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Equivalent Source Method for Calculation of Periodic Commutation Electric Circuits

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Introduction

The aim of the work is to formulate and substantiate the main calculation principles of parameters of cage rotor induction motors supplied from frequency converters. In this case the motor current, rotation speed and other power-related parameters significantly differ from those parameters which are present when the power is supplied from the power grid of the same frequency and the same voltage. This happens because not only one single sine voltage exists at the output of the frequency converter, as in the grid case, but entire spectrum of voltages. Modern frequency converters with IGBT switching keys generate pulse width sine modulation phase voltage. The voltage variation law under the load of active type is shown in Fig. 1 [1].

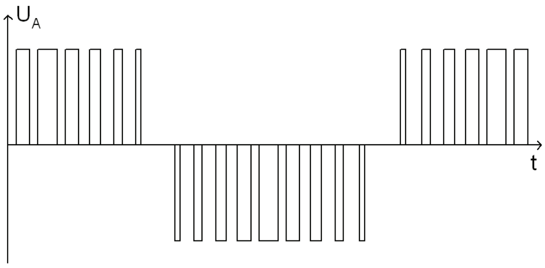

Fig. 1. Phase voltage at the output of the inverter under active load

Since the induction motor is not an active load, magnetic fields are created inside of it and their variation due to the switching excites the induced voltages. Therefore the voltage of the motor clamps at those moments when the respective outputs are switched off is not equal to zero. The magnitude of the induced voltages depends not only on the inverter but also on the parameters and operation mode of the connected motor.

To illustrate this fact we present the current and voltage oscillograms (Fig. 2 and Fig. 3) of two motors from the same manufacturer, with the same settings, of the same power and same number of pole pairs, equally loaded and supplied from the same source, but with different parameters. The first one is the standard "Siemens" motor with aluminum die-casting rotor, and the second one is also manufactured by "Siemens", has an increased energetic efficiency, with the copper die-casting rotor.

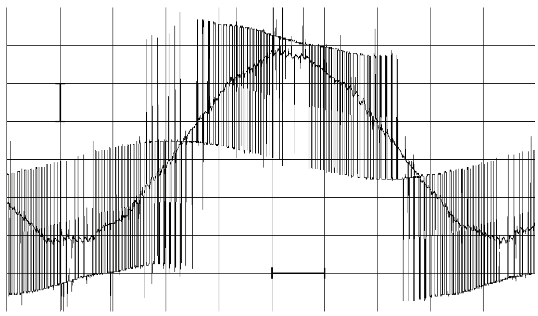

Fig. 2. Phase current and voltage diagrams of the loaded standard 0.75 kW motor supplied from Altivar 16: horizontal segment corresponds to 2,5 ms, vertical – to voltage of 100 V and current of 6 A; zeroes of voltage and current diagrams are not matched

When calculating the characteristics of the motor it is most convenient to use the harmonic analysis. Under assumptions which are typical to the theory of electric machines, it is possible to analyze the operation of separate harmonics independently, i.e. to use the principle of superposition, as in solution of other problems typical to the theoretical works of electric machines. In this way, for example, the characteristics of induction motors are calculated while assessing the higher space harmonics [2]. It is hardly purposeful to analyze the motor operation by assessing the inverter voltage and current distortion factors, as it is done in [3], since the impact of different harmonics on parameters depends not only on their amplitude. For the case of space harmonics it is demonstrated in quite illustrative way in [2]. Most dependencies characterize to the method of harmonic analysis are valid both to space and time harmonics.

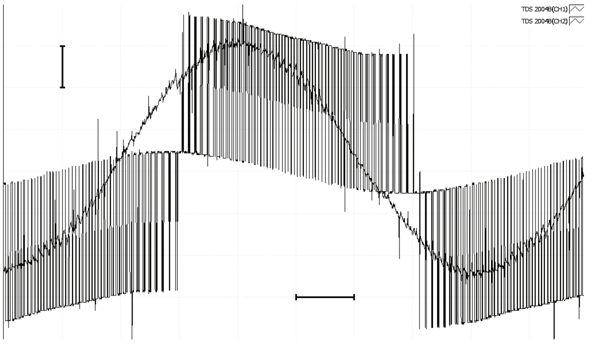

Fig. 3. Phase current and voltage diagrams of the loaded standard 0,75 kW motor with copper rotor winding supplied from Altivar 16: horizontal segment corresponds to 2,5 ms, vertical – to voltage of 100 V and current of 6 A; zeroes of voltage and current diagrams are not matched

However, when using the harmonic analysis method to calculate characteristics of induction motor supplied from the frequency converter, a problem arises. Method of harmonic analysis can be applied to calculate the electric circuit only when the electric circuit configuration is not time-dependent. Meanwhile, the direct voltage sources are periodically switched at the outputs of inverter. The equivalent voltage source method is offered which permits the replacement of the direct voltage source and switching unit with the equivalent alternating voltage source in a non-switched circuit and to use the harmonic analysis method correctly.

If we assume that the alternating voltage source is present instead of the direct voltage source which is placed before the switching unit, and the voltage of which is equal to the voltage of the direct voltage source during time period when the commutator is in a switched-off position, we will have the equivalent non-switched voltage source. During this time, which corresponds to the connected state of the commutator, the voltage of such source would be equal to the voltage of the direct voltage source. The voltage of the equivalent source is equal to the voltage beyond the commutator during the time which corresponds to the connected state of the commutator. If the commutator is in a disconnected state, there is completely no difference what the magnitude of voltage exists there since the current wouldn't flow anyway. If the commutator is in a connected state and the potentials are equal, the current will not flow through the commutator also. Thus all the processes are completely the same both as in case of periodically switched commutator with the direct voltage source present, and as in case of equivalent and constantly connected variable voltage source. The periodic variable voltage of such voltage source can be readily expanded using the Fourier series and to accomplish further calculations with harmonic components. A question may arise: what such calculations are needed for? In order to determine the equivalent variable voltage source for a particular circuit, in fact it is needed to determine the currents in this circuit under the periodic switching. So what else to calculate? However if this periodically switched circuit consists of induction motor, the equivalent circuit of which is also valid for the non-sine shaped voltage system, as it is shown in [4], then it is not enough to calculate only the currents. The dependence between voltage or current, parameters of the equivalent circuit and the motor torque is known only then, when the voltage and current varies according to the sine law. When the voltage spectrum is available, the motor driving torque can be readily calculated.

The equivalent source method

For the purposes of introduction of the equivalent variable voltage source method and in order not to complicate the presentation of the essence, it was analyzed the circuit with the minimal number of elements and a simple switching law. Method is applied when calculating the circuits with the periodic switching of the direct voltage source. The application peculiarities of this method for the purposes of calculation of characteristics of asynchronous motor supplied from the frequency inverter are planned to be analyzed in the future publications.

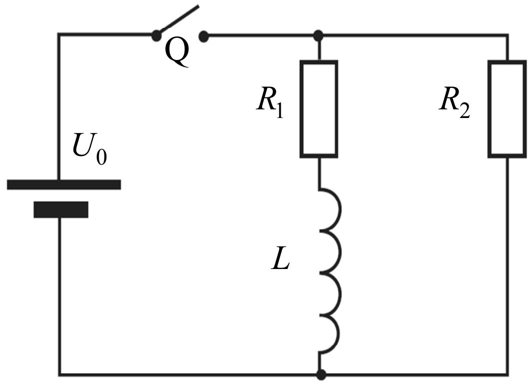

Fig. 4. A simple circuit with periodically switched voltage source

Let's say we have a periodically switched circuit. Assume that in the initial moment of time the switch Q is short-circuited by connecting the source of direct voltage U, and after a time t_1 the source is disconnected. After time t_2 the switch is short-circuited, and so on; over equal time periods t_1 and t_2 the circuit is repeatedly switched on and switched off. It would not be correct to suppose that such circuit could be calculated as non-switched circuit which has the meander pulse source connected to it. The disconnected voltage does not mean that the zero magnitude voltage is available; the voltage beyond the switch, when the switch is disconnected, in general case is not equal to zero and depends on inductance L, resistance values of resistors R_1 and R_2, and on the amount of magnetic field energy accumulated in the inductance, i.e. on the voltage U and time during which the circuit was connected to the source of this voltage. This particular circuit could be replaced using a non-switched alternative with the meander pulse source only in case, when the particular circuit would be not disconnected from the voltage, but the input of the circuit would be switched from the voltage source to the common wire. In fact for this circuit due to the commutation we have two circuits of different configurations. For each of these circuits the transient process after each commutation has to be solved.

A similar circuit is also analyzed in [5], although it is not related to the harmonic analysis; the sine functions in this case are expressed using infinite series.

The current in the branch containing resistor R_2 will be equal to the ratio of voltage U and resistance R_2, when the switch is short-circuited, and it will be of the same magnitude as in the branch containing the inductance, but of different polarity, in those periods of time when the switch will be disconnected. Therefore we will limit our analysis only to the currents and voltages in the branch containing inductance.

The current in this branch when the switch is short-circuited, is described using equation

$$L\frac{\mathrm{d}i}{\mathrm{d}t}+iR_1=U\,. \tag{1}$$

When the switch is switched-off, the equation for the current

$$L\frac{\mathrm{d}i}{\mathrm{d}t}+i(R_1+R_2)=0\,. \tag{2}$$

The characteristic equation of the differential equation for the first case

$$L\alpha+R_1=0 \tag{3}$$

and for the second case

$$L\alpha+R_1+R_2=0\,. \tag{4}$$

The only solutions of these equations, respectively, for the short-circuited switch

$$\alpha_1=-\frac{R_1}{L} \tag{5}$$

and for the disconnected switch

$$\alpha_2=-\frac{R_1+R_2}{L}\,. \tag{6}$$

The general solutions of these equations are respectively $A_1\cdot\mathrm{e}^{-\frac{R_1}{L}t}$ and $A_2\cdot\mathrm{e}^{-\frac{R_1+R_2}{L}t}$. The steady state components are respectively $\frac{U}{R_1}$ for the short-circuited switch and 0 when the switch is disconnected. The complete solutions and integration constants will be found using boundary conditions. During the first time period, when the switch is short-circuited, the current is expressed

$$i_1=\frac{U}{R_1}+A_1\cdot\mathrm{e}^{-\frac{R_1}{L}t}\,. \tag{7}$$

For the second time period when the switch is disconnected the current

$$i_2=A_2\cdot\mathrm{e}^{-\frac{R_1+R_2}{L}t}\,. \tag{8}$$

Since the current can not change its value instantly, at the end of each time period its value is equal to the value it acquires at the beginning of the following time period. Considering this we can write such equation system:

$$\begin{cases}\dfrac{U}{R_1}+A_1\cdot\mathrm{e}^{-\frac{R_1}{L}t_1}=A_2\,;\\[2ex] \dfrac{U}{R_1}+A_1=A_2\cdot\mathrm{e}^{-\frac{R_1+R_2}{L}t_2}\,.\end{cases} \tag{9}$$

Equation system (9) can be written in the following form

$$\frac{U}{R_1}+A_1=\left(\frac{U}{R_1}+A_1\cdot\mathrm{e}^{-\frac{R_1}{L}t_1}\right)\cdot\mathrm{e}^{-\frac{R_1+R_2}{L}t_2}\,. \tag{10}$$

Only one unknown magnitude A_1 remains which is equal

$$A_1=-\frac{\dfrac{U}{R_1}\left(1-\mathrm{e}^{-\frac{R_1\cdot t_2+R_2\cdot t_2}{L}}\right)}{\left(1-\mathrm{e}^{-\frac{R_1\cdot t_1+R_1\cdot t_2+R_2\cdot t_2}{L}}\right)}\,. \tag{11}$$

When A_1 is known, then from equation (9) we determine A_2

$$A_2=\frac{U}{R_1}-\frac{\dfrac{U}{R_1}\left(1-\mathrm{e}^{-\frac{R_1\cdot t_2+R_2\cdot t_2}{L}}\right)}{1-\mathrm{e}^{-\frac{R_1\cdot t_1+R_1\cdot t_2+R_2\cdot t_2}{L}}}\cdot\mathrm{e}^{-\frac{R_1}{L}t_1}=$$

$$=\frac{U}{R_1}\left(1-\frac{\mathrm{e}^{-\frac{R_1\cdot t_1}{L}}-\mathrm{e}^{-\frac{R_1\cdot t_1+R_1\cdot t_2+R_2\cdot t_2}{L}}+1-1}{1-\mathrm{e}^{-\frac{R_1\cdot t_1+R_1\cdot t_2+R_2\cdot t_2}{L}}}\right)=$$

$$=\frac{U}{R_1}\left(\frac{1-e^{-\frac{R_1\cdot t_1}{L}}}{1-\mathrm{e}^{-\frac{R_1\cdot t_1+R_1\cdot t_2+R_2\cdot t_2}{L}}}\right)\,. \tag{12}$$

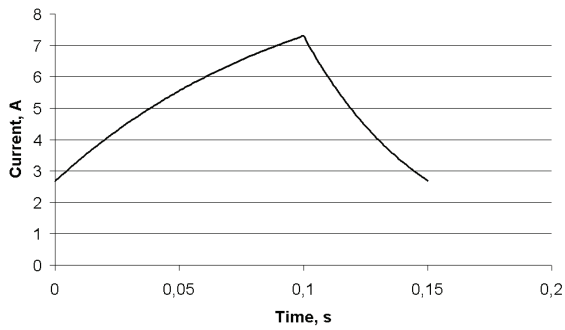

Fig. 5. Variation of the coil current during the commutation period

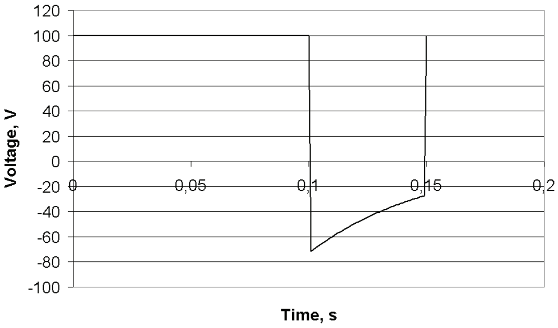

Fig. 6. Variation of the voltage drop over resistor R_2 during the commutation period

Let's calculate a particular example, when $U = 100$ V; $R_1 = R_2 = 10\ \Omega$; $L = 0{,}1$ H and $t_1 = 0{,}1$ s; $t_2 = 0{,}05$ s. At first according to formulas (7), (11), (8) and (12) we calculate the current and inductance when the commutator is switched on and switched off. The calculation results are demonstrated: the current variation over the commutation period is given in Fig. 5, the variation of the voltage drop over resistor R_2 during the commutation period is shown in Fig. 6. The voltage drop over resistor R_2 is the same as the voltage beyond the commutator; it is the general voltage of the circuit. If the alternating voltage source (the variation law of which is illustrated in Fig. 6) would be connected instead of the switched source of direct voltage U, all the currents and voltages in this circuit would remain the same as in the case of the direct switched voltage. But the circuit configuration is unchanged and the application of spectrum analysis method would be correct. If the particular circuit is the equivalent circuit of induction motor, the electromagnetic torques for each harmonic component can be calculated in analogous way as they are calculated for the space harmonics.

Conclusions

The voltage shape at the output of the inverter depends not only on the inverter itself, but also on the load properties.

In those time periods when the transistors of the respective output are closed, the voltage of this output is determined by the energy accumulated in the reactive elements of the respective part of the circuit.

In order to apply the harmonic analysis method for the periodically switched circuit, the switched sources of the direct voltage must be replaced with the equivalent non-switched sources of variable voltage.

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In this paper the equivalent sources method which is intended to calculate the periodically switched electric circuits using harmonic analysis method is described. It is not possible to apply `the harmonic analysis method to analyze such circuits, since the circuit configuration varies over time. When using the offered method, the switched source of direct voltage is replaced using the equivalent source of variable voltage, for which electric currents, voltages power and other electrical magnitudes are completely the same as in the case of the switched direct voltage source; the main difference is that after such substitution the voltage source becomes non-switched. Then the harmonic analysis method can be readily applied for calculation of the input of separate harmonics into the overall result. The essence of the method is illustrated by calculating the equivalent source for a simple electric circuit operated by first law of commutation. Ill. 6, bibl. 5 (in English; abstracts in English and Lithuanian).

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Straipsnyje aprašomas ekvivalentinio šaltinio metodas, skirtas periodiškai komutuojamoms elektrinėms grandinėns skaičiuoti harmoninės analizės metodu. Tiesiogiai taikyti harmoninės analizės metodo tokioms grandinėms negalima, nes grandinės konfigūracija laikui bėgant kinta. Naudojant siūlomą metodą, komutuojamas nuolatinės įtampos šaltinis pakeičiamas ekvivalentiniu kintamosios įtampos šaltiniu, kuriam esant srovės, įtampos, galia ir kiti elektriniai dydžiai būna visiškai tokie pat, kaip ir komutuojamojo nuolatinės įtampos šaltinio, tačiau jis jau nebėra komutuojamas. Tuomet jau galima taikyti harmoninės analizės metodą, apskaičiuojant atskirų harmonikų įtaką suminiam rezultatui. Metodo esmė iliustruojama pagal pirmąjį komutacijos dėsnį apskaičiuojant nesudėtingai elektrinei grandinei ekvivalentišką šaltinį. Il. 6, bibl. 5 (anglų kalba; santraukos anglų ir lietuvių k.).