

# Investigation of the spectrum decomposition technique for estimation of the group velocity Lamb waves

L. Draudvilienė, L. Mažeika

Ultrasound Institute, Kaunas University of Technology,

Studentu Str. 50, Kaunas, LITHUANIA. Phone: +370 37 351162. Fax: +370 37 451489.

E-mail: [ulab@ktu.lt](mailto:ulab@ktu.lt)

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## Abstract

Phase velocity, group velocity and attenuation of guided waves are most important parameters in their applications for non-destructive testing. In previous our articles the investigations technique for measurement of the phase velocity was proposed. However this technique is not applicable for evaluation of the group velocity. The objective of the work presented was to develop a technique which enables estimation of the group velocity taking into account dispersion of the guided waves. As the result of the investigation the technique based on the spectrum decomposition approach was proposed. Performance of the technique is demonstrated on the modelled signal. In the investigation signals propagating in the finite element model of the 2 mm thickness and 200 mm length aluminium plate were used. The asymmetric  $A_0$  mode was excited by adding a shear force to one of the plate end and the  $S_0$  mode - by adding a normal force. The excitation signal was 3 periods, 300 kHz burst with the Gaussian envelope. In general, the investigation demonstrated that, the proposed spectrum decomposition technique enables to reconstruct the segments of the dispersion curves of the group velocity both in the cases of the asymmetric  $A_0$  and the symmetric  $S_0$  modes of Lamb waves in the analyzed frequency range.

**Keywords:** Lamb wave, dispersion, group velocity, spectrum decomposition, finite element.

## Introduction

Application of Lamb waves for non-destructive testing (NDT) is very attractive when inspection of the large structures is needed since they can propagate a long distances (up to 100 m) [1, 2, 3]. The inspection techniques using guided or Lamb waves usually are based on accurate measurements of their parameters: the phase or/and the group velocities and attenuation. However due to dispersive character of guided waves this is not an easy task for any of the signal processing techniques. Most frequently such methods as time - series analysis, frequency analysis and integrated time – frequency analysis are used [4].

In previous our articles [5-7] the technique for measurement of the phase velocity of Lamb wave was proposed and investigated. It was shown that it enables with some uncertainties to reconstruct the segment of the phase velocity dispersion curve.

In general the group velocity can be obtained from the segment of the dispersion curve of the phase velocity. However the relatively small errors of the estimation of phase velocity can lead to essential deviation in the evaluation of group velocity. So, it is important to have independent from the phase velocity measurement technique for group velocity estimation.

So, the objective of presented work was to investigate the spectrum decomposition technique for estimation of the dispersion of the group velocity of Lamb waves.

## The spectrum decomposition technique for group velocity estimation

In general the group velocity is determined using signal envelope measured at several distances. However in this case only one value of the group velocity is obtained, which usually is related to the central frequency of the

spectrum of signal. As the guided waves are dispersive it is important to have the values of the group velocity corresponding to several frequencies and in such a way reconstruct the segment of the dispersion curve of the group velocity. In order to obtain this it was proposed to use the spectrum decomposition approach. The spectrum decomposition technique was already used for investigation of the ultrasound pulse propagation in lossy media [8] and phase velocity estimations [9].

In general, according to the spectrum decomposition technique the frequency spectrum of the signal is multiplied by a transfer function of a special filter (Fig.1) and the signal is reconstructed using the inverse Fourier transform. Using this filtered signal the further signal processing is performed. The filter is shifted in a frequency domain by a selected value and all procedure is repeated. In such a way the result corresponding to different frequencies is obtained. So, it was proposed to use this technique for estimation of the dispersion curve of the group velocity.

The proposed algorithm for estimation of the group velocity of guided waves can be expressed in the following steps:

1. The two signals at different distances are selected  $u_x(t)$  and  $u_{x+\Delta x}(t)$ ;
2. The frequency spectra of these two signals are calculated using the Fourier transform
 
$$U_x(f) = \text{FT}[u_x(t)]$$

$$U_{x+\Delta x}(f) = \text{FT}[u_{x+\Delta x}(t)].$$
3. The frequency spectrums are filtered the using Gaussian filter
 
$$U_{x,F}(f) = U_x(f) \cdot H(f)$$

$$U_{x+\Delta x,F}(f) = U_{x+\Delta x}(f) \cdot H(f)'$$

where  $H(f) = e^{-K(f-f_n)^2}$ ,  $K$  is the coefficient which defines the bandwidth of the filter and  $f_n$  is the central frequency of the filter;

4. The filtered signals are reconstructed using the inverse Fourier transform

$$u_{x,F}(t) = \text{FT}^{-1}[U_{x,F}(f)]$$

$$u_{x+\Delta x,F}(t) = \text{FT}^{-1}[U_{x+\Delta x,F}(f)]$$

5. The envelope of the signal is calculated using the Hilbert transform

$$u_{x,FH}(t) = |\text{Hilbert}[u_{x,F}(t)]|$$

$$u_{x+\Delta x,FH}(t) = |\text{Hilbert}[u_{x+\Delta x,F}(t)]|$$

6. The delay time between two envelopes is estimated using a cross-correlation

$$t_{\Delta x} = \arg \max_t \{ \text{corr}[u_{x,FH}(t), u_{x+\Delta x,FH}(t)] \};$$

7. The group velocity is estimated as

$$c_g = \Delta x / t_{\Delta x}$$

and is related to the central frequency of the spectrum.

The steps 1-7 are repeated for different values of the central frequency of the filter  $H(f)$ . As a result the dependency of the group velocity on a frequency  $c_g(f)$  is obtained in such frequency ranges in which the central frequency of the filter was varied.

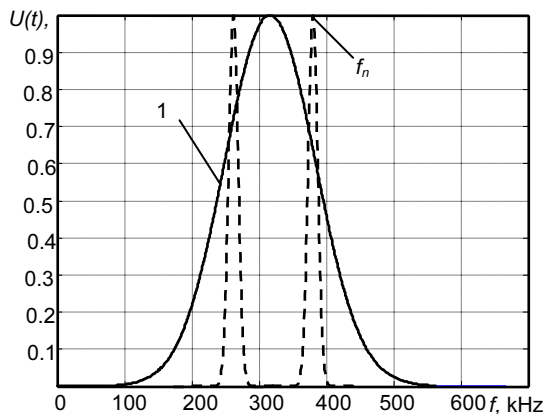


Fig.1. The frequency spectrum of the signal (1) and the frequency response of the filters

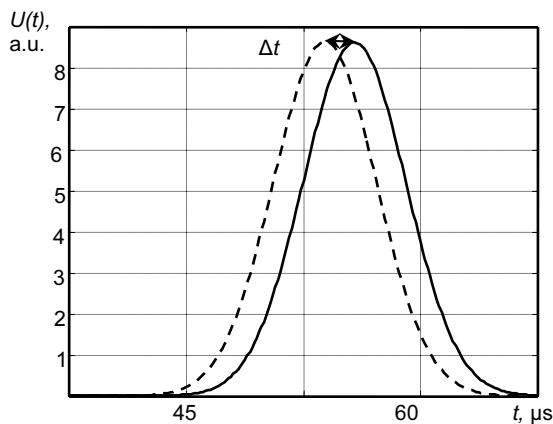


Fig.2. The amplitudes of the different signals

### The modelled signals for investigations

In order to investigate the proposed spectrum decomposition approach for estimation of the group velocity the modelled signals obtained using a 2D finite element model [10] were used. The model identical to the model used in the investigations of the phased velocity measurement was selected [5-7]. The asymmetric  $A_0$  and symmetric  $S_0$  modes of guided waves propagating in a aluminium plate were investigated. The  $A_0$  mode was excited by attaching the tangential force (Fig.3a) and the  $S_0$  mode – by attaching the normal force (Fig.3b) to one of the plate edges. The waveform of the excitation signal was three period burst with the Gaussian envelop and the central frequency 300kHz. The following parameters of the aluminium plate were used in the model: density  $\rho = 2780 \text{ kg/m}^3$ , the Young modulus  $E = 73.1 \text{ GPa}$ , the Poisson's ratio  $\nu = 0.3$ . The sampling step in the spatial domain was  $dx=0.1\text{mm}$  and  $dt=0.15\mu\text{s}$  in the time domain.

The dispersion curves of the group velocity of the asymmetric  $A_0$  and the symmetric  $S_0$  modes of the Lamb waves propagating in 2 mm aluminium plate are presented in Fig.4. The dispersion curves were calculated assuming that the propagation velocities of the longitudinal and shear wave in aluminium are correspondingly  $c_L = 6350\text{m/s}$  and  $c_T = 3100 \text{ m/s}$ .

The obtained B-scan images of the propagating  $A_0$  and  $S_0$  modes waves are presented in Fig.5-6.

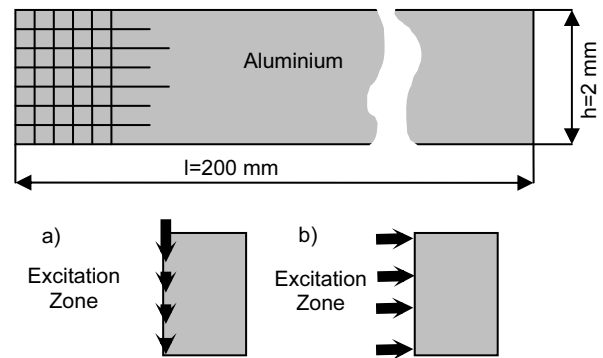


Fig.3. The finite element model for investigation of the  $A_0$  (a) and  $S_0$  (b) modes of the Lamb waves propagating in aluminium plate

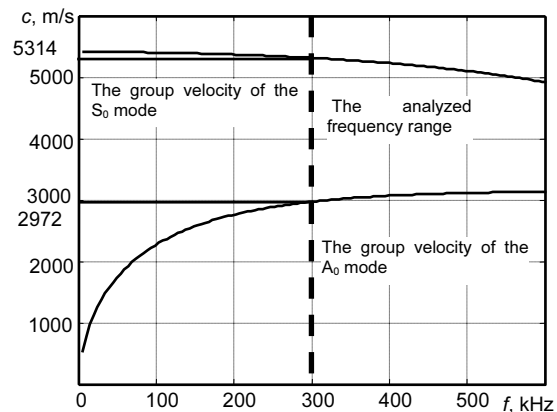


Fig.4. The theoretical dispersion curve of the group velocity asymmetric  $A_0$  and symmetric  $S_0$  modes of the Lamb wave propagating in 2 mm thickness aluminium plate

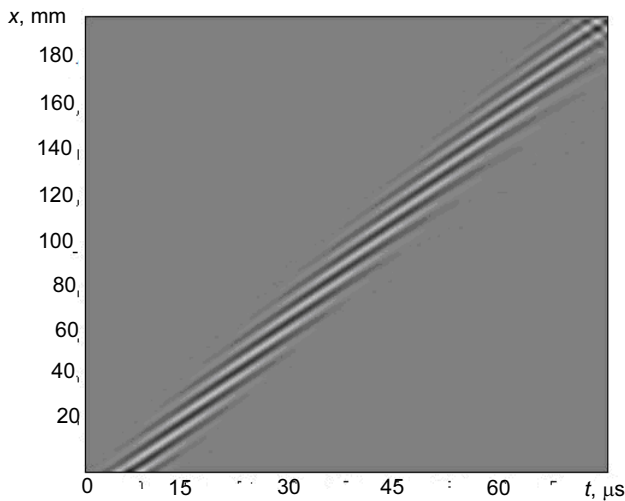


Fig.5. The B-scan image of the normal component of the particle velocity on the surface of the plate in the case of generated  $A_0$  mode

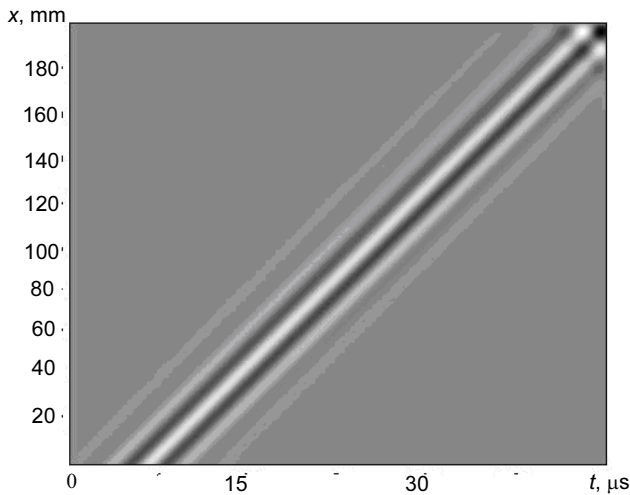


Fig.6. The B-scan image of the normal component of the particle velocity on the surface of the plate in the case of generated  $S_0$  mode

From the B-scan it can be seen that patterns of the  $A_0$  mode and the  $S_0$  mode are different. The pattern of the  $S_0$  is more uniform because phase and group velocities are very similar. In the case of the  $A_0$  mode they are different what is demonstrated by the specific pattern in the B-scan image.

**Demonstration of the proposed group velocity technique on simulated signals**

In order to apply the developed technique for analysis of guided wave signals several parameters should be set. At first the frequency bandwidth of the filter used in the spectrum decomposition technique and a step in the frequency domain by which the filter is step by step shifted should be selected. As can be seen from the frequency spectrum of the propagating  $A_0$  mode signal (Fig.5) it possesses the bandwidth from 100kHz up to 500kHz (using a very low -40dB level). Corresponding to this bandwidth the central frequency of the filter was set in the

ranges of 100-500kHz with the step 20kHz. More complicated is selection of the filter bandwidth. The investigations carried out demonstrated that the scattering of the results or errors of the results essentially depends on these parameters. During these investigations the filter bandwidth was varied in the range of (15-170) kHz according to - 6dB level. The results demonstrated that most optimal bandwidths with respect to the scattering of the group velocity estimation values are different for  $A_0$  and  $S_0$  modes of guided waves. In the case of  $A_0$  mode the best results were obtained using the filter with the bandwidth 75 kHz and in the case of  $S_0$  mode - 170 kHz.

The frequency spectrums of  $A_0$  mode signal and the filter frequency spectrum with the bandwidth 75 kHz (optimal one) are presented in Fig.7. The obtained values of the group velocity enabling to reconstruct the segment of the dispersion curve are presented in Fig.8.

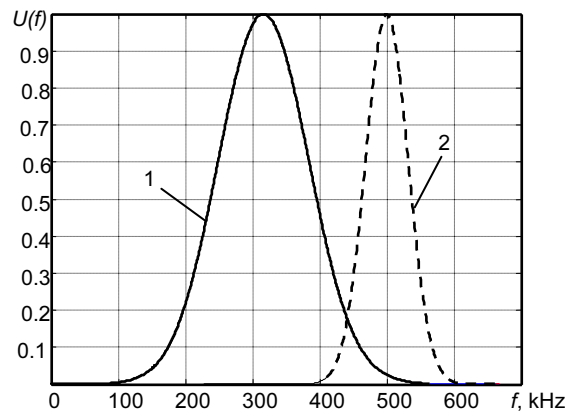


Fig.7. The frequency spectrum of the  $A_0$  mode (1) and the frequency spectrum of the filter (2)

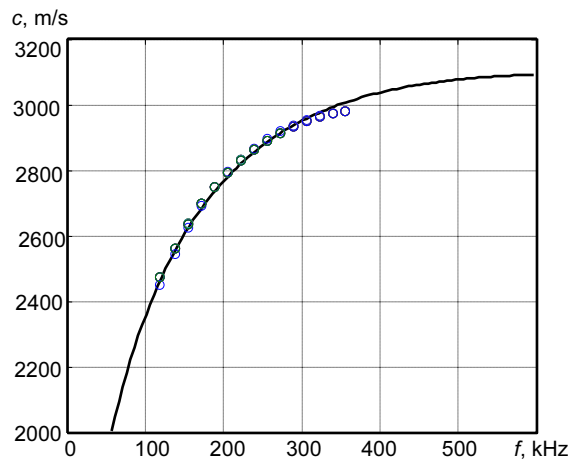


Fig.8. The theoretical dispersion curve (line) of the  $A_0$  mode group velocity and the obtained results (circles) from simulated signals

The frequency spectrum of the optimal filter used for measurement of the group velocity of  $S_0$  mode is presented in Fig.9. The obtained values of the group velocity of  $S_0$  mode are presented in Fig.8. As can be seen in the case of both guided wave modes a reasonable scattering of group wave velocity values is obtained, however some deviation of the theoretical dispersion curve can be observed.

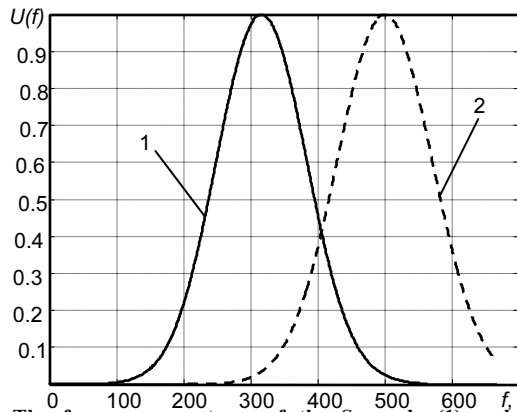


Fig.9. The frequency spectrum of the  $S_0$  mode (1) and (2) the frequency response of the filter

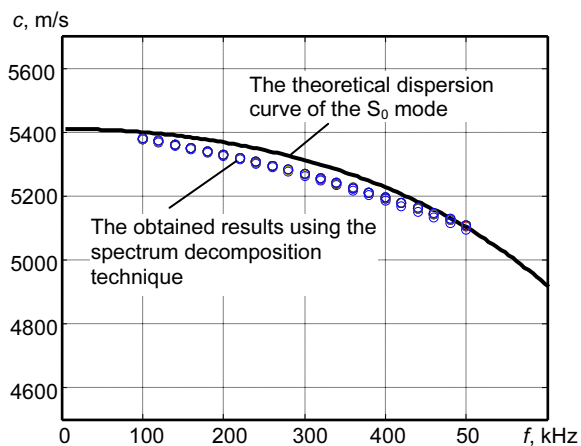


Fig.10. The theoretical dispersion curve of the  $S_0$  mode group velocity and the obtained results using the simulated signals.

## Conclusions

It was proposed to use the spectrum decomposition technique for estimation of the segment of dispersion curves of asymmetric  $A_0$  and symmetric  $S_0$  modes of Lamb waves in the analyzed frequency range. It was demonstrated that by selection of optimal parameters of the filters used in the signal processing the scattering of the results can be reduced essentially.

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L. Draudvilienė L. Mažeika

## Nukreiptųjų ultragarso bangų grupinio greičio matavimas taikant dažnių spektro skaidymo metodą

Reziumė

Lembo bangų naudojimas įvairiose pramonės srityse priklauso nuo šių bangų pagrindinių parametru: grupinio ir fazinio greičių ir slopinimo. Ankstesniuose darbuose atlikus Lembo bangų asimetrinės  $A_0$  ir simetrinės  $S_0$  modų fazinio greičio tyrimus buvo gauta tam tikrų dėsningumų, įgalinančių atkurti fazinio greičio dispersinės kreivės segmentą ir identifikuoti skirtingas modas. Šio darbo tikslas buvo iširti nukreiptųjų ultragarso bangų grupinio greičio matavimo analogiškai atkuriant dispersinės kreivės segmentą galimybes. Grupiniam greičiui matuoti buvo pasiūlytas ir iširtas dažnių spektro skaidymo pagrįstas metodas. Siekiant nustatyti šio metodo galimybes tirti Lembo bangų grupinį greitį, buvo atlikti bangos, sklindančios aliuminio plokštele, matavimai naudojant baigtinių elementų metodu gautus signalus. Tyrimo metu buvo modeliuojamas nukreiptųjų ultragarso bangų  $A_0$  ir  $S_0$  modų sklidimas 2 mm storio aliuminio plokštele. Matavimo rezultatai parodė, kad pasiūlytasis Lembo bangų grupinio greičio duomenų analizės metodas įgalina atkurti skirtingų modų grupinio greičio dispersinių kreivių segmentus nagrinėjamame dažnių diapazone.

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