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# Simulation of Pulling through a Round Hole of Woven and Knitted Yarn Systems

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The behaviour of knitted and woven cotton textile materials during pulling through a circular hole was analysed. Two mathematical simulation models have been formed for simulation of this complicated process. The analysis of the obtained results showed the sufficient precision of shortened epicycloids, Cassini ovals as well as Buto lemniscate mathematical models for modeling the process of knitted material deformation at different stages of the experiment. This notwithstanding, only the shortened epicycloids were sufficient precise for mathematical description of the geometrical shapes of woven fabrics specimens formed at different stages of experiment. The changes in the parameters of simulation models were also investigated.
*Keywords*: materials science, textile, punching, geometrical behaviour, mathematical simulation.

## 1. INTRODUCTION

It is known from the earlier research works that the flat yarn materials (woven or knitted) are more anisotropic compared with the other flat solid materials, e. g. films, foils, similar to textile materials, because of the increased mobility of their structure. The anisotropy of textile material properties leads the advantageous conditions for the formation of complicated shells or for the separate parts of them [1].

In the recent decade, several new methods of punch deformation based on the restricted (controlled) pulling of disc-shaped specimen through the central hole were developed in order to investigate and to evaluate the properties of anisotropic textile materials [2 – 5]. These methods purposefully can be applied to determine the changes of textile hand, friction as well as other specific properties depending on the technological treatment as well as exploitation conditions [6 – 9].

It was shown earlier that during pulling process of the part of a disc-shaped specimen lying in space between the limiting plates changes to wrinkled shell with the outer contour of complicated shapes. During the deformation process the shell with outer contour changing to curve similar to “four-leaved” clover is formed from the disc-shaped specimen cut from woven material. The cavities of “four-leaved” clover are oriented along both warp and weft directions. And, the rises of “four-leaved” clover are located along bias material direction. Whereas, when a disc-shaped specimen cut from knitted material is pulled through a central hole the shell similar to oval with the axles oriented towards the directions of loops courses and wales is formed [10 – 13]. It must be noted that the shape of outer contour of the specimen deformed between the limiting plates is sometimes similar to that of polar diagrams of strip-shaped specimens tensioned uni-axially in the same sector, i. e. in the directions lying between 0º and 90º [13]. Thus, it could be assumed that new pulling method could be used to evaluate the material anisotropy instead of the complicated and labour-intensive uniaxial tension method.

The primary search of mathematical models suitable to simulate the changes of thin yarn-based disc-shaped specimen during deformation process had shown that they could be shortened epicycloids or Cassini ovals [14, 15].

The aim of this research was to find more mathematical models suitable to simulate the geometrical behavior of yarn systems pulled through a central hole as well as to determine the peculiarities of the mathematical parameters’ changes depending on the structural parameters of investigated materials and different deformation stage.

## 2. MATERIALS AND METHODS

Textile materials having two different structures, i. e. woven and knitted (Table 1) were selected for the present study.

The research was performed using KTU-Griff-Tester (Fig. 1) with the disc-shaped specimens the radius $R$ of 56.5 mm. The testing conditions, i. e. the values of the radius of the plate and pad and the distance $h$ between the supporting plate and the pad were chosen according to the thickness of the material $\delta$ (Table 1) as well as the peculiarities of specimen jamming in the hole of the supporting plate and the pad [6]. The changes of the specimen shapes occurring during pulling process at each deformation stage, i. e. changing the specimen deformation height $H$ every other 10.0 mm, were recorded using a digital camera (Fig. 1).

One of the simplest mathematical model is one of the shortened epicycloids (or models of epitrochoids) sufficiently precisely matching the realistic displacements

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**Table 1.** Structure parameters of investigated materials

| Material | Material code | Composition | Wave (knit) type | Thickness $\delta$, mm | Surface density, g/m$^2$ |
|---|---|---|---|---|---|
| Woven | A | 100 % cotton | Plain | 0.29 | 145.4 ±1 |
| Knitted | T | 100 % cotton | Plain jersey | 0.57 | 177.1 ±2 |

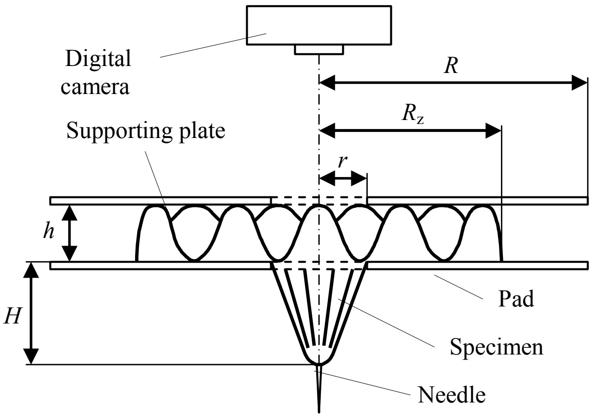



**Fig. 1.** The principal scheme of method applied

of the outer contour of the specimens cut from yarns' systems (woven and knitted materials) and pulled though a central hole [14, 15]. They are plain curves traced by a point attached to a small circle (SC) of radius *a* rolling around the outside of a fixed large circle (LC) of radius 2*a*, where the point is a distance *d* from the center of the small circle, and the centre of small circle moves along the dotted line (Fig. 2). The shape of curve depends on the ratio of circles radiuses. When the radius $R_{DS}$ of LC circle is equal to 4*a*, and the radius $R_{MS}$ of the small circle SC is equal to *a* then the curve is similar to four-leaved clover, and when the radius $R_{DS}$ of LC circle is equal to 2*a*, and the radius $R_{MS}$ of the small circle SC is equal to *a*, then the oval-shaped curve is traced. The parameters of both curves: eccentricity *d*, the constituent *a* of circles radiuses as well as inclination points *W* (Fig. 3) influence the size of traced curve as well as its inclination level.

The parameters *a* and *d* can be calculated using the measured distances from points X and mostly distant point Y to the centre point of the four-leaved curve 0*x* by using the system of equations:

$$\begin{cases} 5a - d = 0\text{X}, \\ 5a + d = 0\text{Y}. \end{cases} \quad (1)$$

The model of „four-leaf clover“ shortened epicycloids can be described using the system of equations:

$$\begin{cases} x = 5a\cos\varphi - d\cos 5\varphi, \\ y = 5a\sin\varphi - d\sin 5\varphi, \end{cases}$$

and then, its equation with polar coordinates is

$$\rho(\varphi) = \sqrt{25a^2 - 10ad\cos 4\varphi + d^2}. \quad (2)$$

It is evident that 0X = $\rho$ (0º), 0Y = $\rho$ (45º).

The distances from the points of shortened epicycloid to its centre point were calculated from the equation $R_z = \rho(\varphi)$.

The coordinate $\varphi$ of inclination point *W* (Fig. 3) is determined from the equation:

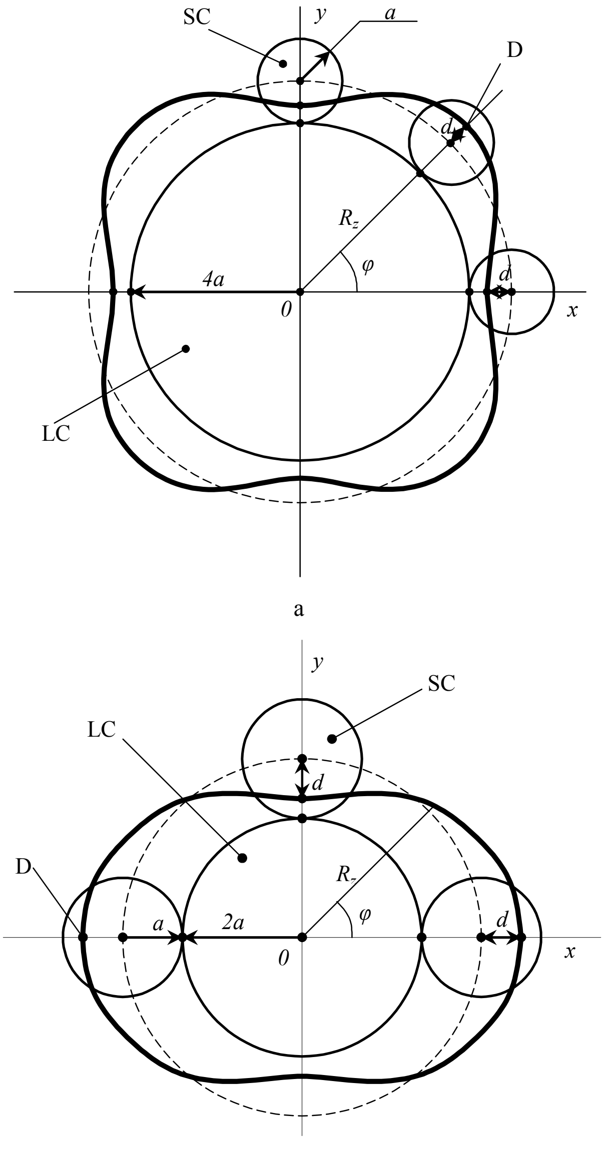



**Fig. 2.** Schemes of the four-leaved (a) and two-leaved (b) shortened epicycloids models

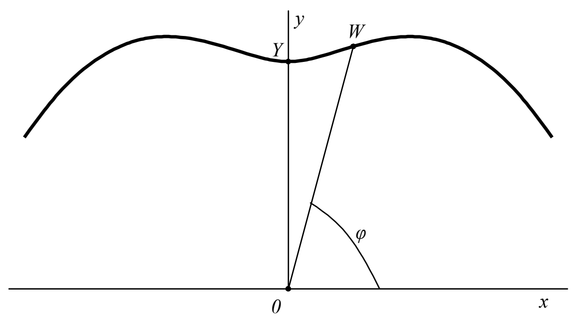



**Fig. 3.** Scheme of the determination of the coordinate $\varphi$ of inclination point *W*

$$\cos 4\varphi = \frac{a^2 + 5d^2}{6ad}. \quad (3)$$

The parameters $a$ and $d$ of "two-leaved" shortened epicycloid were calculated using the measured distances 0X and 0Y, i. e. distances from the point X laying on the curve farthest to its centre (on the axis $0x$) and from the point Y laying on the curve nearest to its centre (on the axis $0y$), respectively, from the equation system:

$$\begin{cases} 3a + d = 0\text{X}, \\ 3a - d = 0\text{Y}, \end{cases} \quad (4)$$

Parametric equations of this shortened epicycloid are the following

$$\begin{cases} x = 3a\cos\varphi - d\cos 3\varphi, \\ y = 3a\sin\varphi - d\cos 3\varphi, \end{cases}$$

Equation of "two-leaf" shortened epicycloid was derived after the substitution of the coordinates $x$ and $y$ with the polar coordinates $x = \rho\cos\varphi$, $y = \rho\sin\varphi$

$$\rho(\varphi) = \sqrt{9a^2 + 6ad\cos 2\varphi + d^2}. \quad (5)$$

From this it is clear, that $0X = \rho\ (0^\circ)$, $0Y = \rho\ (90^\circ)$.

Coordinate $\varphi$ of inclination point $W$ was calculated from the equation:

$$\cos 2\varphi = -\frac{a^2 + 3d^2}{4ad}. \quad (6)$$

For the comparative analysis of the presented mathematical model the shape of outer contour of the specimen was also simulated using the equations of Cassini ovals

$$\left(x^2 + y^2\right) - 2c^2\left(x^2 - y^2\right) = a^4 - c^4$$

and Buto lemniscate [15]:

$$\rho^2 = a^2\cos^2\varphi + b^2\sin^2\varphi.$$

Parameters $a^2$ and $c^2$ of Cassini ovals were calculated from the following equation system:

$$\begin{cases} a^2 + c^2 = 0\text{X}^2, \\ a^2 - c^2 = 0\text{Y}^2. \end{cases} \quad (7)$$

Distances from the points of specimen contour to its centre point or the values of radiuses $R_z$ were calculated from the equation:

$$\rho^2 = c^2\cos 2\varphi + \sqrt{c^4\cos^2 2\varphi + a^4 - c^4}, \quad (8)$$

And the coordinate $\varphi$ of inclination point $W$ was determined from the equation:

$$\cos 2\varphi = -\sqrt{\frac{a^4 - c^4}{3c^4}}. \quad (9)$$

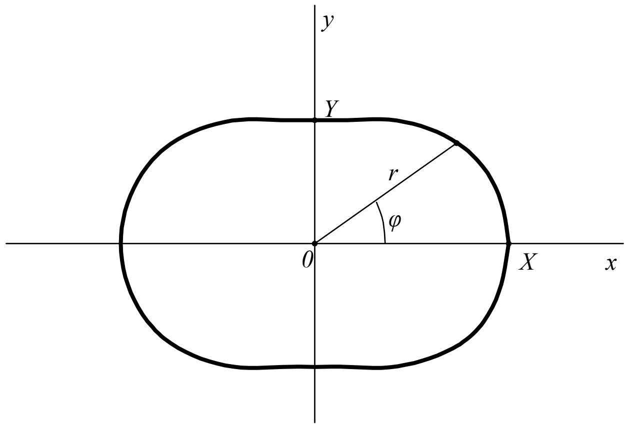



**Fig. 4.** Scheme of Buto lemniscate and Cassini ovals' models, when $c < a < c\sqrt{2}$

Equation of Buto lemniscate model is the following:

$$\rho = \sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}, \quad (10)$$

where $a = 0\text{X} = \rho\ (0^\circ)$, $b = 0\text{Y} = \rho\ (90^\circ)$.

Difference $\Delta$ between the measured and calculated distances was calculated using the following equation:

$$\Delta = \rho_{measured}(\varphi) - \rho_{calculated}(\varphi). \quad (11)$$

## 3. RESULTS AND DISCUSSION

From the results presented in Table 2 can be seen that the difference $\Delta$ between the displacements $R_z$ ° change differently in two adjacent sectors: 0° – 90° and 90° – 180°. It proves the essence of contour asymmetry influenced by the structure parameters of investigated materials, such as repeat or the parameters of material loops.

Visual graphical comparison analysis of the geometrical outer contours of the specimens cut from investigated materials A (Fig. 5) and T (Fig. 6) had shown insignificant differences between their measured and calculated parameters. The shapes of specimens at fifth and sixth deformation stages are small, and the calculated and measured contours differ more significantly. Because of the errors occurring during specimens positioning the measured contours are more angular than the calculated contours having smoother and more formal shapes.

**Table 2.** Differences $\Delta$ between the measured and calculated values of the displacements $R_z$ of woven material A (in mm)

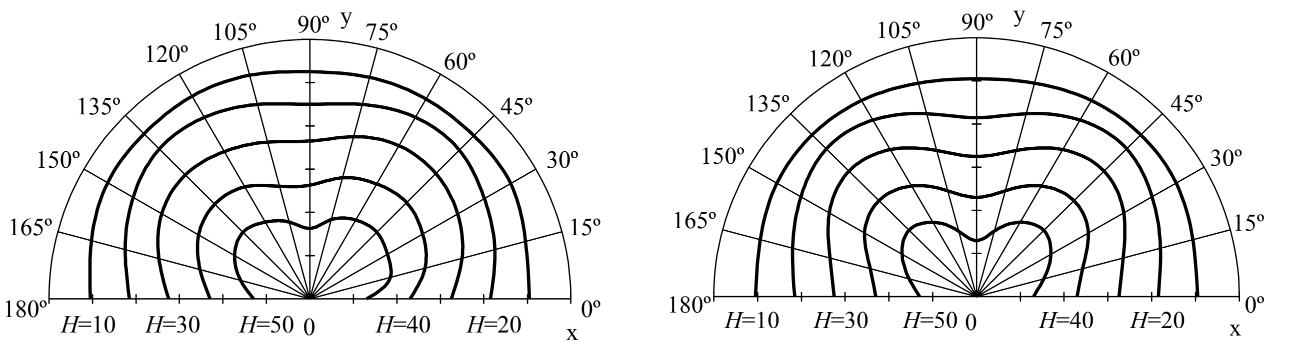
| $H$, mm | $\varphi$, ° | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 |
| 10 | 0 | 0.3 | 0.4 | 0 | 0.9 | 1.8 | 2.0 | 2.1 | 1.4 | 0.5 | 0.9 | 0.8 | 0 |
| 20 | 0 | 0.6 | –0.3 | 0 | 1.7 | 3.1 | 3.5 | 2.9 | 1.2 | 0.5 | 0.2 | 0.6 | 0 |
| 30 | 0 | 1.2 | 0.5 | 0 | 1.8 | 3.7 | 4.0 | 2.5 | 0 | –0.5 | 0 | 1.2 | 0 |
| 40 | 0 | 2.5 | 0.6 | 0 | 1.1 | 3.7 | 3.3 | 1.5 | 0.1 | –0.3 | –0.1 | 1.0 | 0 |
| 50 | 0 | 3.4 | 0.8 | 0 | 1.1 | 3.4 | 3.3 | 2.9 | 0.6 | 0 | 0.6 | 0.9 | 0 |

Comparison analysis of specimen geometry changes (Fig. 3) had shown that the values of both *a* and *d* parameters of shortened epicycloid depend on the deflection *H*. The value of *a* parameter decreases, and the value of *d* parameter increases increasing the *H* deflection. For all of investigated cases *a* parameter is larger than *d* parameter, except the mathematical model parameters of woven material when *H* deflection is equal to 50 mm.

The comparison analysis between the differences Δ of the displacements $R_z$ of both calculated and measured values of specimen outer contour (Table 3) had shown that for the simulation of knitted material behaviour during pulling process mathematical models of shortened epicycloids and Buto lemniscate are more suitable than the model of Cassini oval.

Relations between the parameters of investigated mathematical models, such as *a* and *d* of shortened epicycloids; *a* and *c* of Cassini oval as well as *a* and *b* of Buto lemniscate model and *H* deflection of both woven and knitted material specimens sufficiently precisely described by the linear equation (Fig. 7), but not *d* parameter of the model used to simulate the behavior of woven fabric specimen as well as *c* parameter of the investigated model in case of knitted material.

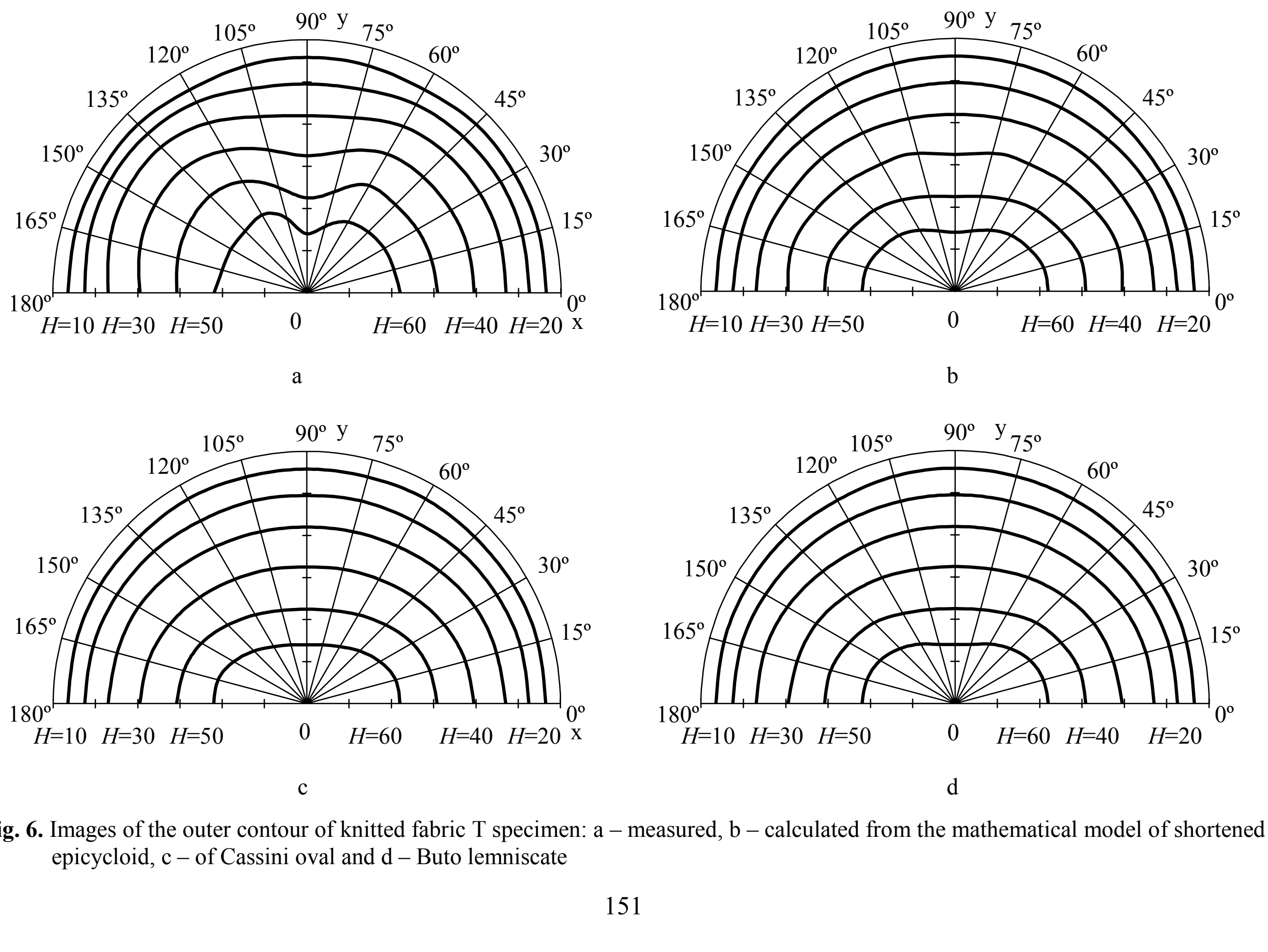
The search of inclination point *W* was made using three mathematical models, such as shortened epicycloids, Cassini oval and Buto lemniscate. But only the simulation using the first method was successful, and showed the importance of inclination point *W*. The simulation of the deformation behavior of A and T investigated materials




a

b

**Fig. 5.** Images of the outer contour of woven fabric A specimen: a – measured, b – calculated from the mathematical model of shortened epicycloid




a

b

c

d

**Fig. 6.** Images of the outer contour of knitted fabric T specimen: a – measured, b – calculated from the mathematical model of shortened epicycloid, c – of Cassini oval and d – Buto lemniscate

**Table 3.** Differences Δ between the measured and calculated geometrical values of the displacements $R_z$ of knitted material T (mm)

| *H*, mm | *φ*, ° | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 |
| Calculated from the model of shortened epicycloid | | | | | | | | | | | | | |
| 10 | 0 | 0 | 0.2 | 0.3 | –0.2 | –0.1 | 0 | –0.1 | –0.2 | 0.3 | 0.2 | 0 | 0 |
| 20 | 0 | 0.2 | 0.7 | 1.3 | 1.0 | 0.3 | 0 | 0.3 | 1.7 | 2.2 | 1.7 | 0.7 | 0 |
| 30 | 0 | 0.3 | 1.2 | 1.9 | 2.2 | 0.7 | 0 | 0.7 | 2.7 | 3.9 | 3.0 | 1.3 | 0 |
| 40 | 0 | –0.1 | 1.1 | 2.3 | 3.1 | 1.5 | 0 | 1.8 | 3.9 | 4.8 | 3.4 | 1.7 | 0 |
| 50 | 0 | –0.3 | 1.1 | 2.5 | 3.2 | 1.4 | 0 | 2.4 | 5.2 | 5.0 | 3.1 | 1.2 | 0 |
| 60 | 0 | –0.6 | 0.5 | 1.6 | 2.6 | 1.3 | 0 | 3.1 | 4.6 | 2.1 | 0.2 | –1.3 | 0 |
| Calculated from the model of Cassini oval | | | | | | | | | | | | | |
| 10 | 0 | 0 | 0.2 | 0.4 | –0.2 | 0 | 0 | 0 | –0.2 | 0.4 | 0.2 | 0 | 0 |
| 20 | 0 | 0.2 | 0.8 | 1.3 | 1.1 | 0.3 | 0 | 0.3 | 1.8 | 2.2 | 1.8 | 0.7 | 0 |
| 30 | 0 | 0.4 | 1.3 | 2.1 | 2.3 | 0.7 | 0 | 0.7 | 2.8 | 4.1 | 3.1 | 1.4 | 0 |
| 40 | 0 | 0 | 1.4 | 2.7 | 3.4 | 1.6 | 0 | 1.9 | 4.2 | 5.2 | 3.7 | 1.8 | 0 |
| 50 | 0 | –0.2 | 1.5 | 3.2 | 3.7 | 1.5 | 0 | 2.5 | 5.7 | 5.7 | 3.5 | 1.3 | 0 |
| 60 | 0 | –0.4 | 1.1 | 2.5 | 3.4 | 1.6 | 0 | 3.4 | 5.4 | 3.0 | 0.8 | –1.1 | 0 |
| Calculated from the Buto lemniscate model | | | | | | | | | | | | | |
| 10 | 0 | 0 | 0.2 | 0.3 | –0.2 | 0 | 0 | 0 | –0.2 | 0.3 | 0.2 | 0 | 0 |
| 20 | 0 | 0.2 | 0.7 | 1.3 | 1.0 | –0.3 | 0 | 0.3 | 1.7 | 2.2 | 1.7 | 0.7 | 0 |
| 30 | 0 | 0.3 | 1.2 | 1.9 | 2.2 | 0.6 | 0 | 0.6 | 2.7 | 3.9 | 3.0 | 1.3 | 0 |
| 40 | 0 | 0.8 | 1.1 | 2.3 | 3.1 | 1.5 | 0 | 1.8 | 3.9 | 4.8 | 3.4 | 2.6 | 0 |
| 50 | 0 | –0.3 | 1.1 | 2.5 | 3.2 | 1.4 | 0 | 2.4 | 5.2 | 5.0 | 3.1 | 1.2 | 0 |
| 60 | 0 | –0.6 | 0.5 | 1.6 | 2.6 | 1.3 | 0 | 3.1 | 4.6 | 2.1 | 0.2 | –1.3 | 0 |

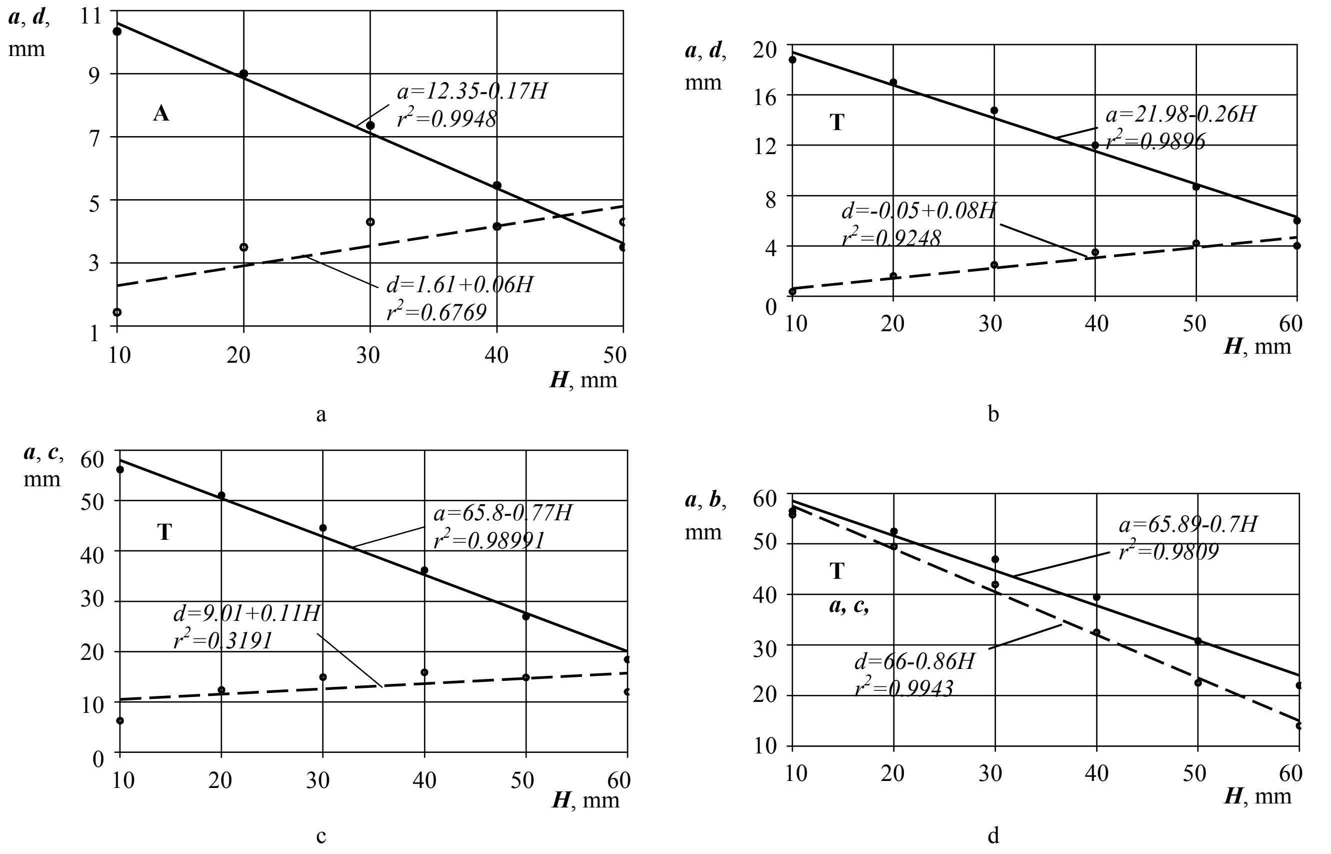



**Fig. 7.** The dependence of the parameters of mathematical models on the specimen deflection *H*: a, b – parameters of shortened epicycloid; c – parameters of Cassini oval; d – parameters of Buto lemniscate model

using other two methods had shown that the inclination point doesn't exist.

**Table 4.** Coordinates of inclination point *W* calculated using an epitrochoid method

| Material code | Deformation stage *H*, mm | Position of inclination point *W* |
|---|---|---|
| A | 10 | doesn't exist |
| | 20 | 10º27′ 79º33′ |
| | 30 | 9º50′ 80º10′ |
| | 40 | 7º56′ 82º04′ |
| | 50 | doesn't exist |
| T | 10 | doesn't exist |
| | 20 | doesn't exist |
| | 30 | doesn't exist |
| | 40 | doesn't exist |
| | 50 | 76º21′ |
| | 60 | 75º59′ |

The position of one inclination point *W* for both investigated materials was near 80º angle (Table 4), and in the case of A woven fabric this point didn't exist at the first deformation stage, i. e. when *H* deflection was equal to 10 mm and the specimen still was approximately disc-shaped as well as at the end of deformation process when *H* deflection was equal to 50 mm, and the break point originated. The displacement of inclination point *W* increases when *H* deflection varies from 20 mm to 40 mm, and conversely, near 10º angle, the displacement of inclination point decreases increasing *H* deflection. The inclination point *W* for knitted material appears only under the large deformations of specimen, i. e. when *H* deflection equals from 50 mm to 60 mm, while up to these deformation stages the outer contour of specimen is oval-shaped and without any inclinations.

## 4. CONCLUSIONS

1. It was determined, that during deformation changing shapes of the outer contour of disc-shaped specimens cut from yarn systems can be simulated using some mathematical models. But the reliability of model depends on material structure, e. g. the shapes of woven material specimens more precisely can be simulated using shortened epicycloids model. While, the shapes of knitted material specimens can be simulated applying the models of shortened epicycloids, Cassini oval and Buto lemniscate. According to the difference Δ, the more suitable are the models of Buto lemniscate model and epitrochoid, and according to the applying simplicity, the best is Buto lemniscate model.
2. It was determined, that the dependencies of the parameters of investigated mathematical models, such as *a* and *d* of shortened epicycloids; a and *c* of Cassini ovals as well as *a* and *b* ones of Buto lemniskate model on the *H* deflection of the both knitted and woven material specimens can be sufficiently precisely described using a linear equation.

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