865. Investigation of vibrations of a two phase material

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Abstract. The model for investigation of vibrations of an elastic structure filled with fluid is proposed. This model is applicable for the analysis of vibrations of soil filled with water and can also be applied for the analysis of vibrations of materials used in various transportation devices. A two-dimensional structure is analyzed and four nodal variables are assumed: displacements of the elastic structure in the directions of the axes of coordinates and displacements of the fluid in the directions of the axes of coordinates. The interaction of the elastic structure and the fluid is assumed through the volumetric strains. This enables to obtain the stiffness and mass matrixes directly applicable for calculation of the eigenmodes of the system. The eigenmode is represented by two figures: the displacements of the elastic structure and the displacements of the fluid. The first eigenmodes of the rectangular structure are obtained and analyzed. An important problem for precise analysis of vibrations of such two-phase systems is the accurate determination of physical parameters used in the analysis. Special experimental procedures are to be designed and implemented for their determination. This is the subject of subsequent investigations and is to be presented in future papers.

Keywords: elastic structure, plane strain, fluid, compressibility, eigenmodes, vibrations, finite elements, penalty method.

Introduction

The model for investigation of vibrations of an elastic structure filled with fluid is proposed. This model is applicable for the analysis of vibrations of soil filled with water and can also be applied for the analysis of vibrations of materials used in various transportation devices.

A two-dimensional structure is analyzed and four nodal variables are assumed: displacements of the elastic structure in the directions of the axes of coordinates and displacements of the fluid in the directions of the axes of coordinates. The interaction of the elastic structure and the fluid is assumed through the volumetric strains. Certainly, in two-phase systems various interactions between the phases are taken into account. But here only this one type of interaction is assumed as it is the most important one for the problems of small vibrations. This enables to obtain the stiffness and mass matrixes directly applicable for calculation of the eigenmodes of the system.

The eigenmode is represented by two figures: the displacements of the elastic structure and the displacements of the fluid. The first eigenmodes of the rectangular structure are obtained and analyzed.

The model for the analysis of vibrations of the investigated two-phase structure is proposed on the basis of the material described in [1-5].

Model for the analysis of vibrations of a two-phase material

Further x and y denote the axes of the system of coordinates. The element has four nodal degrees of freedom: the displacement of the elastic structure in the direction of the x axis denoted as u, the displacement of the elastic structure in the direction of the y axis denoted as v, the displacement of the fluid in the direction of the x axis denoted as u_f, the displacement of the fluid in the direction of the y axis denoted as v_f.

The stresses of the elastic structure assumed as a plane strain problem are expressed as:

$$\{\sigma\} = [D]\{\varepsilon\} - \{\tilde{D}\}\varepsilon_v, \tag{1}$$

where $\{\varepsilon\}$ are the strains of the elastic structure, ε_v is the volumetric strain of the fluid and:

$$[D] = \begin{bmatrix} K+\frac{4}{3}G & K-\frac{2}{3}G & 0 \\ K-\frac{2}{3}G & K+\frac{4}{3}G & 0 \\ 0 & 0 & G \end{bmatrix}, \tag{2}$$

where $K = \frac{E}{3(1-2\nu)}$, $G = \frac{E}{2(1+\nu)}$, E is the modulus of elasticity, ν is the Poisson's ratio and:

$$\{\tilde{D}\} = \begin{Bmatrix} K_i \\ K_i \\ 0 \end{Bmatrix}, \tag{3}$$

where K_i is the bulk modulus of inter-phase interaction.
The pressure in the fluid is expressed as:

$$p = -\rho_f c^2 \varepsilon_v + \{\tilde{D}\}^T \{\varepsilon\}, \tag{4}$$

where ρ_f is the density of the fluid and c is the speed of sound in the fluid.
The mass matrix has the form:

$$[M] = \int \left([N]^T \rho [N] + [\bar{N}]^T \rho_f [\bar{N}] \right) dxdy, \tag{5}$$

where ρ is the density of the elastic structure and:

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & 0 & \dots \\ 0 & N_1 & 0 & 0 & \dots \end{bmatrix}, \tag{6}$$

where N_1, … are the shape functions of the finite element and:

$$[\bar{N}] = \begin{bmatrix} 0 & 0 & N_1 & 0 & \dots \\ 0 & 0 & 0 & N_1 & \dots \end{bmatrix}. \tag{7}$$

The stiffness matrix has the form:

$$[K] = \int \begin{pmatrix} [B]^T [D][B] + [\bar{B}]^T \rho_f c^2 [\bar{B}] + [\tilde{B}]^T \lambda [\tilde{B}] - \\ -[B]^T \{\tilde{D}\}[\bar{B}] - [\bar{B}]^T \{\tilde{D}\}^T [B] \end{pmatrix} dxdy, \tag{8}$$

where λ is the penalty parameter for the condition of non-rotational motion of the fluid and:

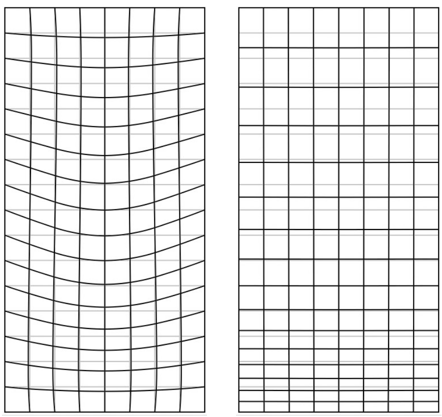
$$\left[B\right] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & \ldots \\ 0 & \frac{\partial N_1}{\partial y} & 0 & 0 & \ldots \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & 0 & \ldots \end{bmatrix}, \tag{9}$$

$$\left[\bar{B}\right] = \begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \ldots \end{bmatrix}, \tag{10}$$

$$\left[\tilde{B}\right] = \begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial y} & -\frac{\partial N_1}{\partial x} & \ldots \end{bmatrix}. \tag{11}$$

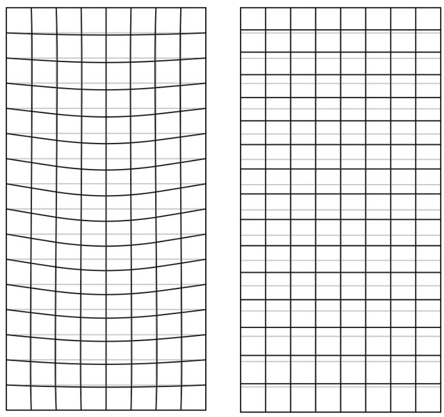
Results of analysis of vibrations of a two-phase material

Length of the structure is equal to 2 m and height of the structure is equal to 4 m. On the right and left boundaries of the structure it is assumed that $u = v = u_f = 0.$ On the lower and upper boundaries of the structure it is assumed that $u = v = v_f = 0.$ The following parameters of the structure are assumed: modulus of elasticity $E = 6\cdot 10^8$ Pa, Poisson's ratio $\nu = 0.3$, density of the material of the elastic structure ρ = 785 kg/m^3, speed of sound in the fluid c = 1524 m/s, density of the fluid ρ_f = 998 kg/m^3, penalty parameter λ = 10^{12} N/m^2, bulk modulus of inter-phase interaction $K_i = 10^6$ N/m^2. The first eigenmodes are presented in Fig. 1, …, Fig. 4.



a) b)

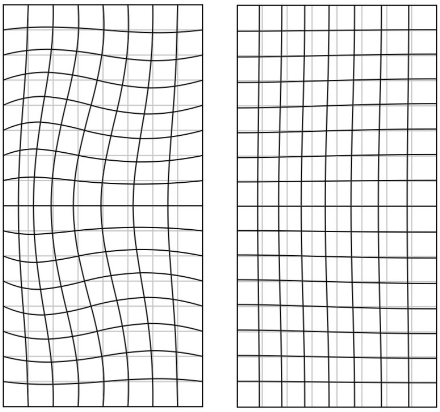
Fig. 1. The first eigenmode: a) elastic structure, b) fluid



a) b)

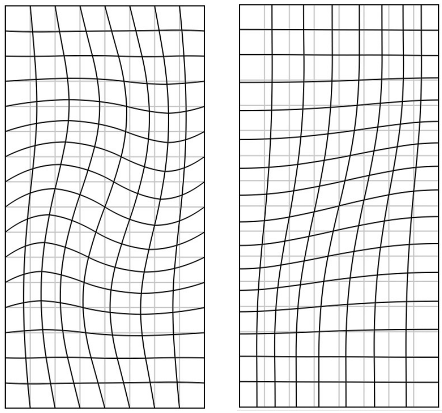
Fig. 2. The second eigenmode: a) elastic structure, b) fluid

One is to have in mind that the scales of representation of displacements of the elastic structure and of the fluid are not the same. For the first eigenmode the displacements of the fluid are multiplied by 1000 with respect to the displacements of the elastic structure. For the second eigenmode the displacements of the elastic structure are multiplied by 100 with respect to the displacements of the fluid. For the third eigenmode the displacements of the fluid are multiplied by 1000 with respect to the displacements of the elastic structure. For the fourth eigenmode the displacements of the fluid are multiplied by 4000 with respect to the displacements of the elastic structure.



a) b)

Fig. 3. The third eigenmode:
a) elastic structure, b) fluid



a) b)

Fig. 4. The fourth eigenmode:
a) elastic structure, b) fluid

Thus in the first, third and fourth eigenmodes the amplitudes of vibration of the fluid are small, while in the second eigenmode the amplitudes of vibration of the elastic structure are small.

Conclusions

The model for investigation of vibrations of an elastic structure filled with fluid is proposed. The two-dimensional structure is analyzed and four nodal variables are assumed: displacements of the elastic structure in the directions of the axes of coordinates and displacements of the fluid in the directions of the axes of coordinates. The eigenmode is represented by two figures: the displacements of the elastic structure and the displacements of the fluid. The first eigenmodes of the rectangular structure are obtained and analyzed.

One is to have in mind that the scales of representation of displacements of the elastic structure and of the fluid are not the same. Thus, in the first, third and fourth eigenmodes the amplitudes of vibration of the fluid are small, while in the second eigenmode the amplitudes of vibration of the elastic structure are small.

An important problem for precise analysis of vibrations of such two phase systems is the accurate determination of physical parameters used in the analysis. Special experimental procedures are to be designed and implemented for their determination. This is the subject of other investigations and is to be presented in other papers.

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