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Analytical model for the investigation of RC slab behaviour under impact load

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1. Introduction

The interaction problem of reinforced concrete (RC) slab with shock impact load leads to more or less efficient methods for solving the motion equation, for example using mathematical model based on the approach derived in HEMP-3D by Wilkins [1] and used of the three-dimensional Lagrange processor in the explicit dynamics capabilities of ANSYS [2]. There are carried out studies for ballistic regimes [3, 4] but need investigations for low velocity situations, which is of most relevance to civil engineering structures. This paper describes investigations of the impact behaviour of RC slabs, subjected to falling weight loads, using analytical model. The main objective of the present study is to find the way to analytically determine displacements of RC slab impacted by falling impactor and analysis of the dynamic processes of reinforced concrete structures under low velocity impacts. The created technique and obtained results of the analytical calculation were compared to the results of numerical calculation from finite element analysis carried out using ANSYS.

2. The analytical model

Interaction of the analytical model of RC slab with an impactor is based on the mathematical model derived by calculation displacements of particles of RC slab under drop-weight impact load in the Cartesian system of axis. The analytical model of interaction of RC slab and an impactor for calculating displacement is shown schematically in Fig. 1. We shall suppose that:

- RC slab is a homogeneous elastic body;
- impactor is absolutely rigid and in equilibrium;
- reinforced concrete density ρ, elasticity modulus E, Poisson's ratio ν and a shear modulus G are known;
- characteristics of RC slab strength and geometric measurements are known;
- rate of external surface force, i.e. drop-weight impact load is known;
- external volume forces (i.e. reinforced concrete mass) are ignored.

Thus we have a heterogeneous system of physical and mechanical parameters of the two body surfaces. We shall analyze the displacements of the RC slab material point, coming through the impact of drop weight and dependence of displacements on the impact load location and parameters.

The structural model was set up in which the four corners of RC slab are fixed. The aim is to calculate the displacement of reinforced concrete points under the impact of drop weight in the known place.

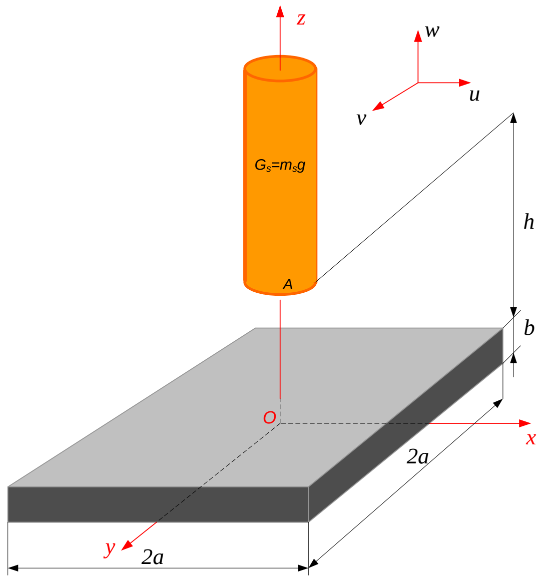

Fig. 1 The analytical model for displacement analysis

3. Mathematical model

The resulting displacements have been analysed in the Cartesian system of axis and mathematical model is based on Hamilton principle [5].

Let's suppose that displacements in Ox direction are u, in Oy direction – v, in Oz direction – w in the Cartesian system of axis. Displacements u, v and w are to be found for the boundary conditions:

- displacements of RC slab particles $u = 0$ at the $x = \pm a$;
- displacements of RC slab particles $v = 0$ at the $y = \pm a$;

and in for the case of initial conditions:

- $u = v = w = 0$ displacements of the RC slab particles of the initial instant $t = 0$ are equal to zero.

Let's suppose that separation functions are

$$u = Uq\,;\ \ v = Vq\,;\ \ w = Wq \qquad (1)$$

where $U = U(x,y,z)$; $V = V(x,y,z)$; $W = W(x,y,z)$; $q = q(t)$.

Functions U, V and W are selected according to the boundary conditions, i.e. they should fit for the body presented in Fig. 1.

$$m\ddot{q}+kq=F \tag{2}$$

where

$$m=\rho\iiint_{(V)}\left(U^2+V^2+W^2\right)dxdydz \tag{3}$$

$$k=2G\iiint_{(V)}\left\{\begin{array}{l}U_x^2+V_y^2+W_z^2+\\ +\dfrac{\nu}{1-2\mu}\left(U_x+V_y+W_z\right)^2+\\ +\dfrac{1}{2}\left[\begin{array}{l}\left(U_y+V_x\right)^2+\left(U_z+W_x\right)^2+\\ +\left(V_z+W_y\right)^2\end{array}\right]\end{array}\right\}dxdydz \tag{4}$$

$$F=\iint_{(S)}\bar{Z}WdS \tag{5}$$

where $\bar{Z}$ is external surface force; S is integration area, i.e. the surface part subjected to external surface forces.

Let's suppose that RC slab impacted by falling impactor is shown schematically in Fig. 1. Applying the principle of work and energy to each point A and O the obtained velocity v_O

$$v_O=\sqrt{2gh} \tag{6}$$

Applying the principle of impulse and momentum to each point A and O the obtained impulse is

$$I=\int_0^t\bar{Z}dt=m_s g\sqrt{\frac{2h}{g}}=m_s\sqrt{2gh} \tag{7}$$

In this case, Eq. (5) can be rewritten

$$\int_0^\tau Fdt=FI \tag{8}$$

where

$$F=\iint_{(S)}WdS \tag{9}$$

Thus, in order to find q, we have to solve the integral differential equation

$$m\frac{dq}{dt}=-k\int_0^\tau qdt+FI \tag{10}$$

where τ is impulse duration.

In this case, the integral differential Eq. (10) is solved approximately by means of the iteration method, for example of the fifth approximation

$$q_{(5)}=\frac{FI}{m}t-\frac{kFI}{6m^2}t^3+\frac{k^2FI}{120m^3}t^5-\frac{k^3FI}{5040m^4}t^7+ +\frac{k^4FI}{362880m^5}t^9-\frac{k^5t^{11}}{39916800m^5} \tag{11}$$

Coefficients of the chosen function U, V and W can be found using Galiorkin method [6]

$$\begin{cases}\int_{-a}^{a}dx\int_{-a}^{a}dy\int_0^b k_1\varepsilon dz=0\\ \int_{-a}^{a}dx\int_{-a}^{a}dy\int_0^b k_2\varepsilon dz=0\\ \int_{-a}^{a}dx\int_{-a}^{a}dy\int_0^b k_3\varepsilon dz=0\end{cases} \tag{12}$$

Knowing m, k, U, V, and W we can calculate approaches for finding the approximate value q according to the Eq. (11) and displacements of RC slab particles

$$u_i=Uq_i;\ v_i=Vq_i;\ w_i=Wq_i \tag{13}$$

Hereby presented, this theoretical study allows to solve approximately integral differential Eq. (10) and to calculate displacement of the material point of RC slab under the action of impactor.

4. Numerical examples

In order to determine the adequacy of the created analytical model, the numerical test was done and the obtained results of the theoretical modeling can be compared to the modeling with ANSYS ones [7]. All geometrical and material parameters of the analytical model were selected as in the real experiment [5]. For this case taking into account the model of a RC slab for displacement analysis (Fig. 1), the geometrical values $a = 0.4$ m, $b = 0.1$ m and $h = 2$ m. Let's suppose that $\rho = 2314$ kg/m^3, $E = 11250$ MPa, $\nu = 0.2$.

Thus functions U, V and W are selected on the basis of the boundary conditions, i.e. they should fit for a room presented in Fig. 1.

$$U=\left(a^2-x^2\right)^3\left(b^2+a^2\right)^2\left(xa^4+k_1xy^2z^2\right)/a^{15} \tag{14}$$

$$V=\left(a^2-y^2\right)^3\left(b^2+a^2\right)^2\left(ya^4+k_2yx^2z^2\right)/a^{15} \tag{15}$$

$$W=\frac{1}{a^{15}}\left(a^2-x^2\right)\left(a^2-y^2\right)\left(a^2+z^2\right)^2\times \times\left(b^2+a^2\right)^2\left(a^3+k_3z^3\right) \tag{16}$$

All parameters for the calculation of Eqs. (14) - (16), (2) and (3) are shown in Table 1.

Table 1

Parameters for the calculation

Parameters	Value
k_1	-282.621
k_2	-282.621
k_3	1.2789
m	77.862
k	5.19×10^{11}

The numerical examples of analytical model were done in the different place of RC slab presented in Fig. 2.

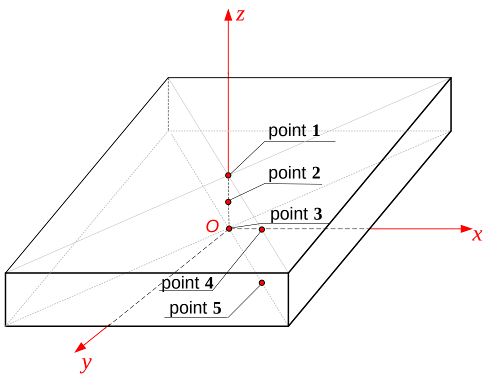

Fig. 2 The principal scheme of the point where displacement was calculated

In order to determine the adequacy of the created analytical model, the model of ANSYS was done and compared with experimental test [7]. Taking into account principal scheme where is displacements of the point calculation (Fig. 2), the obtained results of the analytical and numerical tests in different point of calculation are presented below in Fig. 3 and shows that numerical and analytical by calculated displacement of the point approach one-to-one and in a certain moment of time are equals.

After beyond of this value, calculated displacement of the point becomes different. It can be explained by time factors in the numerical model and numbers of iterations in the analytical model. The modelling with ANSYS using explicit time integration is limited by the CFL

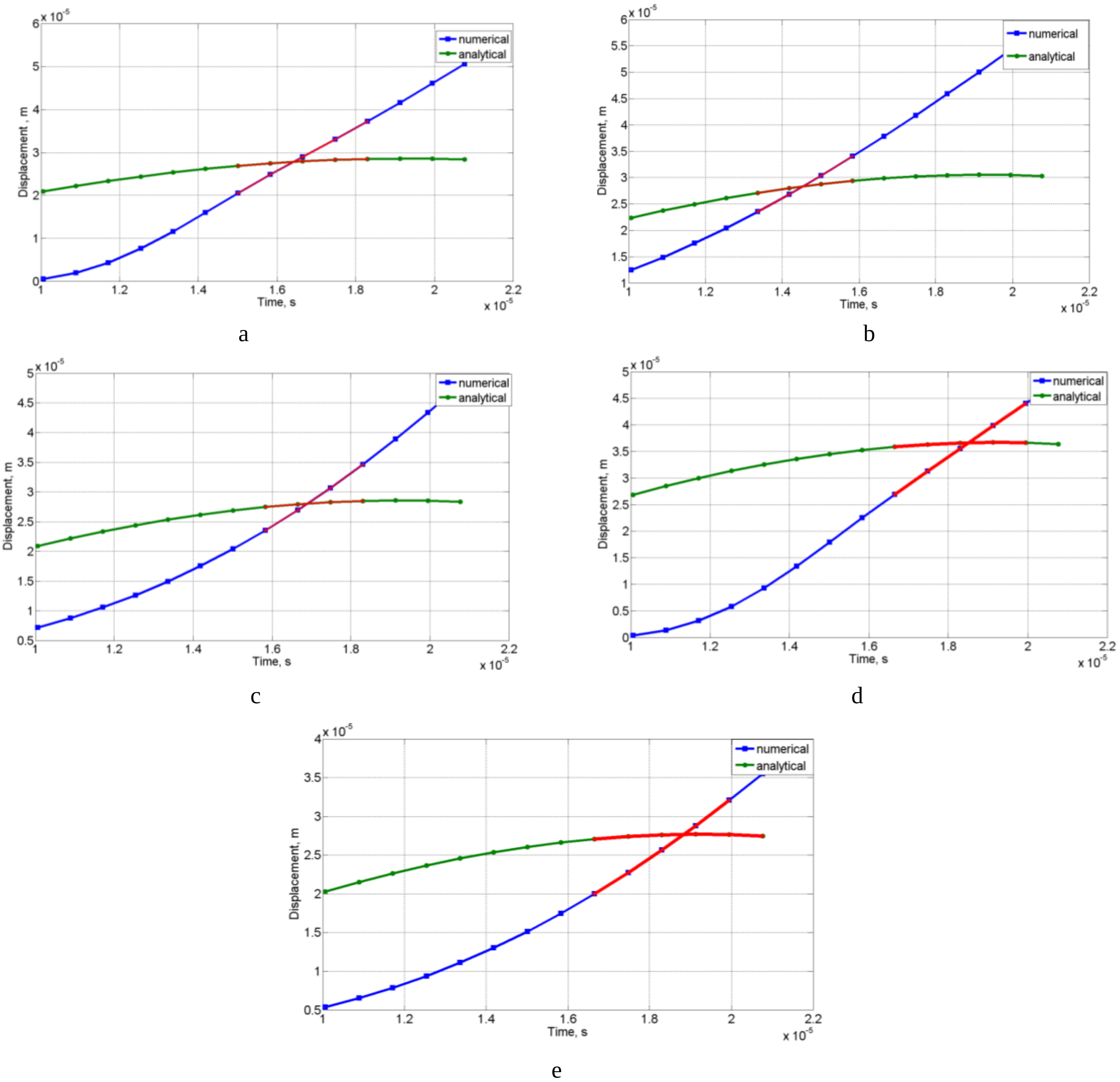

Fig. 3 Analytical and numerical results: a – point 1; b – point 2; c – point 3; d – point 4; e – point 5

(Courant-Friedrichs-Lewy) condition [8]. This condition implies that the time step is limited so that a disturbance (stress wave) cannot travel further than the smallest characteristic element dimension in the mesh, in a single time step. Thus the time step criteria for solution stability is

$$\Delta t \le f\left[\frac{h}{c}\right]_{min} \tag{17}$$

where Δt is the time increment, f is the stability time step factor (0.9 by default), h is the characteristic dimension of an element, c is the local material sound speed in an element.

The modelling using analytical model and obtained results adequacy is limited by approximation level which depends of iteration number in solving Eq. (10). For example 8th iteration is presented in Eq. (18) and is more unwieldy, but it allows to calculating displacement with Eqs. (10) and (13) of point about 0.2×10^{-6} s longer time duration of the shock. It should be noted that RC slab interaction under impact load time duration depends of iteration number of Eq. (10) and must be consider. The accuracy of analysis using analytical method depends of time factors in the calculations and is more suitable for RC slab interaction under very short impulse.

The numerical and analytical solutions of point displacement are approximately equal, for example in this case (Fig. 3, a, b, c) inside of interval of $1.4\times10^{-5}\div1.8\times10^{-5}$ s.

$$q_{(8)} = FI\begin{pmatrix} \frac{t}{m}-\frac{k}{6m^2}t^3+\frac{k^2}{120m^3}t^5- \\ -\frac{k^3}{5040m^4}t^7+\frac{k^4}{362880m^5}t^9- \\ -\frac{k^5}{39916800m^6}t^{11}+ \\ +\frac{k^6}{6227020800m^7}t^{13}- \\ -\frac{k^7}{1.307674368\times10^{12}m^8}t^{15} \end{pmatrix}+$$
$$+\frac{k_T^8}{8.3688\times10^{22}m^8}t^{17}. \qquad (18)$$

5. Conclusions

The proposed and developed 3D analytical method allows the interaction analysis of RC slab with impact load. The analytical method with properly chosen duration time of impact load enables:

1. to calculate approximately the displacements of RC slab particles appeared under impact at a specific place;

2. to create precondition for evaluating dynamical influence on RC slab quality.

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V. Doroševas, J. Vaičiūnas

IMPULSINIO SMŪGINIO POVEIKIO VEIKIAMOS GELŽBETONIO PLOKŠTĖS TYRIMO ANALIZINIS MODELIS

R e z i u m ė

Straipsnyje pateikiamas sukurtas impulsinio smūginio poveikio veikiamos gelžbetonio plokštės tyrimo analitinis modelis. Ištirta 3D analizinio modelio galimybė modeliuoti tam tikroje vietoje veikiančio smogtuvo ir gelžbetonio plokštės sąveiką. Gauti teorinio tyrimo rezultatai buvo lyginami su ANSYS programos aplinkoje atlikto skaitinio tyrimo rezultatais, kurie patikrinti natūriniais eksperimentais. Išnagrinėtas analizinio modelio adekvatumas parodė, kad naudojant sukurtą analizinį metodą ir tinkamai parinkus smūginio poveikio trukmę, galima tirti smogtuvo ir gelžbetonio plokštės sąveikos dinamiką.

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ANALYTICAL MODEL FOR THE INVESTIGATION OF RC SLAB BEHAVIOUR UNDER IMPACT LOAD

S u m m a r y

The analytical model, created for the investigation of RC slab behaviour under impact load, is presented in this paper. The possibility of 3D analytical model to analyze the modeling RC slab impacted by falling impactor at the certain place is investigated. The obtained results of the theoretical experiment were compared to the results of numerical investigation in capabilities of ANSYS, which is adequate to practical experiment. The adequacy analysis of analytical model showed that using the created analytical method with properly chosen duration time of impact load enables to investigate dynamics behaviour of RC slab under impact load.

Keywords: analytical model, impact analysis, falling weight, reinforced concrete.

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