

# 735. Modeling and diagnostics of vertical axis rotary system driven by multi gear drive

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**Abstract.** R & D work related to the technical condition assessment and unexpected failure prevention of the bearings and gearings of vertical rotor system, characterized by particularly large dimensions and mass. Theoretical studies are based on the gear meshing modeling in terms of determined contact stresses. Experiments were carried out by measuring the rotors displacements and vibrations amplitudes, as well as measuring absolute vibrations of bearings. On the basis of experimental and theoretical studies defects on contact surfaces of the gear teeth were determined. Theoretical model of gear's teeth contacting was created and solved by finite element method. The theoretical model enables to assess the disadvantages of vertical gearing teeth contacting process in terms of defected powered gear bearings and to predict development mechanism of defect in order to avoid unexpected failures.

**Keywords:** multi drive system, gear, vibration, modeling, simulation.

## Introduction

Many of the technological equipment in industry for specific processes or due to significant technological equipment size often use a large mass and dimensions vertical axis rotor systems. The vertical axis rotor systems are fundamentally different from horizontal axis rotor systems they do not have a gravity load in a radial direction. Therefore, the vertical axis rotor systems have less supports than horizontal axis rotor systems and the supports are larger and more responsible. Also the stiffness of supports are different, as a result of different supports stiffness the vibration level of the upper bearing is always higher than the lower. When work section of the rotor is overloaded, due to large distances between supports, radial overload occurs in the support, which turn to unacceptably displace the driven section of the rotor system, and that is due to driven mechanism failure.

This article analyzes a particularly large mass and dimension vertical axis rotor system, which is mounted on the three supports. Bearings of an upper and lower supports are designated for axial loads, the middle support bearing is intended to radial loads. The multi gear drive consisting of 10 asynchronous motors that are controlled by frequency inverters is investigated. Driver spur gears get rotational movement through the gearboxes and transfer this movement to single large spur gear. Scientific studies have measured, that due to an overload of the rotor system, the driven shaft has had a defect of upper axial – radial bearing mounted near driven spur gear. Therefore, the axis of shaft was inclined and the end of shaft together with rigidly fixed on it driven spur gear has deviated. Shaft displacement caused the changes in gear meshing conditions of the multi gear drive, consequently working surfaces of the driven gear teeth's has been damaged.

The purpose of this paper is to assess the meshing of the multi gear drive involutes teeth's that drives the particularly large mass and dimension rotor system, when the radial stiffness of the upper rotor support is insufficient. Also, to determine the critical values of the angle of vertical shaft inclination at fixing position of driven spur gear, because when those values are exceeded, working surfaces of the involutes teeth's may be damaged. Spur gears meshing problems of low speed industrial machinery are examined in references [1, 2, 3, 4].

## Experiment

Multi gear driven vertical rotor system consists of ten asynchronous motors 1 controlled by frequency inverters. Each electric motor is connected to a gearbox 2 through the coupling. Spur gears are mounted on the driver shaft of the gearbox 3, number of teeth's  $z_3 = 19$ , module  $m = 20$  mm. Driver spur gears 3, that rotate driven spur gear 4, module  $m = 20$  mm of which number of teeth's is  $z_4 = 235$ . The larger spur gear 4 is rigidly connected to a vertical rotor. Gears 3 and 4 lubricated with liquid oil, feeding liquid oil onto the teeth's of the spur gear 4. Used oil drips down to collection pan located under the great spur gear 4 (Figure 2). Rotor is mounted on the support by the spherical roller thrust bearing 5 (SKF294/E30), the twin spherical bearing 7 (SKF 23096 CA/W33), intended for taking over the main radial load, is located under the work wheel 6.

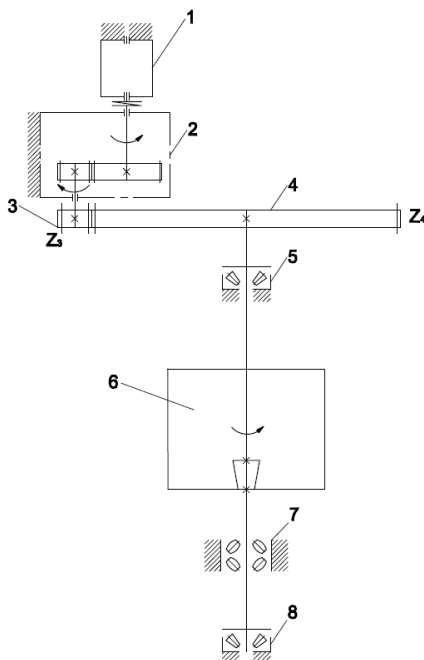


Fig. 1. Diagram of a vertical rotor system under research

The most of the rotor system weight is carried by the bearing 8 (SKF 292/500), mounted in the lower support.

Experimental study was performed with unloaded vertical rotor. The absolute shaft vibrations were measured at the 3 spur gear in a direction perpendicular to rotation axis. Vibrations exited by the spur gear 4, were assessed through measuring the absolute vibrations of the bearing 5 (SKF294/E30) in axial and radial directions. The rotor journal displacements were measured by the great spur gear 4.

The velocity spectrums of the absolute vibrations indicate that 5th bearing is influenced by the vibrations, coming from the gears teeth intermeshing, which is influenced by the damage in involutes gear teeth work surface, coming from a long-term continuous exploitation and the lack of radial stiffness of rotor system supports (Figure 3).

## Theoretical model

After measuring the displacements of multi gear drive rotor journal by the great spur gear 5, the displacement values exceed 1 mm. If an axis of the vertical shaft displaced by the value  $a$ ,

due to insufficient radial stiffness of support 5, shaft axis inclines from a vertical at an angle  $\alpha$ , which is calculated:

$$\alpha = \arctg\left(\frac{a}{b}\right) \quad (1)$$

where:

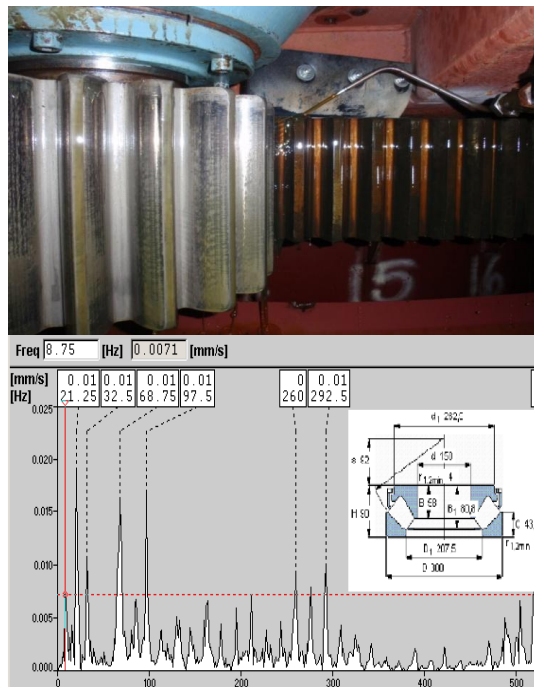
$a$  – measured value of journal displacement, mm;

$b$  – distance from the great spur gear to the top support of the rotor, mm.

Calculated values of the shaft inclination angle  $\alpha$ , indicate that the shaft may incline  $0^\circ 07'$  as shown in Figure 3.

Physical models examining gear teeth as a two parallel cylindrical surface meshing or gear teeth as a two cylindrical surface meshing with two perpendicular axes, are most often used in solving contact problems and analyzing the meshing of the gear teeth.

In the first case, the gear teeth come in contact linearly, in second case when the axes of cylindrical surfaces come in contact perpendicularly, the sphere and surface contact solving method is used.



**Fig. 2.** Driver and driven spur gear exploitation conditions and Vibration velocity spectrum of spur gear intermeshing

When two cylinders with parallel axes come in contact, the maximum contact pressure is calculated:

$$P = \sqrt{\frac{EF}{\pi LR}} \quad (2)$$

where:  $E = \frac{E_1 \cdot E_2}{E_2(1-\nu_1^2) + E_1(1-\nu_2^2)}$ ;  $R = \frac{R_1 \cdot R_2}{R_1 + R_2}$ ;  $F$  – acting force, N;  $L$  – width of the cylinder contact, m.

When two cylindrical surfaces with perpendicular axes come in contact, the contact pressure is calculated:

$$P = \frac{4}{3} \frac{E \cdot \sqrt{R} \cdot \sqrt{d^3}}{\sqrt{Rd}} \quad (3)$$

where:  $E = \frac{E_1 \cdot E_2}{E_2(1-\nu_1^2) + E_1(1-\nu_2^2)}$ ;  $R = \frac{R_1 \cdot R_2}{R_1 + R_2}$ ;  $d$  – penetration, m.

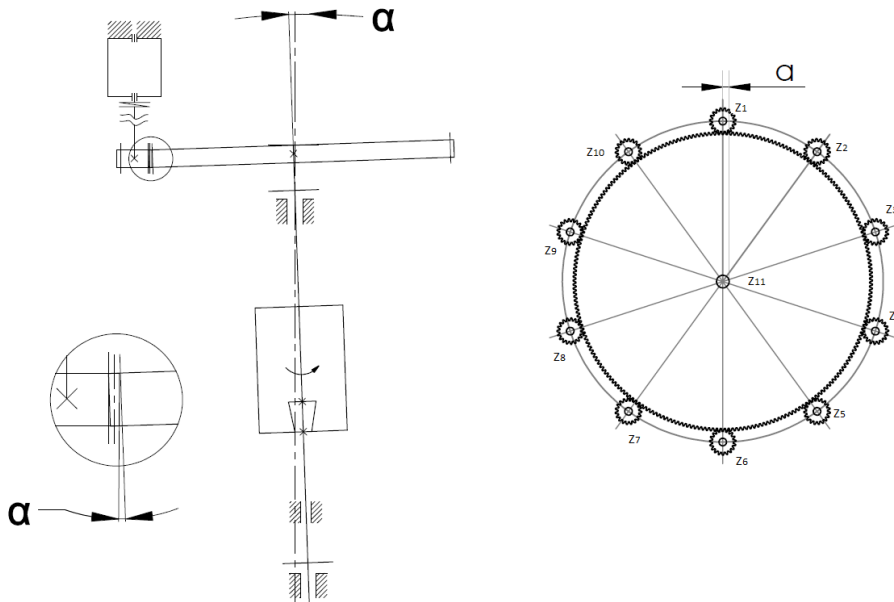


Fig. 3. Physical vertical rotor model

When the axes of contacting cylindrical surfaces form a small angle, the existing contact pressure calculation methods are not appropriate and values are erroneous.

### FEM simulation results

In calculations we used a FEM model (Figure 4) where a driven spur gear is constrained in all directions with only one degree of freedom left (rotation about  $z$  axis), driven spur gear is constrained in all directions with only one degree of freedom left (rotation about  $z$  axis) characterized by high stiffness. This FEM simulation model was used in reference [7].

FEM calculations were carried out in ANSYS software system. In FEM calculations Lagrange augmented contact formulating method was used. CONTA174 and TARGE170 finite elements were used to describe the surfaces in contact. SOLID186 and SOLID187 second order tetrahedral and hexahedral finite elements were used to describe the bodies. The model bodies were mounted in the following order: the driver gear was free to rotate about its axis; other driven gear degrees of freedom were limited. Driver torque was added at the driver gear inner hole; driven gear was also limited in all directions, rotating about driven shaft axis limits attached high stiffness, which offsets driver torque. Gears mashing contact problems using FEM are analyzed in references [5, 6, 7, 8]. Looking at the results of a calculation we see, that the contact stresses, when changing the positions of the spur gears axes evolves very quickly. When the inclination angle of gears axis is  $0.15^\circ$  contact stresses increases almost threefold. This is due to the variation of contact spot. FEM calculation results are given in Table 1.

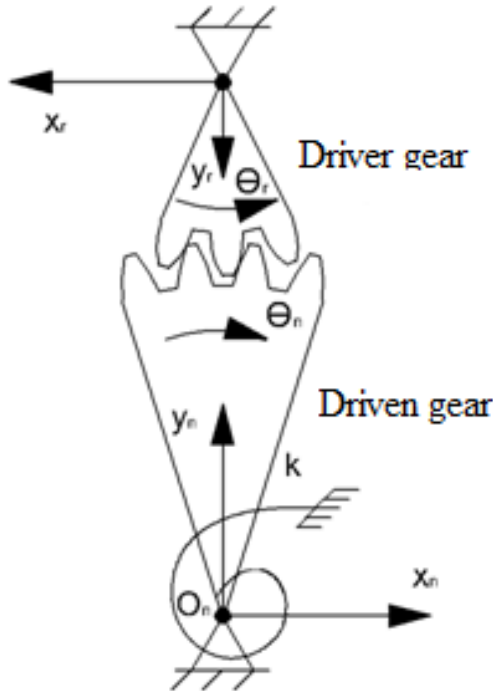


Fig. 4. FEM simulation model

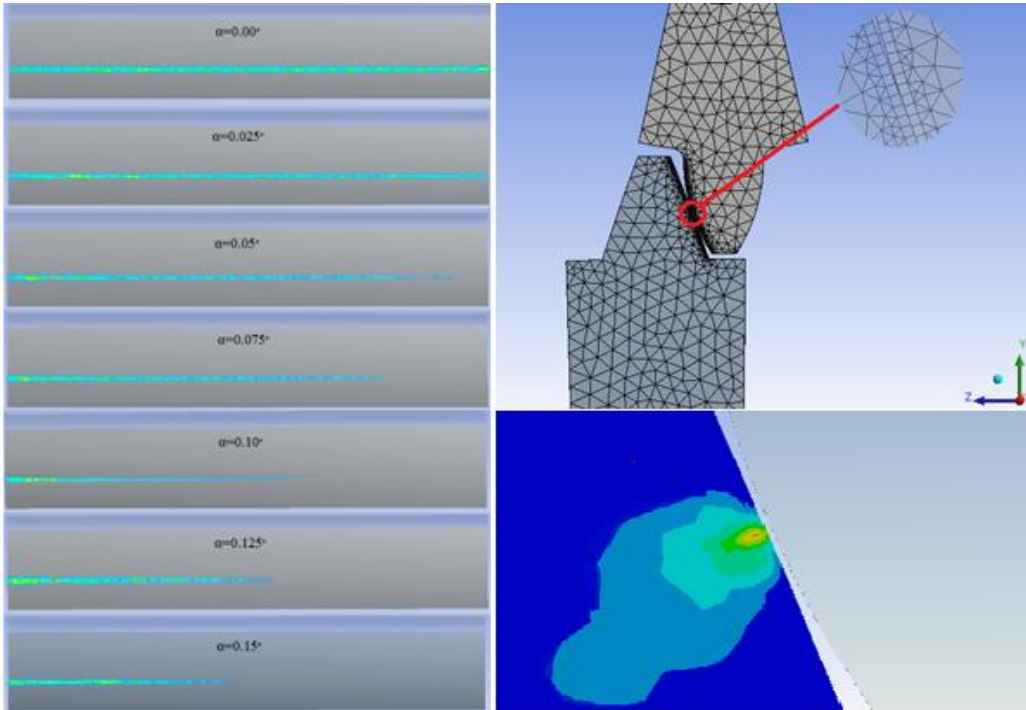
Table 1. FEM calculation results, when the spur gears axis inclination angle is changing from 0,00 to 0,15 degrees

No	Inclination angle $\alpha$ , deg.	Von Misses stress, MPa	Contact stress, MPa	Frictional stress, MPa	Penetration, $m \cdot 10^{-8}$
1	0,000	113,08	243,50	20,95	1,32
2	0,025	149,63	339,44	23,23	1,93
3	0,050	196,05	476,4	27,87	2,57
4	0,075	248,46	548,31	38,33	2,96
5	0,100	289,95	613,67	46,62	3,44
6	0,125	341,16	646,21	56,95	3,78
7	0,150	387,47	685,60	64,29	3,90

When the contact spot goes from the line to the point, the area of contact spot do not change linearly, as well as variations of the surface penetration see Fig. 5. Contact spots and contact stresses changes are shown in the Table 2. The results of calculations of contact with parallel axes of spur gears, the calculations are validated using analytical expressions. Difference between the results of calculations is in no case less than 2%. These Hertz contact mathematical formulations are used to calculate the maximum contact stresses, this formulation used in reference [7]:

$$\sigma_B = \sqrt{\frac{F_N}{abQK_0 \cdot \sin\varphi}}, \quad (4)$$

where:  $F_N$  – normal force at contact, N;  $d$  – the pinch diameter of gear, m;  $b$  – length of contact area, m;  $Q = 2N_v/(N_v + N_r)$  – gear ratio factor,  $N_v$  – number of driver spur gear teeth's,  $N_r$  – number of spur gear teeth's,  $K_0 = (1/E_r + 1/E_v)$ ,  $E_r$  – Young modulus of driver spur gear,  $E_v$  – Young modulus of driven spur gear;  $\varphi$  – contact pressure angle.



**Fig. 5.** Dependence of Contact spot area from shaft inclination angle. Images. Mesh and distribution of stresses at contact

**Table 2.** Dependence of Contact spot area from shaft inclination angle

No.	Inclination angle $\alpha$ , deg.	Contact area, mm <sup>2</sup>	Contact spot, %	Contact stress, MPa	Contact stress compared with normal meshing, %
1	0.000	984.00	100.00	243.50	0.00
2	0.025	773.41	78.60	339.44	39.40
3	0.050	664.29	67.51	476.40	95.65
4	0.075	515.03	52.34	548.31	125.18
5	0.100	445.52	45.28	613.67	152.02
6	0.125	349.91	35.56	646.21	165.38
7	0.150	287.43	29.21	685.60	181.56

Simulating gears meshing with FEM, when varies only radial gap, the stress not develop with that intense as when simulating gears with not parallel axes. The increase of Contact stress exactly calculating using simple, classical, analytical expressions. Simulating gears meshing with FEM, when varies only radial gap, the stress not develop with that intense as when simulating gears with not parallel axes. The increase of Contact stress exactly calculating using simple, classical, analytical expressions.

Simulating gears meshing with FEM, when varies only radial gap, the stress not develop with that intense as when simulating gears with not parallel axes. The increase of Contact stress exactly calculating using simple, classical, analytical expressions. When the material of gears is steel 45 (GOST 1050-88), open-gear, the contact stress allowable  $\sigma_{H \text{ lim}} = 418.18$  MPa. Stress dependence of contact angle of the regression equation estimated aid to critical vertical rotor shaft inclination, due to lack of upper radial bearing stiffness, angle is  $\alpha_{\text{crit}} = 0^{\circ}03'$  then the permissible radial displacement at the driving gear is  $a = 0.3936$  mm.

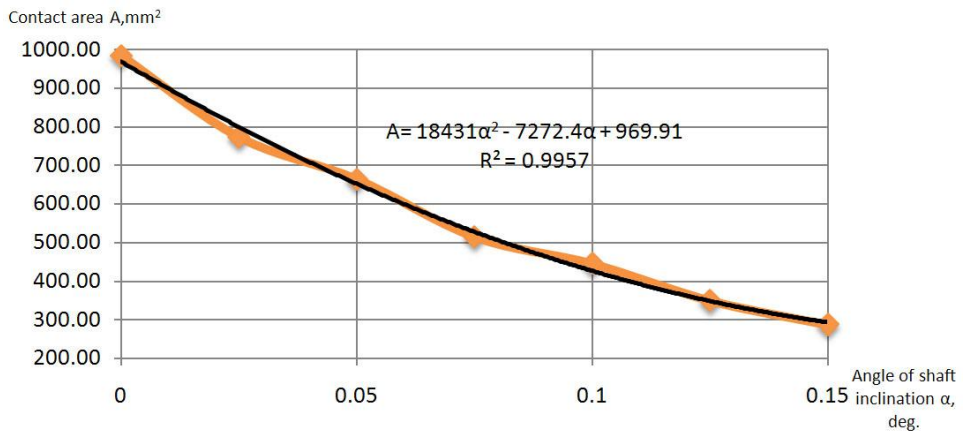


Fig. 6. Dependence of contact spot from rotor inclination angle

### Conclusions and recommendations

Main parameters of irregular involutes gear teeth's meshing and permissible critical meshing parameters used in FEM calculations: permissible critical shaft inclination angle  $\alpha_{crit} = 0^{\circ}03'$ , permissible radial critical shaft deviation from vertical, at the driver spur gear  $a = 0.3936$  mm.

It is estimated that when the spur gears axes are 0.15 deg not parallel, the gears meshing contact stresses grow up almost three times.

In order to protect the driver spur gear working surfaces of the vertical rotor system from potential damage due to system overload and lack of radial stiffness of the upper support, it is recommended to implement a shaft inclination monitoring system.

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