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Comparative Investigation of Feed-Forward Control Algorithms

T. Tekorius, D. Levisauskas
Process Control Department, Kaunas University of Technology, Lithuania,
Studentų str. 50-162, 51368 Kaunas, Lithuania, phone: +370 37 350721, e-mail: tomas.tekorius@ktu.lt

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Introduction

Feed-forward control is an effective way to reject the controlled process disturbances [1, 2]. When the disturbances are measured, control actions can be generated in an attempt to counteract the effects of the disturbances before they have influenced the process.

The most common engineering approach to design the feed-forward controllers is based on using first order plus time delay transfer functions as approximation to describe how the manipulated variable and the disturbance affect the control variable [3]. By applying the engineering design method, the feed-forward controller realization problem arises, when time delay in the disturbance path is less than time delay in process path. In this case, the negative dead-time term in feed-forward controller cannot be implemented, and it is advised simply to ignore this term [3, 4]. In this work, we investigate performance of the feed-forward controllers, in which the negative dead-time term is replaced with the Padé approximation. Performance results of the extended feed-forward controller are compared with those of ordinary steady-state and dynamic feed-forward controllers.

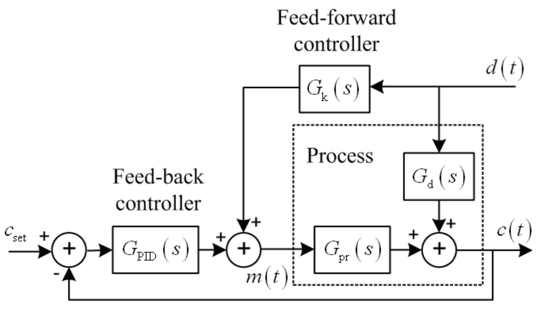

Fig. 1. Feed-forward/feed-back control system (c_{set} – set point, $d(t)$ – disturbance, $m(t)$ – manipulated variable, $c(t)$ – controlled variable)

The feed-forward controllers are usually applied in feed-back/feed-forward control systems, therefore, efficiency of the feed-forward control is investigated by simulation of a performance of the feed-back/feed-forward control system (Fig. 1) with various feed-forward controllers introduced.

Design of feed-forward controller

A simplified engineering method for the feed-forward controller design refers to first order plus time delay approximations of the controlled process dynamics with respect to control and disturbance actions:

$$G_{pr}(s) = \frac{K_{pr}\exp(-\tau_{pr}s)}{T_{pr}s+1}, \quad (1)$$

$$G_d(s) = \frac{K_d\exp(-\tau_d s)}{T_d s+1}, \quad (2)$$

where $G_{pr}(s)$ and $G_d(s)$ are transfer function models with respect to control and disturbance actions, respectively; K_*, T_* and τ_* are resultant gain, time constant and time delay, respectively.

The design formula of the feed-forward controller is [3]

$$G_k(s) = -\frac{G_d(s)}{G_{pr}(s)} = -\frac{K_d}{K_{pr}}\cdot\frac{T_{pr}s+1}{T_d s+1}\exp\left(-\left(\tau_d-\tau_{pr}\right)s\right). \quad (3)$$

The transfer function (3) of the feed-forward controller contains three terms. The first term compensates for the steady-state differences between G_d and G_{pr} paths. The second term (lead/lag) compensates for the differences in time constants between the two paths. The last term is a dead-time compensator that compensates for the differences in dead-time between the two paths. Sometimes the term $(\tau_d-\tau_{pr})$ may be negative, yielding a positive exponent, and cannot be implemented. In present work, we investigate the feed-forward control realized by replacing of dead-time term with the Padé approximation

$$\exp(\tau s) \approx 1+\tau s. \quad (4)$$

In this work, performances of four structures of feed-forward controllers were investigated and compared:

I. Steady-state feed-forward

$$G_{\mathrm{k}}^{\mathrm{I}}(s) = -\frac{K_{\mathrm{d}}}{K_{\mathrm{pr}}}. \quad (5)$$

II. Dynamic feed-forward

$$G_{\mathrm{k}}^{\mathrm{II}}(s) = -\frac{K_{\mathrm{d}}}{K_{\mathrm{pr}}} \cdot \frac{T_{\mathrm{pr}}s+1}{T_{\mathrm{d}}s+1}. \quad (6)$$

III. Modified dynamic feed-forward [3]

$$G_{\mathrm{k}}^{\mathrm{III}}(s) = -\frac{K_{\mathrm{d}}}{K_{\mathrm{pr}}} \cdot \frac{(T_{\mathrm{pr}}+\tau)s+1}{T_{\mathrm{d}}s+1}. \quad (7)$$

IV. Dynamic feed-forward in series with Padé approximation

$$G_{\mathrm{k}}^{\mathrm{IV}}(s) = -\frac{K_{\mathrm{d}}}{K_{\mathrm{pr}}} \cdot \frac{T_{\mathrm{pr}}s+1}{T_{\mathrm{d}}s+1} \cdot (1+\tau s), \quad (8)$$

where $\tau = \tau_{\mathrm{pr}} - \tau_{\mathrm{d}} > 0$.

Simulation results

Performance of the feed-forward control algorithms in a feed-back/feed-forward control system was investigated via numerical simulation implemented in Matlab/Simulink environment. Several transfer function models were generated to represent dynamics of controlled process with respect to control variable and disturbance (Table 1).

Table 1. Transfer function models applied in simulation experiments

$G_*(s)$	**Transfer function**
$G_{\mathrm{pr1}}(s)$	$\frac{1}{(22s+1)}\exp(-3s)$
$G_{\mathrm{pr2}}(s)$	$\frac{1}{(2s+1)(5s+1)(19s+1)}$
$G_{\mathrm{pr3}}(s)$	$\frac{1}{(s+1)^2(15s+1)}\exp(-3s)$
$G_{\mathrm{pr4}}(s)$	$\frac{1}{(2s+1)^2(12s+1)}\exp(-2s)$
$G_{\mathrm{pr5}}(s)$	$\frac{1}{(0.5s+1)(2s+1)(9.5s+1)}\exp(-s)$
$G_{\mathrm{d1}}(s)$	$\frac{1}{(4s+1)(6s+1)}\exp(-0.6s)$
$G_{\mathrm{d2}}(s)$	$\frac{1}{(s+1)^2(6s+1)}\exp(-0.3s)$
$G_{\mathrm{d3}}(s)$	$\frac{1}{(0.5s+1)(s+1)(5s+1)}\exp(-0.2s)$
$G_{\mathrm{d4}}(s)$	$\frac{1}{(0.1s+1)(s+1)(4s+1)}\exp(-0.1s)$

The variants of transfer functions, applied in the simulation experiments of control systems performance, are given in Table 2.

The feed-back controller (PID) was adjusted for disturbance compensation by applying tuning rules developed for Integral of Absolute Error (IAE) criterion [5]. The feed-forward control algorithm (8) in simulation experiments is realized as a lead-lag compensator in series with PD controller, transfer function of which coincides with the Padé approximation term (4).

Simulated performances of the investigated feed-forward control algorithms (5)-(8) are evaluated by calculated values of the IAE criterion.

Table 2. Compensator parameters

Variant No	$G_{\mathrm{pr}}(s)$	$G_{\mathrm{d}}(s)$	**Compensator parameters**			
			$K_{\mathrm{d}}/K_{\mathrm{pr}}$	T_{pr}	T_{d}	τ
1	$G_{\mathrm{pr1}}(s)$	$G_{\mathrm{d1}}(s)$	1	22.00	7.98	0.063
2	$G_{\mathrm{pr1}}(s)$	$G_{\mathrm{d2}}(s)$	1	22.00	6.36	0.95
3	$G_{\mathrm{pr1}}(s)$	$G_{\mathrm{d3}}(s)$	1	22.00	5.29	1.51
4	$G_{\mathrm{pr1}}(s)$	$G_{\mathrm{d4}}(s)$	1	22.00	4.28	2.01
5	$G_{\mathrm{pr2}}(s)$	$G_{\mathrm{d1}}(s)$	1	20.63	7.98	2.85
6	$G_{\mathrm{pr2}}(s)$	$G_{\mathrm{d2}}(s)$	1	20.63	6.36	3.74
7	$G_{\mathrm{pr2}}(s)$	$G_{\mathrm{d3}}(s)$	1	20.63	5.29	4.30
8	$G_{\mathrm{pr2}}(s)$	$G_{\mathrm{d4}}(s)$	1	20.63	4.28	4.80
9	$G_{\mathrm{pr3}}(s)$	$G_{\mathrm{d1}}(s)$	1	15.17	7.98	1.94
10	$G_{\mathrm{pr3}}(s)$	$G_{\mathrm{d2}}(s)$	1	15.17	6.36	2.84
11	$G_{\mathrm{pr3}}(s)$	$G_{\mathrm{d3}}(s)$	1	15.17	5.29	3.39
12	$G_{\mathrm{pr3}}(s)$	$G_{\mathrm{d4}}(s)$	1	15.17	4.28	3.90
13	$G_{\mathrm{pr4}}(s)$	$G_{\mathrm{d1}}(s)$	1	12.74	7.98	2.53
14	$G_{\mathrm{pr4}}(s)$	$G_{\mathrm{d2}}(s)$	1	12.74	6.36	3.42
15	$G_{\mathrm{pr4}}(s)$	$G_{\mathrm{d3}}(s)$	1	12.74	5.29	3.98
16	$G_{\mathrm{pr4}}(s)$	$G_{\mathrm{d4}}(s)$	1	12.74	4.28	4.48
17	$G_{\mathrm{pr5}}(s)$	$G_{\mathrm{d1}}(s)$	1	10.00	7.98	0.19
18	$G_{\mathrm{pr5}}(s)$	$G_{\mathrm{d2}}(s)$	1	10.00	6.36	1.09
19	$G_{\mathrm{pr5}}(s)$	$G_{\mathrm{d3}}(s)$	1	10.00	5.29	1.64
20	$G_{\mathrm{pr5}}(s)$	$G_{\mathrm{d4}}(s)$	1	10.00	4.28	2.15

In the simulation experiments, the IAE criterion values were calculated for the controlled process variants, presented in Table 2, and the investigated feed-forward controllers (I-IV). Performances of the control systems were investigated for compensation step, ramp and sinusoidal type disturbances. Fig. 2 illustrates the simulated responses of control system to the investigated disturbances, corresponding to the controlled process variant No 18 in Table 2. The improvement provided by the feed-forward controller IV is certainly noticeable. Calculation results of the IAE criteria for all investigated variants are given in Fig. 3.

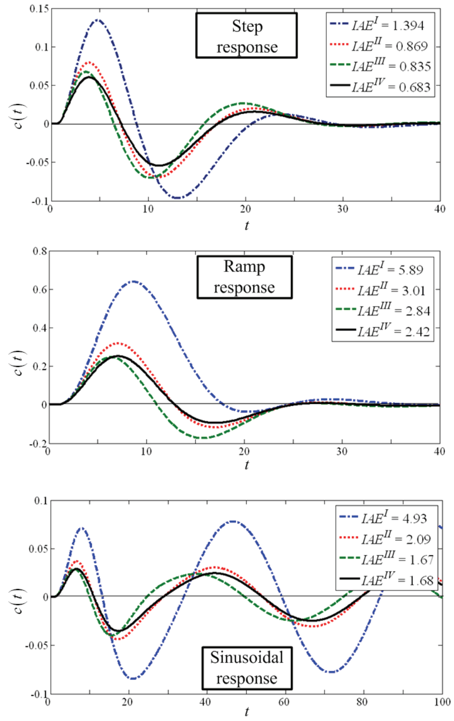

Fig. 2. Comparison of feed-back and feed-forward control to a step, ramp and sinusoidal disturbances (variant No18 in Table 2)

Significance of changes in the control system performance with the investigated feed-forward controllers is assessed by application of the statistical test [6]. The hypothesis is tested if the ratio

$$X = \frac{IAE^{*}}{IAE^{II}} \tag{9}$$

has mean $\mu = 1$.

The used test statistic is

$$T = \frac{\overline{X} - \mu}{S}\sqrt{n}\,, \tag{10}$$

where $\overline{X}$ is the mean of sample of size $n = 20$, and S is the standard deviation of sample.

The critical value of $|T|$ at the significance level $\alpha = 0.05$ is $t_{1-\alpha/2} = 2.093$. If $|T| > t_{1-\alpha/2}$, the difference in control performance results is assumed to be significant. Results of the statistical test are presented in Table 3.

The simulation results demonstrate that the dynamic feed-forward control algorithms (6)-(8) outperform the steady-state control algorithm (5). The extended control algorithm (8) demonstrates somewhat better performance as compared to that of the ordinary (6) and the modified (7) lead-lag control algorithms.

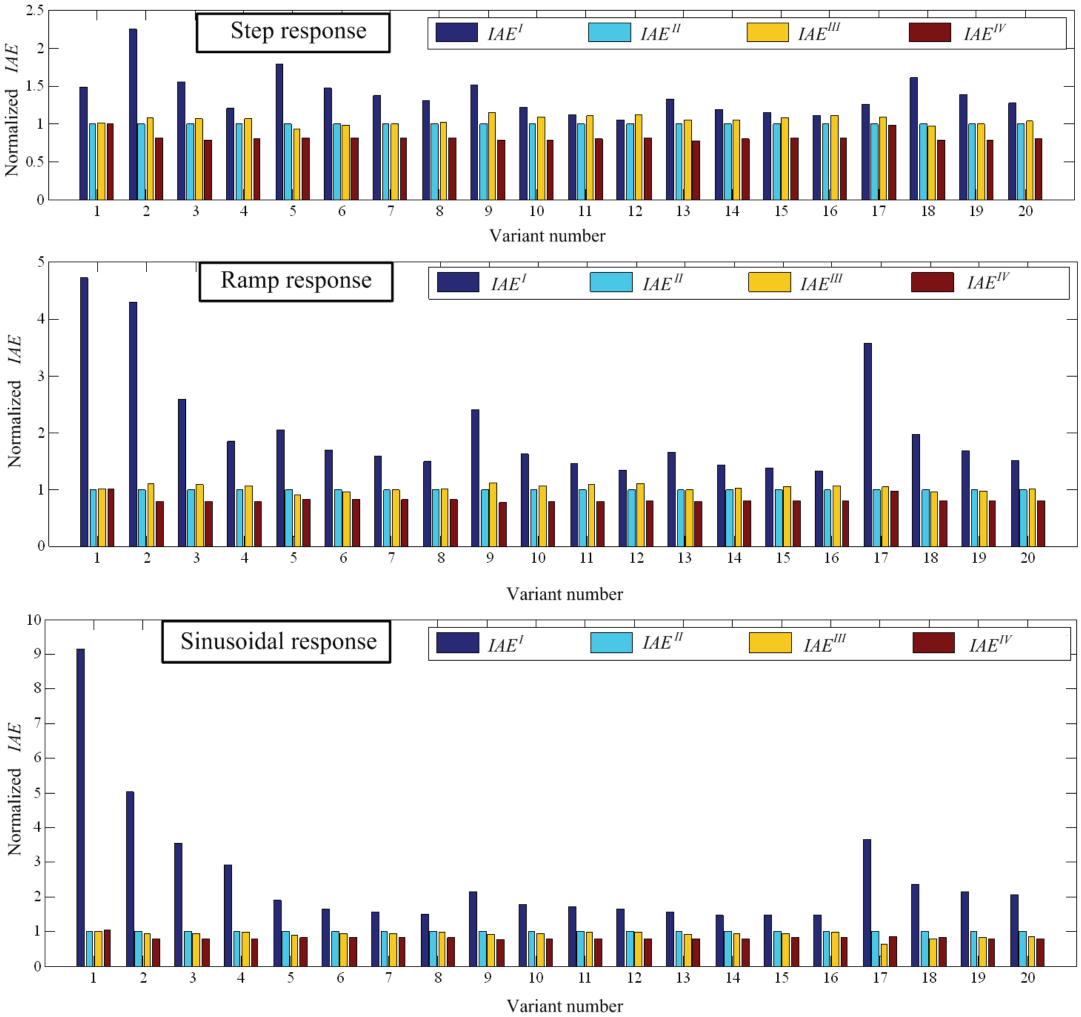

Fig. 3. Simulation results normalized with respect to IAE^{II} value

Table 3. Results of the statistical test

Response type	X	$\overline{X}$	$\lvert T \rvert$	Conclusion
Step	$\frac{IAE^{III}}{IAE^{II}}$	1.047	3.740	$IAE^{III} > IAE^{II}$ significant
	$\frac{IAE^{IV}}{IAE^{II}}$	0.820	13.775	$IAE^{IV} < IAE^{II}$ significant
Ramp	$\frac{IAE^{III}}{IAE^{II}}$	1.029	2.260	$IAE^{III} > IAE^{II}$ significant
	$\frac{IAE^{IV}}{IAE^{II}}$	0.816	13.889	$IAE^{IV} < IAE^{II}$ significant
Sinusoidal	$\frac{IAE^{III}}{IAE^{II}}$	0.913	4.712	$IAE^{III} < IAE^{II}$ significant
	$\frac{IAE^{IV}}{IAE^{II}}$	0.812	15.321	$IAE^{IV} < IAE^{II}$ significant

Conclusions

Feed-forward controllers of different structures are investigated for controlling processes, in which the time delay in the disturbance path is less than that in process path. Four types of control algorithms were investigated via computer simulation of feed-back / feed-forward control system performance at step, ramp and sinusoidal type disturbances. The performance index for comparison the transient responses in control systems was the integral of absolute error. Statistics of the simulation results refers to the control systems performance tests realized for 20 variants of controlled processes with different transfer functions of the disturbance and the process paths. Investigation results show statistically significant improvement of the control system performance with the dynamic feed-forward in series with PD control algorithm compared with the steady-state, the dynamic and the modified dynamic control algorithms. The mean of performance index of the modified dynamic control algorithm is slightly better as compared to the ordinary dynamic feed-forward control, however, in some tests the contrary results are observed.

The dynamic feed-forward controller with the additional PD control action for compensation the negative dead-time is easy realizable and can be usefully applied in the feed-back/feed-forward control systems.

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The paper presents investigation results of four different structures of feed-forward control algorithms applied for controlling processes, in which the time delay in the disturbance path is less than that in process path. The control algorithms were investigated via computer simulation of feed-back / feed-forward control system performance at various type disturbances and dynamical parameters of controlled process. The performance index for comparison of the transient responses in the control systems was the integral of absolute error. The investigation results show statistically significant improvement of the control system performance by using the dynamic (lead-lag) algorithm connected in series with PD control algorithm in a feed-forward controller as compared to the steady-state, the ordinary dynamic and the modified dynamic feed-forward control algorithms. Ill. 3, bibl. 6, tabl. 3 (in English; abstracts in English and Lithuanian).

T. Tekorius, D. Levišauskas Tiesioginio ryšio valdymo algoritmų lyginamasis tyrimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 9(115). – P. 79–82.

Straipsnyje pateikti keturių skirtingų struktūrų tiesioginio ryšio valdymo algoritmų, naudojamų valdyti procesams, kuriuose trikdančiojo poveikio kanalo vėlinimo trukmė yra trumpesnė už valdymo kanalo vėlinimo trukmę, tyrimo rezultatai. Valdymo algoritmai buvo tiriami modeliuojant tiesioginio ir grįžtamojo ryšio valdymo sistemos darbą, veikiant valdomą procesą įvairių tipų trikdantiesiems poveikiams ir esant įvairiems proceso dinaminiams parametrams. Kaip kokybės rodiklis valdymo sistemų reakcijoms palyginti naudojamas absoliučiosios paklaidos integralas. Tyrimo rezultatai rodo statistiškai reikšmingą valdymo sistemos darbo kokybės pagerėjimą tiesioginio ryšio reguliatoriuje naudojant nuosekliai sujungtus dinaminį (spartinimo ir vėlinimo) ir PD valdymo algoritmus, palyginti su nusistovėjusios būsenos, įprastu dinaminiu ir modifikuotu dinaminiu algoritmais. Il. 3, bibl. 6, lent. 3 (anglų kalba; santraukos anglų ir lietuvių k.).