633. Investigation of vibrations of elements of packages with supplementary stiffness elements

A. Kabelkaitė-Lukoševičė[1,a], E. Kibirkštis[1,b], V. Bivainis[1,c], L. Gegeckienė[1,d], L. Ragulskis[2,f]
[1] Kaunas University of Technology, Department of Graphic Communication Engineering,
Studentų 56-350, LT-51424 Kaunas, Lithuania
Phone: +37037300237; **Fax:** +37037451684.
[2] Vytautas Magnus University, Vileikos 8-702, LT-44404 Kaunas, Lithuania.
E-mail: [a] *asta.kabelkaite@ktu.lt,* [b] *edmundas.kibirkstis@ktu.lt,* [c] *vaidas.bivainis@ktu.lt,*
[d] *laura.gegeckiene@ktu.lt,* [f] *l.ragulskis@if.vdu.lt.*

(Received 6 January 2011; accepted 15 May 2011)

Abstract. The paper analyzes the eigenmodes of bending vibrations of ecological paperboard as of a plate by taking into account the additional effect of bending vibrations of a string included in the paperboard. The tension of the string can be used to control the eigenmodes of the analyzed paperboard. Experimental investigations were conducted using time-averaged projection moiré techniques on a specially developed experimental setup. The performed experimental and numerical studies demonstrated that the first eigenmodes of vibrations of the paperboard with supplementary stiffness elements occur at higher frequencies when compared with the conventional paperboard. The obtained eigenmodes enable to determine not only the vibration characteristics of the wall of the package by nondestructive methods, but also the strength parameters of the material of the package. The obtained results are used in the process of design of package elements.

Keywords: element of package, finite elements, supplementary stiffness, plate bending, string, vibrations, control of eigenmodes, experimental setup, projection moiré, time averaging.

Introduction

This study considers the eigenmodes of bending vibrations of paperboard with enhanced ecological qualities as of a plate by taking into account the additional effect of bending vibrations of a string included in the paperboard. The model for the analysis of vibrations of paperboard is proposed on the basis of the material described in [1, 2].

One of the basic requirements from the point of view of ecology concerning the reduction of the amount of packaging materials is to produce packages in such a way that their volume and the mass of the material of the package would be minimized while at the same time ensuring the necessary requirements for the safety of the packed product and also having acceptable consumer satisfaction [3-5]. Among the most important characteristics of the package is its resistance to vertical load. Depending on the expected tensile or compression loads acting on the package, it is necessary to choose paperboard of smaller grammage as this leads to reduction of the package mass [6-10].

Thus, in order to minimize the mass of the package without degrading its mechanical and operational characteristics paperboard of lower grammage and higher stiffness is to be chosen. One of the approaches to increase the supplementary stiffness of the paperboard is the introduction of supplementary elements such as a string into the paperboard. The materials used for packaging may be investigated not only applying conventional methods, but also using the methods of nondestructive testing, such as the geometrical, shadow, projection moiré techniques using stroboscopic or time-averaging approaches [11-19]. The purpose of this paper is to investigate the eigenmodes of the paperboard and of the paperboard with supplementary stiffness elements using the technique of time-averaged projection moiré. This enables to investigate the strength parameters of this composite (modified) material of the package by using the nondestructive approach.

The obtained results are used in the process of design of the elements of packages.

Model for the analysis of paperboard vibrations

Further x, y and z denote the axes of the system of coordinates. The plate bending element has three nodal degrees of freedom: the transverse displacement of the paperboard w and the rotations Θ_x and Θ_y about the axes of coordinates x and y.

The mass matrix of the paperboard has the form:

$$[M]=\int [N]^T \begin{bmatrix} \rho_p h & 0 & 0 \\ 0 & \frac{\rho_p h^3}{12} & 0 \\ 0 & 0 & \frac{\rho_p h^3}{12} \end{bmatrix} [N]\, dxdy, \tag{1}$$

where ρ_p is the density of the material of the paperboard, h is the thickness of the paperboard and:

$$[N]=\begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_1 & 0 & \dots \\ 0 & 0 & N_1 & \dots \end{bmatrix}, \tag{2}$$

where N_i are the shape functions of the two dimensional finite element.

The stiffness matrix of the paperboard has the form:

$$[K]=\int \left([B]^T [D][B] + \left[\bar{B}\right]^T \frac{E_x E_y h}{\left(E_x+E_y+E_x\nu_{yx}+E_y\nu_{xy}\right)1.2} \left[\bar{B}\right] \right) dxdy, \tag{3}$$

where E_x and E_y are the modulus of elasticity of the paperboard, ν_{xy} and ν_{yx} are the Poisson's ratios of the paperboard and:

$$[B]=\begin{bmatrix} 0 & 0 & \frac{\partial N_1}{\partial x} & \dots \\ 0 & -\frac{\partial N_1}{\partial y} & 0 & \dots \\ 0 & -\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \dots \end{bmatrix}, \tag{4}$$

$$[D]=\frac{h^3}{12}\begin{bmatrix} \frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{E_y\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{E_x\nu_{yx}}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & \frac{E_x E_y}{E_x+E_y+E_x\nu_{yx}+E_y\nu_{xy}} \end{bmatrix}, \tag{5}$$

$$\left[\bar{B}\right]=\begin{bmatrix}\frac{\partial N_1}{\partial y} & -N_1 & 0 & \dots \\ \frac{\partial N_1}{\partial x} & 0 & N_1 & \dots\end{bmatrix}. \tag{6}$$

The ecological effect is achieved by taking into account the bending vibrations of a string which is inside the paperboard and is parallel to the y axis.

The mass matrix of the string has the form:

$$\left[\bar{M}\right]=\int\left[\bar{N}\right]^T \rho A\left[\bar{N}\right]dy, \tag{7}$$

where ρ is the density of the material of the string, A is the cross-sectional area of the string and:

$$\left[\bar{N}\right]=\left[\bar{N}_1 \quad 0 \quad 0 \quad \dots\right], \tag{8}$$

where $\bar{N}_i$ are the shape functions of the one dimensional finite element.

The stiffness matrix of the string has the form:

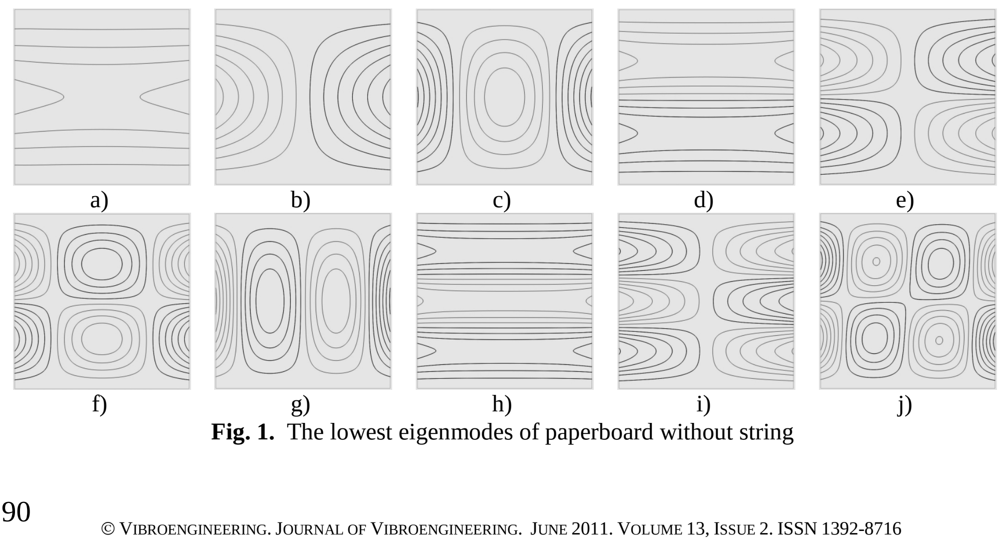
$$\left[\bar{K}\right]=\int[G]^T \sigma A[G]dy, \tag{9}$$

where σ is the longitudinal stress in the string and:

$$[G]=\begin{bmatrix}\frac{d\bar{N}_1}{dy} & 0 & 0 & \dots\end{bmatrix}. \tag{10}$$

Results of vibration analysis

The square piece of paperboard is analyzed. On the lower and upper boundaries all the generalized displacements are assumed equal to zero. The string is located in the center of the paperboard. It is assumed that the modulus of elasticity E_x=5.6×10^9 Pa, Poisson's ratio v_{xy}=0.37, Poisson's ratio v_{yx}=0.12, thickness of the paperboard h=0.000565 m, density of the material of the paperboard ρ_p=708 kg/m^3, force of tension of the string σA=0.1 N, longitudinal density of the material of the string ρA=0.01 kg/m. The first eigenmodes of paperboard without string are presented in Fig. 1. The first eigenmodes of paperboard with string are presented in Fig. 2.



a) b) c) d) e)

f) g) h) i) j)

Fig. 1. The lowest eigenmodes of paperboard without string

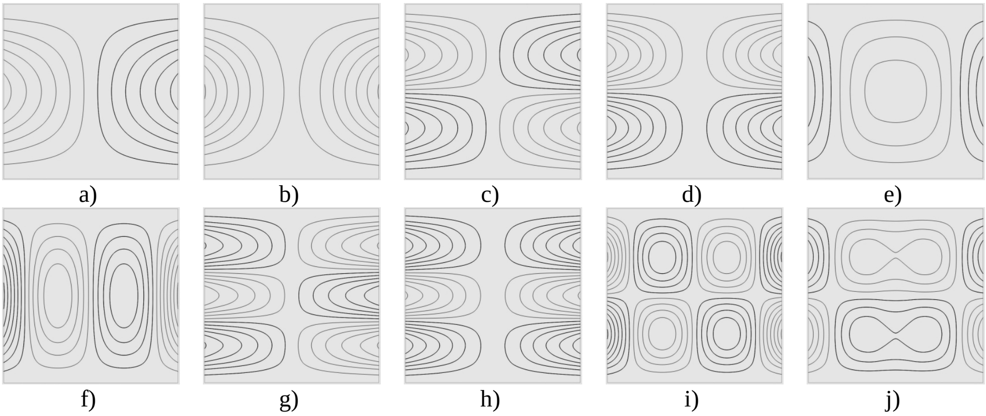


Fig. 2. The lowest eigenmodes of paperboard with string

The presented results indicate that the eigenmodes of the analyzed paperboard without string and the one with the string have substantial differences. This implies that the tension of the string can be used to control the eigenmodes of the analyzed paperboard.

The method and the results of experimental investigation

A special setup for experimental investigation was used in order to determine the dynamical characteristics of the paperboard (Fig. 3a) [20-24]. Diagram of the element of package with supplementary stiffness element is shown in Fig. 3b.

Paperboard MC Mirabell was chosen for the investigation. Technical characteristics of this paperboard are: surface density – 400 g/m^2, thickness – 565 μm. The square piece of paperboard is analyzed: paperboard sheet was 0.2 m in width and 0.2 m in length. The obtained results of experimental study are presented in Fig. 4 and Fig. 5. For the paperboard without string MC Mirabell the image of the first eigenmode is illustrated in Fig. 4a, of the second eigenmode – in Fig. 4b and of the third eigenmode in Fig. 4c. The corresponding images of the first eigenmode of the paperboard with string are presented in Fig. 5a, of the second eigenmode – in Fig. 5b, of the third eigenmode – in Fig. 5c, of the fourth eigenmode – in Fig. 5d and of the fifth eigenmode in Fig. 5e.

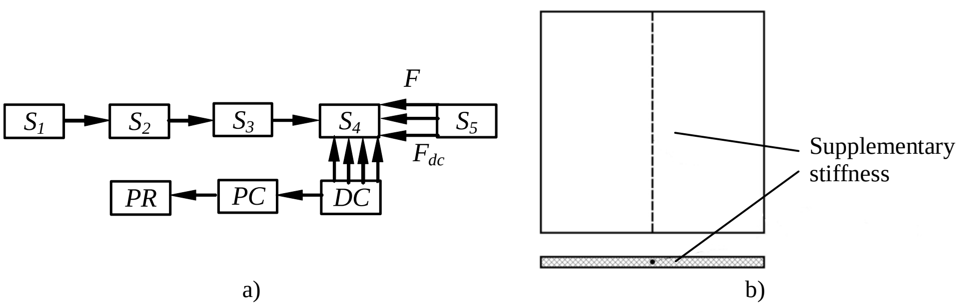
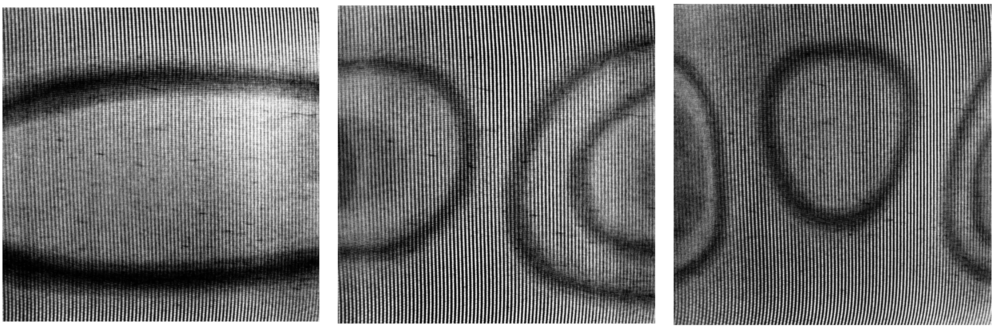
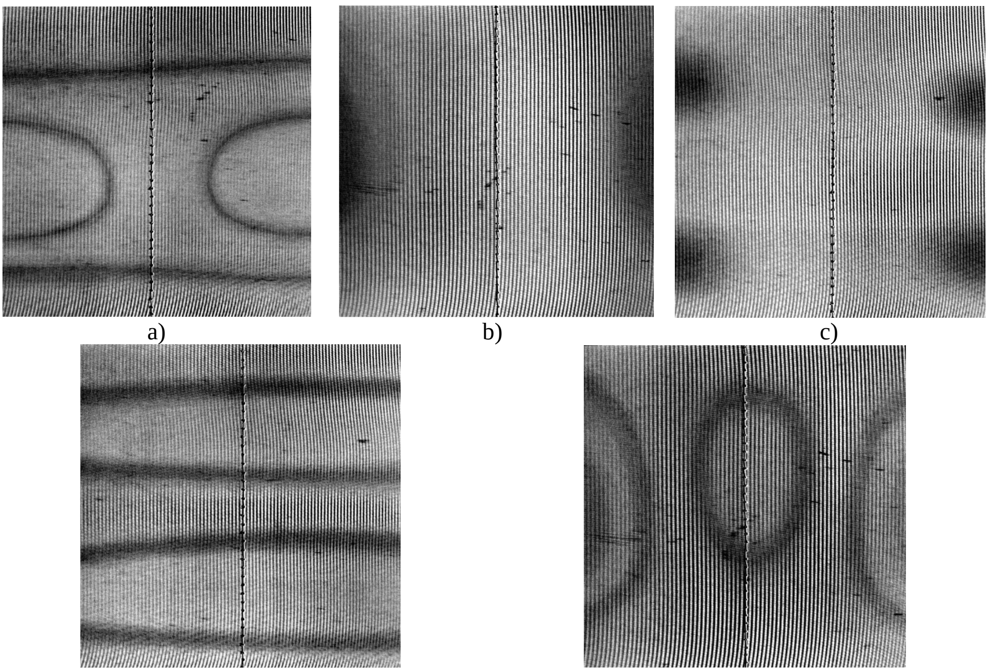


Fig. 3. a) Structural diagram of the setup for experimental investigations: S_1 – signal generator; S_2 – signal amplifier; S_3 – setup for the analysis of vibrations; S_4 – the investigated material; S_5 – source of monochromatic light; DC – digital camera; PC – personal computer; PR – printer; F_{dc} – light flux of the digital camera; F – flux of light projected using the source of light through the grid of the step p at an angle α; b) diagram of the element of package with supplementary stiffness element



a) b) c)

Fig. 4. The eigenmodes of paperboard without string (MC Mirabell 400 g/m^2): a) the first eigenmode, frequency of vibrations 50 Hz; b) the second eigenmode, frequency of vibrations 70 Hz; c) the third eigenmode, frequency of vibrations 92 Hz. Amplitude 3×10^{-4}m, loading force 25.5 N



a) b) c)

d) e)

Fig. 5. The eigenmodes of paperboard with string (MC Mirabell 400 g/m^2): a) the first eigenmode, frequency of vibrations 60 Hz; b) the second eigenmode, frequency of vibrations 106 Hz; c) the third eigenmode, frequency of vibrations 126 Hz; d) the fourth eigenmode, frequency of vibrations 140 Hz; e) the fifth eigenmode, frequency of vibrations 220 Hz. Amplitude 3×10^{-4}m, loading force 25.5 N

Experimental results presented in Fig. 4 and Fig. 5 indicate that for the paperboard with string (Fig. 5) the first, the second and the third eigenmodes take place at higher frequencies if compared with the paperboard without the string (Fig. 4). Also the first, the second and the third eigenmodes of paperboard with string differ from the corresponding eigenmodes of the paperboard without string. From the obtained results it is evident that the introduction of the supplementary stiffness element into the paperboard gave supplementary stiffness to the material of the package. Thus the control of tension of the supplementary stiffness element enables to change the shapes of the eigenmodes of vibrations of the paperboard.

Conclusions

The model for the analysis of vibrations of paperboard with string is proposed. The eigenmodes of bending vibrations as of a plate are evaluated by taking into account the bending vibrations of a string located inside the paperboard. Presented results reveal that the eigenmodes of the analyzed paperboards without string and with string have substantial differences. Thus the tension of the string can be used to control the eigenmodes of the analyzed paperboard.

The performed experimental and numerical investigations demonstrated that the first, the second and the third eigenmodes of vibrations of the paperboard with supplementary stiffness elements (in this case with a tensioned string) occur at higher frequencies when compared with the conventional (unmodified) paperboard.

The obtained eigenmodes in the process of design of packages enable to determine not only the vibration characteristics of the wall of the package by nondestructive methods, but also the strength parameters of the material of the package such as Poisson's ratios and modulus of elasticity. This enables to make the design of such packages simpler and easier.

References

[1] **Zienkiewicz O. C.** The Finite Element Method in Engineering Science. Moscow: Mir, 1975.

[2] **Bathe K. J.** Finite Element Procedures in Engineering Analysis. New Jersey: Prentice-Hall, 1982.

[3] European Parliament and Council Directive 94/62/EC on Packaging and Packaging Waste // Official Journal of the European Union.

[4] Directive 2004/12/EC of the European Parliament and of the Council of 11 February 2004 Amending Directive 94/62/EC on Packaging and Packaging Waste // Official Journal of the European Union. No. L 47, 18/02/2004. p. 26 - 31.

[5] **Kibirkštis E., Lebedys A., Kabelkaitė A., Havenko S.** Experimental Study of Paperboard Package Resistance to Compression. - Mechanika. - Kaunas: Technologija, 2007, no.1 (63), p. 36 - 41.

[6] **Beldie L., Sandberg G., Sandberg L.** Paperboard Packages Exposed to Static Loads - Finite Element Modeling and Experiments // Packaging Technology and Science. - John Wiley & Sons. - 2001. Vol. 14, p. 171 - 178.

[7] **Kabelkaitė A., Kibirkštis E., Ragulskis L.** Investigation of Compression of Packages // Journal of Vibroengineering. Vilnius: Vibromechanika. 2008, Vol. 10, no. 1, p. 104 - 113.

[8] **Hoffmann J.** Compression and Cushioning Characteristics of Moulded Pulp Packaging // Packaging Technology and Science. - John Wiley & Sons. - 2000. Vol. 13, p. 211 - 220.

[9] **Han J., Min Park J.** Finite Element Analysis of Vent / Hand Hole Designs for Corrugated Fibreboard Boxes // Packaging Technology and Science. - John Wiley & Sons. - 2007. Vol. 20, p. 39 - 47.

[10] **Bivainis V., Kibirkštis E., Lebedys A.** Experimental Study of Paperboard Crease Weight to Mechanical Characteristics of Paperboard Packages. – Mechanika - 2008. Proceedings of the 13th International Conference. - Kaunas: Technologija, 2008, p. 62 - 66.

[11] **Cloud G.** Optical Methods in Experimental Mechanics. Part 21: Shadow Moire // Experimental Techniques. Bethel. - 2006, Vol. 30, no. 2, p. 15 -18.

[12] **Han C.-W., Han B.** High Sensitivity Shadow Moire Using Nonzero - Order Talbot Distance // Experimental Mechanics, August 2006, Volume 46, number 4, p. 543 - 554.

[13] **Kadooka K., Kunoo K., Uda N., Ono K., Nagayasu T.** Strain Analysis for Moire Interferometry Using the Two - Dimensional Continuous Wavelet Transform // Experimental Mechanics, March 2003, Volume 43, number 1, p. 45 - 51.

[14] **Naganuma S., Tagawa N., Minagawa A., Moriya T.** Determination of Object Shape and Reflectance Based on Moiré System Using Multiple Light Sources and a Reflection Model // Systems and Computers in Japan, June 2006, Volume 37, issue 6, p. 32 - 43.

[15] **Cloud G.** Optical Methods in Experimental Mechanics. Part 22: Projection Moire // Experimental Techniques. Bethel. - 2006, Vol. 30, no. 4, p. 15 - 18.

[16] **Ragulskis M., Maskeliūnas R., Saunorienė L.** Identification of In - Plane Vibrations Using Time Average Stochastic Moire // Experimental Techniques. Bethel. - 2005, Vol. 29, issue 6, p. 41 - 45.

[17] **Cloud G.** Optical Methods in Experimental Mechanics. Part 18: Geometric Moiré Phenomena and Simulations // Experimental Techniques, July 2005, Volume 29, issue 4, p. 15 - 18.

[18] **Ragulskis M., Saunorienė L.** Applicability of Optical Geometric Differentiation for Time - Average Geometric Moire // Strain, August 2006, Vol. 42, issue 3, p. 173 - 179.

[19] **Ratnam M. M., Lim J. H., Abdul Khalil H. P. S.** Study of Three - Dimensional Deformation of a Pallet Using Phase - Shift Shadow Moire and Finite - Element Analysis // Experimental Mechanics, February 2005, Volume 45, number 1, p. 9 - 17.

[20] **Kabelkaitė A., Kibirkštis E., Ragulskis L., Dabkevičius A.** Analysis of Vibrations of Paper in a Printing Device // Journal of Vibroengineering. - Vilnius. – 2007. Vol. 9, no. 1, p. 41 - 50.

[21] **Kibirkštis E., Kabelkaitė A., Dabkevičius A., Ragulskis L.** Investigation of Vibrations of a Sheet of Paper in the Printing Machine // Journal of Vibroengineering. - Vilnius. – 2007. Vol. 9, no. 2, p. 40 - 44.

[22] **Kibirkštis E., Kabelkaitė A., Dabkevičius A., Ragulskis L.** Investigation of Vibrations of Packaging Materials // Journal of Vibroengineering. - Vilnius. – 2008. Vol. 10, no. 2, p. 225 - 235.

[23] **Kibirkštis E., Kabelkaitė A., Dabkevičius A., Bivainis V., Ragulskis L.** Non - Destructive Diagnostics of Uniformity of Loading of the Sheet of Paper // Ultrasound. Vol. 64, no. 3, 2009, p. 24 - 28.

[24] **Kibirkštis E.,Bivainis V.,Ragulskis L.,Dapkevičius A.** Investigation of compression of cylindrical packages. – Mechanika. –Kaunas: Technologija,2009,No3(77),p.47-53.