

670. Numerical study of cantilevers with non-uniform width for enhancing the performance of vibration-driven micropower generator based on piezoelectric conversion

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Abstract. This study is dedicated to investigation of rectangular- and trapezoidal-shaped cantilevers for achieving improved efficiency of the piezoelectric micropower generation. The developed finite element model of a unimorph piezotransducer with a proof-mass at the tip is used to examine how different cantilever shapes and proof-mass dimensions influence stress distribution, dynamic response and voltage output of the microgenerator. Numerical results indicate that cantilevers with increasingly triangular shape permit markedly larger kinematic excitation magnitudes and generate slightly larger voltages for a comparable deflection level.

Keywords: energy harvesting, piezoelectric micropower generator, unimorph transducer, finite element model, dynamics, trapezoidal cantilever, triangular shape, strain uniformity.

1. Introduction

Rapid advances in wireless technology and low-power electronics have led to the deployment of autonomous wireless sensor networks for various applications including environmental and structural health monitoring, gas and chemical sensing, motion and explosive detection, etc. Current wireless devices are designed to employ electrochemical batteries for powering. However the use of batteries can become increasingly troublesome because of their limited lifespan and inadmissibly high cost related to their replacement in hardly accessible and remote locations (battery replacement costs \$80–\$500 including labor and it exceeds the price of a sensor [1]). These issues together with the emerged concept of self-powered MEMS-based sensing devices have triggered active research of micropower harvesting technologies during the last 10 years or so. Different harvesting approaches are considered but vibrational energy has gained the most of attention due to its widespread availability. Vibration-to-electricity converters are usually based on electromagnetic, electrostatic and piezoelectric transduction. The latter is regarded to be the most promising for realization of piezoelectric micropower generators (PMPGs) due to relatively easy microfabrication, simple geometry and fewer peripheral components resulting in lower device cost [2-3].

Cantilever-type PMPGs attracted the largest attention in the research community. They are very efficient when the driving vibration matches their resonant frequency. However when they are not operated in resonance the output power diminishes by orders of magnitude [4]. Therefore such PMPGs are highly suitable only for applications with well-defined excitation frequency.

A lot of studies have been conducted on PMPGs over the past decade however their efficiency is still very low. State-of-the-art PMPG prototypes currently harvest only about 5% of the available power [5]. Thus, considerable research work is still needed in different fields in order to improve efficiency of the microgenerators. In terms of materials research, current

efforts are directed towards development of piezoelectric materials with higher coupling efficiency and low parasitic damping. Most popular piezoelectric materials in energy harvesting are PZT, ZnO and PVDF [6]. On the electronics side, improvements in performance of ac/dc conversion circuitry and on-board storage are essential. And, finally, in the structural domain boosting of charge density is required therefore optimized geometric designs should be sought in order to enhance the utilization of piezoelectric materials.

This paper deals with investigation of geometric aspects of PMPG design with focus on simulation of cantilevers with variable width profile in order to examine influence of shape on stress distribution as well as dynamic and electrical characteristics of the microgenerator.

2. Finite element model of a cantilever-type PMPG

The design of the considered PMPG is based on a unimorph cantilevered piezoelectric transducer with a proof-mass attached at the free end (Fig. 1 and Table 1). The supporting cantilever layer and the proof-mass are made from silicon, while PZT-5A is used for piezoelectric layer, which is positioned on the top of the supporting layer and is poled along the thickness direction resulting in a transverse (“ d_{31} ”) operation mode. It is assumed that the piezoceramic layer is sandwiched between two planar electrodes, which provide a uniform voltage along the piezoceramic surface. Uniformity of electric field is assumed because the electrodes are considered to be perfect conductors, while the piezoceramics is characterized by the uniform dielectric constant. The electrodes are assumed to be sufficiently thin so as they exert negligible effect on the mechanical properties of the structure (they only facilitate the electromechanical coupling in the piezo-layer). The thickness of the passive and active layers is kept constant in the course of simulations while the variable parameters include cantilever width along its length, aspect ratio of the cantilever section and dimensions of the proof-mass. The shape of the cantilever structure is varied from a rectangular one to a nearly triangular one (Fig. 1(b)) with intermediate trapezoidal shapes resulting in a total of 6 different shapes as indicated in Table 1.

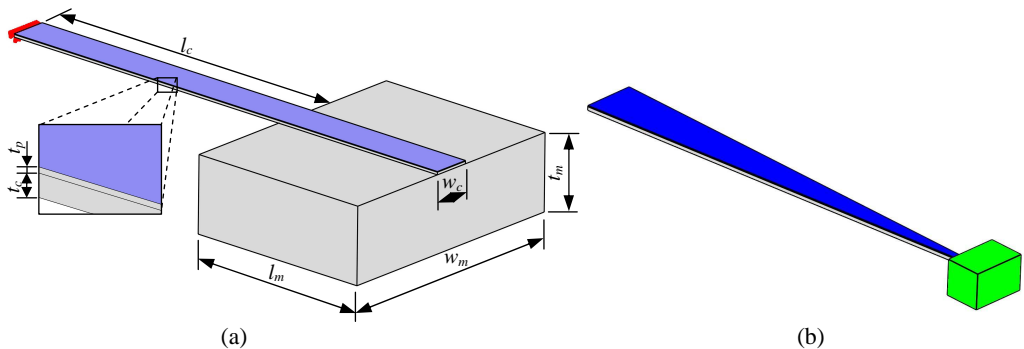


Fig. 2. (a) General schematic of a finite element model of a unimorph piezoelectric cantilever-type micropower generator with a proof-mass at the free end. Zoomed area illustrates bi-layer structure of the cantilever consisting of the passive Si layer t_c ($E_{Si}=200$ GPa, $\nu_{Si}=0.33$, $\rho_{Si}=2330$ kg/m³) and piezoactive PZT-5A layer t_p ($\rho_p=7750$ kg/m³). (b) Visualization of the finite element model of micropower generator with trapezoidal-shaped cantilever structure

Cantilever configuration in PMPGs is preferred since: a) for a given force input it produces relatively high average axial strain, which directly determines the electric field (and power, which is a function of electric field squared); b) for a given generator size a relatively low resonant frequency is achieved (which may be further reduced by an additional proof-mass that

is mounted at the free end of the cantilever structure); c) it is relatively ease to realize it in a microfabrication process.

Finite element model of the PMPG was realized in Comsol Multiphysics 3.5a [7]. For the complete description of the developed finite element model the readers are referred to the earlier publication of the authors [8].

Table 1. Geometric parameters of the simulated rectangular-shaped and trapezoidal-shaped piezoelectric micropower generators

Cantilever shape No.	Larger base width	Smaller base width	Length of cantilever section l_c	Thickness of passive Si layer t_c	Thickness of PZT-5A layer t_p
	μm				
1 (the most triangular)	570	30	2500	20	5
2	540	60			
3	450	150			
4	400	200			
5	350	250			
6 (rectangular)	300	300	Variable		

3. Numerical study of influence of cantilever geometry on harvesting performance

In the case of traditional rectangular-shaped cantilevers only the clamped region is characterized by a maximum stress and contributes the most to generation of the electrical energy, while the predominant remaining part of the piezoelectric film produces little effect. Uniformity of induced electric potential throughout the length of the structure is preferable since the equipotential surface leads to a constant voltage. In this case the whole piezoelectric volume equally contributes to the mechano-electrical transduction making it more efficient. Therefore, consideration of various alternatives for improvement of design geometry is necessary in order to increase PMPG performance. First of all, it is reasonable to optimize cantilever geometry with respect to maximization and homogenization of stress in the piezoelectric material taking into account that the area of the material is the design constraint because it determines the size (weight) and cost of the PMPG. Bending strain is proportional to the beam curvature, which is the second spatial derivative of the displacement function. This implies that generation of the electric field is the most productive when the slope of the cantilever is monotonic, which is the case only in the fundamental mode, where the curvature never alternates its sign. Meanwhile, for the stress distribution to acquire a constant value the bending moment and area moment of inertia must have an identical dependency in x direction. To this aim the application of triangular shape is considered in this paper. It is known from theory that in this case the surface electric energy density is three times larger in comparison to the case with rectangular cantilever. However, in performing the considered design improvement one must be cautious and avoid overstraining, i.e. the stress inside the piezoelectric material must not exceed the yield stress. Thus, with triangular-shaped cantilever structure it is essential to ensure that the material is uniformly stressed to values that are well below the yield point.

Fig. 2 illustrates computed stresses in the cantilevers of various shapes presented as a function of position in x direction. These stress curves allow to judge about the corresponding strain distribution. Strain curves are proportional to the amount of electrical energy that is generated along the length of the cantilever, while the amount of charge induced at some point along its length is proportional to the product of strain and width of the corresponding section. Simulations results in Fig. 2(b) demonstrate that the increasingly triangular trapezoidal geometry is characterized by a flatter stress profile over the length of the structure: it allows to distribute

the stress in the lateral direction of the cantilever much more evenly, such that maximum strain is induced at the predominant part of the unimorph structure. However, numerical results in Fig. 2(c) reveal that the triangular shape of the cantilever for stress homogenization is only valid for beam with a larger proof-mass at the end because in this case the inertia of the mass significantly exceeds that of the cantilever section and the microgenerator may be represented as a beam loaded with a force acting on its end.

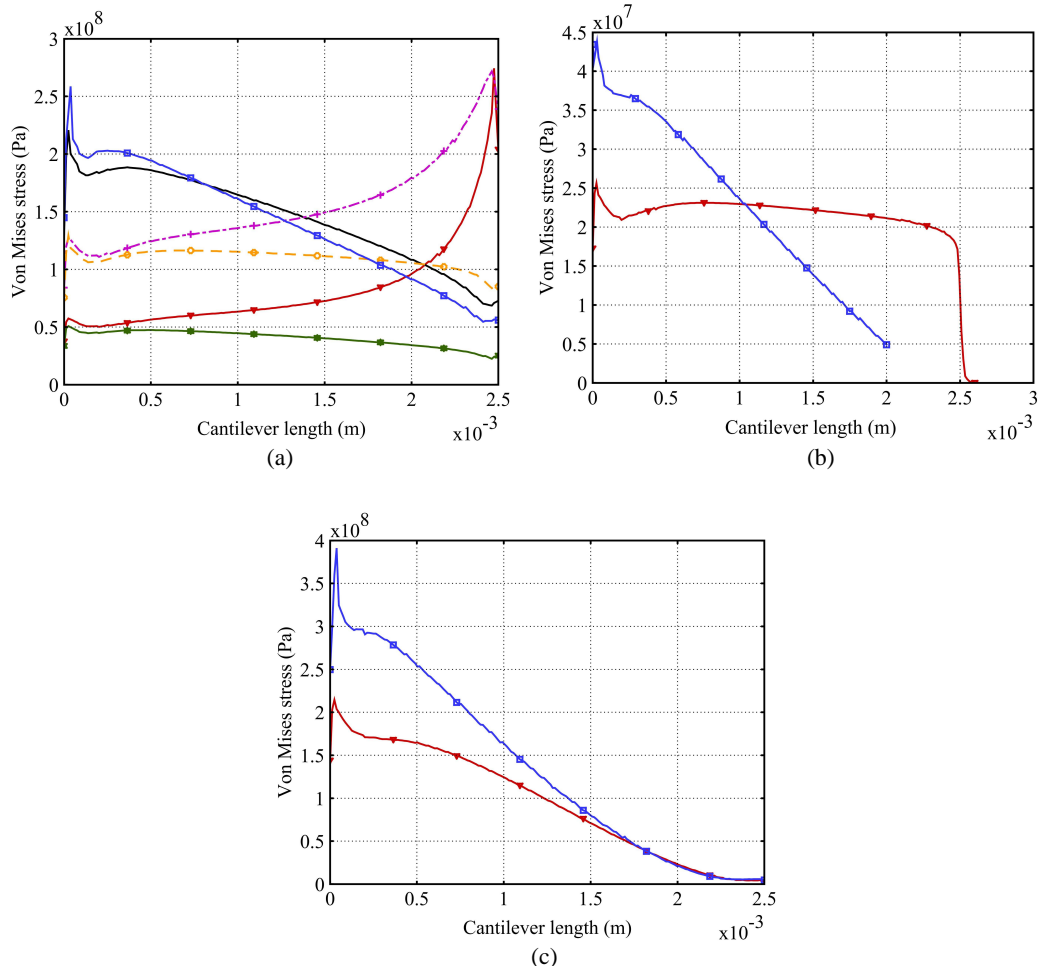


Fig. 2. Distribution of stress throughout the length of the cantilever section of the piezoelectric transducer at resonance obtained from harmonic analysis: (a) for the case when fundamental frequency is $f_1 \approx 170$ Hz, proof-mass/beam-mass ratio is in the range of 60-70; (b) for the case when fundamental frequency equals $f_1 \approx 2300$ Hz and proof-mass/beam-mass ratio is 0.9. (c) Stress distribution obtained from modal analysis for the case when proof-mass/beam-mass ratio equals zero. (Curves: shape 1 – red solid line ∇ , shape 2 – magenta dash-dot line $+$, shape 3 – yellow dashed line \circ , shape 4 – green solid line \star , shape 5 – black solid line, shape 6 – blue solid line \square)

A series of frequency response analyses were performed with rectangular-shaped (No.6 in Table 1) and the most triangular trapezoidal-shaped (No.1) cantilevers in order to determine the influence of the geometry on dynamic and electrical response of the PMPG. During these simulations all the parameters were not modified except for the shape of the cantilever, i.e. the

lateral dimensions (aspect ratio) of the rectangular- and trapezoidal-shaped cantilevers were adjusted so as they occupy identical volume and have the same fundamental frequency of about 2300 Hz (a constant area of the piezoelectric layer ($7.5 \times 10^{-7} \text{ m}^2$) was maintained). The size of the proof-mass was constant and selected so as the proof-mass/beam-mass ratio is equal to 0.9. Simulated responses in Figs. 3(a-b) reveal that for the same excitation magnitude of $a=1g$ the trapezoidal-shaped cantilever produces 3.58 times less displacement and output voltage. Figs. 3(c-d) illustrate these characteristics for the case when rectangular-shaped cantilever is excited with $a=1g$, while the trapezoidal one with $a=3.58g$. These results indicate that the trapezoidal-shaped cantilever tolerates a considerably larger excitation amplitude and generates slightly larger open-circuit voltage (Fig. 3(d)) for the comparable deflection level (Fig. 3(c)).

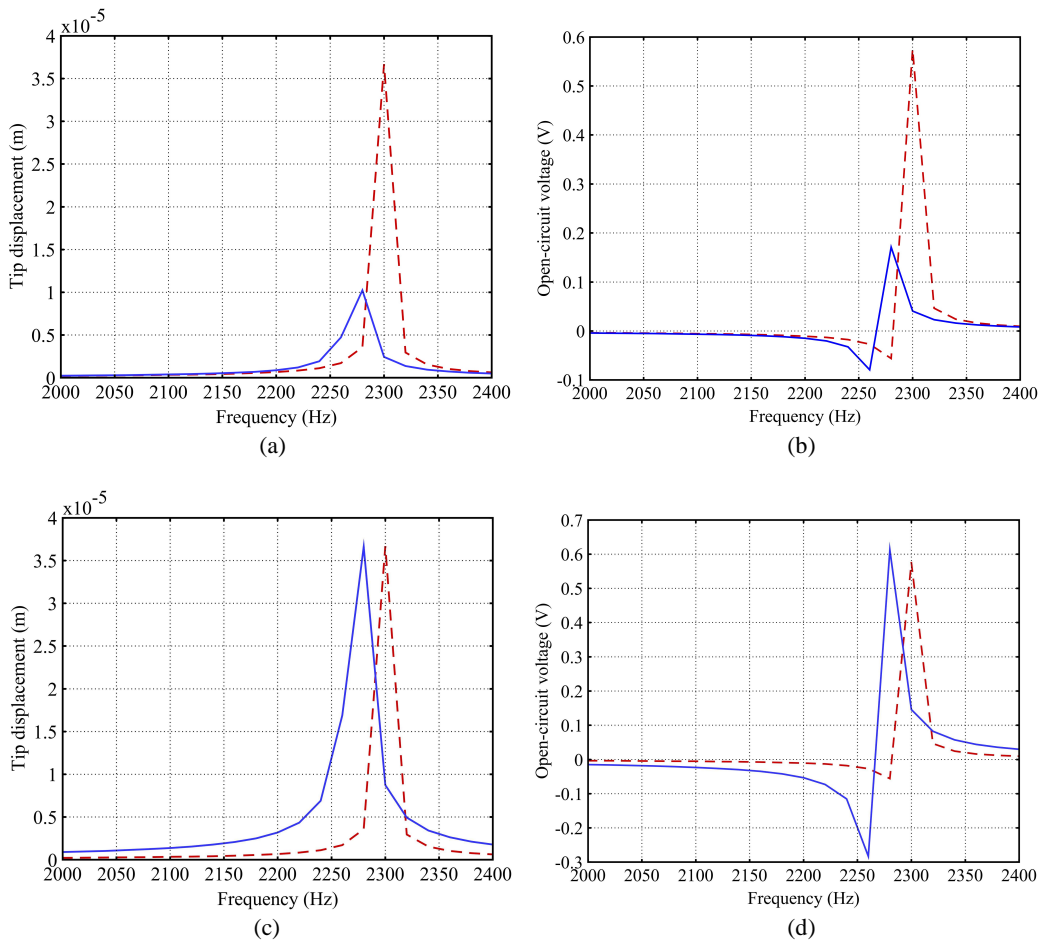


Fig. 3. Amplitude-frequency characteristics (a-c) and corresponding voltage characteristics (b-d) of rectangular-shaped cantilever No.6 (red dashed line) and trapezoidal-shaped cantilever No.1 (blue solid line), obtained in the vicinity of their fundamental frequency (~ 2300 Hz) when: (a-b) both cantilevers are excited with identical acceleration magnitude of $a=1g$; (c-d) cantilever No.6 is excited with $a=1g$, while No.1 – $a=3.58g$

Increase in voltage output is also observed in the results of transient simulations (Fig. 4), which were performed by assuming a damping ratio of approximately 0.005 (research results in [9] indicate that this is a reasonable assumption). Therefore, for the Rayleigh damping model a

coefficient $\beta=1 \times 10^{-5}$ was applied during transient analysis in Comsol. Simulated time responses in Fig. 4 obtained for rectangular- and trapezoidal-shaped cantilever structures are not completely concurrent due to slight difference (<1%) in fundamental frequency of the two different structures (~ 2300 Hz).

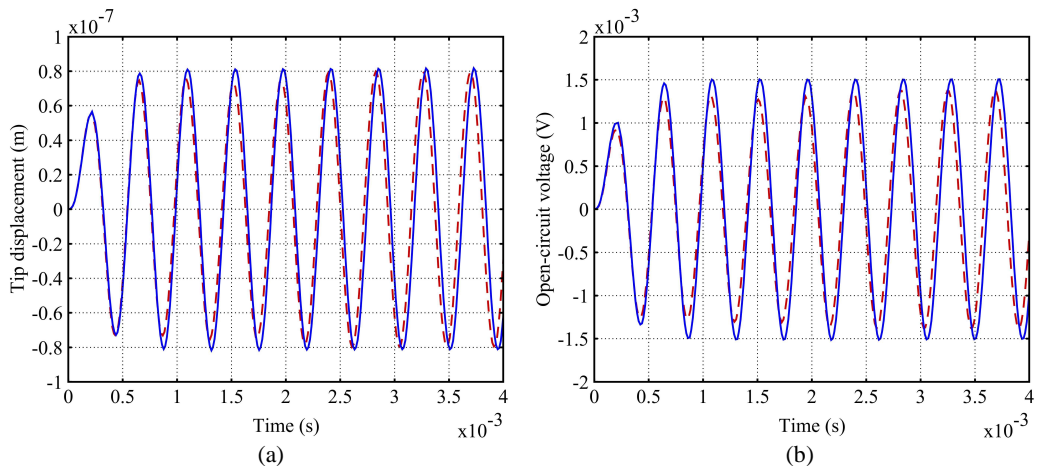


Fig. 4. Time response of tip displacement (a) and generated open circuit voltage (b) during sinusoidal kinematic excitation of the microgenerators with acceleration magnitude of $a=1g$ for rectangular-shaped cantilever (dashed red line) and $a=3.58g$ for trapezoidal-shaped cantilever No.1 (blue solid line). Excitation frequency corresponds to the fundamental frequency of the microgenerators (~ 2300 Hz).

The obtained result, which indicates that a trapezoidal-shaped cantilever needs a higher force to achieve the same deflection as the rectangular one, is attributed to different mass distribution in the considered structures: the distance of the center of gravity to the clamped edge is only a third of the total length for the trapezoidal-shaped cantilever and half of that length for the rectangular one, which leads to an increased bending stiffness of the former. At equal deflections the trapezoidal-shaped cantilever generates only a slightly higher voltage (Fig. 4), which implies that for the considered case the benefits due to strain uniformity are nearly cancelled out by the "unfavorable" mass distribution leading to a less efficient conversion of excitation energy into the mechanical one in the case of the trapezoidal shape.

Another series of frequency response simulations were carried out in a completely different PMPG design scenario: a much larger proof-mass/beam-mass ratio of 60-70 was used in order to model a piezoelectric microgenerator with a more practically applicable fundamental frequency of 170 Hz (in this case the height of the proof-mass was slightly adjusted for each shape in order to maintain the same fundamental frequency). Numerical results obtained for this case (Fig. 5) are different from the previous one: comparable deflections and voltage outputs are achieved for the same excitation amplitude of $a=1g$ for all the considered cantilever shapes. The previously observed effect of higher tolerance for excitation amplitude does not manifest here. It could be rationalized that in this case the inertia of the proof-mass is so much larger with respect to the inertia of the cantilever section that the aforementioned differences in mass distributions between the rectangular- and trapezoidal-shaped cantilevers do not have appreciable influence on the overall dynamic response of the PMPG.

Meanwhile, the results in Fig. 2(a) reveal highly variable stress distributions throughout the length for different cantilever shapes. For the most triangular shapes (No.1 and No.2) a considerable increase in stress value is observed towards the cantilever-proof-mass junction place. This suggests that gradual widening of the cantilever towards its end would be a

reasonable shape modification: it would reduce the stress to the safer levels and because of the larger width the generated charge would not be diminished (or even slightly increased resulting in a larger PMPG efficiency).

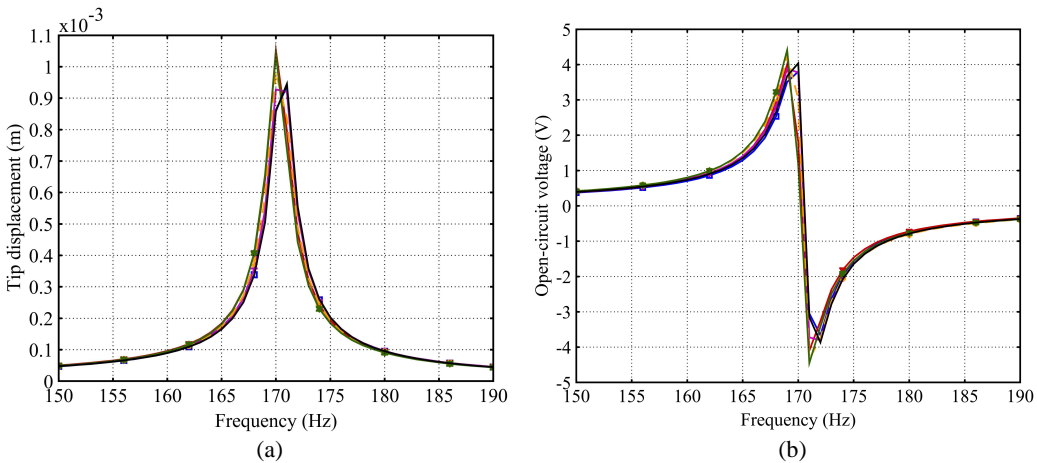


Fig. 5. Amplitude-frequency characteristics (a) and the corresponding voltage characteristics (b) of microgenerators with cantilevers of different shapes, obtained in the vicinity of their fundamental frequency (~ 170 Hz) when excited with identical acceleration of $a=1g$. (Curve notations are identical to those of Fig. 2).

In addition, it could be also mentioned that previous research results on cantilever dynamics [10] suggest that in the considered case of very large proof-mass/beam-mass ratios the changes in modal behavior may also occur and additionally influence the electrical response of the PMPG. Considerable enlargement of the size and weight of a proof-mass may eventually alter the boundary conditions of the structure: with increasingly heavier proof-masses the structure will gradually transform from a cantilever-type to a fixed-fixed beam. This implies that the magnitude of the fundamental mode of the cantilever structure will be constantly decreasing in this case until the limiting case is reached when this mode is entirely suppressed and is ultimately converted into a fundamental mode of a fixed-fixed beam, which slope is no longer monotonic and may therefore lead to a considerable charge cancellations inside the piezoceramic material. This aspect is of particular importance for the case when PMPG under random or impact-type excitations is considered since higher vibrations modes are induced in this case and must, therefore, be taken into account when developing the piezoelectric configuration of the microgenerator.

Conclusions

Results of the research may be summarized as follows.

Conducted numerical study demonstrated that cantilevers with increasingly triangular shape are highly beneficial in providing improved stress uniformity over the length of the structure for the case when the weight of the proof-mass is similar to that of the cantilever section. It is safe to assume that this would also remain valid for somewhat larger proof-mass/beam-mass ratios (up to 5 at least). It is obvious that for the considerably larger ratios of 60-70 the effect of much larger proof-mass is no longer advantageous and even may become detrimental for increasingly triangular shapes. These findings indicate that the current design trend of incorporating larger and heavier proof-masses in micro-scale energy harvesters (attempting to achieve sub-100 Hz

resonant frequencies) has to be approached with care in order to avoid negative consequences in terms of electrical performance of a PMPG. In addition, in the case of very large and heavy proof-masses the attenuation of the fundamental vibration mode of the cantilever and amplification of detrimental vibration modes with non-monotonic slopes should also be taken into consideration, particularly when dealing with broadband excitations that are characteristic to many real-life ambient vibration sources.

Cantilevers with increasingly triangular shape are capable of withstanding markedly larger base excitation magnitudes. However, the generated electrical output is only marginally increased for the comparable cantilever deflection levels. This outcome is governed by a complex relationship between the opposing effects of increased stress homogeneity and structural stiffness in a trapezoidal-shaped cantilever. It suggests that in order to achieve a meaningful gain in PMPG electrical output a careful geometry optimization procedure should be performed in terms of both cantilever and proof-mass dimensions taking into account maximum permissible stress levels at cantilever clamping and cantilever–proof-mass junction places.

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