523. Switching leg method for trajectory planning of mobile piezorobot

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Abstract. This paper presents switching leg method of trajectory planning of mobile piezorobots. Deduction of motional simultaneous equations for this kind of robots is presented, which describe point-to-point motion by given function. Simulated results indicate the existence of critical values of dimensions that influence the motion of the microrobot. Preliminary experimental results prove the feasibility of proposed mathematical model.

Keywords: mobile piezorobot, trajectory planning method.

Introduction

Trajectory planning in two-dimensional space of mobile piezorobot with power actuators was analyzed. Mobile piezorobots are devices capable of manipulating objects of small mass in a limited space but with very high accuracy (0.1 μm) [1, 2].

Trajectory planning (also called kinodynamic motion planning) is one of principal elements of robot motion planning. Base components of planning are [3, 4]: *state* – position and orientation, velocity, structure of robot; *time* – all planning problems involve a sequence of decisions that must be applied over time; *actions* – generate actions that manipulate the state; *goal state* – during the actions, robot must reach a final set position; *a criterion* – action selection depends on desired criteria: feasibility, optimality, speed.

Different trajectory planning methods are used for autonomous robots. Generally are used splines [5, 6, 7, 8]: cubic, trigonometric, exponential and β-splines. Spline functions are formed by joining polynomials together at fixed points. Most popular of them are cubic splines. They have two features [9]: are paths of minimal curvature and are easy to generate online.

Clothoids curves [10, 11] are applied as well. Using clothoid curves it is also possible to produce smooth trajectories also with smooth changes in curvature. Clothoids allow smooth transitions from a straight line to a circle arc or vice versa [7].

All these methods are generating trajectory through a set of prescribed points, called way points. In our research moving function is given. Mobile piezorobot cannot rotate and dynamically change its direction according to the given function. Only one power actuator is active at a time. Thus mobile piezorobot moving path constitutes a straight line and has only three possible directions, which depend on active power actuator. In this paper we analyze mobile piezorobot moving trajectory and consider which power actuator must be active to move on.

Mobile piezorobot model

Mobile piezorobot model is designed from piezoceramic plate with three magnetized metallic cylinders, which are attached to piezoceramic plate (Fig. 1a). Converter contact zone is top surface of three cylinders, which at the time of excitation are moving in elliptical trajectory.

Electrodes cover all bottom space of the ring and are divided in three equal segments (Fig. 1b). Such division of electrodes enables excitation of slider motion in any direction as well as rotary motion.

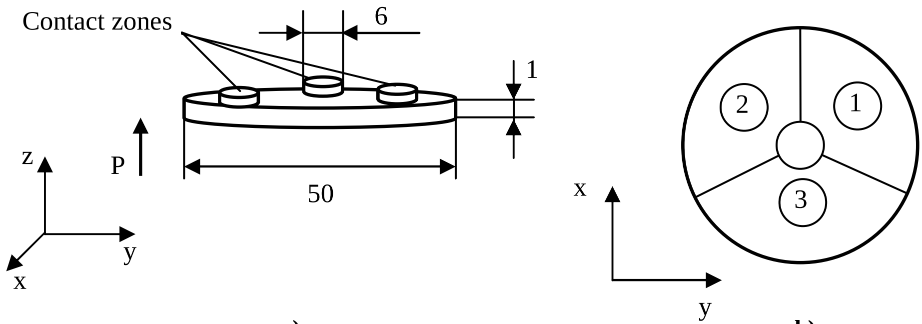

Fig. 1. Construction of mobile piezorobot: a), b) electrode positions

Piezoconverter is fabricated from piezoceramic material of type PZ 27. Polarization vector is routed along the ring thickness. Contact cylinders are made from steel.

For piezoconverter excitation three electrodes exciting schemes are employed, when exciting one electrode sector at a time, which is generating rectilinear movement. If phases of voltage in each electrode are different 120 degrees, then running wave is induced, which generates rotary movement. Harmonic variation with 40 V amplitude is used as excitation voltage (Fig. 2).

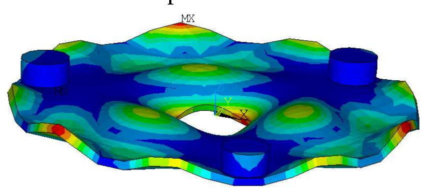

Fig. 2. Distribution of piezoconverter amplitudes at 25.0 kHz excitation

Electrodes can be divided into a greater number of sectors, but excitation principle should remain the same.

Trajectory planning algorithm

Requirements for piezorobot movement:

1. Move given by function $\psi = f(x, y)$. Function ψ must be continuous at each point

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a,b), \text{ where } a \in [x_0; x_n],\ b \in [y_0; y_n].$$

Its derivatives $\dfrac{\partial f}{\partial x}$ at those points must be continuous too.

2. Only one power actuator is active at a time.
3. Mobile piezorobot cannot rotate.

4. Determined angle α between first power actuator and x axis.
5. Determined maximum deflection ε from function $f(x, y)$, which cannot exceed the mobile piezorobot center (Fig. 3).
   So $g^{*} < \psi < g^{**}$, where g^{*} and g^{**} marginal coordinates.

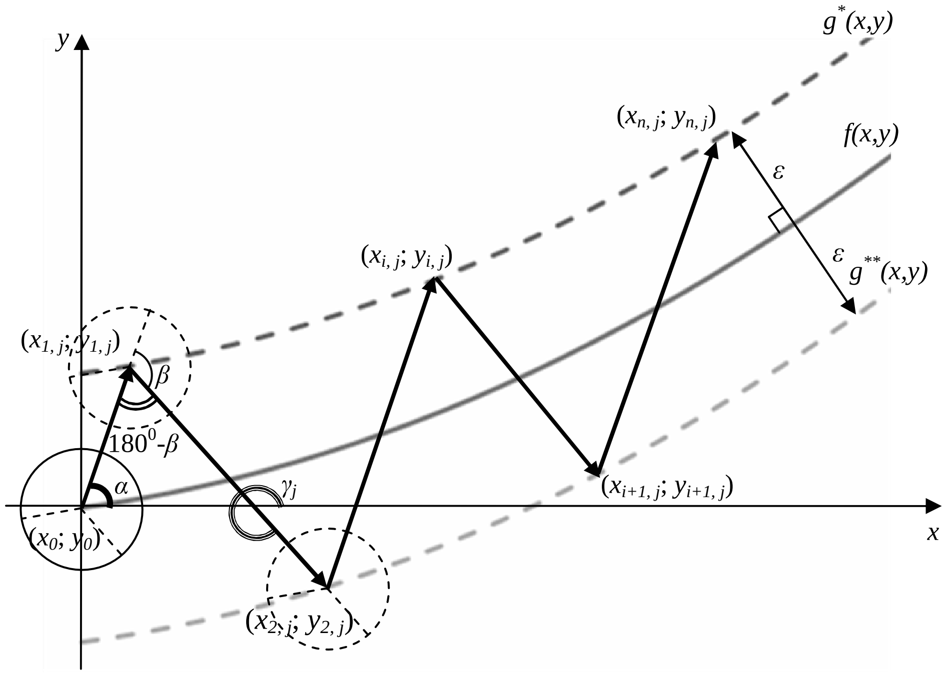

Fig. 3. Mobile piezorobot motion trajectory by given function *f(x,y)*

To find the coordinates of limit, it is necessary to solve simultaneous equations:

$$\begin{cases} y_{gi} - y_{fi} = -\dfrac{1}{\left.\dfrac{\partial f}{\partial x}\right|_{x=x_{fi}}}\left(x_{gi} - x_{fi}\right); \\ \varepsilon = \sqrt{(x_{gi} - x_{fi})^2 + \left(y_{gi} - y_{fi}\right)^2}, \end{cases} \tag{1}$$

where $(x_{fi}; y_{fi})$ point at function *f(x, y)*, $(x_{gi}; y_{gi})$ marginal coordinates, ε maximum deflection, i = 0, 1, 2 …, n function's point number.

First equation is at point x_{fi} normal equation, and the second equation is distance between two points $(x_{fi}; y_{fi})$ and $(x_{gi}; y_{gi})$.

From (1) we can find marginal coordinates:

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$$g_i^{*}(x_i, y_i) = \begin{cases} x_{gi} = -\dfrac{\varepsilon \cdot \left.\dfrac{\partial f}{\partial x}\right|_{x=x_{fi}}}{\sqrt{\left(\left.\dfrac{\partial f}{\partial x}\right|_{x=x_{fi}}\right)^2 + 1}} + x_{fi}; \\ y_{gi} = \dfrac{\varepsilon}{\sqrt{\left(\left.\dfrac{\partial f}{\partial x}\right|_{x=x_{fi}}\right)^2 + 1}} + y_{fi}, \end{cases} \quad (2)$$

$$g_i^{**}(x_i, y_i) = \begin{cases} x_{gi} = \dfrac{\varepsilon \cdot \left.\dfrac{\partial f}{\partial x}\right|_{x=x_{fi}}}{\sqrt{\left(\left.\dfrac{\partial f}{\partial x}\right|_{x=x_{fi}}\right)^2 + 1}} + x_{fi}; \\ y_{gi} = -\dfrac{\varepsilon}{\sqrt{\left(\left.\dfrac{\partial f}{\partial x}\right|_{x=x_{fi}}\right)^2 + 1}} + y_{fi}. \end{cases} \quad (3)$$

If mobile piezorobot is constructed with m power actuators, angles between axes of actuators are:

$$\beta = \frac{360^0}{m}. \quad (4)$$

Then angles between each power actuator and x axis are:

$$\gamma_j = \alpha + \beta(j-1), \quad (5)$$

where $j = 1, 2, ..., m$ is power actuators number.

To describe the movement of the mobile piezorobot, it is necessary to determinate these types of movement directions:

1. General direction of movement with regard to x axis is $d_x = 1$ (x value is increasing) or $d_x = -1$ (x value is decreasing). A value is selected depending on target position.
2. Function directions between points (x_{fi}; y_{fi}) and (x_{fi+1}; y_{fi+1}) are:

$$d_{xi} = \begin{cases} 1, & \text{if } x_{fi+1} - x_{fi} \geq 0 \\ -1, & \text{if } x_{fi+1} - x_{fi} < 0 \end{cases}, \quad (6)$$

$$d_{yi} = \begin{cases} 1, & \text{if } y_{fi+1} - y_{fi} \geq 0 \\ -1, & \text{if } y_{fi+1} - y_{fi} < 0 \end{cases}, \quad (7)$$

where $i=1, 2, 3, \ldots, n$ is function point number.

Distance between current piezorobot position and coordinates at function *f(x, y)* is equal to:

$$\varepsilon^2 = \left(x_i - x_{fi}\right)^2 + \left(y_i - f(x_{fi})\right)^2, \tag{8}$$

where $(x_i; y_i)$ mobile piezorobot coordinates.

So point x_{fi} at function *f(x,y)* will be minimum distance from mobile piezorobot position:

$$\left((x_i - x_{fi})^2 + \left(y_i - f(x_{fi})\right)^2\right)' \mathrm{d}x_{fi} = 0. \tag{9}$$

Solving equation (9) yields coordinate x_{fi}.

Coordinate x_{fi+1} is equal to:

$$x_{fi+1} = x_{fi} + d_x s_i, \tag{10}$$

where $s_i \geq \varepsilon$ is step moving through function.

3. Power actuators directions depend on angle between power actuator and *x* axis:

$$x_j = \begin{cases} 1, & \text{if } 0^0 \leq \gamma_j \leq 90^0 \vee 270^0 \leq \gamma_j \leq 360^0 \\ -1, & \text{if } 90^0 < \gamma_j < 270^0 \end{cases}, \tag{11}$$

$$y_j = \begin{cases} 1, & \text{if } 0^0 \leq \gamma_j \leq 180^0 \\ -1, & \text{if } 180^0 < \gamma_j < 360^0 \end{cases}. \tag{12}$$

4. Possible mobile piezorobot moving directions. Microrobot movement straight line can intersect functions $g^*(x, y)$ and $g^{**}(x, y)$. Thus $x_{gi} = x_{i+1,j}$, $y_{gi} = y_{i+1,j}$ where $(x_{i+1,j}; y_{i+1,j})$ mobile piezorobot movement coordinates. Then simultaneous equations are solved with each function $g^*(x, y)$ and $g^{**}(x, y)$:

$$\begin{cases} y_{i+1,j} - y_{i,j} = k_j\left(x_{i+1,j} - x_{i,j}\right), \\ \begin{pmatrix} x_{i+1,j} \\ y_{i+1,j} \end{pmatrix} = g_i^*\left(x_i, y_i\right), \\ f\left(x_{fi}, y_{fi}\right) = 0, \end{cases} \tag{13}$$

and

$$\begin{cases} y_{i+1,j} - y_{i,j} = k_j\left(x_{i+1,j} - x_{i,j}\right), \\ \begin{pmatrix} x_{i+1,j} \\ y_{i+1,j} \end{pmatrix} = g_i^{**}\left(x_i, y_i\right), \\ f\left(x_{fi}, y_{fi}\right) = 0, \end{cases} \tag{14}$$

where k_j coefficient of straight line direction:

$$k_j = \tan\gamma_j, \tag{15}$$

From (12) and (13) simultaneous equations one can evaluate possible movement points $(x_{i+1,j}; y_{i+1,j})$. Then possible mobile piezorobot movement directions are as follows:

$$r_{xi,j} = \begin{cases} 1, & \text{if } x_{i+1,j} - x_{i,j} \geq 0 \\ -1, & \text{if } x_{i+1,j} - x_{i,j} < 0 \end{cases}, \tag{16}$$

$$r_{yi,j} = \begin{cases} 1, & \text{if } y_{i+1,j} - y_{i,j} \geq 0 \\ -1, & \text{if } y_{i+1,j} - y_{i,j} < 0 \end{cases}, \quad (17)$$

For selecting right moving point, conditions must be met:

1. point must be on the same moving direction as power actuator $r_{xi,j}=x_j$ and $r_{yi,j}=y_j$;
2. point must be on the same moving direction as function $r_{xi,j}=d_{xi}$ and $r_{yi,j}=d_{yi}$;
3. maximum distance from current mobile piezorobot position.

In some situations there is no point at which these conditions are satisfied. Then it is necessary to choose coordinate that is invert moving direction and it must be minimum distance from current mobile piezorobot position.

Results of analysis

Mobile piezorobot with three power actuators (m = 3) moving trajectory by given different functions is analyzed. Limit ε = 0.1. Trajectory, when function $y=x^2$, $\alpha=90^0$ is presented in Fig.3a, when function $y=\sqrt{x}$, $\alpha=45^0$ is presented in Fig. 3b, function $y=\sin x$, $\alpha=45^0$ is presented in Fig. 3c, when function $y=\sin^3 x$, $\alpha=60^0$ is presented in Fig. 3d. Power actuator number, which must be active, is presented in Table 1.

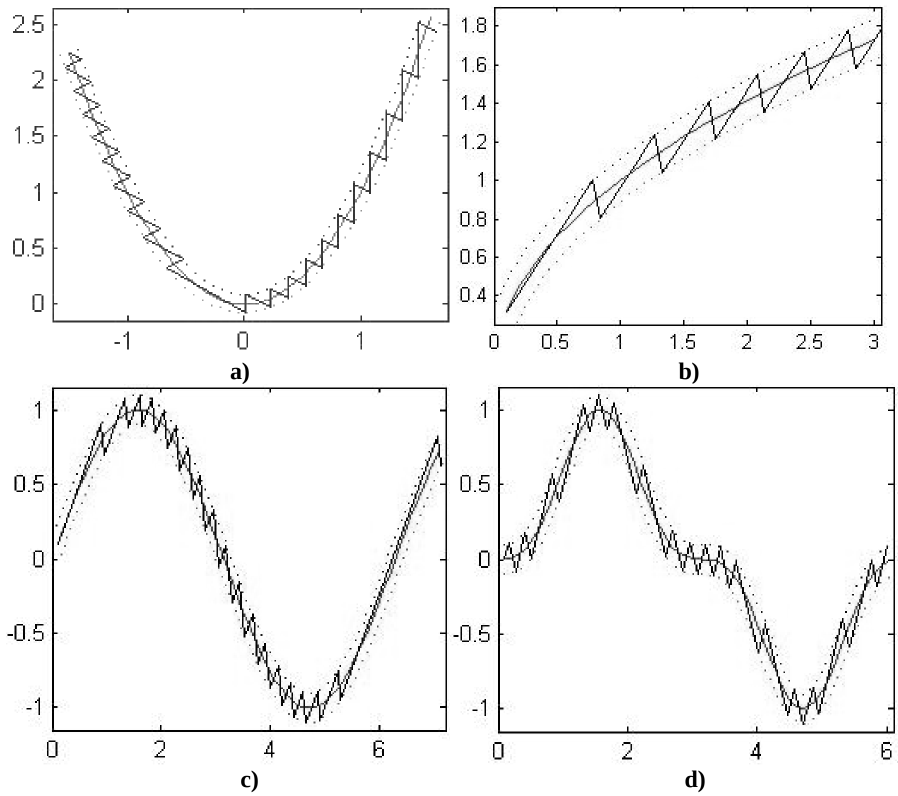

Fig. 4. Mobile piezorobot movement trajectory, when ε=0.1: a) function $y=x^2$, $\alpha=90^0$, b) function $y=\sqrt{x}$, $\alpha=45^0$, c) function $y=\sin x$, $\alpha=45^0$, d) function $y=\sin^3 x$, $\alpha=60^0$

Table 1. Power actuator number, which must be active to move on

$y = x^2$ $\alpha = 90^0$				$y = \sqrt{x}$ $\alpha = 45^0$		$y = \sin x$ $\alpha = 45^0$						$y = \sin^3 x$ $\alpha = 60^0$			
i	*j*	*i*	*j*	*i*	*j*	*i*	*j*	*i*	*j*	*i*	*j*	*i*	*j*	*i*	*j*
1	3	19	3	1	1	1	1	19	1	37	1	1	1	19	1
2	2	20	1	2	3	2	3	20	3	38	3	2	3	20	3
3	3	21	3	3	1	3	1	21	1			3	1	21	1
4	2	22	1	4	3	4	3	22	3			4	3	22	3
5	3	23	3	5	1	5	1	23	1			5	1	23	1
6	2	24	1	6	3	6	3	24	3			6	3	24	3
7	3	25	3	7	1	7	1	25	1			7	1	25	1
8	2	26	1	8	3	8	3	26	3			8	3	26	3
9	3	27	3	9	1	9	1	27	1			9	1	27	1
10	2	28	1	10	3	10	3	28	3			10	3	28	3
11	3	29	3	11	1	11	1	29	1			11	1	29	1
12	2	30	1	12	3	12	3	30	3			12	3	30	3
13	3	31	3	13	1	13	1	31	1			13	1	31	1
14	2	32	1	14	3	14	3	32	3			14	3	32	3
15	3	33	3	15	1	15	1	33	1			15	1	33	1
16	2	34	1			16	3	34	3			16	3	34	3
17	3	35	3			17	1	35	1			17	1	35	1
18	2	36	1			18	3	36	3			18	3		

Analysis results indicate that switching point quantity *n* is dependent on values of dimensions ε and α. For testing this dependency, function $y=x^2$ was selected, while start position and final point in every test was the same. Switching point quantity *n*, when changing limit ε = (0.08; 0.1; 0.12) and angle $\alpha = (45^0; 60^0; 90^0)$, is presented in Fig. 5.

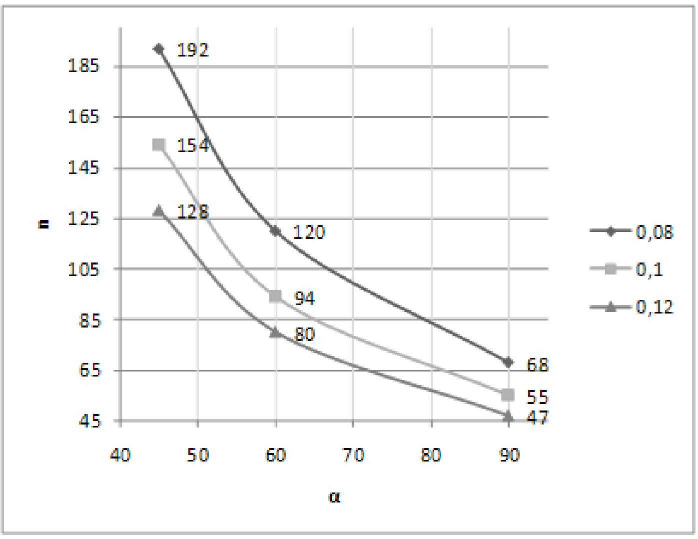

Fig. 5. Switching point's quantity *n* dependency on dimensions ε and α

These results demonstrate that switching point quantity is inversely proportional to moving limit and angle between power actuator and *x* axis. Dependence is observed to be nonlinear.

Conclusions

Moving (but not rotating) mobile piezorobot trajectory planning method is presented. The developed method is based on simultaneous equations solution and selection of correct movement point.

Reported results confirm the feasibility of proposed mathematical model. Research results demonstrate that switching point quantity is inversely proportional to moving limit and angle between first power actuator and x axis. For generation of optimal trajectory these dimensions must be considered.

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