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Modeling of Nonlinear Circuit using Volterra Series

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Introduction

Distortion is a key issue in the design of many types of circuits. In modern circuits linearity is a very important parameter, especially with the strict modern telecommunication system standards The method of Volterra series is the most widely method to analyze the nonlinear circuits [1, 2]. In order to get a low signal distortion, a great focus is made on investigation of nonlinearities The Volterra series method of distortion analysis is presented in the analysis of a common emitter circuit.

For analysis of nonlinear systems differential equations representing nonlinear, frequency and parametrical performances of the system are applied. Unlike numerical simulations which give no information about the source of the distortion, closed form expressions for distortion components in terms of circuit parameters can be found using Volterra series.

In this paper we modeling nonlinearities of a common emitter circuit, analyze their influence on signal distortions, obtain general solutions of Volterra kernels appropriate for engineering calculations. To determine nonlinear behaviour is used the Volterra series, the most widely used method to analyze the nonlinear behavior of analog circuits.

Model and analysis methodology

The Volterra series for a circuit is generally represented as a summation of operators [3]

$$y(t)=H(x(t))=H_1(x(t))+H_2(x(t))+\ldots+H_n(x(t))+\ldots, \quad (1)$$

where

$$H_n(x(t))=\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty}h_n(\tau_1\ldots\tau_n)x_i(t-\tau_1)\ldots x_n(t-\tau_n)d\tau_1\ldots d\tau_n . \quad (2)$$

The Laplace transform of the Volterra kernel $h_n(t_1,\ldots,t_n)$ is represented by $H_n(p_1,\ldots,p_n)$, $x(t)$ is the input, $y(t)$ the output.

Using the method of Volterra series of distortion analysis a nonlinear system is composed of linear,square,cube,etc.subsystems connected in parallel (Fig. 1).

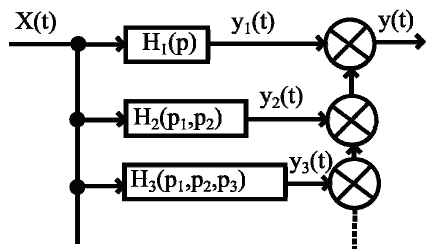

Fig. 1. Model nonlinear system (H_i – appropriate kernel Volterra series)

The first subsystem is linear. The output signal of this system is the following:

$$y_1=\int_{-\infty}^{\infty}h_1(t-\tau_1)x(\tau_1)d\tau_1=\int_{-\infty}^{\infty}h_1(\tau_1)x(t-\tau_1)d\tau_1 \quad (3)$$

or a view is as follows

$$Y_1(p)=H_1(p)x(p), \quad (4)$$

where p – complex variable.

An output signal of the square subsystem is:

$$y_2(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(t-\tau_1, t-\tau_2)\prod_{i=1}^{2} x(\tau_i)d\tau_i =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)\prod_{i=1}^{2} x(t-\tau_i)d\tau_i, \qquad (5)$$

$$Y_2(p_1, p_2) = H_2(p_1, p_2)\prod_{i=1}^{2} X(p_i) \ . \qquad (6)$$

An output signal of the cube subsystem is:

$$y_3(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_3(t-\tau_1, t-\tau_2, t-\tau_3)\prod_{i=1}^{3} x(\tau_i)d\tau_i =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_3(\tau_1, \tau_2, \tau_3)\prod_{i=1}^{3} x(t-\tau_i)d\tau_i, \qquad (7)$$

$$Y_3(p_1, p_2, p_3) = H_3(p_1, p_2, p_3)\prod_{i=1}^{3} X(p_i). \qquad (8)$$

Only three harmonics will be evaluated as already third harmonic in amplifier is very low.

If a amplifier receives the following signal

$$x(t) = U\cos\omega t = \frac{1}{2}U\left(e^{j\omega t} + e^{-j\omega t}\right), \qquad (9)$$

change constant component of a output signal is defined by analyzing a square system [4]

$$i_2(t) = \frac{1}{2}U^2 \operatorname{Re}(H_2(jw, -jw)) +$$

$$+\frac{1}{2}U^2 \left|H_2(jw, jw)\right| \cdot \cos(2wt + \varphi_{2w}), \qquad (10)$$

where φ_{2w} – phase of the second harmonic.

Change in the first harmonic is figured out by analyzing a cube system

$$i_3(t) = \frac{3}{4}U^3 \left|H_3(jw, jw, -jw)\right| \cos(wt + \varphi_w) +$$

$$+\frac{1}{4}U^3 \left|H_3(jw, jw, jw)\right| \cos(3wt + \varphi_{3w}), \qquad (11)$$

where φ_{3w} – phase of the third harmonic.

Analytic solutions

The first step befor calculating a Volterra series is to expand the circuit nonlinearities in Taylor series.

The main nonlinearities that determine the signal distortion of amplifier are nonlinear junction currents of an emitter and collector and their nonlinear capacitances [4].

Fig. 2 presents an equivalent circuit for signal distortion analysis of amplifier.

Most analytical amplifier models[5]express the emitter current following

$$i_e = I_{e_0}\left(\exp(\gamma_e u_e) - 1\right), \qquad (12)$$

where u_e – junction voltage, φ_T – temperature potencial, α – collector current transmission coefficient, i_k –current of collector, I_{eT} – heat current

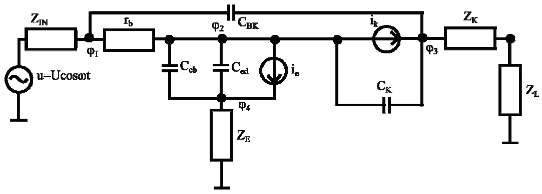

Fig. 2. Equivalent circuit for distortion analysis of amplifier

After expanding (12) in a Taylor series, the following is obtained

$$i_e = I_{e_0} + \lambda_1 u_e + \lambda_2 u_e^2 + \lambda_3 u_e^3, \qquad (13)$$

where I_{e_0} – current constant component;

$$\lambda_1 = G_e; \ \ \lambda_2 = \frac{1}{2}G_e^2 / I_{e_0}; \ \ \lambda_3 = \frac{1}{6}G_e^3 / I^2{}_{e_0}. \qquad (14)$$

Nonlinear generator current flowing through the junction capacitance of an emitter is as follows:

$$i_{c_e} = \beta_1\, du_e/dt + \beta_2\, du_e^2/dt + \beta_3\, du_e^3/dt, \qquad (15)$$

$$\begin{cases} \beta_1 = \dfrac{\tau_{\alpha FT} G}{\sqrt{2}\,\varphi_T} + v(\varphi_T - U)^{-\frac{1}{2}}, \\ \beta_2 = \dfrac{\tau_{\alpha FT} G^2}{2\sqrt{2}\, I_{e_0}\varphi_T} - \dfrac{1}{4}v(\varphi_T - U)^{-\frac{3}{2}}, \\ \beta_3 = \dfrac{\tau_{\alpha FT} G^3}{6\sqrt{2}\, I^2{}_{e_0}\varphi_T} + \dfrac{1}{8}v(\varphi_T - U)^{-\frac{5}{2}}, \end{cases} \qquad (16)$$

where $\tau_{\alpha T}, v_e$ – physical parameters of integrated transistor.

Nonlinear generator current flowing through the collector capacitance

$$i_{C_K} = \gamma_1\, du_k/dt + \gamma_2\, du_K^2/dt + \gamma_3\, du_K^3/dt, \qquad (17)$$

where U_K – junction voltage of a collector;

$$\gamma_1 = V_K U_K^{-\frac{1}{3}}; \ \gamma_2 = \frac{1}{6}V_K U_K^{-\frac{4}{3}}; \ \gamma_3 = \frac{2}{27}V_K U_K^{-\frac{7}{3}}, \qquad (18)$$

where V_K – constant.

Variable component of an output current is

$$i_K = (L_1 - pD_1)u_e + (L_2 - pD_2)u_e^2 + (L_3 - pD_3)u_e^3, \qquad (19)$$

where L_1, L_2, L_3 and D_1, D_2, D_3 – coefficients dependent on circuit parameters and frequency.

After evaluation of amplifier nonlinearities for equivalent circuit formed equations system:

$$\begin{cases} \frac{u-\overline{\varphi}_1}{Z_{IN}} + pC_{KB}(\overline{\varphi}_3 - \overline{\varphi}_1) = \frac{\overline{\varphi}_1 - \overline{\varphi}_2}{r_b}, \\ \frac{\overline{\varphi}_1 - \overline{\varphi}_2}{r_b} + (L - pN)(\overline{\varphi}_2 - \overline{\varphi}_4) + \gamma p(\overline{\varphi}_3 - \overline{\varphi}_2) = \lambda(\overline{\varphi}_2 - \overline{\varphi}_4) + \beta p(\overline{\varphi}_2 - \overline{\varphi}_4), \\ \gamma p(\overline{\varphi}_3 - \overline{\varphi}_2) + (L - pN)(\overline{\varphi}_2 - \overline{\varphi}_4) + pC_{KB}(\overline{\varphi}_3 - \overline{\varphi}_1) = \frac{-\overline{\varphi}_3}{Z_{ap} + pL_K + r_K}, \\ \beta p(\overline{\varphi}_2 - \overline{\varphi}_4) + \lambda(\overline{\varphi}_2 - \overline{\varphi}_4) = \frac{-\overline{\varphi}_4}{Z_e}, \end{cases} \quad (20)$$

where $\overline{\varphi}_1, \overline{\varphi}_2, \overline{\varphi}_3, \overline{\varphi}_4$ – appropriate note potentials, Z_{IN} – input impedance, Z_L – load impedance, Z_e – emitter impedance, C_{KB} – feedback capacitance, C_K – output capacitance, r_b – base impedance, Z_K – output impedance.

Adressing the equations system by method of Volterra series first found Volterra kernels of linear subsystem,then of square subsystem,after evalution of linear kernells,afterwords of cube subsystem,after evalution kernels of linear, squar subsystems.

Kernels of a linear system will be referred to as $H_{11}(p)$, $H_{12}(p)$, $H_{13}(p)$ and $H_{14}(p)$. They correspond to the units of an equivalent circuit diagram and are figured out from a matrix equation

$$\begin{pmatrix} H_{11}(p) \\ H_{12}(p) \\ H_{13}(p) \\ H_{14}(p) \end{pmatrix} = W^{-1}(p) \begin{pmatrix} \frac{1}{Z_{IN}} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (21)$$

where $W(p)$ – conductance matrix.

$$W(p) = \begin{pmatrix} \frac{1}{Z_{IN}} + pC_{KB} + \frac{1}{r_b'} & -\frac{1}{r_b'} & -pC_{KB} & 0 \\ \frac{1}{r_b'} & \frac{1}{r_b'} - (L_1 - pN_1) + p\gamma_1 + p\beta_1 + \lambda_1 & \gamma_1 p & (L_1 - pN_1) - \lambda_1 - p\beta_1 \\ -pC_{KB} & (L_1 - pN_1) + p\gamma_1 & p\gamma_1 + pC_{KB}\frac{1}{Z_K + Z_L} & -(L_1 - pN_1) \\ 0 & \lambda_1 + p\beta_1 & 0 & -p\beta_1 - \lambda_1 - \frac{1}{Z_e} \end{pmatrix}. \quad (22)$$

Two-dimensional kernels of IT diagram units will be marked respectively $H_{21}(p_1, p_2)$, $H_{22}(p_1, p_2)$, $H_{23}(p_1, p_2)$ and $H_{24}(p_1, p_2)$, and obtained from the (23).

After figuring out two-dimensional kernel $H_{23}(p_1, p_2)$ from (23), dependences of a constant component of an output current in a frequency range shall be established taking into account both the parameters of an equivalent diagram and outer elements.

$$\begin{pmatrix} H_{21}(p_1,p_2) \\ H_{22}(p_1,p_2) \\ H_{23}(p_1,p_2) \\ H_{24}(p_1,p_2) \end{pmatrix} = W^{-1}(p_1 + p_2) \times$$

$$\times \begin{pmatrix} 0 \\ (L_2 - (p_1+p_2)N_2)\prod_{i=1}^{2}(B_1(p_i) - D_1(p_i)) - \lambda_2 \prod_{i=1}^{2}(B_1(p_i) - D_1(p_i)) - \\ -\beta_2(p_1+p_2)\prod_{i=1}^{2}(B_1(p_i) - D_1(p_i)) + \gamma_2(p_1+p_2)\prod_{i=1}^{2}(C_1(p_i) - B_1(p_i)) \\ (L_2 - (p_1+p_2)N_2)\prod_{i=1}^{2}(B_1(p_i) - D_1(p_i)) - \gamma_2(p_1+p_2)\prod_{i=1}^{2}(C_1(p_i) - B_1(p_i)) \\ \lambda_2 \prod_{i=1}^{2}(B_1(p_i) - D_1(p_i)) + \beta_2(p_1+p_2)\prod_{i=1}^{2}(B_1(p_i) - D_1(p_i)) \end{pmatrix}. \quad (23)$$

Then change in a constant component of a output current after figuring out a two-dimensional kernel $H_{23}(j\omega, -j\omega)$, according to (10), is the following [4]

$$\Delta I_{K_0} = \frac{U^2 H_{23}(j\omega, -j\omega)}{2(R_L + r_k)}. \quad (24)$$

Fig. 3 presents dependences change a constant component of a IT output current $\gamma_{K0} = \Delta I_{K0}/I_{K0}$ on magnitude and frequency of an input signal $m = U/\varphi_T$.

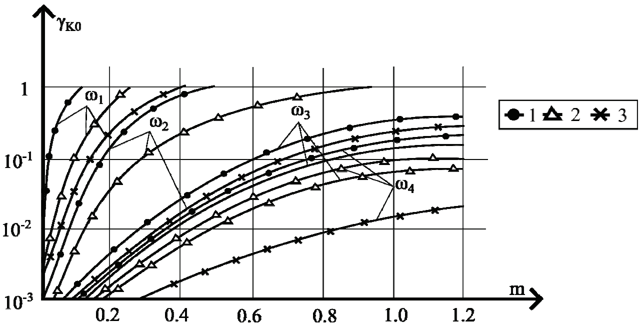

Fig. 3. Dependences of change a constant component of a output current γ_{K0} on an input signal *m* ($\omega_1 = 0{,}1\omega_{T_0}$, $\omega_2 = 0{,}25\omega_{T_0}$, $\omega_3 = 0{,}375\omega_{T_0}$, $\omega_4 = 0{,}5\omega_{T_0}$; 1 – $I_{e0} = 1$mA, 2 – $I_{e0} = 5$mA, 3 – $I_{e0} = 10$mA)

Thus, application of the method of Volterra series allows defining dependences of a output current on the input signal parameters and evaluating the influence of parameters of a amplifier itself and outer elements on the distortion of the signals transferred.

Similarly, change first harmonic output currrent depending on nonlinearities in a amplifier is defined. The first harmonic is observed at the output of linear and cube systems. In order to establish the first harmonic at the output of a cube system, three-dimensional kernels shall be calculated. A matrix equation that is used to figure out the aforementioned kernels is the following

$$\begin{pmatrix} H_{31}(p_1,p_2,p_3) \\ H_{32}(p_1,p_2,p_3) \\ H_{33}(p_1,p_2,p_3) \end{pmatrix} = W^{-1}(p_1+p_2+p_3)\times$$

$$\times\begin{pmatrix} 0 \\ (L_3-(p_1+p_2+p_3)N_3)\prod_{i=1}^{3}(B_1(p_i)-D_1(p_i))-\lambda_3\prod_{i=1}^{3}(B_1(p_i)-D_1(p_i))- \\ -\beta_3(p_1+p_2+p_3)\prod_{i=1}^{3}(B_1(p_i)-D_1(p_i))+\gamma_3(p_1+p_2+p_3)\prod_{i=1}^{3}(C_1(p_i)-B_1(p_i)) \\ \\ (L_3-(p_1+p_2+p_3)N_3)\prod_{i=1}^{3}(B_1(p_i)-D_1(p_i))-\gamma_3(p_1+p_2+p_3)\prod_{i=1}^{3}(C_1(p_i)-B_1(p_i)) \\ \\ \lambda_3\prod_{i=1}^{3}(B_1(p_i)-D_1(p_i))+\beta_3(p_1+p_2+p_3)\prod_{i=1}^{3}(B_1(p_i)-D_1(p_i)) \end{pmatrix}. \quad (25)$$

After calculating three-dimensional kernel $H_{33}(p_1,p_2,p_3)$ from (11), change output current first harmonic may be calculated

$$\Delta I_{K1}=\frac{3}{4}U^3\left|H_{33}(j\omega_1,j\omega_1,-j\omega_1)\right|/\left|Z_K+Z_L\right|. \quad (26)$$

Fig. 4 presents dependences of change the first harmonic output current $\gamma_{K1}=\Delta I_{K1}/I_{K1}$ of on magnitude and frequency on an input signal $m=U/\varphi_T$.

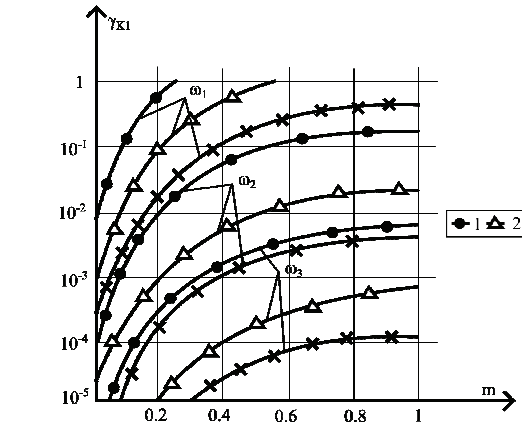

Fig. 4. Dependences of change the first harmonic of an output current γ_{K1} on input signal m ($\omega_1=0{,}1\omega_{T_0}$, $\omega_2=0{,}25\omega_{T_0}$, $\omega_3=0{,}5\omega_{T_0}$, $1-I_{e_0}=1mA$, $2-I_{e_0}=5mA$, $3-I_{e_0}=10mA$)

Conclusions

For modeling nonlinearities of nonlinear circuit the method of Volterra series has been offered. Unlike numerical simulations,closed form expressions for distortion components in terms of common emitter circuit parameters has be found..The mathematical model establisched here let to calculate Volterra kernels of the respective linear, square and cube systems and presents dependences of change output current component on magnitude of an input signal within a frequency range.

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Voterra series method was used to analyze signal distortions. During application of this method there is no need to form differential equations, which characterize parametric and frequency properties of non-linear circuits. Volterra series method was applied to analyze the nonlinearities of common emitter circuit. Modeling of nonlinearities was performed. Fundamental components of signal distortions were evaluated. Equation system in operator form was written for non-linear circuit, after solving which, the analytical expressions of Volterra series kernels were obtained. Laplace transform of kernels was used to determine the magnitudes of signal distortion components. The output signal constant component and first harmonic variations in dependence on the parameters of the input signal over a frequency range were presented. Ill. 4, bibl. 5 (in English; abstracts in English and Lithuanian).

J. Anilionienė, R. Anilionis, D. Andriukaitis. Netiesinės grandinės modeliavimas naudojant Volterra eilutes // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 4(120). – P. 43–46.

Netiesinei grandinei modeliuoti taikomas Volterra eilučių metodas, kuris, palyginti su kitais metodais, analizuojančiais netiesines sistemas su kintamais parametrais, suteikia daugiau informacijos apie iškraipymų šaltinius; nereikia spręsti sudėtingų diferencialinių lygčių. Bendro emiterio schemos iškraipymų analizei pritaikytas Volterra eilučių metodas. Įvertinus pagrindinius netiesiškumus, ekvivalentinei stiprintuvo schemai sudaryta operacinės formos lygčių sistema. Gautos analitinės išraiškos Volterra eilučių branduoliams skaičiuoti. Pateiktos išėjimo signalo pastovios dedamosios ir pirmosios harmonikos pokyčių priklausomybės nuo įėjimo signalo dydžio dažnių diapazone. Il. 4, bibl. 5 (anglų kalba; santraukos anglų ir lietuvių k.).