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Fracture of laminated rectangular bar after buckling

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1. Introduction

The delamination of the composites depends on its matrix and changes mechanical characteristics of reinforced elements during deformation. The mechanical behavior of laminated composites during compression is the case when the bending moment appears besides the axial forces. The thread experiences normal stresses and shear stresses [1-3]. Similar works were done while analyzing interfaces of I-beam shelves and walls [4] columns [5] beams [6], and cases of bar buckling depending on their geometry [6, 7, 9]. J. Brewer and P. Langace, M. Fenske and A. Vizzini [9 - 11] suggested the measuring criteria of delamination. Authors [11, 12] were solving the problems of composite fracture. However, the problem of investigating composite delamination remains topical, because the investigations and evaluations of thread remain difficult.

Composite fracture measuring elasticity characteristics for separate layers is analyzed by E. Saouma [13], Z. Gürdal [14]. With mechanical characteristics of separate layers known the measuring of composite fracture is possible. This allows selecting optimal lamination materials while producing bars of significant resistance.

2. Delamination of laminated bars during buckling

In case of buckling, Fig. 1 according to Euler's formula, the critical buckling force is presented as follows

$$F_{cr} = \frac{4\pi^2\left(EI_{ef}\right)}{L^2} \qquad (1)$$

where F_{cr} is critical buckling force; E is modulus of elasticity; L is length of bar; I_{ef} is minimum moment of inertia.

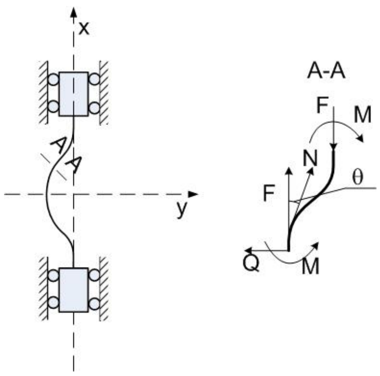

Fig. 1 Bar buckling scheme.

The important characteristic of material is composite modulus of elasticity E_C. It is calculated in the following way [15]

$$E_C = \frac{2t_v E_v + 2t_m E_m + t_f E_f}{t} \qquad (2)$$

where t is thickness of a layer, indexes v, m and f mean cover, thread and filling respectively.

The modulus of elasticity E_v is accepted as resin. Also the composite E_c is received experimentally.

The limitary shear stresses τ_{lim} are calculated in the following way [15]

$$\tau_{lim} = \frac{1}{2} sin2\theta\, \sigma_Y \qquad (3)$$

where σ_Y is yield stress; θ is angle of the layers with regard to stretching axis.

The lateral displacement is calculated as follows [16]

$$w = \frac{w_{max}}{2}\left(\cos\frac{2\pi x}{L} - 1\right) \qquad (4)$$

where w is lateral displacement; x is coordinate in the longitudinal direction of the bar; w_{max} is maximum lateral displacement in the middle part of the bar during delamination.

Thus, when the plate is compressed by F force, the transverse forces Q are obtained in the following way [16]

$$Q = F\sin\theta = F\sqrt{\frac{tan^2\theta}{tan^2\theta + 1}} \qquad (5)$$

where

$$tan\,\theta = \frac{dw}{dx} \qquad (6)$$

In order to evaluate composite strength, various criteria are applied. One of the simplest is Tresca criterion, which evaluates normal stresses and shear stresses [10]

$$\frac{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}{2} \le \tau_{lim} \qquad (7)$$

where σ_x is normal stresses; τ_{xy} is shear stresses.

It is important that normal stresses σ_y in the direction of axis y and shear stresses τ_{yz} on the plane yz are quite small and may not be considered.

Then

$$\tau_{xy} = \frac{dM}{dx}\frac{\int_A E(y) y dA}{bE_{eff}} = Q\frac{\int_A E(y) y dA}{bE_{eff}} \tag{8}$$

where A is area of cross-section; M is bending moment; E_{eff} is elasticity modulus of the laminated bar.

Normal stresses are calculated in the following way

$$\sigma_x = \frac{F\cos\theta}{A\left(V_v + nV_f\right)} \tag{9}$$

where V_v and V_f are volumes of resin and reinforced elements, and n is the ratio of elasticity moduli of reinforcement and matrix.

According to the strength criterion of Mises [15]

$$\sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \sigma_Y \tag{10}$$

where σ_Y is yield stress.

Authors of this paper apply polynomial strength criteria [18]

$$F(\sigma_1, \sigma_2, \tau_{12}) = R_{11}\sigma_1^2 + R_{22}\sigma_2^2 + S_{12}\tau_{12}^2 = 1 \tag{11}$$

where $\tau_{12} = \dfrac{\sigma_1 - \sigma_2}{2}$, R and S are constants, σ_1, σ_2 are principal stresses.

R and S constants are found from the boundary conditions:

$$\left.\begin{array}{l} F(\sigma_1 = \bar{\sigma}_1,\ \sigma_2 = 0,\ \tau_{12} = 0) = 1 \\ F(\sigma_1 = 0,\ \ \sigma_2 = \bar{\sigma}_2,\ \tau_{12} = 0) = 1 \\ F(\sigma_1 = 0,\ \sigma_2 = 0,\ \tau_{12} = \bar{\tau}_{12}) = 1 \end{array}\right\} \tag{12}$$

Then the Eq. (11) is as follows

$$\left(\frac{\sigma_1}{\bar{\sigma}_1}\right)^2 + \left(\frac{\sigma_2}{\bar{\sigma}_2}\right)^2 + \left(\frac{\tau_{12}}{\bar{\tau}_{12}}\right)^2 = 1 \tag{13}$$

The stresses $\bar{\sigma}_1, \bar{\sigma}_2$ are obtained as

$$\left.\begin{array}{l} \bar{\sigma}_1 = \bar{\sigma}_1^+,\ \text{if}\ \ \sigma_1 > 0\ \text{or}\ \ \bar{\sigma}_1 = \bar{\sigma}_1^-,\ \text{if}\ \ \sigma_1 < 0 \\ \bar{\sigma}_2 = \bar{\sigma}_2^+,\ \text{if}\ \ \sigma_2 > 0\ \text{or}\ \ \bar{\sigma}_2 = \bar{\sigma}_2^-,\text{if}\ \ \sigma_2 < 0 \end{array}\right\} \tag{14}$$

When the strength criterion is put in the form [17]

$$\begin{gathered} f(\sigma_1, \sigma_2, \tau_{12}) = R_1\sigma_1 + R_2\sigma_2 + R_{11}\sigma_1^2 + \\ + R_{22}^2\sigma_2^2 + S_{12}\tau_{12}^2 = 1 \end{gathered} \tag{15}$$

the boundary conditions

$$\left.\begin{array}{l} f(\sigma_1 = \bar{\sigma}_1^+, \sigma_2 = 0,\ \tau_{12} = 0) = 1,\ \text{if}\ \ \sigma_1 > 0 \\ f(\sigma_1 = -\bar{\sigma}_1^-, \sigma_2 = 0,\ \tau_{12} = 0) = 1,\text{if}\ \ \sigma_1 < 0 \\ f(\sigma_1 = 0, \sigma_2 = \bar{\sigma}_2^+,\ \tau_{12} = 0) = 1,\ \text{if}\ \sigma_2 = 0 \\ f(\sigma_1 = 0, \sigma_2 = -\bar{\sigma}_2^-,\ \tau_{12} = 0) = 1,\text{if}\ \ \sigma_2 < 0 \\ f(\sigma_1 = 0, \sigma_2 = 0,\ \tau_{12} = \bar{\tau}_{12}) = 1 \end{array}\right\} \tag{16}$$

We write

$$\begin{gathered} \sigma_1\left(\frac{1}{\bar{\sigma}_1^+} - \frac{1}{\bar{\sigma}_1^-}\right) + \sigma_2\left(\frac{1}{\bar{\sigma}_1^+} - \frac{1}{\bar{\sigma}_1^-}\right) + \\ + \frac{\sigma_1^2}{\bar{\sigma}_1^+\bar{\sigma}_1^-} + \frac{\sigma_2^2}{\bar{\sigma}_2^+\bar{\sigma}_2^-} + \left(\frac{\tau_{12}}{\bar{\tau}_{12}}\right)^2 = 1 \end{gathered} \tag{17}$$

According to the experimental tests [17] strength criterion Eq. (17) corresponds the experimental results better than criterion Eq. (13) and even more precisely than criteria Eqs. (7) and (11).

However, the polynomial strength criteria show formal approximation of experimental data in the coordinates of principal axes. These criteria become more complex in other coordinates. Therefore, the tensoric strength criteria are applied. For example, when the orthotropic material moves from the principal axes 1 and 2 to the turned axes 1′ and 2′ at the angle $\varphi = 45^\circ$, the strength criterion is presented in the following way

$$\begin{gathered} f(\sigma_1, \sigma_2, \tau_{12}) = R_1\sigma_1 + R_2\sigma_2 + R_{11}\sigma_1^2 + \\ + R_{12}\sigma_1\sigma_2 + R_{22}\sigma_2^2 + S_{12}\tau_{12}^2 = 1 \end{gathered} \tag{18}$$

When the boundary conditions are applied to obtaining constants, based on the Eq. (16), we obtain

$$\begin{gathered} f(\sigma_1, \sigma_2, \tau_{12}) = \left(\frac{1}{\bar{\sigma}_1^+} - \frac{1}{\bar{\sigma}_1^-}\right)\sigma_1 + \\ + \left(\frac{1}{\bar{\sigma}_2^+} - \frac{1}{\bar{\sigma}_2^-}\right)\sigma_2 + \frac{\sigma_1^2}{\bar{\sigma}_1^+\bar{\sigma}_1^-} + \\ + R_{12}\sigma_1\sigma_2 + \frac{\sigma_2^2}{\bar{\sigma}_2^+\bar{\sigma}_2^-} + \left(\frac{\tau_{12}}{\bar{\tau}_{12}}\right)^2 = 1 \end{gathered} \tag{19}$$

This criterion differs from the criterion Eq. (17) because new constant R_{12} cannot be obtained, according to the conditions of Eq. (16).

Acording the tensoric criterion [18], which is presented in the following way

$$m_1\sigma_i + m_2\sigma_0 \le \sigma_{Y,\mu} \tag{20}$$

where m_1, m_2 are constants materials; $\sigma_{Y\mu}$ is strength limit at μ_σ stress state; σ_i is intensity of stresses (when σ_x is used and τ_{xy}, $\sigma_i = \dfrac{1}{\sqrt{2}}\sqrt{\sigma_x^2 + 3\tau_{xy}^2}$); $\sigma_0 = \dfrac{\sigma_1 + \sigma_2}{3} = \dfrac{\sigma_x}{3}$ is average stress.

Parameter of stress state

$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{2\sigma_2 - \sigma_1}{\sigma_1} = -1$$

(at σ_x and τ_{xy}), i. e. $\sigma_1 = \sigma_{Y,t}$ while stretching, and while compressing when σ_3 stress is used, $\mu_\sigma = +1$ and $\sigma_3 = \sigma_{Y,c}$.

Then criterion Eq. (20) is presented in the following way

$$\frac{1}{\sqrt{2}} m_1 \sqrt{\sigma_x^2 + 3\tau_{xy}^2} + m_2 \frac{\sigma_x}{3} \le \sigma_{Y,c} \tag{21}$$

With criterion Eq. (21) given in nonlinear form

$$m_3\left(\sigma_x^2 + 3\tau_{xy}^2\right) + m_4\sigma_x^2 \le \sigma_{Y,c}^2 \tag{22}$$

we obtain

$$\left(m_3 + m_4\right)\sigma_x^2 + m_3\tau_{xy}^2 \le \sigma_{Y,c}^2 \tag{23}$$

In order to solve the delamination problem of a composite, authors of the paper apply strength criterion Eq. (23). Considering Eqs. (3) and (23), the strength criterion is presented in the following way

$$\sigma_x^2\left(m_3 + m_4 + \frac{1}{2} m_3 \sin^2 2\theta\right) \le \sigma_{Y,c}^2 \tag{24}$$

With the angle $\theta = 45^\circ$, we obtain net shear and $\sigma_x = \tau_{lim} = \dfrac{\sigma_{Y,c}}{2}$, and in the case when the angle is $\theta = 0$, we obtain axial compression and $\sigma_x = \sigma_{Y,c}$. Then the constants m_3 and m_4 in the Eq. (23) must be calculated using these equations

$$\begin{cases} m_3 + m_4 + \frac{1}{4} m_3 = 2 \\ m_3 + m_4 = 1 \end{cases} \tag{25}$$

From where $m_3 = 4$; $m_4 = -3$.

Thus, the strength criterion Eq. (23) is presented in the following way

$$\sigma_x^2\left(1 + \sin^2 2\theta\right) \le \sigma_{Y,c}^2 \tag{26}$$

or

$$\sigma_x \le \sqrt{\frac{\sigma_{Y,c}^2}{1 + \sin^2 2\theta}} \tag{27}$$

Applying strength criterion in buckling the following value is calculated

$$\sigma_x \le \sqrt{\frac{\sigma_{cr,b}^2}{1 + \sin^2 2\theta}} \tag{28}$$

where $\sigma_{x,b}$ is buckling stresses; $\sigma_{cr,b}$ is critical buckling stresses.

However, in order to observe fracture case while buckling the following values are necessary as $\sigma_{cr,b} = \sigma_{Y,c}$. That way considering Eq. (9) after taking buckling force from the Eq. (1) and performing the operations, the following formula is obtained

$$L_{cr}^4 = \frac{16\pi^4\left(EJ_{ef}\right)^2\left(1 + \sin^2 2\theta_{cr}\right)\cos^2\theta_{cr}}{A^2\left(V_r + nV_f\right)^2\sigma_{Y,c}^2} \tag{29}$$

This formula determines the relation between values of length L_{cr} and shear angle θ_{cr} with straight bar or bar made from composite being buckled.

3. Regularities of spreading interlayer fracture

Interlayer of laminar material suffers normal σ_{yy} and tangential τ_{xy} stresses in Fig. 2.

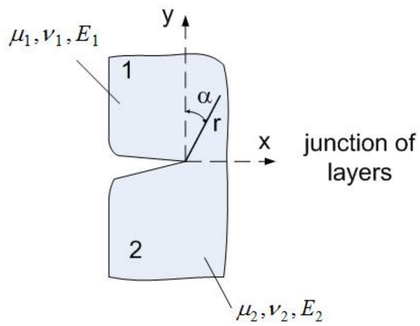

Fig. 2 Fracture geometry at layer junction

Referring to studies of Victor E. Saouma [13], in case of flat deformation relative fracture energy G is calculated as follows

$$G = \frac{\left(1/\overline{E}_1 + 1/\overline{E}_2\right)\left(K_1^2 + K_2^2\right)}{2\cosh^2(\pi\varepsilon)} \tag{30}$$

where

$$\overline{E}_1 = E_1/\left(1 - \nu_1^2\right),\ \overline{E}_2 = E_2/\left(1 - \nu_2^2\right) \tag{31}$$

E_1, E_2 are moduli of layer elasticity; ν_1, ν_2 are Poisson's ratios for the layer; K_1, K_2 are intensity ratios for layer stresses; ε is variable calculated as follows

$$\varepsilon = \frac{1}{2\pi} \ln\left(\frac{1-\beta}{1+\beta}\right) \tag{32}$$

β is parameter of elasticity loss calculated as follows

$$\beta = \frac{\mu_1(1 - 2\nu_2) - \mu_2(1 - 2\nu_1)}{2\left[\mu_1(1 - \nu_2) + \mu_2(1 - \nu_1)\right]} \tag{33}$$

where μ_1, μ_2 are shear moduli for layers.

Stress intensity ratios K_1 and K_2 calculated as follows [18]

$$K_1 = \left(\frac{\sigma_x\left[\cos(\varepsilon \log 2a) + 2\varepsilon \sin(\varepsilon \log 2a)\right]}{\cosh(\pi\varepsilon)} + \right.$$

$$+\frac{\tau_{xy}\left[sin\left(\varepsilon log2a\right)-2\varepsilon cos\left(\varepsilon log2a\right)\right]}{cosh\left(\pi\varepsilon\right)}\Bigg)\sqrt{a} \tag{34}$$

$$K_2=\left(\frac{\tau_{xy}\left[cos\left(\varepsilon log2a\right)+2\varepsilon sin\left(\varepsilon log2a\right)\right]}{cosh\left(\pi\varepsilon\right)}-\right.$$

$$\left.-\frac{\sigma_x\left[sin\left(\varepsilon log2a\right)-2\varepsilon cos\left(\varepsilon log2a\right)\right]}{cosh\left(\pi\varepsilon\right)}\right)\sqrt{a} \tag{35}$$

Marked $cos\left(\varepsilon\, log2a\right)=B$; $2\varepsilon\, sin\left(\varepsilon\, log2a\right)=C$; $sin\left(\varepsilon\, log2a\right)=D$; $2\varepsilon cos\left(\varepsilon log2a\right)=H$; $cosh\left(\pi\varepsilon\right)=J$.

The following equations are presented

$$K_1=\frac{\sigma[B+C]+\tau[D-H]}{J}\sqrt{a} \tag{36}$$

$$K_2=\frac{\tau[B+C]-\sigma[D-H]}{J}\sqrt{a} \tag{37}$$

Considering that buckling presents critical stresses calculated after the Eq. (30)

$$\sigma_x=\sigma_c=\sqrt{\frac{\sigma_{cr,b}^2}{1+sin^2 2\theta}}$$

Tangential stresses calculated after Eq. (3)

$$\tau_{xy}=\tau_c=\frac{1}{2}sin\,2\theta\sigma_x.$$

Eqs. (36) and (37) presented as follows

$$K_{1,c}=\left(\frac{\sqrt{\frac{\sigma_{cr,b}^2}{1+sin^2 2\theta}}\left[B+C\right]}{J}+\frac{\frac{1}{2}sin2\theta\sigma_x\left[D-H\right]}{J}\right)\sqrt{a} \tag{38}$$

$$K_{2,c}=\left(\frac{\frac{1}{2}sin2\theta\sigma_x\left[B+C\right]}{J}-\frac{\sqrt{\frac{\sigma_{cr,b}^2}{1+sin^2 2\theta}}\left[D-H\right]}{J}\right)\sqrt{a} \tag{39}$$

Therefore, values G_c considering Eq. (30) are obtained as follows

$$G_c=\frac{\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}}{2J^2}\times\left(\left(\frac{\sqrt{\frac{\sigma_{cr,b}^2}{1+sin^2 2\theta}}\left[B+C\right]+\frac{1}{2}sin2\theta\sigma_x\left[D-H\right]}{J}\sqrt{a}\right)^2+\right.$$

$$\left.+\left(\frac{\frac{1}{2}sin2\theta\sigma_x\left[B+C\right]-\sqrt{\frac{\sigma_{cr,b}^2}{1+sin^2 2\theta}}\left[D-H\right]}{J}\sqrt{a}\right)^2\right) \tag{40}$$

After certain operations in Eq. (40) the following equation is obtained

$$G_c=\frac{1}{2J^4}\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)a\left[\left(B+C\right)^2+\left(D-H\right)^2\right]\times$$

$$\times\left(\frac{\sigma_{cr,b}^2}{1+sin^2 2\theta}+\frac{1}{4}sin^2\theta\sigma_x^2\right) \tag{41}$$

Eq. (40) has a short form presented as the following equation

$$G_c=Z\left(\frac{\sigma_{cr,b}^2}{1+sin^2 2\theta}+\frac{1}{4}sin^2\theta\sigma_x^2\right)a \tag{42}$$

where

$$Z=\frac{\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)\left[\left(B+C\right)^2+\left(D-H\right)^2\right]}{2J^4} \tag{43}$$

Further, dependence of fracture energy on angle θ is analyzed.

Several edge cases are:

$\theta=0^{\circ}$, $G_c=Z\sigma_{cr,b}^2 a$;

$\theta=30^{\circ}$, $G_c=Z\left(\frac{\sigma_{cr,b}^2}{1.25}+\frac{1}{16}\sigma_x^2\right)a$;

$\theta=45^{\circ}$, $G_c=Z\left(\frac{\sigma_{cr,b}^2}{1.5}+\frac{1}{8}\sigma_x^2\right)a$.

4. Determination of strength and fracture characteristics

In order to perform experimental tests, the composite bar of thickness 12 mm was chosen. Laminated by $t_v=0.5$ mm cover, resin thickness $t_m=2$ mm, and fiberglass thickness $t_f=7$ mm. This makes relative volume of filling $V_f=0.62$, and one of matrix V_r is 0.35. Modulus of elasticity are the following: filling is $E_f=45$ GPa, resin is $E_m=11$ GPa, cover $E_v=E_m=11$ GPa. Thus, total modulus of elasticity received from the Eq. (2) makes

$E = 30.89\,\text{GPa}$. E_f and E_m proportion is $n = E_f / E_m = 4.09$. According to ASTM D 638, sample width is 12.7 mm, $\sigma_{Y,c} = 3000\,\text{MPa}$.

Cross-section area is
$A = 152.4 \cdot 10^{-6}\,\text{m}^2$.
Area moment of inertia

$$I_{ef} = I_{\min} = \frac{bh^3}{12} = 2.048 \cdot 10^{-9}\,\text{m}^4.$$

Strength limit of compression
$\sigma_{cr,c} = \sigma_{Y,c} = 3000\,\text{MPa}$

and $EI_{ef} = 63\,\text{N}\cdot\text{m}^2$.

Entered the values of experimental and calculated parameters into the formula (29) the following is obtained:

$$L_{cr} = 1.28\sqrt[4]{1+sin^2 2\theta_{cr}}\,\sqrt{cos\theta_{cr}} \tag{44}$$

Table
Dependencies of critical delamination angles and plate lengths

No.	θ_{cr}, degrees	L_{cr}, m
1	0	1.28
2	5	1.286
3	10	1.308
4	28	1.3705
5	30	1.39
6	32	1.367
7	40	1.103
8	45	1.076

According to Table, maximum critical length of the bars is received with the delamination angle 30^0.

With this angle maximum resistance stratification is obtained, and minimum resistance stratification with $\theta = 45^\circ$.

Minimum critical length given by $\theta = 45^\circ$, and critical value of fracture energy are applied in this case. Further, fracture regularities are analyzed.

With $\theta = 45^\circ$ L_{cr} Eq. (42) presents

$$G_c = Z\left(0.6666\sigma_{cr,b}^2 + 0.125\sigma_x^2\right)a \tag{45}$$

with $\sigma_x = \sigma_{cr,b}$,

$$G_c = 0.792 Z\sigma_{cr,b}^2 a \tag{46}$$

Consequently, critical value of fracture energy is described by material characteristics Z and $\sigma_{cr,b}$, that depends on fracture length a. $G_c - a$ dependence for analyzed glass plastic bar presented in Fig. 3.

The obtained dependences allow measuring critical values of relative energy with various approximate thread lengths and active stress known. Therefore, practical observing thread length and known active stresses allows foreseeing after critical energy value if the fracture spreads further causing construction failure or the thread remains constant (Fig. 4)..

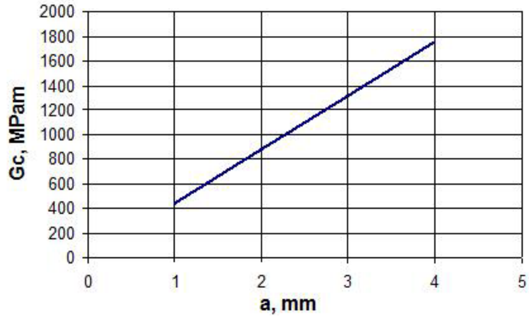

Fig. 3 Dependence of critical fracture energy on thread length

With $G_c = const$ stresses σ_x depend on thread a in Fig. 4.

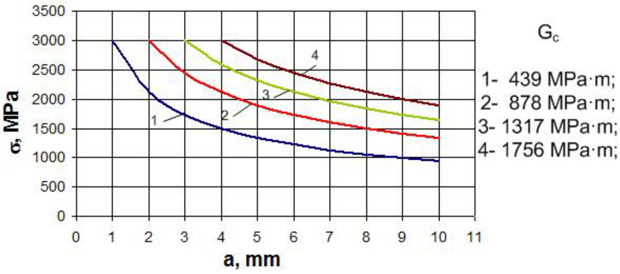

Fig. 4 Dependence of fracture stresses on thread length

5. Conclusions

1. Delamination of composite constructional elements is determined by normal and shear stresses in the thread.

2. Strength criteria used to evaluate composite strength are too complex because of big number of constants and their difficult determination.

3. The nonlinear strength criterion suggested by the author in case of complex state of stresses allows obtaining engineeringly simple dependency between critical delamination angles and critical bar lengths at buckling.

4. According to the experimental and calculation data, minimum critical length of the bar at buckling is obtained with the delamination angle 45°.

5. Measuring elasticity characteristics for separate layers critical fracture energy is calculated after suggested formulas.

6. Having critical values of fracture energy further possibilities of fracture are foreseen after thread length and stresses.

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LAMINUOTO STAČIAKAMPIO STRYPO IRIMAS PO KLUPDYMO

R e z i u m ė

Kompozitinių konstrukcinių elementų atsisluoksniavimas nustatomas pagal normalinius ir šlyties įtempius. Darbe autoriai siūlo taikyti netiesinį stiprumo kriterijų esant sudėtingiems įtempiams. Po klupdymo laminuoto strypo plyšio plitimas aprašomas sudėtiniais irimo kriterijais. Parodoma irimo energijos priklausomybė nuo sluoksnių mechaninių charakteristikų bei tarpsluosknio pasipriešinimo šlyčiai. Gauti teoriniai sprendiniai patvirtinti eksperimentiniais tyrimais, nagrinėjant stikloplastikį.

A. Ziliukas, A. Malatokiene

FRACTURE OF LAMINATED RECTANGULAR BAR AFTER BUCKLING

S u m m a r y

The delamination of composite constructional elements is determined by normal stresses and shear stresses. The non-linear strength criterion is suggested by the authors in case of complex state of stress. Fracture of laminate bar after buckling is described by mixed fracture spreading regularities. Dependence of fracture energy on mechanical characteristics of the layers and shear resistance of the interlayer is presented. The obtained theoretical values are based on experimental investigation of glass plastic.

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