ISSN 1392 - 1207. MECHANIKA. 2013 Volume 19(1): 5-11

The elastoplastic concrete strain influence on the cracking moment and deformation of rectangular reinforced concrete elements

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crossref http://dx.doi.org/10.5755/j01.mech.19.1.3619

1. Introduction

When analysing the states of stresses of flexural reinforced concrete elements in the stages of failure, the operation of tensile concrete usually is not estimated [1-2] but its operation has the effect on the cracking moment and stiffness before the crack opening. The expressions of stress-strain relationship of compression concrete are defined by the rules of EC2 (Eurocode 2) as well as in literal sources [3-7]; however there are few sources about the stress-strain relationship of tensile concrete, especially in flexural elements.

The description and evaluation of compression concrete stress-strain relationship of flexural reinforced concrete elements when calculating their stiffness in different states is a complex problem which is usually solved using simplified methods. As the research shows [8], when calculations are made using the method of ultimate limit states, the coefficient values of concrete stress sheet have a significant effect on the change of the results according to different methods, and it is even more difficult to define the concrete stress-strain relationship before cracking.

One more important value subject to the concrete stress sheet is a limit relative height of compressive zone. In *EC2* standards, this value depends only on reinforcement grade for lower class concrete in flexural reinforced concrete elements, i.e. concrete limit strain does not depend on its strength class in this case. However, when concrete class varies from C8/10 to C50/60 (more than 6 times), the concrete modulus of elasticity increases more than 1.5 times. If concrete strain characteristics change and reinforcement characteristics remain the same (in the same class), a limit relative height of compressive zone also changes. It can be concluded that a concrete stress-strain curve varies but, according to EC2, concrete stress diagram coefficients λ and η [8] are constant for the concrete of examined classes. In this case the construction technical regulations (STR) [2] evaluate concrete strength when calculating a limit relative height of compressive zone. On the ground of the research presented in literal sources [9-10], it can be stated that concrete limit strains also depend on concrete strength. Although this article does not discuss the ultimate limit state and concrete stress diagram in the failure stage; however the principle of description of concrete stresses is important.

For uncracked section, it is more important to know the tensile stress-strain relationship because the compressed concrete operates elastically in most cases. But the investigation of relationship of concrete tensile stress-strain is more complicated than the investigation of compression stress. EC2 does not describe that relationship and STR specifies the rectangular tensile stress diagram for the calculation of cracking moment. It is difficult to describe the relationship of concrete tensile stress and strain. Thus, the limit strain values of tensile concrete are used in some methods [11, 12].

One more method of problem solving (having evaluated the modulus of elasticity and the strain of concrete) is the description of stress-strain relationship of compression and tensile concrete on the ground of the relation of elastic and elastoplastic concrete strain λ_c and λ_{ct}. With regard to the experimental results [13], coefficient λ_c varies from 1 to 0.15 for compression concrete and coefficient λ_{ct} is equal to 0.5 for tensile concrete, approaching a short-term strength. Since coefficient λ_c depends on loading time and extent (when time and loading extent increase, λ_c decreases), it is taken that these coefficients are equal for both tensile and compression concrete $\lambda_{ct} = \lambda_c = 0.5$. In this case it is accepted that with the increase of loading, tensile and compression concrete change from the elastic to the plastic stage gradually, i.e. the coefficients vary gradually from $\lambda_{ct} = \lambda_c = 1.0$ to 0.5. Having accepted these initial conditions, it is possible to express the stress-strain relationship of concrete and to describe the equations of equilibrium of flexural reinforced concrete elements. The direct solution of the obtained equation system is quite complex; however, the solution can be found by the method of approximation. It is also possible to calculate the height of compressive zone and concrete strains using iterative procedure and dividing the element section into layers. Moreover, the solution can be found using the finite element method (FEM) by means of changing layers into finite bar elements. With the sufficient number of layers, the error is minor.

2. Concrete stress-strain relationship

Having accepted that the elastic part of both tensile and compression concrete ranges up to $0.4f_{ct}$ and $0.4f_c$ the limit strains of elastic zone for tensile concrete $\varepsilon_{ct,el} = 0.4 f_{ct}/E_c$ and for compression concrete $\varepsilon_{c,el} = 0.4 f_c/E_c$. According to the classic model, the coefficient of elastoplastic concrete strains which evaluates the relation of elastic and total strains is equal to $\lambda_{ct,lim} = \varepsilon_{ct,el}/\left(\varepsilon_{ct,el} + \varepsilon_{ct,pl}\right) = 0.5$ at the time of crack opening. The same coefficient of elastoplastic concrete strains can also be taken for compressive concrete zone, i.e. $\lambda_{c,lim} = 0.5$. Thus, in the elastic zone the coefficient $\lambda_{ct} = \lambda_c = 1$ and when stresses (or strains) exceed the elasticity limit, the coefficient decreases until it becomes equal to 0.5 (when limit stresses are reached). On the assumption that the coefficient λ_{ct} changes gradually from the elasticity limit to the concrete tensile strength limit, it can be expressed as follows:

$$\lambda_{ct} = 1 - 0.5\frac{\varepsilon_{ct,i} - \varepsilon_{ct,el}}{\varepsilon_{ct,lim} - \varepsilon_{ct,el}}, \qquad (1)$$

where $\varepsilon_{ct,lim}$ is limit strain of tensile concrete that conforms to tensile strength, i.e.:

$$\varepsilon_{ct,lim} = \frac{f_{ct}}{\lambda_{ct,lim} E_c} = \frac{2 f_{ct}}{E_c}. \qquad (2)$$

Having the accurate data of stress-strain relationship and the strain modulus of tensile and compression concrete, it is possible to describe the variation of the modulus of strain more accurately when elastoplastic strains are applied in concrete. Having such relationship between stresses and strains of compression concrete, it can be compared with relationship according to EC2, and can be illustrated in the following way (Fig. 1).

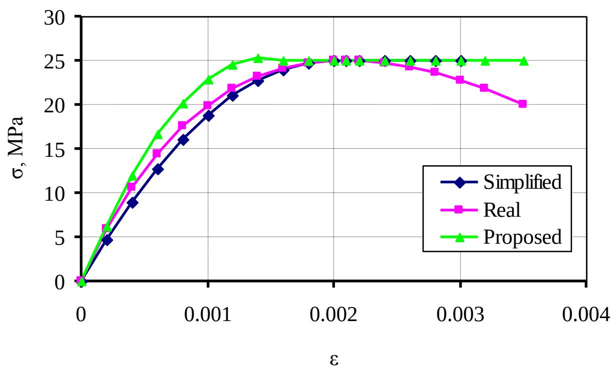

Fig. 1 Stress-strain curves: *1* - curve of flexural element calculated according to the proposed method; *2* - curve calculated according to EC2; *3* - simplified curve calculated according to EC2

The variation of coefficient λ_c according to EC2 and the proposed model is presented in Fig. 2.

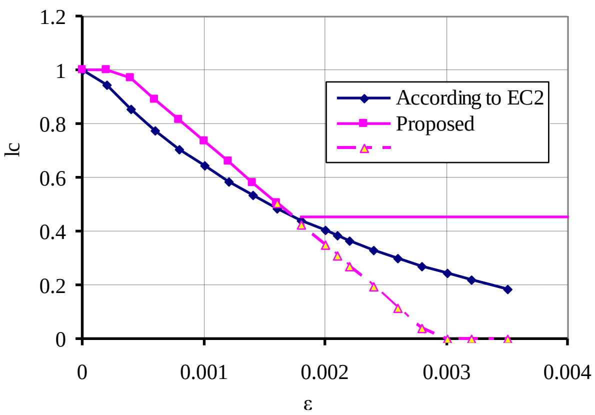

Fig. 2 The relationship of elastoplastic strain coefficient of compression concrete to relative strains according to the proposed and EC2 methods

3. The state of stresses when concrete operates elastically in the compressive zone

In most cases, the strains in the compressive zone are elastic before the crack opening. When loading approaches the limit of element cracking, Hooke's law also holds in the tensile zone from the neutral axis to elastic strain limit $\varepsilon_{ct,el}$ and the stress-strain relationship becomes nonlinear from $\varepsilon_{ct,el}$ to ε_{ct} (Fig. 3).

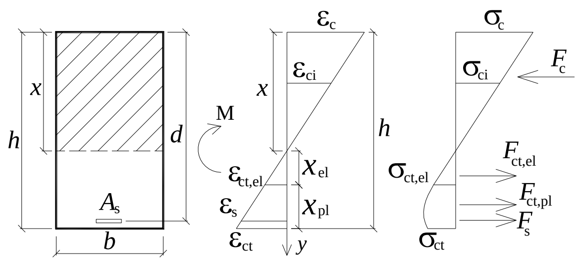

Fig. 3 Strains and stresses of element when compressive concrete zone operates elastically

Equality $\sigma_{ct} = \lambda_{ct} E_c \varepsilon_{ct}$ holds in this zone. Having stress-strain relationships in these three zones, we can write the equations of equilibrium of force projections and bending moments; therefore, we can obtain the compressive zone height of flexural element and the strains and stresses of edge layers. Thus, the resultant of rectangular section element in the compressive concrete zone following the assumption of flat sections is calculated as follows:

$$F_c = -\int_0^x bE_c \varepsilon_{ci} dy = -0.5 bE_c \varepsilon_c x, \qquad (3)$$

where $\varepsilon_{ci} = \dfrac{\varepsilon_c y}{x}$.

The resultant in the elastic part of tensile zone is:

$$F_{ct,el} = \int_0^{x_{el}} bE_c \varepsilon_{ci} dy = 0.5 bE_c \varepsilon_{ct,el} x_{el} = 0.5 bE_c \frac{\varepsilon_{ct,el}^2}{\varepsilon_c} x, \qquad (4)$$

where $x_{el} = \dfrac{\varepsilon_{ct,el} x}{\varepsilon_c}$ and $\varepsilon_{ci} = \dfrac{\varepsilon_{ct,el} y}{x_{el}}$.

The resultant in the elastoplastic part of tensile zone is calculated as follows:

$$F_{ct,pl} = \int_0^{x_{pl}} bE_c^* \varepsilon_{ci} dy. \qquad (5)$$

Having evaluated that $E_c^* = E_c \lambda_{ct} = E_c \times \left(1 - 0.5\dfrac{\varepsilon_{ci} - \varepsilon_{ct,el}}{\varepsilon_{ct,lim} - \varepsilon_{ct,el}}\right)$ and having indicated $1 + \dfrac{\varepsilon_{ct,el}}{\varepsilon_c} = k$ (then $x_{pl} = h - x - x_{ct,el} = h - kx$), we obtain:

$$F_{ct,pl} = \int_0^{h-kx} bE_c \left(1 - 0.5\frac{\varepsilon_{ci} - \varepsilon_{ct,el}}{\varepsilon_{ct,lim} - \varepsilon_{ct,el}}\right) \varepsilon_{ci} dy. \qquad (6)$$

Strains in elastoplastic zone can be expressed as follows:

$$\varepsilon_{ci} = \varepsilon_{ct,el} + \left(\varepsilon_{ct} - \varepsilon_{ct,el}\right)\frac{y}{h-kx}. \tag{7}$$

Then the resultant is calculated:

$$F_{ct,pl} = \int_0^{h-kx} bE_c \left(1 - 0.5\frac{\left(\varepsilon_{ct,el} + \left(\varepsilon_{ct} - \varepsilon_{ct,el}\right)\frac{y}{h-kx}\right) - \varepsilon_{ct,el}}{\varepsilon_{ct,lim} - \varepsilon_{ct,el}}\right)\left(\varepsilon_{ct,el} + \left(\varepsilon_{ct} - \varepsilon_{ct,el}\right)\frac{y}{h-kx}\right)dy. \tag{8}$$

Having evaluated that $\varepsilon_{ct} = \frac{\varepsilon_c(h-x)}{x}$ and having solved an integral expression, we obtain:

$$F_{ct,pl} = bE_c\left(h-kx\right)\times\left[0.5\varepsilon_{ct,el} + \frac{\varepsilon_c\left(h-x\right)}{2x} - \left(\frac{0.5}{\varepsilon_{ct,lim} - \varepsilon_{ct,el}}\right)\times\left(\frac{\varepsilon_c^2\left(h-x\right)^2}{3x^2} - \frac{\varepsilon_c\varepsilon_{ct,el}\left(h-x\right)}{6x} - \frac{1}{6}\varepsilon_{ct,el}^2\right)\right]. \tag{9}$$

Finally, the resultant in tensile reinforcement:

$$F_s = \varepsilon_s E_s A_s = \frac{\varepsilon_c(d-x)}{x}E_s A_s \tag{10}$$

Bending moments around neutral element axis are calculated in an analogical manner. Bending moment from the resultant of compressive concrete zone:

$$M_c = \int_0^x bE_c\varepsilon_{ci}ydy = \frac{1}{3}bE_c\varepsilon_c x^2. \tag{11}$$

Bending moment from the resultant of elastic part of tensile concrete zone:

$$M_{ct,el} = \int_0^{x_{el}} bE_c\varepsilon_{ci}ydy = \frac{1}{3}bE_c\frac{\varepsilon_{ct,el}^3}{\varepsilon_c^2}x^2. \tag{12}$$

Bending moment from the resultant of elasto-plastic part of tensile concrete zone:

$$M_{ct,pl} = bE_c\left(h-kx\right)\left[\begin{array}{l} 0.5\varepsilon_{ct,el}\left(h-x\right) + \frac{\varepsilon_{ct,el}^2 x}{2\varepsilon_c} - \frac{0.5\varepsilon_{ct,el}x}{\varepsilon_c\left(\varepsilon_{ct,lim} - \varepsilon_{ct,el}\right)}\times \\ \times\left(\frac{\varepsilon_c^2\left(h-x\right)^2}{3x^2} - \frac{\varepsilon_c\varepsilon_{ct,el}\left(h-x\right)}{6x} - \frac{\varepsilon_{ct,el}^2}{6}\right) + \frac{\varepsilon_c\left(h-x\right)\left(h-kx\right)}{3x} + \frac{\varepsilon_{ct,el}\left(h-kx\right)}{6} - \\ - \frac{0.5}{\varepsilon_{ct,lim} - \varepsilon_{ct,el}}\times\left(\frac{\varepsilon_c^2\left(h-x\right)^2\left(h-kx\right)}{4x^2} - \frac{\varepsilon_c\varepsilon_{ct,el}\left(h-x\right)\left(h-kx\right)}{6x} - \frac{\varepsilon_{ct,el}^2\left(h-kx\right)}{12}\right) \end{array}\right]. \tag{13}$$

Bending moment from the resultant of tensile reinforcement:

$$M_s = \varepsilon_s E_s A_s\left(d-x\right) = \frac{\varepsilon_c\left(d-x\right)^2}{x}E_s A_s. \tag{14}$$

Having the expressions of resultants and bending moments produced by them, we find unknown values x and ε_c from the equation system:

$$\begin{cases} F_c + F_{ct,el} + F_{ct,pl} + F_s = 0; \\ M_c + M_{ct,el} + M_{ct,pl} + M_s = M. \end{cases} \tag{15}$$

Basically, the calculation of strains, are made analogically when compressive concrete zone operates elastically and plastically, but equation members F_c and M_c become more complex functions in this case.

4. The cracking moment when concrete operates elastically in the compressive zone

The calculation of cracking moment is simpler because of known maximum tensile strain value $\varepsilon_{ct} = \varepsilon_{ct,lim}$ before a crack opens. In this case, the value λ and the elastoplastic strain could be expressed in another form:

$$\lambda = 1 - 0.5\frac{y}{h-kx}; \tag{16}$$

$$\varepsilon_{ci} = \varepsilon_{ct,el} + \left(\varepsilon_{ct,lim} - \varepsilon_{ct,el}\right)\frac{y}{h-kx}. \tag{17}$$

So, the resultant values of the compression zone and elastic tensile zone are the same and the resultant of elastoplastic tensile zone after some changes according to Eqs. (16) and (17) will be:

$$F_{ct,pl} = bE_c\left(h-kx\right)\left[\frac{5\varepsilon_{ct,el}}{12} + \frac{\varepsilon_{ct,lim}}{3}\right]. \tag{18}$$

The resultant of tensile reinforcement is:

$$F_s = \frac{\varepsilon_{ct,lim}\left(d-x\right)}{h-x}E_s A_s. \tag{19}$$

The compression zone height can be expressed from the forces equilibrium equation:

$$Ax^2 + Bx + C = 0\,, \tag{20}$$

where $A = e_1 - 0.5\varepsilon_{ct,lim} + \dfrac{\varepsilon_{ct,el}}{\varepsilon_{ct,lim}}\left(0.5\varepsilon_{ct,el} - e_1\right)$;

$$B = \frac{\varepsilon_{ct,el}}{\varepsilon_{ct,lim}}\left(2e_1 h - \varepsilon_{ct,el} h\right) - 2e_1 h - \frac{\varepsilon_{ct,lim}\alpha_s A_s}{b}\,;$$

$$C = \frac{\varepsilon_{ct,el}}{\varepsilon_{ct,lim}}\left(0.5\varepsilon_{ct,el} h^2 - e_1 h^2\right) + \frac{\varepsilon_{ct,lim}\alpha_s A_s d}{b} + e_1 h^2\,;$$

$$e_1 = \left(\frac{5\varepsilon_{ct,el}}{12} + \frac{\varepsilon_{ct,lim}}{3}\right).$$

When the compression zone height is known, the cracking moment is calculated as follows:
:

$$M_{crc} = bE_c\left(h - kx\right)\left[\frac{3\varepsilon_{ct,el}x_{el}}{4} + \frac{\left(\varepsilon_{ct,lim} - \varepsilon_{ct,el}\right)x_{el}}{3} + \frac{\varepsilon_{ct,el}\left(h - kx\right)}{3} + \frac{5\left(\varepsilon_{ct,lim} - \varepsilon_{ct,el}\right)\left(h - kx\right)}{24}\right] +$$
$$+ \frac{1}{3}bE_c\varepsilon_c x^2 + \frac{1}{3}bE_c\frac{\varepsilon_{ct,el}^3}{\varepsilon_c^2}x^2 + \frac{\varepsilon_c\left(d - x\right)^2}{x}E_s A_s\,. \tag{21}$$

Section dimensions (*b*, *h*, *d*), the amount of reinforcement A_s, the properties of concrete ($\varepsilon_{ct,el}, \varepsilon_{ct,lim}, E_c$) and reinforcement ($E_s$) have the main influence on the cracking moment.

5. Layer calculation method of a flexural element with rectangular section

The solution of equations system (15) is complex because the equations in both first and second system expressions are not linear. Therefore, it is more convenient to find unknown values *x* and ε_c using iterative procedure and dividing a member into separate equal layers (Fig. 4).

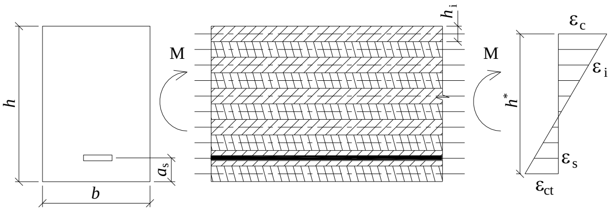

Fig. 4 The division of flexural element into layers

In this case, when making equations of equilibrium it is more convenient to express the strains of layers according to the scheme of Fig. 5 because when the location of neutral axis is unknown it is difficult to find how many layers are included into compress and tensile zones.

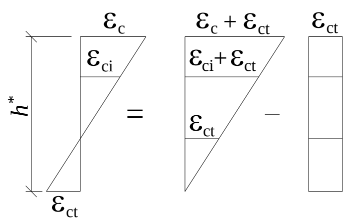

Fig. 5 The description scheme for the strain of *i* layer

According to the mentioned scheme, the strain of *i* layer can be found from the following equation:

$$\frac{\varepsilon_c + \varepsilon_{ct}}{h^*} = \frac{\varepsilon_{ci} + \varepsilon_{ct}}{h_i\left(i - 1\right)} \rightarrow \varepsilon_{ci} = \frac{\left(\varepsilon_c + \varepsilon_{ct}\right)\left(h_i\left(i - 1\right)\right)}{h^*} - \varepsilon_{ct}\,, \tag{22}$$

where h_i is the height of *i* layer, *i* layer number.

Having expressed the strain of outer layer of tensile zone $\varepsilon_{ct} = \dfrac{\varepsilon_c\left(h - x\right)}{x}$, we obtain:

$$\varepsilon_{ci} = \varepsilon_c\frac{h_i\left(i - 1\right)}{h^*} - \varepsilon_c\frac{h^* - x}{x}\left(1 - \frac{h_i\left(i - 1\right)}{h^*}\right). \tag{23}$$

Then, the height of compressive zone can be expressed from the equation of equilibrium of force projections:

$$\sum_{i=1}^{n} E_{ci}A_{ci}\varepsilon_{ci} + E_s A_s\varepsilon_s = 0\,, \tag{24}$$

where $E_{ci} = \lambda_{ci}E_c$, $A_{ci} = bh_i$.

Having evaluated expression (23) and the fact that $\varepsilon_s = \varepsilon_c\dfrac{a_s}{h^*} - \varepsilon_c\dfrac{h^* - x}{x}\left(1 - \dfrac{a_s}{h^*}\right)$, we can get the proportion $H = \dfrac{h^* - x}{x}$ from expression (24):

$$H = \frac{\displaystyle\sum_{i=1}^{n} E_{ci}A_{ci}\frac{h_i\left(i - 1\right)}{h^*} + E_s A_s\frac{a_s}{h^*}}{\displaystyle\sum_{i=1}^{n} E_{ci}A_{ci} - \sum_{i=1}^{n} E_{ci}A_{ci}\frac{h_i\left(i - 1\right)}{h^*} + E_s A_s - E_s A_s\frac{a_s}{h^*}}\,, \tag{25}$$

where a_s is distance from the edge of tensile concrete to the weight centre of tensile reinforcement.

Having obtained value *H* from the equation of equilibrium of bending moments, we can calculate the strain of compressive zone edge in the following way:

$$\varepsilon_c\left[\sum_{i=1}^{n} E_{ci}A_{ci}\frac{\left(h_i\left(i - 1\right)\right)^2}{h^*} - \sum_{i=1}^{n} E_{ci}A_{ci}Hh_i\left(i - 1\right) + \sum_{i=1}^{n} E_{ci}A_{ci}\frac{\left(h_i\left(i - 1\right)\right)^2}{h^*} + E_s A_s\frac{a_s^2}{h^*} - E_s A_s H a_s + E_s A_s H\frac{a_s^2}{h^*}\right] = M\,; \tag{26}$$

$$\varepsilon_c = \frac{M}{\sum_{i=1}^{n} E_{ci} A_{ci} \frac{\left(h_i(i-1)\right)^2}{h^*}(1+H) - \sum_{i=1}^{n} E_{ci} A_{ci} H h_i (i-1) + E_s A_s \frac{a_s^2}{h^*}(1+H) - E_s A_s H a_s} . \tag{27}$$

In this case, it is not necessary to solve equation system but the solution is repeated in order to make layer Eci more precise.

This iterative solution can also be written in a matrix form:

$$\boldsymbol{F} = \boldsymbol{E}\varepsilon . \tag{28}$$

If we divide element into 6 equal layers and mark the bottom tensile layer with number “1”, and the reinforcement weight centre is between the axes of layers “1”and “2”, then matrix $\boldsymbol{E}$ is equal to:

$$\boldsymbol{E} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ E_{c1}A_{c1} & E_{c2}A_{c2} & E_{c3}A_{c3} & E_{c4}A_{c4} & E_{c5}A_{c5} & E_{c6}A_{c6} & E_s A_S \\ 0 & E_{c2}A_{c2}h_i & 2E_{c3}A_{c3}h_i & 3E_{c4}A_{c4}h_i & 4E_{c5}A_{c5}h_i & 5E_{c6}A_{c6}h_i & E_s A_S a_s \\ (h_i - a_s) & a_s & 0 & 0 & 0 & 0 & -h_i \end{bmatrix} . \tag{29}$$

Displacement vector $\boldsymbol{\varepsilon} = \{\varepsilon_{c1} \quad \varepsilon_{c2} \quad \varepsilon_{c3} \quad \varepsilon_{c4} \quad \varepsilon_{c5} \quad \varepsilon_{c6} \quad \varepsilon_s\}^T$ and force vector is calculated - $\boldsymbol{F} = \{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad M \quad 0\}^T$.

When element is divided into quite small parts, the weight centre of tensile reinforcement will probably coincide with the axis of one of the layers. Let us suppose that it will coincide with the axis of layer “2”. Then, member $\boldsymbol{E}(5, 2)$ of matrix $\boldsymbol{E}$ will be equal to $(E_{c2}A_{c2} + E_s A_s)$ and member $\boldsymbol{E}(6, 2)$ – to $(E_{c2}A_{c2} + E_s A_s)h_i$. In this case, members of the final line and final column as well as vector members $\boldsymbol{\varepsilon}(7)$ and $\boldsymbol{F}(7)$ will be equal to 0.

It is convenient to solve the discussed element divided into layers using the (FEM). In this instance, each layer will correspond to a bar element and stiffness of that layer (Fig. 6). In order to obtain a flat section strain, the external bending moment is added to the element with high stiffness which has the following characteristics $E_{st} I_{st}$. Also the calculation can be made with reinforcement in the compression zone. In this case, the solution is the same as in the case of tensile reinforcement.

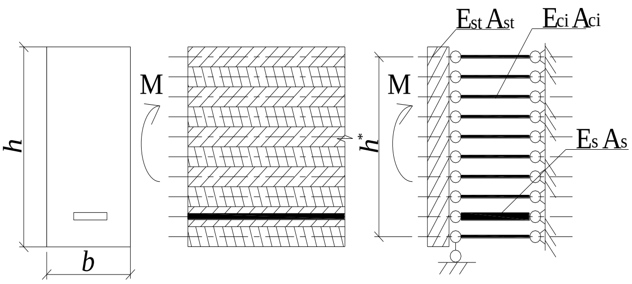

Fig. 6 Calculation model of the finite element method

Having calculated a particular element of rectangular section (Eq. 15), the results of expression members are obtained similar according to expressions (3-4), (9-10) and (11-14), the layer method and FEM. The element calculation results (b = 0.2 m, h = 0.21 m, d = 0.185 m, f_{ck} = 30 MPa, f_{ct} = 2 MPa, A_s = 5 cm^2, E_c = 30 GPa , E_s = 200 GPa, $\varepsilon_{ct,el}$ = 2.667e-5, $\varepsilon_{ct,lim}$ = 1.333e-4), when 3 kNm bending moment is applied, are presented in Table.

The stress and strain relationship of the examined element when bending moment is equal to 3 kNm is presented in Fig. 7.

Table

The comparison of calculation results of presented example according to different methods

	F_c, kN	$F_{ct,el}$, kN	$F_{ct,pl}$, kN	F_s, kN	M_c, kNm	$M_{ct,el}$, kNm	$M_{ct,pl}$, kNm	M_s, kNm
By (3-4), (9-10) and (11-14) expressions	21.308	3.589	13.222	4.500	1.553	0.107	0.999	0.341
By “Layer method” and “FEM” (21 layers)	21.350	2.957	13.885	4.51	1.553	0.079	1.027	0.341
By “Layer method” and “FEM” (189 layers)	21.309	3.633	13.176	4.500	1.553	0.109	0.997	0.341

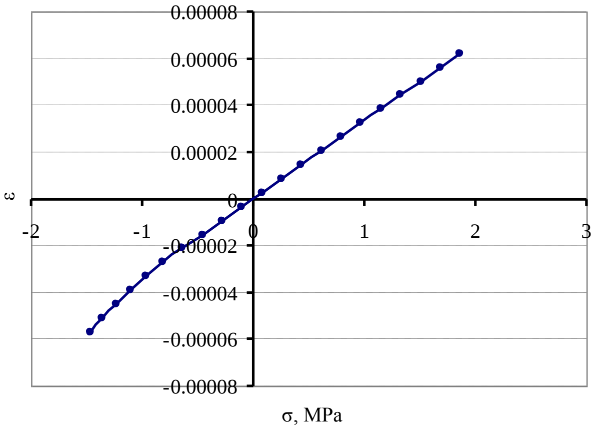

Fig. 7 The stress-strain relationship ("–" means tensile) of the examined element when bending moment is equal to 3 kNm

6. The comparison of the results of different methods

Comparing the proposed model with other methods laid down in STR, EC2, ACI [14] and the method presented in [12], different height of an element with the constant width 0.2 m was chosen. The differences of results could be seen in Fig 8. The comparison of the proposed method with EC2 and ACI codes is complicated because the influence of reinforcement is ignored. The cracking moment depends only on section dimensions in these codes.

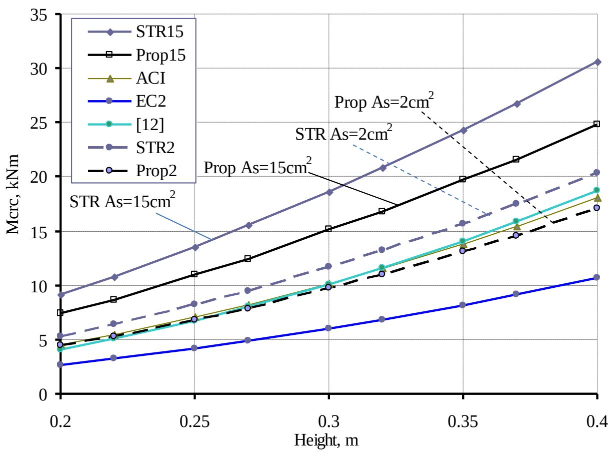

Fig. 8 The dependence of cracking moment on the height of element when the width is 0.2 m.

The relationship between the cracking moment and reinforcement amount was calculated according to STR and the proposed model (Fig. 9).

To find out the influence of such parameters as the element dimensions and the reinforcement amount on the cracking moment, the rectangular section with different dimensions and reinforcement amount was calculated. The width varies from 0.1 m to 0.3 m, height – 0.2 m – 0.45 m and the cross-section of reinforcement – 2.5 cm^2 – 20 cm^2. In order to eliminate the influence of width, values M_{crc}/b and I_{eff}/b were compared. The comparison of the average values of cracking moment calculated by expression (21) and STR is shown in Fig. 10, where differences vary from 5 to 15%. It can be seen from this figure that the influence of a section dimensions on the character of cracking moment variation is quite similar. The area of reinforcement to the inertia moment is not estimated in ACI and EC2 codes. Thus, the comparison with STR and the proposed method is not quite precise.

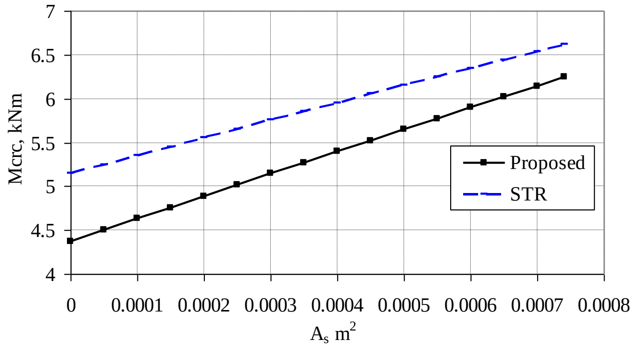

Fig. 9 The dependence of cracking moment on the cross-section area of tensile reinforcement

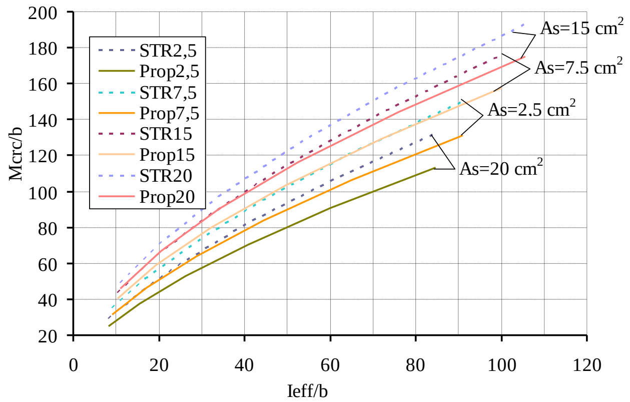

Fig. 10 The relationship of cracking moment and section dimensions by STR and the proposed model

For model testing, the cracking moment of element (b = 0.2 m, h = 0.5 m, d = 0.46 m, A_s = 14.7 cm^2, C25/30, S400) was calculated and it was equal to 44.195 kNm. The cracking moment calculated according to STR is equal to 54.4 kNm, EC2 – 21.4 kNm; and ACI - 28.3 kNm. This quite significant difference of proposed method result compared with EC2 and ACI occurs because of the amount of tensile reinforcement.

7. Conclusions

It is convenient to calculate the cracking of flexural elements by applying the coefficients of elastoplastic strain of tensile and compression concrete. Moreover, using the proposed calculation model, it is not difficult to evaluate the coefficients of elastoplastic strain that are different for tensile and compression concrete, i. e. when $\lambda_{ct} \neq \lambda_c$. In this case, concrete strengths that have essential effect on the limit strains when describing the cracking moments are also evaluated.

Since the stress sheet of tensile zone is not rectangular, the obtained values of cracking moments are slightly lower than those of classic calculation methods. The solutions of chosen iterative method of layers are quite precise and simple, therefore it is possible to avoid solving of more difficult integral equations. The FEM can also be used for the solution.

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TAMPRIAI PLASTINIŲ DEFORMACIJŲ ĮTAKA GELŽBETONINIŲ STAČIAKAMPIO SKERSPJŪVIO ELEMENTŲ PLEIŠĖJIMO MOMENTUI IR DEFORMACIJOMS

R e z i u m ė

Darbe atliktas lenkiamų gelžbetoninių elementų pleišėjimo momento skaičiavimas atsižvelgiant į betono tampriai plastinį būvį. Betono deformacijų modulis plyšio atsivėrimo metu apskaičiuotas remiantis tampriai plastinių deformacijų koeficientu. Atliekant skaičiavimus buvo laikoma, kad betono tampriai plastinių deformacijų koeficientas įtempiams artėjant prie maksimalių tiek tempiamai, tiek gniuždomai zonai yra vienodas. Naudojantis pateiktu modeliu galima apskaičiuoti lenkimo momento, neviršijančio pleišėjimo momento, sukeltas nesupleišėjusio elemento skerspjūvio įtempių ir deformacijų reikšmes. Šiuo atveju laikoma, kad įtempiams viršijus tamprumo ribą tampriai plastinių deformacijų koeficientas iki ribinio, kai pasiekiami maksimalūs įtempiai, mažės tolygiai. Sudarytų integralinių lygčių sprendimui supaprastinti galima taikyti sluoksnių ar baigtinių elementų metodus. Siūlomas metodas patogus tuo, kad juo naudojantis nesunku nustatyti galimus betono tampriai plastinių koeficientų pokyčius.

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THE ELASTOPLASTIC CONCRETE STRAIN INFLUENCE ON THE CRACKING MOMENT AND DEFORMATION OF RECTANGULAR REINFORCED CONCRETE ELEMENTS

S u m m a r y

The calculation of cracking moment of flexural reinforced concrete elements is made in the article by evaluating the elastoplastic state of concrete. At the time of crack opening, the concrete strain modulus is calculated on the ground of the coefficient of elastoplastic strains. It is accepted in the calculations that, when stresses reach maximal values, the coefficient of concrete elastoplastic strains is the same for both tensile and compressive zone. The presented model can be used to calculate the stress and strain values of uncracked element section with regard to bending moment not exceeding cracking moment. In this case, it is taken that when stresses exceed elasticity limit the coefficient of elastoplastic strain will decrease gradually to the limit value if maximal stresses are reached. In order to simplify the solution of formed integral equations, the method of layers and the finite element method are proposed for solving. The proposed method is convenient because it can easily be used to evaluate the potential changes of concrete elastoplastic coefficients.

Keywords: reinforced concrete, layer modeling, stresses, elastoplastic strain, cracking moment.

Received December 13, 2011
Accepted January 16, 2013