

## **An Electrostatic Approach to the Numerical Simulation of Combined Corona and Electrostatic Field in Three-Electrode System**

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### **Introduction**

Two-electrode systems consisting of ionizing electrode, such as wire, needle or the other one with small radius of surface curvature, and a non-ionizing electrode are often used in many applications of corona field. Multi-electrode systems are of more complicated structure and more complex methods are needed for their analysis.

Numerical analysis of combined corona-electrostatic electric fields in wire-to-plane electrode system supplemented by a cylindrical non-ionizing electrode is presented in [1]. This third electrode is associated with the wire electrode in the dual corona-electrostatic arrangement. Electric field changes the sign in a point situated between the wire and cylindrical electrode. Electric field lines originating from the wire electrode will concentrate on a smaller region at the surface of the plane electrode, as compared with the case of a single wire element. Mathematical model of the field is similar to those commonly employed for the study of plain corona fields in wire-to-plane electrode systems.

Numerical investigation of the electric field in the domain with no free charges using the Multi Quadratic and Differential Quadrature methodology as a new meshless method is developed in [2]. Poisson equation includes a constant source term which does not affect the solution and therefore this technique can be used to solve Laplace and Poisson equations in the fields of complex geometries. But it isn't suitable to solve the corona field system of equations comprising of Poisson and non-linear charge conservation equation.

Results of the analysis of electric field in a cylindrical triode corona charger used for charging aerosol nanoparticles are presented in [3]. Cylindrical corona charger consists of corona wire, perforated inner cylinder and grounded outer cylinder with coaxial position of all three electrodes. The inner zone of corona discharge and the outer particle charging zone are related with space

charge distribution and there is no zone without space charges. The field is analyzed as unipolar corona field in all the computational area.

Potential distribution within a pipe through which charged gas is transported is analyzed in [4] by using Bessel functions. Boundary conditions at the surface between space charge and no charge regions are the equality of potentials and their derivatives normal to the surface. Uniform distribution of space charge is assumed in this analysis. This distribution is far from that in corona field, but the mentioned boundary conditions may be used for analysis of combined corona- and electrostatic fields.

Electric field of corona triode [5] is analyzed as the one comprising of two components. The first one, generated by the space charge, is governed by the Poisson equation with uniform Dirichlet boundary conditions and is solved using the finite element method. The second one, produced by the potentials of conductors, is described by the Laplace equation with non-uniform Dirichlet boundary conditions. It is solved by boundary element method. Computation of charge distribution on characteristic lines is performed by using the method of characteristics. There are no regions without the space charge and the method is the same as the one used for analysis of two-electrode corona field.

Electrostatic field of roll-type separator consisting of grounded drum, corona wire and a tubular non-ionizing high-voltage electrode [6] is analyzed by the charge simulation program based on boundary-element method. The field under study can be divided into region without space charges and the one with distributed space charges. Boundary conditions between these regions are of specific interest, but they are not determined there.

Computations and experiments are carried out in the [7, 8] for a two-dimensional three-electrode system consisting of wire parallel to plane and a non-ionizing cylinder placed in opposite side of the wire. This configuration is characterized by the existence of singular point on the axis of symmetry where the electric field takes zero value, and

this is a major difficulty for which none of the existing algorithms is able to provide a solution. To overcome this difficulty initial computational region  $D(z)$ ,  $z = x+jy$ , is transposed to a domain  $D(\gamma)$ ,  $\gamma = \xi+j\eta$ , with simply defined boundary conditions by using a conformal transform

$$\gamma = \ln[(z+1)/(z-1)]. \quad (1)$$

This enables to use finite difference method with a simple quadratic grid and to define numerically regions with space charge distribution and with no space charge.

We develop there a concept of existence of the singular point with zero field strength on the axis of symmetry and of separatrix related to this point and dividing the field into region having no space charge and a region with space charge.

### Problem formulation

We analyze the electrostatic field in three-electrode system consisting of two parallel non-ionizing plane electrodes and a corona wire between them (Fig. 1).

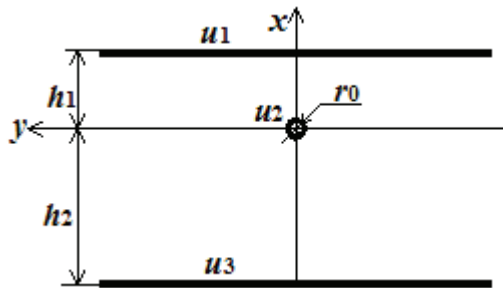


Fig. 1. Configuration of the electrode system under study

Complex potential of the field

$$w = u + jv = f(z) = f(x + jy) \quad (2)$$

is defined as the superposition of the field in the electrode system “a wire between two parallel grounded plates” and the homogeneous electrostatic field [9]. Potential function is the real part of the complex potential (2)

$$u(x, y) = k_\tau \cdot \ln S_0 + u_1 + E_0 \cdot (h_1 - x) \quad (3)$$

and the field-line function is the imaginary part of the function (2)

$$v(x, y) = k_\tau \cdot \varphi - E_0 \cdot y, \quad (4)$$

where

$$E_0 = (u_3 - u_1)/(h_1 + h_2), \quad (5)$$

$$k_\tau = \frac{u_1 + E_0 \cdot h_1}{k_{\text{geom}}}, \quad (6)$$

$$k_{\text{geom}} = 0,5 \cdot \ln \frac{\sin^2(k_h h_1) \cdot \sin^2[k_h(r_0 + 2h_2)]}{\sin^2(k_h r_0) \cdot \sin^2[k_h(h_1 + 2h_2)]}, \quad (7)$$

$$k_h = \frac{\pi}{2(h_1 + h_2)}, \quad (8)$$

$$S_0 = \sqrt{\frac{\sin^2(k_h h_1) + \text{sh}^2(k_h y)}{\sin^2[k_h(x + 2h_2)] + \text{sh}^2(k_h y)}}, \quad (9)$$

$$\varphi = \arctan[\cot(k_h x) \cdot \tanh(k_h y)] - \arctan[\cot(k_h(x + 2h_2)) \cdot \tanh(k_h y)]. \quad (10)$$

Meanings of the symbols used in the equations (3)–(10) are the following:  $u_1$ ,  $u_2$ ,  $u_3$  are the electrode potentials,  $r_0$  is the radius of the wire,  $h_1$  and  $h_2$  are the distances from the center of the wire to the upper and lower planes, correspondingly.

Equation of the separatrix can be derived from the field-line function (4) equality to zero

$$k_\tau \cdot \varphi - E_0 \cdot y = 0. \quad (11)$$

### Analysis of separatrix boundary condition

Results of analysis of the field separatrix correspond to the following values of field parameters:  $r_0 = 0,025$  mm,  $h_1 = 10$  mm,  $h_2 = 40$  mm,  $u_1 = -10000$  V,  $u_2 = 0$ ,  $u_3 = -5000$  V. Coordinates of separatrix points calculated by using (6)–(11) formulas are given in Table 1 for the step  $\Delta x = 1$  mm.

Table 1. Coordinates of points of separatrix

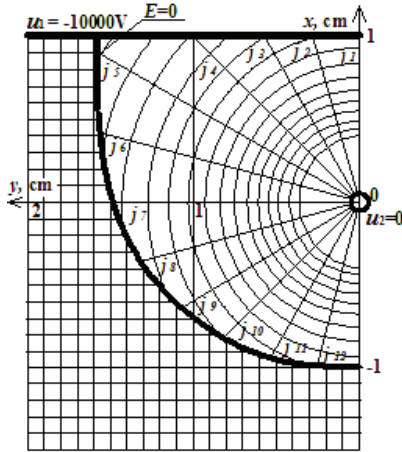
x, mm	y, mm	x, mm	y, mm
10,0	16,4	-10,0	0
9,0	16,4	-9,0	3,2
8,0	16,3	-8,0	7,8
7,0	16,3	-7,0	9,5
6,0	16,2	-6,0	10,0
5,0	16,0	-5,0	11,1
4,0	15,7	-4,0	12,6
3,0	15,4	-3,0	13,2
2,0	15,1	-2,0	14,1
1,0	14,9	-1,0	14,8
0	14,8	0	14,8

The part of the field area under study is presented in Fig. 2. Lower plain electrode of potential  $u_3$  isn't shown here. There are two regions of the shown field area: the region on the left of separatrix, comprising the flux of field lines between electrodes 1 and 3 with no space charge, and the region on the right of separatrix with field lines directed from the wire to the upper plane electrode and with the distributed space charge. The first region corresponds to Laplacian field, and the second one is occupied by corona field. The field in the first region is quasi-homogeneous, the grid in Cartesian coordinates can be used for solution of Laplace equation [10]. Potentials of the points given in Table 1 and calculated by using equations (3) and (6)–(9) are given in Table 2.

Table 2. Potentials of separatrix at the points of Laplacian grid

x, mm	u, V	x, mm	u, V
10,0	-10000	1,0	-7723
9,0	-9744	0	-7483
8,0	-9491	-1,0	-7355
7,0	-9238	-2,0	-7085
6,0	-8989	-3,0	-6799
5,0	-8735	-4,0	-6564
4,0	-8492	-5,0	-6239

$x, \text{ mm}$	$u, \text{ V}$	$x, \text{ mm}$	$u, \text{ V}$
3,0	-8299	-6,0	-5978
2,0	-8083	-7,0	-5848



**Fig. 2.** Areas of the field with and without space charges

Polar grid is used in [10] for computation of corona field in wire-to-plane electrode system similar to that located in the area to the left of separatrix in Fig. 1. Position of points on separatrix coinciding with points on lines of polar grid denoted from  $j = 5$  to  $j = 13$  can be determined from the system of equations:

$$\begin{cases} \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}, \\ y = x \cdot \tan \frac{(j-1)\pi}{m}, \end{cases} \quad (12)$$

where  $j$  is a number of polar grid line which one of points is situated on the separatrix,  $m$  is a polar grid size index,  $m = \pi / \alpha_0$ ,  $\alpha_0$  is the central angle between  $j_i$  and  $j_{i+1}$  lines. Index  $m = 12$  for the grid shown in Fig. 2,  $\alpha_0 = 15^\circ$ . Coordinates  $x_1, y_1$  and  $x_2, y_2$  determine the position of separatrix points neighbor to the point of polar grid line positioned on separatrix. For example, neighbor points are of  $x_1 = 10,0 \text{ mm}$ ,  $y_1 = -16,45 \text{ mm}$  and  $x_2 = 9,0 \text{ mm}$ ,  $y_2 = -16,41 \text{ mm}$  for the line  $j = 5$ .

Coordinates of points common to the separatrix and lines of polar grid corresponding to the values of  $j$  from 5 to 12 are given in Table 3. Potentials of that points determined by using the equation (3) and (6)–(9) are also given here.

**Table 3.** Coordinates and potentials of points common to the separatrix and lines of polar grid

$j$	$x, \text{ mm}$	$y, \text{ mm}$	$u(x,y), \text{ V}$
5	9,4	16,4	-9805
6	4,0	16,0	-8511
7	0	14,9	-7483
8	-3,5	13,0	-6675
9	-6,1	9,9	-6112
10	-7,8	7,5	-5623
11	-8,6	4,9	-5490
12	-9,2	2,3	-5115

Other boundary conditions of the field under study are given in [11].

Numerical analysis of an electrostatic field in the system under study is carried out by using the finite difference method in MATLAB environment. Laplace equation is solved as a test problem by using the Cartesian coordinate system with the grid size  $\Delta = 1 \text{ mm}$ . Values of potentials at the several points of separatrix are compared in the Table 4 with the potential values of numerical analysis at the neighbor nodes of Cartesian grid. All potential values given in Table 4 are negative. Numerical values of potentials corresponding to points of separatrix are determined by linear interpolation of potential values at neighbor nodes. Differences of compared values are equal to 0 at the ends of separatrix and are less than 1 % at the middle points of this line.

**Table 4.** Comparison of separatrix potentials with numerical values at the grid nodes

Potentials of separatrix			Potentials of neighbor grid nodes			Differ - ences, %
$x, \text{ mm}$	$y, \text{ mm}$	$ u , \text{ V}$	$x, \text{ mm}$	$y, \text{ mm}$	$ u , \text{ V}$	
			9,0	17,0	9754	
9,0	16,4	9744	9,0	16,0	9739	0
			8,0	17,0	9509	
8,0	16,4	9491	8,0	16,0	9481	0
			0	15,0	7561	
0	14,9	7483	0	14,0	7424	0,7
			-8,0	8,0	5596	
-8,0	7,8	5533	-8,0	7,0	5438	0,7
-10,0	0	4996	-10,0	0	4996	0

Discussed data are the result of analysis of the electrostatic field. Computation of corona field in the region with bounds formed by straight lines  $x = 10 \text{ mm}$ ,  $y = 0$  and separatrix can be performed under assumption that the shape of separatrix doesn't depend on the presence of the space charge in the region. This assumption corresponds to the conclusion of [8] that the space charge density is 0 in all points of separatrix. This is in agreement with the results of [1] where presence of the point with zero field strength and zero space charge density on the symmetry axis is stated. The mentioned statement is similar to Deutsch assumption successfully used for analysis of corona fields in various electrode systems [12], although some restrictions of this assumption were made in [13] and it was criticized in [14]. Our hypothesis of independence of separatrix position on the space charge density is not such rigorous as the Deutsch assumption because the component of electric field produced by space charge is much smaller than the field of electrodes [5].

## Conclusions

Numerical analysis of combined corona and electrostatic field can be performed for two related regions: a region without and with space charge. If the separatrix between the regions is known from the analysis of the electrostatic field boundary condition for potential

can be determined easily. The field in the region without space charge can be assumed as quasi-homogeneous and the finite-difference method in Cartesian coordinates can be used to solve the Laplace equation. The field in the region related to emitting electrode is strongly inhomogeneous with distributed space charge. Analysis of the field in this region can be performed by numerical solution of corona field equations in polar coordinates. Assumption of independence of the separatrix position on the space charge must be used. Analytical and numerical results of the separatrix parameters determination coincide well, maximum differences between the values of potentials doesn't exceed 0,7 %

## References

1. **Caron A., Dascalescu L.** Numerical modeling of combined corona–electrostatic fields // *Journal of Electrostatics*, 2004. – Vol. 61. – Iss. 1. – P. 43–55
2. **Jalaal M., Soleimani S., Domairry G., Ghasemi E., Bararnia H., Mohammadi F., Barari A.** Numerical simulation of electric field in complex geometries for different electrode arrangements using meshless local MQ–DQ method // *Journal of Electrostatics*, 2011. – Vol. 69. – Iss. 3. – P. 168–175.
3. **Intra P.** Corona discharge in a cylindrical triode charger for unipolar diffusion aerosol charging // *Journal of Electrostatics*, 2012. – Vol. 70. – Iss. 1. – P. 136–143.
4. **Asano K.** Electrostatic potential and field near the boundary between space charge and no charge regions within a cylindrical pipe // *Journal of Electrostatics*, 2010. – Vol. 68. – Iss. 2. – P. 132–137.
5. **Deng X., Adamiak K.** The electric corona discharge in the triode system // *IEEE Transactions on industry applications*, 1999. – Vol. 35. – No 4. – P. 767–773.
6. **Rafiroiu D., Morar R., Atten P., Dascalescu L.** Premises for the mathematical modeling of the combined corona–electrostatic field of roll–type separators // *IEEE Transactions on industry applications*, 2000. – Vol. 36. – No 5. – P. 1260–1266.
7. **Dumitran L. M., Atten P., Notinger P. V., Dascalescu L.** Computational and experimental study of ionic space charge generated by combined corona–electrostatic electrode systems // *IEEE Transactions on industry applications*, 2006. – Vol. 42. – No 2. – P. 378–384.
8. **Dumitran L. M., Dascalescu L., Atten P., Notinger P. V.** 2–D corona field computation in configurations with ionizing and non–ionizing electrodes // *Journal of Electrostatics*, 2006. – Vol. 64. – Iss. 3–4. – P. 176–186.
9. **Mikšta J., Žebrauskas S.** Analysis of the electric field in three–electrode system with corona discharge // *Materials science. – Wrocław*, 1990. – Vol. 16. – No. 1–3. – P. 287–292.
10. **Marčiulionis P., Žebrauskas S.** Numerical analysis of electric wind in corona field // *Electronics and electrical engineering. – Kaunas: Technologija*, 2011. – No. 10(116). – P. 15–18. DOI: 10.5755/j01.eee.116.10.871.
11. **Barčiauskas R., Žebrauskas S.** Boundary conditions for combined corona and electrostatic field analysis in multi–electrode systems // *Proceedings of the 6<sup>th</sup> International Conference on Electrical and Control Technologies ECT'2011. – Kaunas: Technologija*, 2010. – P. 264–267.
12. **Ianconescu R., Sohar D., Mudrik M.** An analysis of the Brown–Biefeld effect // *Journal of Electrostatics*, 2011. – Vol. 69. – Iss. 6. – P. 512–521.
13. **Amoruso V., Lattarulo F.** Deutsch hypothesis revisited // *Journal of Electrostatics*, 2005. – Vol. 63. – Iss. 6–10. – P. 717–721.
14. **Jones J. E., Davies M.** A critique of the Deutsch assumption // *Journal of Physics, D: Appl. Phys.*, 1992. – No 25(12). – P. 1749–1759.

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**R. Barčiauskas, J. A. Virbalis, S. Zebrauskas. An Electrostatic Approach to the Numerical Simulation of Combined Corona and Electrostatic Field in Three-Electrode System // *Electronics and Electrical Engineering. – Kaunas: Technologija*, 2012. – No. 6(122). – P. 105–108.**

If there are more than two electrodes in the corona field system the area occupied by the field can be divided into two regions: a region with electric flux between ionizing and non-ionizing electrodes and a region with electric flux between non-ionizing electrodes. The first region contains a distributed space charge and analysis of the field in this region must be performed by solving the system of corona field differential equations. The field in the second region is quasi-homogeneous, has no space charge and it is sufficient to solve the Laplace equation for field analysis. The subject of this paper is using of separatrix equation dividing the field area into two regions for numerical simulation of the field in the electrode system “a wire between two parallel plates”. If the equation of the separatrix is known it is easy to determine boundary conditions on this line. Results of this approach are checked by numerical solution of Laplace equation in all the field area. Ill. 2, bibl. 14, tabl. 4 (in English; abstracts in English and Lithuanian).

**R. Barčiauskas, J. A. Virbalis, S. Žebrauskas. Trijų elektrodų sistemos jungtinio elektrostatinio ir vainikinio išlydžio elektrinio lauko skaitinio modeliavimo elektrostatinis pagrindimas // *Elektronika ir elektrotechnika. – Kaunas: Technologija*, 2012. – Nr. 6(122). – P. 105–108.**

Jei vainikinio išlydžio elektrodų sistemą sudaro daugiau negu du elektrodai, visą lauko sritį galima padalyti į dvi skirtingų parametrų dalis. Pirmąją dalį sudaro elektrinis srautas tarp jonizuojančiųjų ir nejonizuojančiųjų elektrodų su paskirstytais erdviniais krūviais. Skaitmeniškai modeliuojant šią lauko dalį reikia spręsti vainikinio išlydžio elektrinio lauko diferencialinių lygčių sistemą. Antrąją lauko dalį sudaro elektrinis srautas tarp nejonizuojančiųjų elektrodų be erdvinų krūvių. Šis laukas yra kvazivienalytis ir jį analizuojant pakanka išspręsti Laplaso lygtį. Šiame straipsnyje elektrodų sistemos „laidas tarp dviejų lygiagrečių plokštumų“ skaitmeniniam lauko analizės modeliui sudaryti naudojama separatrix tarp dviejų minėtų lauko dalių lygtis. Žinant separatrix lygtį, nesunku apibrėžti šios ribos kraštinę sąlygą. Šios sąlygos nustatymo elektrostatinio metodo rezultatai palyginti su skaitiniu Laplace'o lygties sprendiniu visai nagrinėjamo lauko sričiai. Il. 2, bibl. 14, lent. 4 (anglų kalba; santraukos anglų ir lietuvių k.).