiniMax multifactor model, investor.*

Introduction

The research problem. From the 6-7 decades of the twentieth century for the experienced investor there were known values, coefficients and investment portfolio optimization models that were able to estimate the efficiency of the investment portfolio in conditions of assumed risk and expected return. Today there are a lot of investment portfolio optimization models that can form the portfolio for the different types of investors, which are ready to take different risk with the different level of the profit. Majority of the models help investors to take solution using mathematical calculations and they are able to take into account the period of the investment, assumed risk and expected return, the capital of the investment and etc.

General rule of the investment states that investor always prefers investment portfolio with a risk that gives the higher level of expected return. But the main problem is that different investors take different interest to the risk and return levels. One group of the investors is ready to invest into risky assets and expect higher level of the profit, meanwhile others take assets, that give expected return just higher than the level of inflation. This shows that selection of the investment portfolio is based on the individual investors' property and other external factors.

There is distinguished fundamental and technical stock market analysis (Valakevicius, 2008). Yhe author of this work agrees with opinion of Brentani C. (2004), analyzing the market of investment there should be performed both, fundamental and technical analyses. After these analyses the investor will have information enough to take a decision.

The research object are historical data of the companies quoted in Vilnius investment assets market, return and historical changes of Lithuanian macroeconomic indicators. *The aim of the research* is to compare different types of portfolio evaluation and optimization models, also to distinguish the most conservative investment portfolio between the models and analysis performed during the research. *The methods of the research* were mean-risk investment portfolio optimization models Markowitz, Mean Absolute Deviation, MiniMax, multifactorial analysis, comparative analysis.

Expected return and risk of investment portfolio

One of the most important characteristics of investment portfolio is that it always has a risk. It means that there is a loss related to investment probability. Risk is one of the two main factors which affect formation of investment portfolio. Most often risk is associated with unfavorable economical and negative micro environment changes. Investor should not have only this approach to risk. If risk would be associated only with negative results, investment would be irrational for him. Investor understands risk as opportunity for additional return. Investors approach to risk, related to investment to assets, could be defined as assets return deviation from expected return.

In assets portfolio theory factors causing risk are assigned to systematic (non-diversifiable risk) and unsystematic (diversifiable risk) (Valentinavicius, 2004). Systematic and unsystematic risks compose general risk.

Unsystematic risk can be avoided using, one of the approaches – diversification of portfolio (Markowitz, 1952, 1956), which is being used in this paper. The increase of investment portfolio assets types and numbers decreases unsystematic risk exponentially so general risk decreases too (see Figure 1). Diversifiable risk could not be reduced absolutely. Systematic risk cannot be reduced at all.

W. F. Sharpe in his work "Capital Asset Prices - A Theory of Market Equilibrium under Conditions of Risk" (1964) was the first to separate systematic and unsystematic risk and he suggested a market model. In this model he defined two characteristics of assets:

- coefficient α, which characterizes expected return, when market return is equal to 0. Investors' goal is to keep the value of α greater than 0;
- coefficient β, which shows sensitivity of assets return to market return changes.

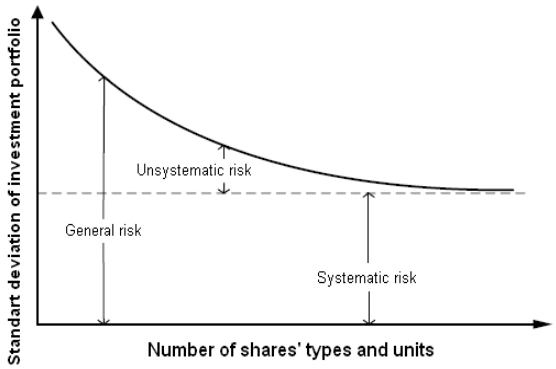

Figure 1. Investment portfolio risk dependence on the size of portfolio

Publications of H. Markowitz, W. F. Sharpe and other scientists in the middle of twenties started an open and wide discussion about investment portfolio risk and expected return. While many scientists agree to estimate expected return using mean of shares return, their opinions on risk evaluation differs. In 1966 W. F. Sharpe in his work "Mutual Fund Performance" introduced investment portfolio measure, which estimated portfolio behavior as excess return per one unit of risk. At first it was called as reward-to-variability measure and now it is known as Sharpe coefficient. In research of the model W. F. Sharpe raised two main questions. a) what is the best measure of portfolio risk? and b) what is the equilibrium between portfolio risk ant expected return? W. F. Sharpe in this work agreed with Markowitz (1952) suggestion to estimate assets portfolio risk using standard deviation, assuming that data dispersion exists. It should be noted that both scientists were using normally distributed returns.

In 1959 publication "Portfolio Selection: Efficient Diversification of Investments" Markowitz tried changing standard deviation to semi deviation, which would measure deviation below mean of return. Using this suggestion in 1991 Sortino and van der Meer changed standard deviation with semi deviation risk measure. After two years Sortino coefficient was derived, this changed Sharpe coefficient by replacing risk measure with semi deviation. While Sharpe measure gives importance to assets return volatility, Sortino coefficient differentiates volatility to up and down appearing shares returns. Upper changes are welcomed and are not assigned to volatility.

One more widely used risk measure is value-at-risk (VaR). By definition VaR is corresponding a loss distribution percentile with confidence level α, investment portfolio α-VaR is the least amount of money ς that with probability γ loss will be less than ς. Popularity of VaR risk measure was determined by its easy interpretation (Puelz, 1999).

VaR disadvantage in applying it to discrete distributed data was the reason scientists (Martin *et al.,* 2003) improved this model and derived a new measure CVaR - *Conditional Value-at-Risk.* In general, including discrete distributed data, CVaR is defined as a mean of VaR and loss exceeding VaR. CVaR measure has more positive properties than VaR: CVaR risk function is more informative, subadditive and convex (Rockafeller & Uryasev, 2000). Geometrical interpretation of VaR and CVaR is illustrated in Figure 2.

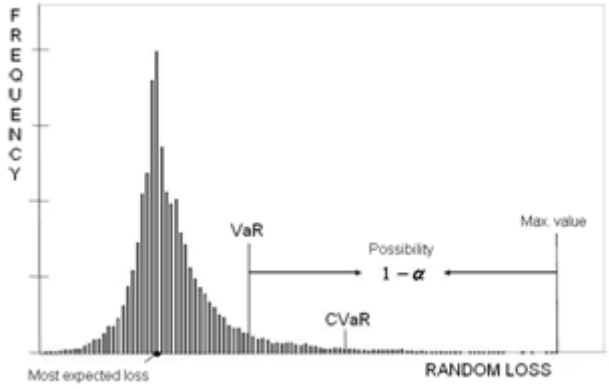

Figure 2. Geometric interpretation of VaR and CVaR

Discussion on statistics, which can estimate expected return and risk of investment portfolio, raised a new problem of evaluating return and risk in complex. Performance of investment portfolio measures was one of the solutions – by defining statistics, which are used to estimate risk and expected return, performance of the investment portfolio could be expressed as a ratio of expected return and risk statistic.

The development of investment portfolio formation models

Mean-risk analysis. The formation of investment portfolio is a part of investment process. In this part there are selected shares of investment portfolio and the proportion of capital to each share. There is also another very important task of investment portfolio formation part – in case of individual investor's demand form the investment portfolio with acceptable level of risk and expected return. Mean-risk models solve the main task of the formation of investment portfolio. They use mathematical optimization procedures to find the best portfolio that meets investors demand and diversify investment portfolio among chosen assets. Mean-risk models are criticized for the importance

they take for historical return data and do not take account for the macroeconomic environment. Skeptics of mean-risk models are sure that it is not enough to analyze just historical return data, because macro economical changes have influence on the investment and cannot be ignored. Factorial investment portfolio analysis is the portfolio monitoring part, which helps investors to review investment portfolio in the context of region macroeconomic trend.

One period investment portfolio formation problem is the solution of selection between random numbers with a highest level of expected return and a lowest level of risk.

Classical mean-risk model was created by H. Markowitz in 1952. He suggested using standart deviation as a ratio of risk and, in condition that returns of shares are distributed by normal distribution, he described efficient frontier. Model is described by:

$$\min \quad \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{1}$$

with conditions: $$\sum_{i=1}^{n} x_i \cdot r_i = r_p \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{3}$$

Model minimizes the variance (1) provided that (2) the sum of all share's return is equal to the average investor's portfolio rate attributed to the r_p, and (3) portfolio weighting amount is equal to one.

Scientists are trying to suggest more alternative mean-risk models, which are realized by linear programming methods. In 1988 Konno and Yamazaki showed that Mean Absolute Deviation (MAD) risk ratio is equivalent to variance model ratio. They stated that MAD model is a good substitute for Markowitz model retaining all its advantages: if m_t describes absolute portfolio deviation from the mean time t of r_{11}, r_{12}, ..., r_{nT} , than simplified MAD model is:

$$\min \ MAD = \frac{1}{T} \cdot \sum_{t-1}^{T} m_t \tag{4}$$

where $$m_t = \left| r_{it} - \mu_i \right| \cdot x_i \tag{5}$$

with conditions $$\sum_{i-1}^{n} x_i \cdot \mu_i = r_p \tag{6}$$

$$\sum_{i-1}^{n} x_i = 1 \tag{7}$$

Konno ir Yamazaki (1991) argue that MAD is a good substitute for Markowitz model retaining all its advantages. They provide strong arguments for the following:

a) MAD model formulation has no requirement for covariance matrix of returns;

b) simplified solving of linear system compared to the square - larger problems can be solved more efficiently and faster;

c) MAD portfolios typically have fewer shares - this reduces the transaction costs of changing the portfolio.

Markowitz model has not been widely used by investors in practice; it was more a theoretical basis for the development of modern portfolio theory and such methods as MAD. The main disadvantages of the Markowitz portfolio was, that its practical application was created for normally distributed data, complex calculation of the covariance matrix and difficult square programming method. Simaan (1997) argue that MAD partially solves these shortcomings in the Markowitz model, because it does not require the calculation of covariance matrix and is easier to program.

The investment portfolio of MiniMax (Young, 1998) model picks n shares appealing of shares return in historical data through the all historical observations $t=1,...,T$. Minimal risk in historical data is assigned as risk ratio. M_p describes the minimum portfolio return received during the observation period:

$$M_p = \min_t \sum_{i-1}^{N} x_i \cdot r_{it} \tag{8}$$

With this characteristic Minimax model is:

$$\max MM = M_p \tag{9}$$

with conditions: $$\sum_{i-1}^{n} x_i \cdot \mu_i = r_p \tag{10}$$

$$\sum_{i-1}^{n} x_i = 1 \tag{11}$$

Model idea is to maximize this ratio till it reaches chosen expected return level. Another alternative portfolio selecting interpretation of MiniMax model is minimizing the maximal loss of historical observations. MiniMax model's results can vary a lot in case one historical value comes out.

Markowitz and MAD methods are criticized on the grounds that they treat the mean positive and negative deviations, while investors desire for large positive deviations, but not negative. In other words, Markowitz and MAD models are looking for shares that have smaller deviations from the mean, regardless of whether the positive or negative deviations from the mean.

Young (1998) in his work "*A Minimax Portfolio Selection Rule with Linear Programming Solution*" shows that MiniMax model is ideal for investors making an investment decision separating the positive and negative deviations of returns. In oher words, Minimax is most appropriate model compared to MAD and Markowitz for investors that seek to evade downside-risk.

The author identifies Minimax model is not appropriate if the investor has little historical return data.

Main motives why scientists keep on searching for new mean-risk models are: 1) new models are easier programmed compared with Markowitz square programming method; 2) there are more alternative risk measures which estimate risk more correctly with real shares return data than Markowitz model.

Scientists understand the importance of new model improvement, because it is known that technical abilities to analyze new models are changing, also there are changes in the process in financial market. The main task of portfolio optimizations is to raise the accuracy of forecast and to minimize investment risk.

Factor analysis. The great influence on investment portfolio formation and portfolio performance theory was

performed by works of W. F. Sharpe: "*A Simplified Model for Portfolio Analysis*" (1963), "*Capital Asset Prices - A Theory of Market Equilibrium Under Conditions of Risk*" (1964), "*Mutual Fund performance*" (1966) and other publications. He suggested to split the main investment risk into systematic and unsystematic risk and created analysis of risk model - Capital Asset Pricing Model (CAPM). CAPM tries to find the link between each share's expected return and risk: in condition of the balance in market, the expected return of share is linearly proportional to systematic risk. Analysis of this model gives a conclusion that with a highest level of systematic risk, investor expects higher level of return.

Indeed, W. F. Sharpe's CAP model is one factor model, which started a discussion in literature on factor models. Coefficient β estimates systematic investment portfolio risk using expected return of portfolio, variation and shares covariance. W. F. Sharpe suggested a coefficient, which shows share sensitivity to investment portfolio market. It is assumed, that total investment market exchange coefficient β equals 1. This means that the higher β coefficient, the more risky investment asset is. If β coefficient is positive then the expected return of the share is rising the same way as market profit rises. If β coefficient is negative, share's profit is rising opposite way than market profit is rising.

Capital Asset Pricing Model is criticized because there is just one factor in this model. Scientists state that shares return depends not only on market change index but also on other indexes and there are lots of them in region economics. This means, that the portfolio return should be described with multifactor model.

Multifactor model, where the stock return depends on two factors can be represented graphically (see Figure 3).

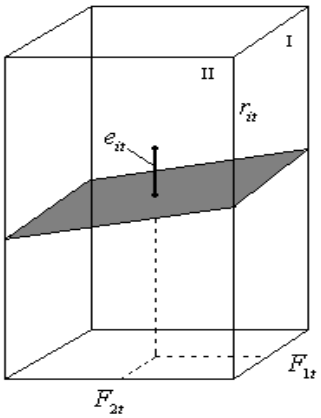

Figure 3. Geometric interpretation of the two-factor model

1th rectangular parallelepiped wall depicts the stock *i* linear dependence on factor's F_{1t} growth over the period *t*, while 2d wall depicts the stock *i* linear dependence on factor's F_{2t} growth over the period *t*. This creates a plane r_{it} – stock *i* return in the period *t*. The plane tilt angles b_{i1} and b_{i2} expresse the sensitivity of stock *i* to factors F_{1t} and F_{2t}.

Stock *i* return dependence on two factors equation:

$$r_{it} = a_i + b_{i1} \cdot F_{1t} + b_{i2} \cdot F_{2t} + e_{it} \quad (12)$$

where e_{it} - specific return of stock *i* in the period *t*;

a_i – return of stock *i*, when factors F_{1t} and F_{2t} are equal to zero.

Here stock *i* dispersion in two-factor model is:

$$\sigma_i^2 = b_{i1}^2 \cdot \sigma_{F1}^2 + b_{i2}^2 \cdot \sigma_{F2}^2 + 2 \cdot b_{i1} \cdot b_{i2} \cdot \mathrm{cov}(F_{1t}, F_{2t}) + \sigma_{ei}^2 \quad (13)$$

Also covariance of two stocks *i* and *j* is:

$$\sigma_{ij} = b_{i1} \cdot b_{j1} \cdot \sigma_{F1}^2 + b_{i2} \cdot b_{j2} \cdot \sigma_{F2}^2 + (b_{i1} \cdot b_{j2} + b_{i2} \cdot b_{j1}) \cdot \mathrm{cov}(F_{1t}, F_{2t}) \quad (14)$$

Factor models evaluate main systemic forces that influence shares return. There is a case in factor model that two or more shares have correlated returns i.e., they all together respond to selected factors in model in the same way. In these models there is a consideration that factor, which is not selected as part of model and influences share return, is called specific share return. Specific share return does not correlate to another share specific return.

Factor models are powerful instrument to estimate share return, to form investment portfolio and also to monitor it. Models are simple to use if it is needed to calculate variation, covariance of each share, to plot the Markowitz efficient frontier. Factor model can be used to calculate sensitivity of investment portfolio characteristics to each factor changes (Nedzveckas & Rasimavicius, 2000, 2001). This is a very important task to select the factors that should be inserted to factor model. There should be selected those factors whose changes mainly influence shares return, and what is the level of share's sensitivity to factor's changes. The right selection of factors is a very complicated process and can vary in different regions and markets, because, for example, factors which work in developed capital markets will not work in poorly developed ones. Selecting factors in new and unknown markets is a long process. For the purposes of factor analysis it is very important to approximate the regression equation with a higher volume of data that the calculations are more accurate. Even the choice is made to factors that have large-scale historical data, there is always the risk that future stock returns will behave and react differently to selected economic and social indicators.

Factor models are universal, because they can calculate investment of portfolio's expected return not just with macroeconomic, but also with microeconomic indicators. Investor should choose the number of factors he wants to use in the model. Bigger number of factors does not mean the optimality of calculation – the number of factors should be optimal to compute the return of investment portfolio in case of complicated calculations or time input.

When level of financial markets got higher and the number of financial tools grew, investors realized, that objective and theory oriented diversified investment portfolio can much decrease the level of investment risk on expected return purpose. That was the main stimulus for the development of modern portfolio theory of a 20th century and which is popular and developed till today. This work is based on this methodology.

Data and methodology

According to the literature analysis and recommend-dations, authors suggested to perform investment portfolio optimization and evaluation research in five steps:

1. Shares selection;
2. Analysis of share's return;
3. Investment portfolio optimization using mean-risk models;
4. Investment macroeconomic environment analysis;

5. Investment portfolio evaluation using multifactor model;
6. Analysis of investment portfolio statistic characterristics;
7. Analysis of the priority shares in mean-risk investment portfolios;
8. Comparison and discussion of analysis results.

The research started with the selection of shares. Attention was payed to the 20 shares with a highest positive skewness (Sharpe, 1966; Beedles, 1979) coefficient from Vilnius exchange market. Also there were calculated other statistic characteristics for each selected share: mean, standard deviation and kurtosis. Diagrams were drawn of each share return, price and turnover changes in observations. Kolmogorov-Smirnov test was used to check whether returns of selected shares are distributed by Normal distribution.

Three mean-risk optimization models – Markowitz, MAD and MiniMax - were used to form the portfolios with selected shares. In the fourth step the analysis of the macroeconomic and social investment environment was performed. The aim of this step was to select the indicators that mainly influence return and price of investment. Also calculations of correlation between indicator changes and shares' return were performed. After linear dependence analysis, 8 indicators were selected, which were used in forming multifactor model.

In the fifth step the approbation of multifactor model to the evaluation of investment portfolio profit and risk was performed. The purpose of the approbation was to evaluate the formed mean-risk models using the selected macroeconomic and social indicators. In addition there were also run through each mean-risk portfolio characteristics and in detail analyzed priority shares. In this step the research tried to pick out the characteristic values, that are specific for priority shares, and also the indicators, that have the main influence to priority shares.

Through a comparison of the results, a discussion on each portfolio's conservativeness was made with aim to distinguish the most conservative model and make some recommendations for moderate investor.

Empirical research

This work's research included he shares of 20 companies and a research of fundamental indicators would be too complicated. That's why it was decided to use only economic and social development indicators, which could influence shares return or price.

16 indicators of economical and social development were chosen after macroeconomical analysis of literature on banks and other financial institutions. But after analysis of linear dependence between shares return and indicators changes there were chosen only 8 indicators for multifactor analysis: VILIBOR interbank interest rate, consumer prices index, producer price index, construction expenses price index, trade balance, foreign direct investments, gross domestic product and inflation.

Each formed mean-risk portfolios' historical returns and prices were calculated using selected shares' returns and prices. There was simulated a research that in 2006.01.02 there were invested 1 EUR to each mean-risk portfolio. It is accurate to use portfolio prices calculated in this way, but in portfolio dynamic analysis it is right, because return is a comparative characteristic which does not depend on size of investments capital. Calculation results of portfolio characteristic are showed in Table 1.

Calculations showed that MAD and MiniMax models formed portfolios with similar mean, accordingly 0,0001481 and 0,0001404. Markowitz portfolio historical data returns have the lowest mean – 0,0001275. Markowitz and MAD portfolios have similar and low standard deviations of historical data, accordingly 0,01053560 and 0,0108585.

Table 1

Statistic characteristics of investment portfolios

Portfolio	Mean	Standard deviation	Skewness	Kurtosis
Markowitz	0,0001275	0,0105360	0,5521596	9,182817
MAD	0,0001481	0,0108585	0,9208017	16,13344
MiniMax	0,0001404	0,0121969	0,5021598	3,547564

MiniMax portfolio has the highest mean, also the highest standard deviation – 0,0121969. Calculations lead to the conclusion, that owner of MiniMax portfolio should take a high risk level for high expected return.

MAD portfolio has the highest skewness coefficient which means that deviation of this model is usually not big, but positive. Also this means, that standard deviation of this portfolio is determined by small positive deviation, not by random pop-up values. Markowitz portfolio compared with other portfolio is very moderate, because low level of risk gives ensured small return.

It should be noted that all the models have formed the positive mean portfolios. The standard deviations of these portfolios show, that diversification of risk have been achieved – all portfolios have standard deviation lower than selected shares standard deviation individually. This achievement has its own "price" – all the portfolios have lower mean than each selected shares separately.

Multifactorial analysis results showed that created model is not statistically reliable, so the authors decided to ignore results of this analysis and to perform additional analysis with portfolio priority shares.

Priority shares in Markowitz model are 41,41 % and in MAD 59,5 % of all capital. This leads to the conclusion, that Markowitz portfolio is more diversified. This conclusion can also be supported by the fact, that Markowitz portfolio is composed from 18 pieces of shares and MAD portfolio from 15 pieces of shares.

The greatest difference between Markowitz and MAD portfolios is that MAD is giving more investment capital to AB "TEO LT" shares. AB "TEO LT" shares were included using these models, because they have the least standard deviation and in estimating risk it is the priority indicator for these models. After looking at AB "TEO LT" shares in macroeconomic context by using correlation between these shares and economical, social indicators' changes, we see that these shares mostly depend on consumer confidence indicator and the cost of construction expenses price index changes. Negative influence is caused by VILIBOR index and bruto wage index. Prognoses of indicators with positive influence are good so we can expect growth of AB "TEO LT" shares price in the following years.

Shares of AB "Klaipedos nafta" are also stable and they take the second position after AB "TEO LT" by using standard deviation statistic. Mean is not very high though it is positive. This is why shares of this company were included to Markowitz and MAD portfolios and were given 13,50 % and 14,40 % of capital respectively. Correlation between shares of this company and economical, social indicators' analysis results shows that shares of AB "Klaipedos nafta" has strong linear relation with VILIBOR, unemployment level, product export and central government abroad debt. Linear relation to other indicators is not so strong.

Markowitz gave 9,00 % and MAD 8,80 % and 9,40 % of shares respectively to companies from the same sector - AB "Rokiskio suris" and AB "Pieno zvaigzdes". Both of these companies belong to one of the strongest industries – milk processing industry. Shares of same sectors are decreasing diversification of portfolio however sector is stable and choice of these shares can be assessed positively. After looking at correlation between share prices of AB "Rokiskio suris", AB "Pieno zvaigzdes" and economical, social indicators you can see that both shares have strong positive linear relation with confidence of consumers price indicator. It is also visible that shares' prices of AB "Pieno zvaigzdes" are more linearly dependent on interbank interest rates and balance between import/export. Shares' prices of AB "Rokiskio suris" are more sensitive to unemployment level and average monthly bruto wage.

Structure of investment portfolio formed using MiniMax model is very different from other models. In this model priority is given to other shares. In this portfolio priority shares take 41,30 % from all shares – least from all mean-risk portfolios. The greatest part of MiniMax investment capital is given to AB "Gubernija" shares which have large dispersion around mean. AB "Gubernija" is one of the three companies which have shares with big standard deviation. This is the reason why shares of this company were chosen as MiniMax method is trying to pick such shares, which have large positive deviation from mean. These shares have long tails i.e. have values which are located very far from the mean. Shares of AB "Gubernija" also have large linear relation with social and economical indicators. Shares' prices of this company have strong positive correlation with EURIBOR, consumers' confidence level, construction expenses price index, and have negative influence from rising unemployment level, average bruto wage, trade balance and central government abroad debt. This sensitivity to economical and social indicators could be treated as a negative feature as it is hard to make prognosis for this type of shares.

Other priority shares of MiniMax portfolio AB "Kauno energija", AB "Lietuvos elektrine" and AB "Utenos trikotazas" were given 7,50 % of investment capital each. These shares have similar large standard deviation. AB "Utenos trikotazas" shares are the only in this portfolio with negative mean. These shares could be chosen because of one very high value of all observations. Shares of AB "Lietuvos elektrinės" are different from others priority shares because they don't have strong linear relation ($|\rho| \geq 0{,}6$) with economical and social indicators. Shares of AB "Kauno energija" and AB "Utenos trikotazas" have strong linear relation with EURIBOR, consumers' confidence level, unemployment level, average bruto wage, constructions expenses price index and balance between export/import. Shares of AB "Utenos trikotazas" also have strong linear relation with central government abroad debt. After looking at structure of MiniMax portfolio the conclusion could be made that this model could be used by aggressive investors who expect to take advantage of high returns' deviations. This conclusion shows that this model is very different from Markowitz and MAD models.

According to analysis of shares and assets, MAD portfolio is most suitable for conservative investment strategy. MAD portfolio is the least diversified as its greatest part of investment capital (29,90 %) is given to AB "TEO LT" shares and it eliminated most of the picked shares. It consists of 15 shares. It was decided that number of shares is sufficient and the advantage of this portfolio is low administration expenses. Also portfolio with fewer shares could be analyzed easier and capital restructurization would be easier too. MAD portfolio is more conservative as it eliminated shares of AB "Snoras", AB "Panevezio statybos trestas", AB "Linas", AB "Dvarcioniu keramika" and AB "Zemaitijos pienas" as standard deviation indicators of these shares is high. Comparing MAD and Minimax portfolios, MAD is more conservative because it has a small number of return values that pop-up in historical data.

Results of portfolio return analysis showed, that MAD portfolio would have had the highest return among all three mean-risk models. Also it would have had low coefficient of standard deviation – only 3 % higher than Markowitz at the same time return would be 16 % higher than Markowitz portfolio. Also MAD has the highest coefficient of skewness, it means, that MAD portfolio typically has positive historical returns. Analysis of investment macroeconomic and social environment showed, that MAD is attractive for conservative oriented investor, because he has the lowest sensibility to environment among three formed portfolios. This means, that it is easier to forecast portfolio performance in the future for the investor, he can take more attention to the technical historical analysis, which is done by computer, and pay attention just to few macroeconomic indicators' changes and prognosis.

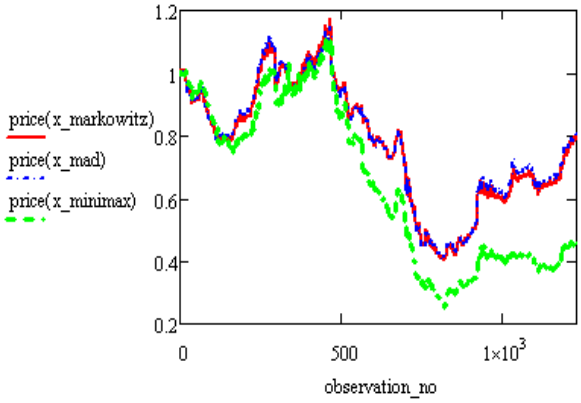

Figure 4. Prices of portfolios changes in period 2006.01.02-2010.01.03

Figure 4 illustrates dynamics of portfolio prices, when there is simulated investment of 1 EUR for 4 year investment period to each portfolio. Figure shows, that all

portfolio's price would be fallen, but MiniMax would have the biggest loss in this investment. Also calculation results show, that MiniMax would have very dynamic price changes, meanwhile MAD and Markowitz portfolios would have similar price trend.

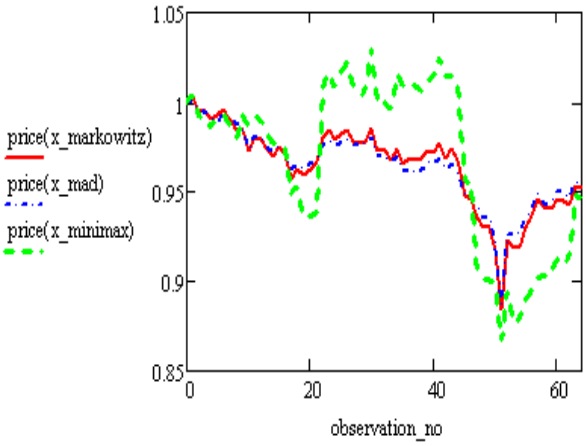

Figure 5. Prices of portfolio changes in period 2011.01.03-2011.04.01

There were also performed an analysis of each portfolio price changes after the date of used historical data end with a stimulated situation of 1 EUR investment to each portfolio. Analysis results are shown in Figure 6. There are all portfolio prices in similar level at the end of the period, i.e. at 2011.04.01. Although within four month period MiniMax portfolio had the highest dynamics in price changes. Figure 5 confirms analysis results - MiniMax portfolio is more acceptable for speculators, but not for long term investment oriented investors. This is because, speculator would take advantage of big positive portfolio changes and sell part of the portfolio in time after price fall.

Conclusions

Literature on investment portfolio formation showed that the main problem of forming the investment portfolio is to find appropriate statistic characteristics that can evaluate portfolio risk and expected return correctly.

There were AB "Klaipedos nafta" shares with the highest mean coefficient and AB "TEO LT" with the lowest standard deviation between selected shares. In general Vilnius exchange market trade is very static, most returns of the shares are equal to zero and returns are not normally distributed.

Formed Markowitz and MAD portfolios gave the highest priority to AB "TEO LT" shares, accordingly 19,91 proc. and 29,90 proc., and MiniMax gave 18,80 proc. of investment capital to AB "Gubernija" shares.

Analysis of linear dependence between macroeconomic indicator changes and share's return, also share's price showed that there is a more significant dependence between prices and indicator changes than between returns and indicator changes. The most significant linear dependence is between share's return and VILIBOR, Consumers prices index, Price index of sold production, Price index of constructions expenses, Trade balance, Foreign direct investment index, GPD and inflation index indicators changes.

Analysis results of portfolio historical return data showed that optimization of mean-risk portfolios achieved diversification purpose – standard deviation of each portfolio would be lower than any selected share, but means of each portfolio would be also lower than any of selected shares.

Aprobed multifactor model analysis results showed that multifactor model was not statistically reliable and do not meet the main condition.

Analysis of priority shares showed, that Markowitz and MAD portfolios are more suitable for conservative investors than MiniMax, also they depend less on macroeconomic and social changes. According to analysis of shares and assets, the authors distinguished MAD portfolio as most suitable for conservative investment strategy. MAD portfolio has lower administration expenses than other portfolios. Also portfolio with fewer shares could be analyzed easier and capital restructurization would be easier too. MAD portfolio is more conservative as it eliminated shares of AB "Snoras", AB "Panevezio statybos trestas", AB "Linas", AB "Dvarcioniu keramika" and AB "Zemaitijos pienas" as standard deviation indicators of these shares is high.

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Investicinio portfelio optimizavimas ir vertinimas

Santrauka

Šiandieniniame pasaulyje vis svarbesniu tampa investavimas į vertybinius popierius. Akcijos biržos duomenys pasaulyje tapo vienu iš ekonomikos rodiklių, kuris vaizdžiai iliustruoja esamą ekonomikos padėtį. Pagrindiniu mokslinių tyrimų objektu, prognozuojant finansinius investicinius portfelius, tapo diskusija apie tai, kokie dydžiai gali išmatuoti portfelio riziką ir laukiamą grąžą, bei kaip makroekonominiai ir socialiniai įvykiai ir veiksniai sąlygoja vertybinių popierių grąžos kitimą.

Vertybinių popierių kotiruotės atspindi šalies ekonomikos, taip pat ūkio šakos ir pačios įmonės padėtį, todėl vien techninės akcijų grąžų analizės nepakanka. Mokslininkai pasiūlė daug metodų kaip statistiškai susieti grąžas bei kainas ir ekonominius bei socialinius rodiklius.

Šio *darbo tikslas* – ištirti vertybinių popierių portfelių formavimo ir vertinimo modelius bei jų taikymą formuojant vertybinių popierių portfelį Vilniaus vertybinių popierių rinkoje, taip pat išskirti konservatyviausią investavimo strategiją atitinkantį modelį.

Tyrimo metodai: vidurkio-rizikos vertybinių popierių portfelių optimizavimo metodai Markowitz, MAD, MiniMax, daugiafaktorinė analizė, lyginamoji analizė.

Vertybinių popierių portfelių optimizavimo ir vertinimo tyrimas buvo įvykdytas penkiais etapais:

1. Akcijų atranka;
2. Akcijų grąžų analizė;
3. Vertybinių popierių portfelių formavimas vidurkio-rizikos metodais;
4. Investavimo makroaplinkos įvertinimas;
5. Vertybinių popierių portfelių vertinimas daugiafaktorinės analizės metodu;
6. Vertybinių popierių portfelių skaitinių charakteristikų analizė;
7. Vertybinių popierių portfelių prioritetinių akcijų analizė;
8. Atliktų analizės rezultatų palyginimas ir aptarimas.

Tyrimas buvo pradėtas nuo vertybinių popierių atrankos. Iš visų, Vilniaus vertybinių popierių rinkoje kotiruojamų akcijų, buvo atsirinktos 20 akcijų, turinčių didžiausius asimetrijos koeficientus. Taip pat šiame etape buvo apskaičiuojamos atrinktų akcijų grąžų skaitines charakteristikos: vidurkis, standartinis nuokrypis ir eksceso koeficientas. Peržvelgus šias charakteristikas pastebėta, kad didžiausiu vidurkiu pasižymi AB „Klaipėdos nafta“ akcijos, o mažiausiu standartiniu nuokrypiu AB „TEO LT“ akcijos. Nagrinėjant Vilniaus akcijų rinkos duomenis nustatyta, kad akcijų prekyba pasižymi statiškumu, didelę dalį akcijų gražų sudaro nuliai bei grąžos nėra pasiskirsčiusios pagal normalųjį dėsnį.

Vertybinių popierių portfelių formavimas buvo vykdomas pasitelkiant vidurkio-rizikos metodus. Atrinktoms 20-čiai akcijų bus suformuoti vertybinių popierių portfeliai šiais trimis metodais: Markowitz, MAD ir MiniMax. Markowitz ir MAD portfeliai didžiausią kapitalo dalį skyrė AB „TEO LT“ akcijoms, atitinkamai po 19,91 proc. ir 29,90 proc., o Minimax – AB „Gubernija“ akcijoms, 18,80 proc. investicinio kapitalo.

Vėliau atlikta investavimo aplinkos ekonominė ir socialinė analizė, norint išskirti svarbiausius ekonominius ir socialinius rodiklius, lemiančius vertybinių popierių portfelių grąžas ir kainas. Atrinktiems rodikliams buvo atlikta jų pokyčių statistinės tiesinės priklausomybės nuo akcijų grąžų analizė, taip išskirti daugiausia akcijų grąžas lemiantys rodikliai ir jų pokyčiai. Pastebėta, kad didžiausia tiesinė priklausomybė yra tarp akcijų kainų ir makroekonominių rodiklių. Tiesinė priklausomybė tarp akcijų grąžų ir makroekonominių rodiklių pokyčių yra silpnesnė. Išskirti didžiausią tiesinę priklausomybę su akcijų grąžomis turintys makroekonominiai rodiklių pokyčiai: vilibor, VKI, GKI, SSKI, prekybos balansas, TUI, BVP, infliacija.

Atliktas daugiafaktorinės analizės metodo pritaikymas vertybinių popierių portfelių pajamingumo ir rizikos vertinimui, siekiant įvertinti sumodeliuotus vidurkio-rizikos portfelius. Aprobavus modelį, jis bus pritaikytas jau suformuotų vidurkio-rizikos portfelių vertinimui, tačiau pastebėta, kad statistiškai šis modelis nėra pagrįstas, jis netenkina jam keliamų prielaidų.

Papildomai buvo peržvelgtos kiekvieno suformuoto vertybinių popierių portfelio skaitinės charakteristikos, bei išsamiau išanalizuotos vertybinių popierių portfelių prioritetinės akcijos. Siekiant kuo tiksliau atlikti tyrimą, buvo stengiamasi išskirti prioritetinėms akcijoms būdingas skaitines charakteristikas ir jas lemiančius makroekonominius rodiklius. Analizės metu pastebėta, kad suformuoti vertybinių popierių portfeliai pasiekė diversifikavimo tikslą – jų istorinių grąžų standartiniai nuokrypiai yra mažesni už visų atrinktų akcijų, tačiau tai sumažino ir jų grąžų istorinius vidurkius – jų grąžos yra mažesnės nei atskirų akcijų vidutinės grąžos. Prioritetinių akcijų analizė išskyrė, kad Markowitz ir MAD portfeliai yra konservatyvesni nei MiniMax, bei jų prioritetinės akcijos yra mažiau priklausomos nuo ekonomikos pokyčių.

Remiantis suformuotų portfelių akcijų grąžų skaitinėmis charakteristikomis, atliktų investavimo aplinkos ir faktorinės analizės rezultatais buvo išskirtas MAD vertybinių popierių portfelių formavimo metodas kaip geriausiai atitinkantis konservatyvią investavimo strategiją. MAD portfelio administravimo išlaidos būtų mažiausios dėl mažesnio akcijų kiekio portfelyje. Ši savybė taip pat suteiktų portfelio savininkui lankstumo ir paprastumo atliekant portfelio stebėjimą ir analizę bei greitumo priimant sprendimus dėl portfelio kapitalo perskirstymo tarp akcijų. MAD porfelis tapo daug konservatyvesniu už kitus nagrinėtus portfelius todėl, kad jis eliminavo akcijas, kurios pasižymėjo vienu aukščiausiu standartiniu nuokrypiu.

Raktažodžiai: *vidurkio-rizikos modeliai, portfelio optimizavimas, portfelio vertinimas, vidutinis absoliutinis nuokrypis (MAD), Markowitz, MiniMax daugiafaktorinis modelis, investuotojas.*

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