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Analysis of the zero-crossing technique in relation to measurements of phase velocities of the Lamb waves

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Abstract

The delay time estimation using zero-crossing technique is widely used in ultrasonic measurements. The measurements of the phase velocity of guided waves are more complicated due to the dispersion. Application of zero-crossing technique for the measurements of the phase velocities is more complicated due to changes of the waveform of the signals and limited ranges of the measurements base. The objective of the work presented was to investigate in more details the influence of different parameters of the zero – crossing technique on measurement of the phase velocity of A_0 mode of the Lamb waves. Using the signals obtained from the finite element modelling of Lamb waves in a 2 mm thickness aluminium plate it was demonstrated that an insufficient sampling frequency can lead to the errors in the phase velocity and corresponding frequency estimations. On the other hand it was shown also, that in order to obtain a higher equivalent sampling frequency it is reasonable to exploit interpolation. The optimal parameters of the zero-crossing technique and necessary steps for phase velocity measurement are presented also.

Keywords: Lamb wave, interpolation, phase velocity, measurement uncertainties.

Introduction

The Lamb waves are used in various non – destructive testing (NDT) tasks: to detect corrosion, the detection of non – homogeneities or defects and the estimation of the elastic properties of materials. One of the most important parameters in these tasks is a propagation velocity. However, to measure the propagation velocity of the guided waves is complicated due to dispersion and infinite number of modes. In most cases the propagation velocity of guided waves is estimated using measurement of the propagation time. Different techniques, such as signal maximum position in the time domain, the zero–crossing technique [1, 2, 3], cross-correlation [4] are known and used in practice for the delay time evaluation. Most accurate are the cross-correlation and the zero-crossing techniques used in many ultrasonic applications such as distance measurements or ultrasonic gas or liquids flow measurements. As was demonstrated in [1], the zero–crossing technique in application to guided waves gives several advantages including possibility to reconstruct segment of the phased velocity dispersion curve. However some scattering of the results obtained by different techniques was observed. One of the reasons for that can be the errors caused by the uncertainties of the zero-crossing technique.

So, the objective of the work presented was to investigate in more details the influence of different parameters of the zero – crossing technique on measurement of the phase velocity of the A_0 Lamb wave mode.

The zero-crossing technique

The zero-crossing technique is one of the methods enabling to evaluate the delay time of propagating waves. The main idea of this technique is that using some threshold level the half period of the signal exceeding this level is determined (Fig.1).

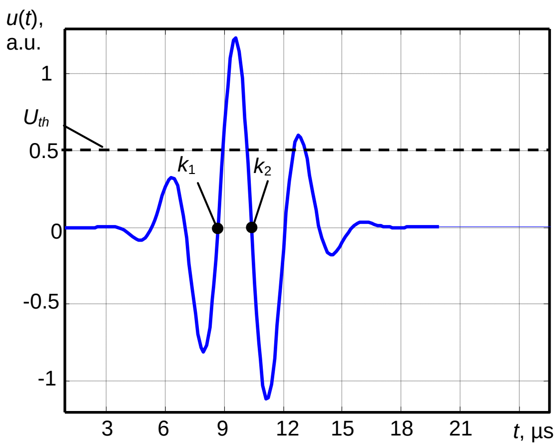


Fig.1. **Illustration of main idea of zero-crossing technique**

In the second step the time instance at which the signal crosses the zero level is estimated. The main advantage of such approach is that this zero-crossing instant in some ranges does not depend on the amplitude of the signal. In guided wave applications when not one, but several zero-crossing instants are determined it enables to reconstruct the segment of the phase velocity dispersion curves [1]. This is a very important feature of the technique, because no other delay time measurement techniques enable to do this. However, in order to apply it correctly all peculiarities of the method should be investigated, possible sources of the errors analysed and the expected level of them estimated. The zero crossing technique many years was widely used even in an analog electronics for ultrasonic measurements and later was well known in the numerical implementations. However when the signal is digitized many questions arise related to different implementations. The approach based on just selection of the closest to zero sample of the signal possesses a relatively low accuracy. The zero crossing point can be determined also using a linear interpolation between two closest to the zero points.

However in the case of noisy signals it will lead to the errors. A more accurate technique is based on the polynomial approximation, but the optimal order of polynomial and the number of the samples of the signal used in approximation should be determined. So, the task of the presented investigation is to analyse the influence of all parameters of the technique on a final accuracy of the phase velocity measurements.

The delay time measurement using the zero-crossing technique with application of the approximation is performed in the following steps:

1. The threshold level U_{th} is defined;

2. The first sample of the signal, which exceeds the threshold level is found (Fig.2)

$$n_1 = \min\{\arg[u(t_n) > U_{th}]\}, \quad (1)$$

where $u(t_n)$ is the digitized signal, $n = 1 \div N_s$, N_s is the total number of samples in the signal;

3. The samples with the smallest absolute amplitude are determined in backward and front directions (in time)

$$k_1 = \arg\{\min[|u(t_k)|]\}, k = n_1 - \frac{N_T}{2} \div n_1, \quad (2)$$

$$k_2 = \arg\{\min[|u(t_k)|]\}, k = n_1 \div n_1 + \frac{N_T}{2}, \quad (3)$$

where N_T is the number of samples per period;

4. The samples in the zones of the zero-crossing are approximated using the M order polynomial $P^M(t_k)$, where $k = \left(k_1 - \frac{N_A}{2}\right) \div \left(k_1 + \frac{N_A}{2}\right)$ for the zero-crossing zone in the backward direction and $k = \left(k_2 - \frac{N_A}{2}\right) \div \left(k_2 + \frac{N_A}{2}\right)$ for the zero-crossing zone in the front direction, N_A is the number of samples used in the approximation; as the result the two polynomial are obtained $P_1^M(t_k)$ and $P_2^M(t_k)$;

5. The equations

$$\begin{aligned} P_1^M(t_k) = 0, \\ P_2^M(t_k) = 0, \end{aligned} \quad (4)$$

are solved and the zero – crossing instances t_1, t_2 are determined;

6. The steps 2 - 5 are repeated, just in the step 2 the ranges of the variable n are different $n = k_2 + \frac{N_T}{4} \div N_s$, and the zero-crossing instants t_3, t_4 of the next period of the signal are obtained. The process is continued until the next period of the signal which does not exceeds the threshold level. The obtained set of the zero-crossing instances t_1, t_2, t_3..... is used for reconstruction of the dispersion curve segment according to the algorithm defined in [1].

As can be seen all parameters can affect the accuracy of the delay time determination or to cause errors. The most important are:

1. The sampling interval in the time domain dt;

2. The degree of a polynomial used for approximation;

3. The number of samples N_T per period T of the signal;

4. The number of points N_A used for approximation.

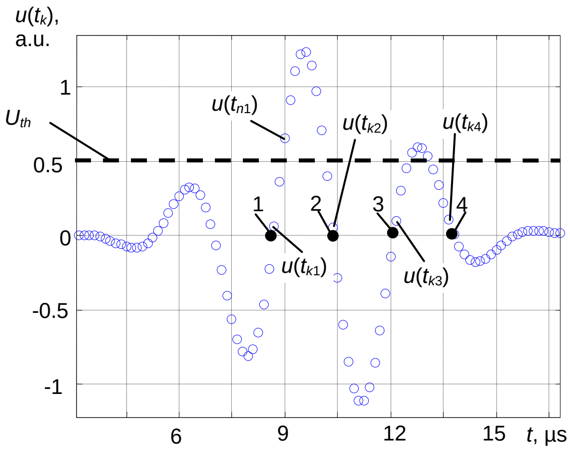


Fig.2. **The several zero-crossing point of digitized signals**

Some of the parameters can be defined just from general considerations. The sampling interval dt affects the final result directly – the smaller sampling interval enables more accurately to determine the zero-crossing instant. In general, if the sampling step is small enough, the approximation even can not be needed. On the other hand if the signal is noisy the approximation can reduce errors. Too small sampling step (over sampling) leads to a very long signals and as a consequence to non-efficient usage of the memory and the processing time of the computer or the measurement instrument.

The degree of the polynomial which should be used for approximation can be determined just from a following consideration. The third order polynomial probably is the best one enabling to imitated sine shape of the signal in the zero-crossing zone. The higher order polynomials are capable to approximate more complex waveforms, however due to this feature they will be more affected by presence of the noise and as a consequence will lead to bigger errors in the delay time determination.

The number of samples per period and the number of samples used in the approximation are interrelated. It can be stated that the approximation should be performed in a steep part of the signal including the zero-crossing point (Fig.3). If the points corresponding to maximums or minimums of the signal will be included into the approximation it will lead to bigger errors as these points are more affected by the noise. In the case of guided wave signals, the amplitudes of different periods in the burst changes due to the dispersion (Fig.4). This is another reason to avoid approximation of the larger part of the signal including zones of the maximums and the minimums. On the other hand, the bigger number of samples used in the approximation enables to reduce the influence of the noise due to integration effect. In order to investigate in more details influence of these parameters on scattering of the results, the presented below investigations were carried out.

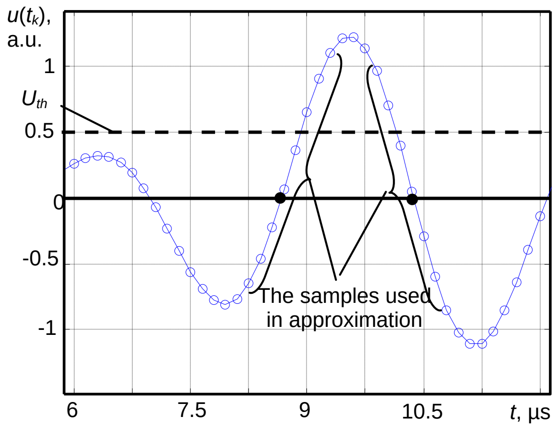


Fig.3. **The parts of the signal used in the approximation**

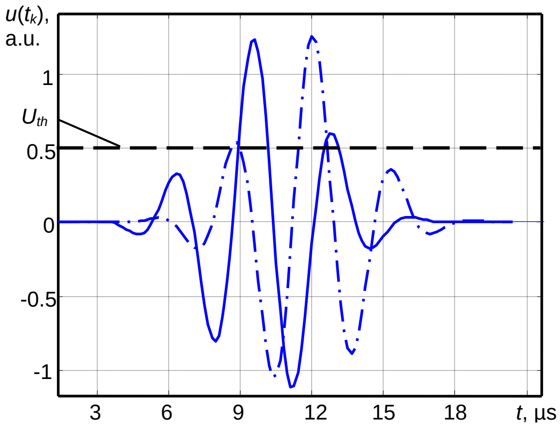


Fig.4. **The signals of A_0 mode at distance 10 mm (solid line) and 15 mm (dashed line) from the edge of the plate.**

The finite element model and signals used in analysis

The zero-crossing technique was investigated on the modelled signals obtained using a finite element method. Propagation of the asymmetric A_0 of Lamb wave mode in the 2 mm thickness and 200 mm length aluminium plate was simulated. The parameters of the aluminium plate were used in the model are: the density $\rho = 2780$ kg/m^3, the Young modulus $E = 71.78$ GPa, the Poisson's ratio $\nu = 0.3435$. The sampling step in the spatial domain was $dx = 0.1$ mm and $dt = 0.15$ µs in the time domain. The A_0 mode was excited by applying a tangential force to one of the plate edges (Fig.5).

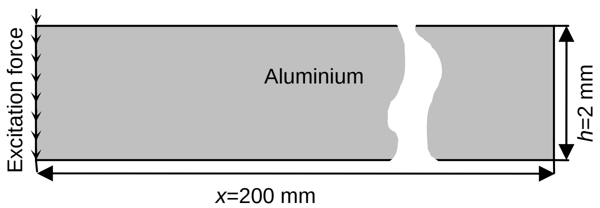


Fig.5. **The finite element model used for investigation of propagation of the A_0 mode Lamb wave in an aluminium plate**

The waveform of the excitation force was 3 period, 300 kHz burst with the Gaussian envelope. Propagation of the guided wave was modelled in the time interval 100 µs. The normal component of the particle velocity corresponding to the top surface of the plate was used as the set of the signals for analysis. The B-scan image of these signals is presented in (Fig.6).

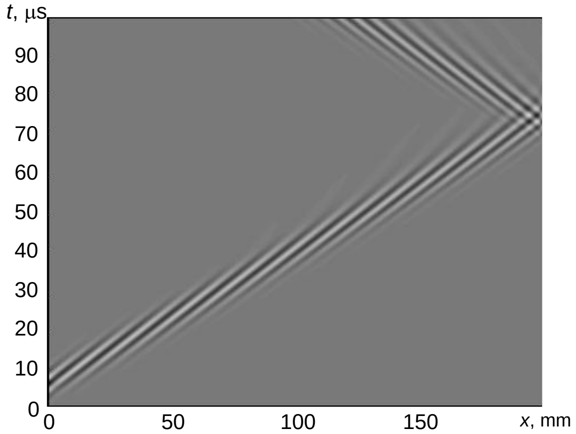


Fig.6. **The B-scan image of the normal component of the particle velocity on the surface of the plate**

It can be estimated that each period of the signal consists of 22 samples. The time interval between maximum and minimum of the period contains 11 samples of the signal. If to exclude the maximal and minimal samples, as it was discussed above, it will lead to 9 samples, which can be exploited in the approximation using the third order polynomial.

Determination of the phase velocity and the frequency

In order to investigate how the number of samples per period affects the final results of the delay time determination three different cases were analysed: the original non-interpolated signals obtained with the sampling interval 0.15µs, the signal with one additional interpolated point between two samples and with two interpolated points. The additional points were obtained using the linear interpolation. The number of samples per period and the equivalent sampling frequency are presented in Table 1.

Table 1. **The number of samples per period and the equivalent sampling frequency in analyzed cases**

Number of the interpolated points between original samples	0	1	2
Samples per period	22	44	66
Equivalent sampling frequency [kHz]	6,67	13,3	20

The calculated delay times $t_1(x)$, $t_2(x)$, $t_3(x)$, $t_4(x)$ in the case of non-interpolated signals are presented in Fig. 7, where four parallel lines can be observed with some sharp changes ("jumps"). These changes are explained by the fact that waveform of the propagating wave changes due to the presence of the dispersion. The first period of the burst increases in amplitude and 'moves' inside. At the same time the new first period is growing. This can be observed in the B-scan image (Fig. 6) and in two signals measured at different distances (Fig. 4). However, in Fig. 7 only general changes of the measured delay times can be observed. More important is the delay time difference $\Delta t_k(x) = t_k(x+dx) - t_k(x)$ measured between two neighbouring positions, because the phase velocity of the propagating wave at some point x can be calculated according to

$$c_{ph}(x) = \frac{dx}{t_m(x+dx) - t_m(x)}. \quad (5)$$

The delay time difference $\Delta t_1(x)$ calculated for the first zero-crossing point in the case of original signals is presented in Fig. 8. The duplicated lines show a quite large scattering of the results. The question is: is it possible to reduce this scattering can not be reduced using interpolated signals? In order to get the answer the same delay time difference was calculated using interpolated signals. The obtained results are presented in Fig. 9. They demonstrate that the scattering of the results is reduces essentially.

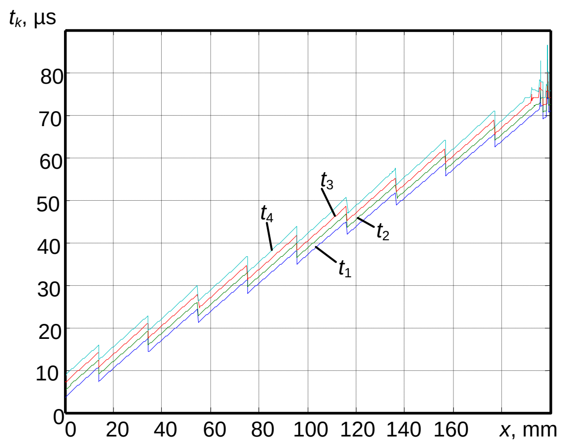


Fig.7. **The measured zero-crossing instance versus distance in the case of non-interpolated signals**

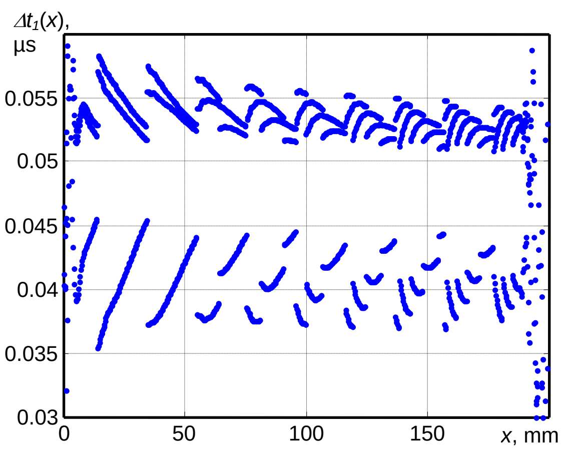


Fig.8. **The delay time difference $\Delta t_1(x)$ calculated for first zero-crossing point in the case of original signals.**

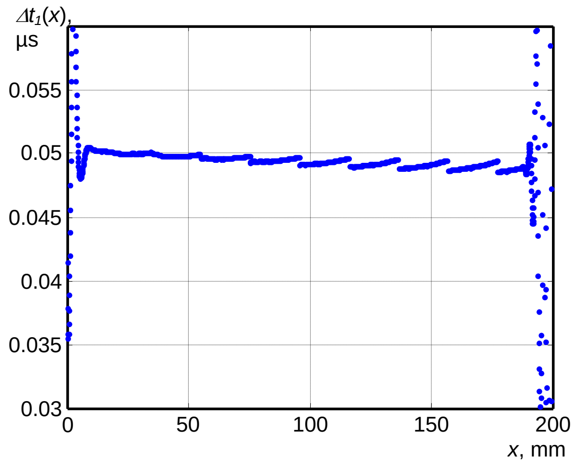


Fig.9. **The delay time difference $\Delta t_1(x)$ calculated for the first zero-crossing point in the case interpolated signals (two interpolated point between each two samaples) .**

In the next stage of the phased velocity estimation the mean value of the time difference is calculated for each interval between the sharp "jumps":

$$\overline{\Delta t_k}(\overline{x_m}) = \frac{1}{N_m} \sum_{x=x_{m,1}}^{x_{m,2}} \Delta t_k(x), \quad (6)$$

where $[x_{m,1}; x_{m,2}]$ are the intervals of the measurement data between two "jumps", $m = 1 \div M$, M is the total number of the intervals without "jumps", $\overline{x_m} = \frac{x_{m,1} - x_{m,2}}{2}$ is the mean value of the distance of the interval without "jumps" and N_m is the number of the measurements points in the interval. Using the average delay time values the average phase velocity is calculated

$$\overline{c_{ph,k}}(\overline{x_m}) = \frac{dx}{\overline{\Delta t_k}(\overline{x_m})}, \quad (7)$$

The obtained average values of the phase velocity in the case of original and interpolated signals demonstrate (Fig.10) that the difference between them is approximately 5m/s or 0.25%.

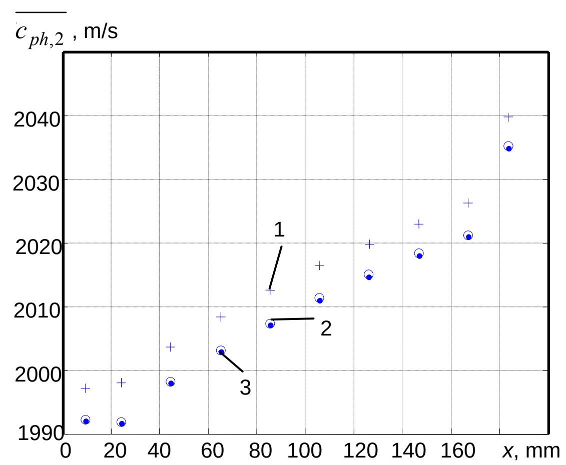
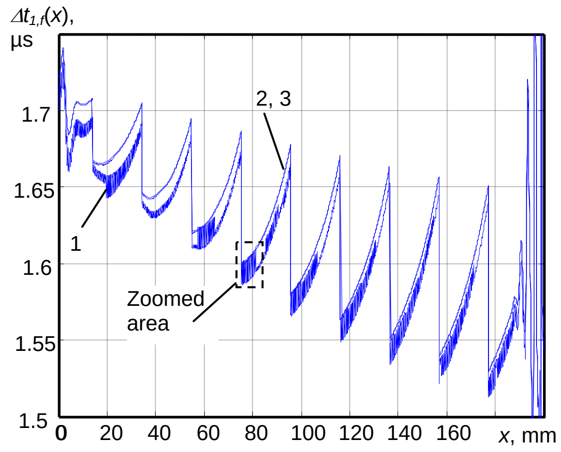


Fig.10. **The phase velocity of A_0 Lamb wave mode estimated using the second zero-crossing point in the case of: 1 - original signals; 2 - one interpolated point between samples; 3 – two interpolated points between samples**

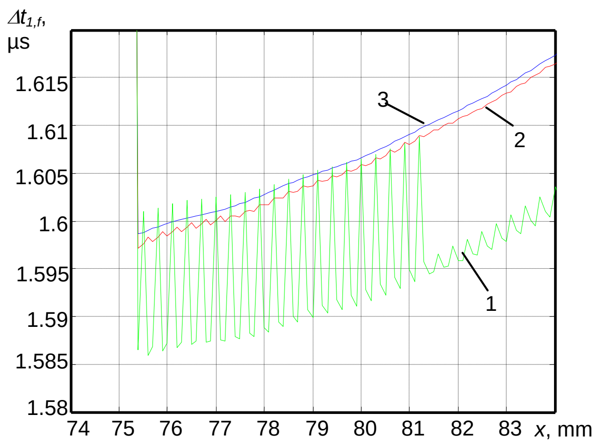
However these phase velocity values should be related to some frequencies, because the phase velocity of the guided waves is frequency dependent. This relation is determined using the duration of each half period in the burst. The duration of each half period in the burst can calculated using the same zero-crossing instants t_k:

$$\Delta t_{k,f}(x) = t_{k+1}(x) - t_k(x). \qquad (8)$$

The obtained duration of the first half period at different distances is presented in Fig.11. It can be seen that in the case of non-interpolated signals the scattering of the results is quite big. However it can be reduced essentially using interpolated signals. This can be clearly observed in the zoomed part presented in Fig.11b



a



b

Fig.11. **The duration of the first half period of the signal $\Delta t_{1,f}(x)$ versus distance (a) and the zoomed part (b): 1 –in the case of non-interpolated signals; 2, 3 – in the case of one and two interpolated point between each pair of samples**

The average durations of the half period in each interval between "jumps" are estimated according to

$$\overline{\Delta t_{k,f}}\left(\overline{x_m}\right) = \frac{1}{N_m} \sum_{x=x_{m,1}}^{x_{m,2}} \Delta t_{k,f}(x). \qquad (9)$$

Then the equivalent frequencies of the each half period in the burst are

$$\overline{f_k}\left(\overline{x_m}\right) = \frac{1}{2 \cdot \overline{\Delta t_{k,f}}\left(\overline{x_m}\right)}. \qquad (10)$$

The frequencies of the first and the second periods in the burst can be obtained according

$$\overline{f_{12}}\left(\overline{x_m}\right) = \frac{\overline{f_1}\left(\overline{x_m}\right) + \overline{f_2}\left(\overline{x_m}\right)}{2}, \quad \overline{f_{23}}\left(\overline{x_m}\right) = \frac{\overline{f_2}\left(\overline{x_m}\right) + \overline{f_3}\left(\overline{x_m}\right)}{2}. \qquad (11)$$

The equivalent frequencies $\overline{f_{12}}\left(\overline{x_m}\right)$ of the first period in the burst versus a distance for non-interpolated and interpolated signals are presented in Fig.12. It can be observed that there is difference between them approximately equal to 3kHz or 1%. This is a quite big error in accurate measurements.

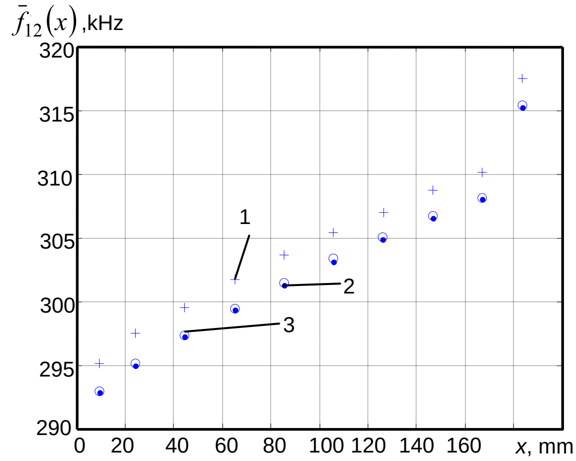


Fig.12. **The estimated equivalent frequency of the first period in the burst in the case of: 1 – non-interpolated signals; 2 - one interpolated point between samples; 3 – two interpolated points between samples**

As the result of the entire algorithm the set of the phase velocities $\overline{c_{ph,k}}\left(\overline{x_m}\right), k = 1 \div 4$, for each zero crossing point and the set of frequencies $\overline{f_k}\left(\overline{x_m}\right), k = 1 \div 3$ for each interval between zero-crossing points are obtained. As the phase velocities are obtained using some zero–crossing point which is between two intervals it is reasonable to analyse only the phase velocities obtained using middle zero-crossing points $\left(\overline{c_{ph,2}}\left(\overline{x_m}\right), \overline{c_{ph,3}}\left(\overline{x_m}\right)\right)$ and relate them to the equivalent frequencies of the first and the second period in the burst. These two sets of results can be presented as the segment of dispersion curve (Fig. 13 and 14).

In the same figures the theoretical dispersion curve is denoted by the solid line. Neglecting the observed earlier difference between the phase velocities and the frequencies determined using non-interpolated and interpolated signals there is no essential difference with respect to theoretical curve, just some shift along the dispersion curve.

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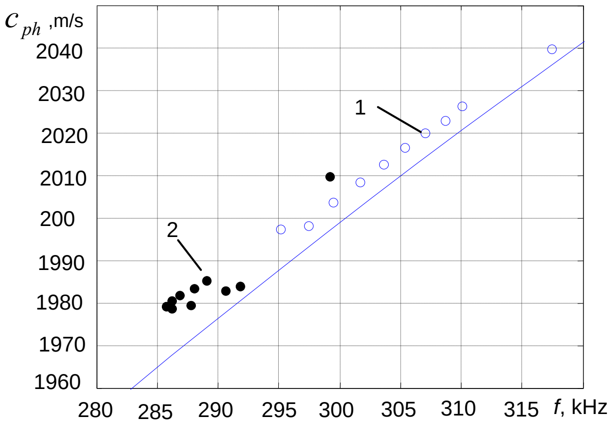

Fig.13. **The theoretical dispersion curve (solid line) and the phase velocity versus frequency obtained using non-interpolated signals: 1-** $\overline{c_{ph,2}}$, **2 -** $\overline{c_{ph,3}}$,

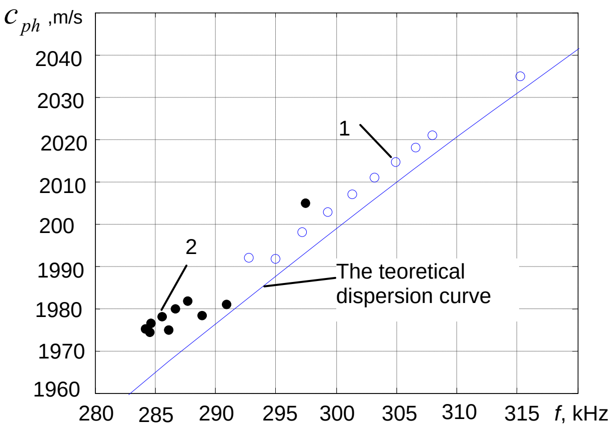

Fig.14. **The theoretical dispersion curve (solid line) and the phase velocity versus frequency obtained using two interpolated points between each pair of samples: 1-** $\overline{c_{ph,2}}$, **2 -** $\overline{c_{ph,3}}$

Conclusions

The analysis of the delay time measurement using the zero-crossing technique has demonstrated that:

1. In the case when a high accuracy is needed, the sampling frequency, enabling to obtain at least 40 points per period of the signal should be used. In the case of lower sampling frequencies the interpolation can be exploited in order to obtain additional sampling points;
2. In the case of measurements at multiple positions the averaging essentially reduces scattering of the results even if relatively low sampling frequencies have been used. However still some error can be expected in estimation of phase velocity and frequency. In the analysed case of the A_0 Lamb wave mode it was in the range of 0.25% for phase velocity and 1% for frequency estimations.
3. In the analysed case of the A_0 mode presence of two errors in some sense compensates each other and no essential differences between results obtained using the original and the interpolated signals were observed with respect to theoretical dispersion curve. However, it does not mean that it can be extrapolated for the case of other guided wave modes

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Signalo perėjimo per nulį metodo nukreiptųjų bangų faziniam greičiui matuoti tyrimas

Reziumė

Naudojant Lembo bangas tiek neardomuosiuose ultragarsiniuose bandymuose (NDT), tiek medžiagos savybėms nustatyti (NDE), svarbu tiksliai išmatuoti šių bangų greitį. Šiam tikslui naudojami įvairūs suvėlinimo laiko matavimo metodai. Vienas iš jų paremtas sklindančio signalo perėjimo per nulį laiko momento matavimu. Šio darbo tikslas buvo detaliai ištirti Lembo bangų faziniam greičiui matuoti naudojamo metodo tikslumą ir parametrus. Tyrimo metu buvo naudojami baigtinių elementų metodu gauti Lembo bangų A_0 modos signalai. Buvo tiriama perėjimo per nulį metodu gautų matavimo rezultatų sklaidos priklausomybė nuo signalo diskrečiųjų taškų skaičiaus per periodą, aproksimuojamo polinomo laipsnio ir aproksimuotų naudojamų taškų skaičiaus. Nustatyta, kad labiausiai tinka aproksimacija trečiojo laipsnio polinomu, o didžiausią įtaką rezultatų sklaidai turi taškų skaičius per periodą. Jo neįvertinus arba jį netinkamą parinkus, rezultatų sklaida gali labai padidėti.

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