800. Vibrations, stability and compression of elements of packages made from corrugated board

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Abstract. The paper considers vibrations, stability and compression of a package element made from corrugated board. The analysis is based on a model of a structure consisting from two-dimensional beams taking into account the orthotropic properties of the corrugated board. The problem of initial stability was solved and the first eigenmodes of stability were obtained. Performed experimental study allowed to determine the dependence of the vertical compression force on the geometrical parameters of the multilayered board before the development of plastic deformations in the board. The results of investigation are used for the design of package elements.

Keywords: element of package, corrugated board, beam, compression, stability, vibrations, eigenmodes, finite elements, experimental investigations.

Introduction

Packages made from multilayered board are used for transportation and storage of various products as well as for their protection. Thus one of the basic requirements for the packages is strength. Main mechanical properties of materials are strength and resistance to external effects. They characterize the ability of materials to resist various loads and not to degrade.

A number of scientific investigations of multilayered board have been performed with the purpose of choosing the optimal structure of the used material, the thickness of the internal layer, wave type and other parameters [1-4].

In the research papers [5-9] the resistance to compression in various directions of the chosen type of multilayered board by the finite element method (FEM) was analyzed and the optimal shapes of boxes of multilayered board were modeled.

Vibrations and stability of a package element made from corrugated board were analyzed in this paper. The model of a structure consisting from two dimensional beams was used. Orthotropic qualities of the corrugated board were taken into account and the problem of initial stability was solved. As a result, the first eigenmodes of stability were obtained. The model for the analysis of stability of corrugated board is proposed on the basis of the material described in [10-12].

The obtained results are used in the process of design of package elements.

Model for the analysis of stability of corrugated board

Further x, y and z denote the axes of the system of coordinates. The element of a two dimensional beam in the plane xOz has three nodal degrees of freedom: the displacement in the direction of the x axis denoted as u, the displacement in the direction of the z axis denoted as w and the rotation about the y axis denoted as Θ_y.

The values of u, w, Θ_y in the element are represented as:

$$\begin{Bmatrix} u \\ w \\ \theta_y \end{Bmatrix} = [N]\{\delta\}, \tag{1}$$

where $\{\delta\}$ is the vector of generalized nodal displacements and:

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_1 & 0 & \dots \\ 0 & 0 & N_1 & \dots \end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the one-dimensional finite element.

Further the displacements in the direction of the longitudinal axis of the beam s and in the direction of the axis perpendicular to s and located in the plane xOz are denoted as $\bar{u}$ and $\bar{w}$. Thus:

$$\begin{Bmatrix} \bar{u} \\ \bar{w} \\ \theta_y \end{Bmatrix} = [T]\begin{Bmatrix} u \\ w \\ \theta_y \end{Bmatrix}, \tag{3}$$

where:

$$[T] = \begin{bmatrix} \dfrac{dx}{d\xi} \Big/ \dfrac{ds}{d\xi} & \dfrac{dz}{d\xi} \Big/ \dfrac{ds}{d\xi} & 0 \\ -\dfrac{dz}{d\xi} \Big/ \dfrac{ds}{d\xi} & \dfrac{dx}{d\xi} \Big/ \dfrac{ds}{d\xi} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

where ξ is the local coordinate of the finite element and:

$$\frac{ds}{d\xi} = \sqrt{\left(\frac{dx}{d\xi}\right)^2 + \left(\frac{dz}{d\xi}\right)^2}. \tag{5}$$

The following notation is introduced:

$$[\bar{N}] = \begin{bmatrix} [\bar{N}_1] \\ [\bar{N}_2] \\ [\bar{N}_3] \end{bmatrix} = [T][N]. \tag{6}$$

The derivatives of u, w, Θ_y are represented as:

$$\begin{Bmatrix} \dfrac{du}{ds} \\ \dfrac{dw}{ds} \\ \dfrac{d\theta_y}{ds} \end{Bmatrix} = [N']\{\delta\}, \tag{7}$$

where:

$$[N'] = \begin{bmatrix} \frac{dN_1}{ds} & 0 & 0 & \dots \\ 0 & \frac{dN_1}{ds} & 0 & \dots \\ 0 & 0 & \frac{dN_1}{ds} & \dots \end{bmatrix}. \tag{8}$$

It is assumed that:

$$\begin{Bmatrix} \frac{d\bar{u}}{ds} \\ \frac{d\bar{w}}{ds} \\ \frac{d\theta_y}{ds} \end{Bmatrix} = [T] \begin{Bmatrix} \frac{du}{ds} \\ \frac{dw}{ds} \\ \frac{d\theta_y}{ds} \end{Bmatrix}. \tag{9}$$

The following notation is introduced:

$$\left[\bar{N}'\right] = \begin{bmatrix} \left[\bar{N}_1'\right] \\ \left[\bar{N}_2'\right] \\ \left[\bar{N}_3'\right] \end{bmatrix} = [T][N']. \tag{10}$$

The stiffness matrix of the model of orthotropic paper as a beam has the form:

$$[K] = \int \begin{pmatrix} \left[\bar{N}_1'\right]^T \left[\frac{E_s}{1-\nu_{sy}\nu_{ys}} h\right] \left[\bar{N}_1'\right] + \left[\bar{N}_3'\right]^T \left[\frac{E_s}{1-\nu_{sy}\nu_{ys}} \frac{h^3}{12}\right] \left[\bar{N}_3'\right] + \\ + [B]^T \left[\frac{E_s E_y h}{\left(E_s + E_y + E_s\nu_{ys} + E_y\nu_{sy}\right)1.2}\right][B] \end{pmatrix} ds, \tag{11}$$

where E_s and E_y are the modulus of elasticity of the beam, ν_{sy} and ν_{ys} are the Poisson's ratios of the beam, h is the thickness of the beam and:

$$[B] = \left[\left[\bar{N}_2'\right] + \left[\bar{N}_3\right]\right], \tag{12}$$

$$E_s = E_y \frac{\nu_{sy}}{\nu_{ys}}. \tag{13}$$

The matrix of supplementary stiffness of the model of orthotropic corrugated board as a beam has the form:

$$[K_\sigma] = \int \left[\bar{N}_2'\right]^T M_\sigma \left[\bar{N}_2'\right] ds, \tag{14}$$

where:

$$M_\sigma = \left[\frac{E_s}{1-\nu_{sy}\nu_{ys}} h\right] \left[\bar{N}_1'\right] \{\delta\}. \tag{15}$$

Analysis of stability of corrugated board

The structure consists from a lower straight beam, an upper straight beam and four periods of curvilinear beam between them. Length of the structure is 0.2 m and the distance between the lower and upper beams is 0.02 m. On the left and the right ends of the structure all the generalized nodal displacements are assumed equal to zero, except the displacements of the nodes on the right end of the structure in the direction of the x axis are assumed equal to - 1. The following parameters of the corrugated board are assumed: modulus of elasticity $E_y = 0.34\times10^9$ Pa, Poisson's ratio $\nu_{sy} = 0.4$, Poisson's ratio $\nu_{ys} = 0.14$, thickness $h = 0.0001$ m. The first eigenmodes of stability are presented in Fig. 1.

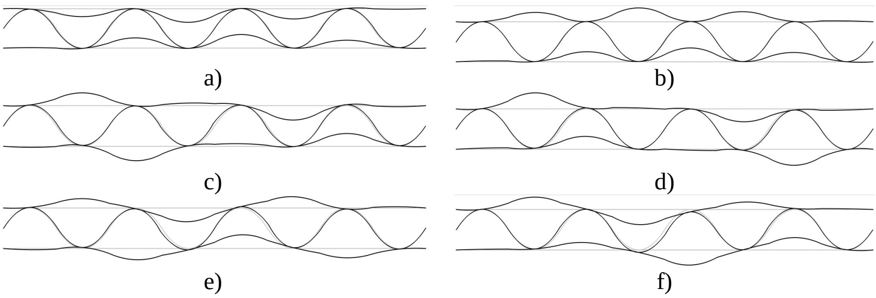

Fig. 1. The first eigenmodes of stability of corrugated board:
a) the first eigenmode, b) the second eigenmode, f) the sixth eigenmode

In practical applications the first eigenmode of stability is particularly important. Higher eigenmodes are of more complicated character and thus lead to displacements of the structure which are difficult to predict.

Model for the analysis of vibrations of corrugated board

The mass matrix of the model of orthotropic paper as a beam has the form:

$$[M] = \int [\bar{N}]^T \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \rho h & 0 \\ 0 & 0 & \dfrac{\rho h^3}{12} \end{bmatrix} [\bar{N}] ds, \tag{16}$$

where ρ is the density of the material of the beam.

The stiffness matrix of the model of orthotropic paper as a beam has the form:

$$[K] = \int \left(\begin{array}{l} [\bar{N}_1']^T \left[\dfrac{E_s}{1-\nu_{sy}\nu_{ys}} h \right] [\bar{N}_1'] + [\bar{N}_3']^T \left[\dfrac{E_s}{1-\nu_{sy}\nu_{ys}} \dfrac{h^3}{12} \right] [\bar{N}_3'] + \\ + [B]^T \left[\dfrac{E_s E_y h}{(E_s + E_y + E_s\nu_{ys} + E_y\nu_{sy})1.2} \right] [B] + [\bar{N}_2']^T M_\sigma [\bar{N}_2'] \end{array} \right) ds. \tag{17}$$

Analysis of vibrations of corrugated board

The structure consists from a lower straight beam, an upper straight beam and eight periods of curvilinear beam between them. Length of the structure is 0.2 m and the distance between the lower and upper beams is 0.01 m. On the left and the right ends of the structure all the generalized nodal displacements are assumed equal to zero, except the displacements of the nodes on the right end of the structure in the direction of the x axis are assumed equal to 0.02 m. The following parameters of the paper are assumed: modulus of elasticity $E_y = 0.34\times10^9$ Pa, Poisson's ratio $\nu_{sy} = 0.4$, Poisson's ratio $\nu_{ys} = 0.14$, thickness $h = 0.0001$ m, density of the material $\rho = 785$ kg/m^3. The first eigenmodes are presented in Fig. 2.

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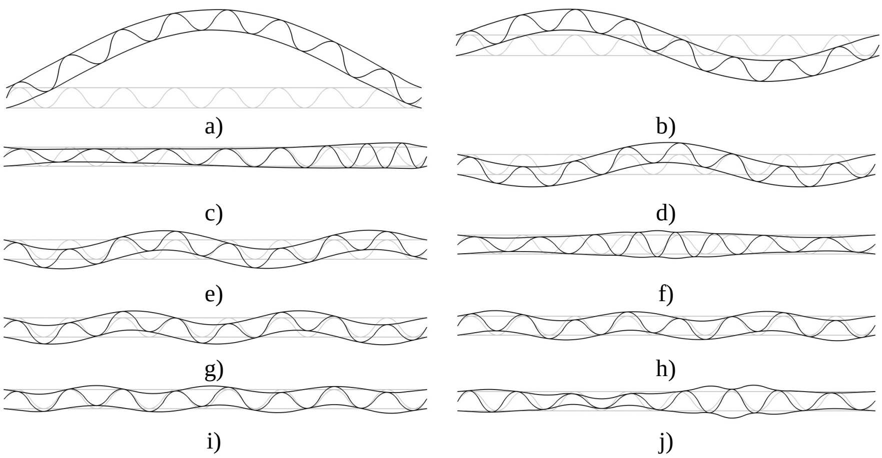


Fig. 2. The first eigenmodes of paper of special type with static tension:
a) the first eigenmode, b) the second eigenmode, j) the tenth eigenmode

The eigenmodes for the problem without static tension are presented in Fig. 3.

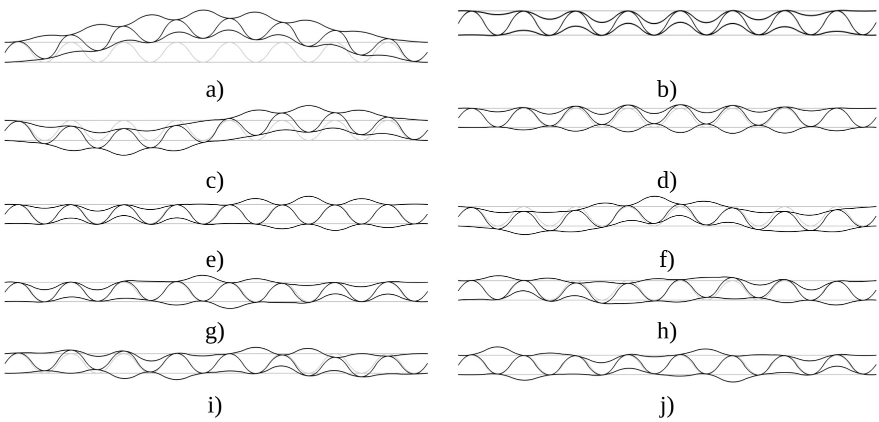


Fig. 3. The first eigenmodes of paper of special type without static tension:
a) the first eigenmode, b) the second eigenmode, j) the tenth eigenmode

From the presented results it is observed that the eigenmodes of the analyzed paper of special type are influenced by the static tension. The eigenmodes of tensioned paper of special type have much smaller local distortions and thus are preferable in many practical applications.

Vibrations and stability of multilayered board were analyzed with the purpose of appropriate optimal choice of type of multilayered board, its thickness and number of layers for the future package of the product.

By extending the investigations presented previously, further follows the vertical compression of the plane of the multilayered board until the plastic deformation of multilayered board starts. In the investigation three types of multilayered board are analyzed (B, C and E), which differ by the thickness of layers, the height of wave and the step of wave.

Method of experimental investigations, devices and materials used

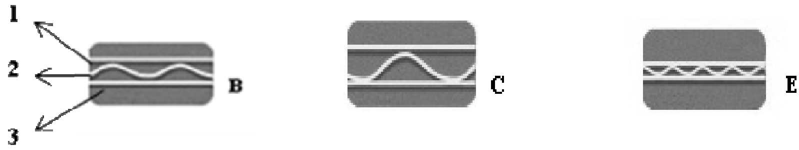
The consistence and nomenclature of materials used in the investigations are presented in Table 1. All the samples before starting the investigations were aclimatized during 7 day time period.

Table 1. Technical characteristics of multilayered board used in the investigations

GK profile	Type	Consistence*	Step of the wave λ, mm	Total thickness h, mm	Thickness of GK layers, mm		Highth of wave h_G, mm	Grammature, g/m^2
					Number of layer	Thickness, mm		
B	13B	FR-FR-FR	6.5	2	1	0,23	1,54	272
					2	0,25		
					3	0,23		
	14B	TB-FR-TB	6.5	3	1	0,35	2,3	376
					2	0,38		
					3	0,35		
	14BW	WT-FR-TB	6.5	3	1	0,35	2,3	390
					2	0,35		
					3	0,35		
	15BT	K-FR-K	6.5	3	1	0,43	2,14	398
					2	0,45		
					3	0,43		
	15BW	WK-FR-K	6.5	3	1	0,45	2,1	418
					2	0,45		
					3	0,45		
	16BW	WK-SC-K	6.5	3	1	0,48	2,04	466
					2	0,5		
					3	0,48		
C	14C	TB-FR-TB	8	4	1	0,65	2,7	411
					2	0,68		
					3	0,65		
	14CT	K-FR-FR	8	4	1	0,72	2,56	408
					2	0,75		
					3	0,72		
	15C	K-FR-K	8	4	1	0,75	2,5	451
					2	0,78		
					3	0,75		
	15CTT	TB-FR-TB	8	4	1	0,68	2,64	445
					2	0,7		
					3	0,68		
E	13E	TB-FR-TB	3.5	1	1	0,2	0,6	386
					2	0,23		
					3	0,2		
	14E	K-FR-K	3.5	1	1	0,18	0,64	355
					2	0,2		
					3	0,18		
	14EWW	WK-FR-WK	3.5	1	1	0,15	0,7	408
					2	0,18		
					3	0,15		

Notes: * FR – recycled fluting; TB – brown colored testliner; K – kraftliner or testliner with kraft top; WK – white top kraftliner; WT – white top testliner; SC – semi-chemical fluting.

From the data presented in Table 1 it is observed that the multilayered board which was used in the investigations has three layers, has different profiles, is of 13 different types, consistencies and grammatures. Photographs of the profiles of the layers of the investigated multilayered board are presented in Figs. 4 and 5.



a) b) c)

Fig. 4. General view of profiles of multilayered board: a) profile of multilayered board of B type, 1 – 3 denote the numbers of layers; b) profile of multilayered board of C type; c) profile of multilayered board of E type

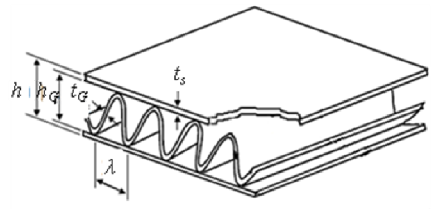


Fig. 5. Main geometric parameters of the multilayered board: h – total thickness of multilayered board, mm; h_G – wave height, mm; t_S – layer thickness, mm; t_G – wave thickness, mm; λ – wave step, mm

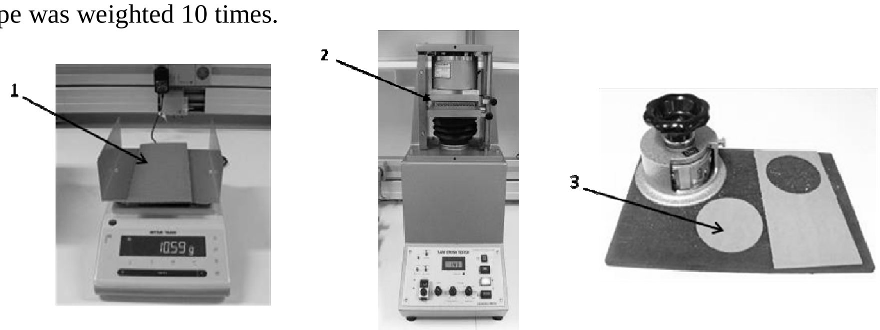
The multilayered board of type B (see Fig. 4a) is a three layered board with thickness 2.5 – 3.0 mm. This type of multilayered board can not be noted for high resistance to compression, thus it is used for production of packages of lightweight objects for which there are no requirements for looking very beautiful.

The multilayered board of type C (see Fig. 4b) is a three layered board with thickness 3.0 – 4.0 mm. The multilayered board of this type is used most often, because due to good technical characteristics it is widely applied in the field of production of various boxes.

The micro-multilayered board of type E (see Fig. 4c) is a three layered board with thickness 1.0 – 2.0 mm. Due to comparatively small height of the wave of the middle layer and small step of the wave this type of multilayered board can not be noted for high strength, but it has an even surface. Because of this fact it is usually used for production of packaging of lightweight materials which are required to look beautiful.

The main geometrical parameters of multilayered board are presented in Fig. 5.

The measurements of grammature of multilayered board were performed with weighing device Mettler Toledo (Model MS 1602S, error of measurement ±0.01 g), see Fig. 6a. Sample of each type was weighted 10 times.



a) b) c)

Fig. 6. Devices used in the process of experimental investigations: a) weighing device Mettler Toledo (Model MS 1602S), b) L&W Crush Tester device, c) L&W FCT Cutter device; 1, 2 and 3 – tested samples

Flat Crush Test (FCT) was performed. In this test the resistance to compression (N) and the strength of wave of the sample of multilayered board are determined. The investigation was performed with L&W Crush Tester. The main technical characteristics of the device are presented in Table 2 and general view of the device is shown in Fig. 6b.

Table 2. Technical characteristics of the device L&W Crush Tester

Precision of measurement of the compression force	±0.01, N
Range of measurement	50 – 5000 N
Velocity of compression	1 – 50 mm/min

When performing the investigations of the process of compression the constant velocity of compression $v = 2\cdot10^{-4}$ m/s (12.5 mm/min) was chosen.

For the investigation of compression of a plane (FCT) samples of the diameter of 79.8 mm (having an area of 50 mm^2) were cut from the multilayered board. For this purpose the L&W FCT Cutter device was used. General view of this device is presented in Fig. 6c.

Experimental results

Results of compression of the plane of multilayered board obtained during the investigation of the Flat Crush Test (FCT) (maximum compression force before plastic deformations take place in the material) are presented in Table 3 and Fig. 7.

Table 3. Results obtained during the investigation of compression of the plane of multilayered board (FCT)

Profile of multilayered board	Type of multilayered board	Maximum compression force, after exceeding of which plastic deformation of multilayered board starts, N
B	13B	450
	14B	563
	14BW	582
	15BT	695
	15BW	663
	16BW	702
C	14C	524
	14CT	593
	15C	674
	15CTT	706
E	13E	1608
	14E	1688
	14EWW	1635

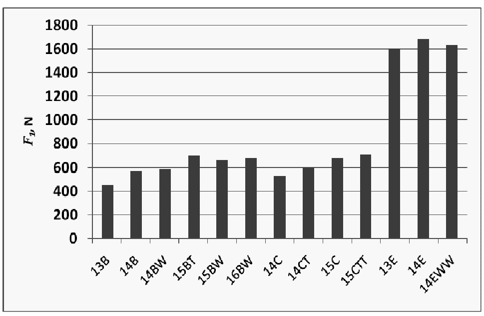


Fig. 7. Values of the compression force F_1 of the three layered multilayered board (profiles B, C and E) depending on the type of multilayered board

From Table 3 and Fig. 7 one can note that the value of the compression force F_1 depends on the type of the multilayered board. The compression force F_1 of the three layered board usually reaches up to 700 N, but there are several exceptions: for the multilayered board of type E the force F_1 increases almost to 1700 N (for example for the multilayered board of the type 14E). This can be explained by the fact that the waves of multilayered board of the type E are the smallest ones and thus for this case the resistance to compression increases substantially.

Further the stages of deformation for different moments of loading of the multilayered board are presented in Fig. 8.

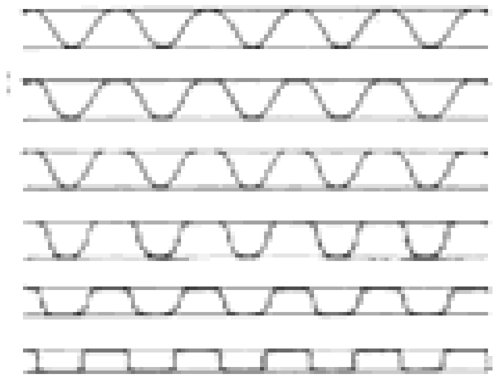


Fig. 8. Stages of deformation of the multilayered board of profile C for plane vertical compression

Fig. 9 provides the characteristic of maximum compression force F_1 from which plastic deformation starts. This force is equal to 695 N for the multilayered board of type 15BT. Here the maximum compression force is very sensitive to the type of multilayered board and its wave height.

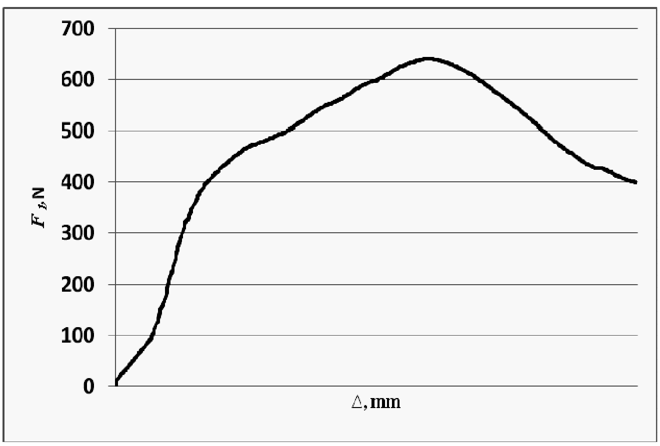


Fig. 9. Dependence of deformation Δ from the compression force F_1 for a three layered board having the profile of B type (of type 15BT from Table 1)

Conclusions

1. Results of investigation of compression of the plane of multilayered board indicated that the maximum compression force depends on the type of multilayered board, on the wave character and on the geometric parameters of the board.

2. The maximum compression force F of the three layered board varies from 700 N to 1700 N. This can be explained by the fact that the character of the wave has substantial effect on the results: with the wave becoming smaller the resistance to compression increases (the maximum compression force of the multilayered board of the type 14E in the region of elastic deformations F = 1688 N).

3. Vibrations, stability and compression of an element of package made from corrugated board are investigated by using the model of a structure consisting from two dimensional beams. In the numerical model of the beam orthotropic properties of the corrugated board are introduced. The problem of initial stability is solved and thus the first eigenmodes of stability are determined.

4. The obtained first eigenmode of stability is of a regular character, while higher eigenmodes are of more complicated character and thus lead to displacements of the structure which are difficult to predict.

5. The obtained results are used in the process of design of package elements.

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References

[1] **Urbanik T. J.** Review of buckling mode and geometry effects on postbuckling strength of corrugated containers. PVP, Development, Validation, and Application of Inelastic Methods for Structural Analysis and Design, ASME, Vol. 343, 1996, p. 85-94.

[2] **Lellep J., Majak J.** Nonlinear constitutive behavior of orthotropic materials. Mechanics of Composite Materials, 2000, Vol. 36(4), p. 261-266.

[3] **Rahman A., Urbanik T. J., Mahamid M.** Moisture diffusion through a corrugated fiberboard under compressive loading: its deformation and stiffness response. PPP 2002, Progress in Paper Physics, Finger Lake, N. Y., 2002, p. 85-88.

[4] **Saliklis E. P., Urbanik T. J., Tokyay B.** Bilinear modeling of cellulosic orthotropic nonlinear materials. Journal of Pulp and Paper Science, Vol. 29(12), December 2003, p. 407-411.

[5] **Urbanik T. J., Saliklis E. P.** Finite element corroboration of buckling phenomena observed in corrugated boxes. Wood and Fiber Science, Vol. 35(3), 2003, p. 322-333.

[6] **Saliklis E. P., Urabanik T. J., Tokyay B.** A Special Orthotropic Plate Theory Suitable for Use with ANSYS. FPL (USDA – Forest Products Laboratory) Publication, 2000.

[7] **Rahman Adeeb A.** Finite element buckling analysis of corrugated fiberboard panels. ASME Symposium on 2003 International Paper Physics Conference, 281 Cellulosic Materials, AMD – Vol. 221, 1997, p. 87-92.

[8] **Urbanik T. J., Saliklis E. P.** Comparison of postbuckling model and finite element model with compression strength of corrugated boxes. Proceedings of the 2002 Progress in Paper Physics Seminar, 2002 September 8-13, Syracuse, N. Y., Rochester, Rochester Institute of Technology, 4 p.

[9] **Gilchrist A. C., Suhling J. C., Urbanik T. J.** Nonlinear finite element modeling of corrugated board. ASME Joint Applied Mechanicals and Materials Division Meeting, AMD – Vol. 231/MD – Vol. 85, Mechanics of Cellulosic Materials, 1999, p. 101-106.

[10] **Zienkiewicz O. C.** The Finite Element Method in Engineering Science. Moscow: Mir, 1975.

[11] **Bathe K. J.** Finite Element Procedures in Engineering Analysis. New Jersey: Prentice-Hall, 1982.

[12] **Castro J., Ostoja – Starzewski M.** Elasto – plasticity of paper. International Journal of Plasticity, Vol. 19, 2003, p. 2083-2098.