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Modeling of Level of Defects in Electronics Systems

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Introduction

In companies that utilize modern technologies databases of manufactured production and technological processes are constantly maintained and complemented. Data is often read and set automatically. By using computer networks and database control and analysis systems, information can be transmitted in real-time and can be used in decision making processes at any time, intermediately or in the final stage of manufacture. Thus, there is a possibility to use information not only from the current, but also from the previous stages of manufacture. That can increase the quality of electronics systems (ES). Complex ESs are defined in technical documentation as an entire series of parameters, the values of which determine the level of quality. Parameters can be differentiated according to their importance regarding the implementation of purpose functions. International standard ISO-2859-0 recommends to divide parameters into two – A, B – or three – A, B, C – classes (groups). Here A – most important or significant parameters, and B, C – secondary or less significant parameters. Such classification of parameters is convenient when analyzing problems of multiparametric product quality control. When imitational modeling is applied, stochastic models of quality level are required for separate parameters, their groups and for entire ES [1, 2, 5].

The continuous operational control main probabilities characteristics modeling techniques for multilevel electronics systems is analyzed in [1]. These are useful when separate independent parameters defect level probabilities distributions are set (known) for chosen (selected) control schematics place. Denied electronics systems streams goes back to production process for regeneration and electronics systems classification rules in different control levels are similar, when electronics system classification first and second type errors are not denied by different parameters (good is denied or bad is accepted as good). Offered to use approximated models instead of exact whole electronics system defect level probabilities density transformed models because of complicated process of integration.

Models of control quality are described in [4]. Here a method is offered for synthesis of stochastic distributions of defectivity levels of multiparametric ES with interindependent parameters. This synthesis can be performed in groups of parameters or for entire product according to known distributions of defectivity levels of separate parameters. For practical applications it is advisable to differentiate average defectivity levels of separate parameters according to selected defectivity level of entire product, when ratio between defectivity levels in separate groups is selected or according to needed dispersion of parameters (selected variation coefficient).

Methods of control quality

Analyzing double level control schematics shown in Fig. 1 [2, 3, 7], where electronics system is described as l-dimensional controlled parameters vector i=1-l

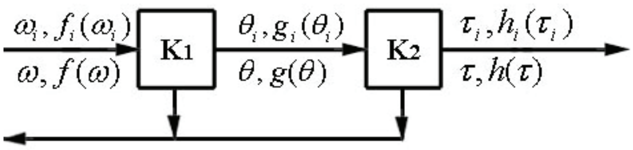

Fig. 1. Multiparameter electronics systems double level control and its probabilities characteristics

Assume, that product is characterized by l–dimensional random vector of independent parameters $X = (X_1, X_2, \dots, X_l)$ Probability of defective ES θ_i(in respect of the i-th parameter) is a random quantity (r.q.) with its density $g_i(\theta_i)$, distribution function $G_i(\theta_i)$, and main numerical characteristics: mean $E\theta_i = \mu_i$ and dispersion $V\theta_i = \sigma_i^2$ where $i = 1,2,\dots,l$. For practical application we are going to use additional dispersion characteristics: standard deviation $\sigma_i = \sqrt{V\theta_i}$ and variation coefficient $v_i = \frac{\theta_i}{\mu_i}$. Probability of good electronics systems η_i according to one parameter is $\eta_i = 1 - \theta_i$ and is also characterized by density $\varphi_i(\eta_i)$ distribution function $\Phi_i(\eta_i)$, mean $E\eta_i = \bar{\mu}_i$ and dispersion $V\eta_i = V\theta_i = \sigma_i^2$. The following relation formulas are valid: $\bar{\mu}_i = 1 - \mu_i$, $g_i(\theta_i) = \varphi_i(1-\theta_i)$, $G_i(\theta_i) = 1 - \Phi_i(1-\theta_i)$. The probability of defective product for the entire electronics systems r.q. θ and probability of good electronics systems r.q. η are calculated as $\eta = \prod_{i=1}^{l} \eta_i$, $\theta = 1 - \eta = 1 - \prod_{i=1}^{l}(1-\theta_i)$. Since η_i are inter-independent r.q., the following equation is valid: $\varphi(\eta) = \prod_{i=1}^{l} \varphi_i \eta_i$ [1, 6, 7]

The main problem is to keep the needed quality of multilevel electronics system. The process has to be checked by between-operational quality control in the end of the production process.

In Fig. 1: ω_i, θ_i, τ_i – defected product probabilities by i-th parameter before first level K1, before and after second level K2. These values are accidental with densities $f_i(\omega_i)$, $g_i(\theta_i)$, $h_i(\tau_i)$ and distribution functions $F_i(\omega_i)$, $G_i(\theta_i)$, $H_i(\tau_i)$. Respectively ω, θ, τ – the analogical characteristics for product by all of the controlled l parameters. For the future analysis good electronics system probabilities by i-th parameter are needed: $\xi_i = 1 - \omega_i$ $\eta_i = 1 - \theta_i$ $\kappa_i = 1 - \tau_i$ – with accidental value densities:

$$\ddot{\varphi}_i(\xi_i) = f_i(1-\xi_i) \tag{1}$$

$$\begin{cases} \varphi_i(\eta_i) = g_i(1-\eta_i), \\ \dot{\varphi}_i(\kappa_i) = h_i(1-\kappa_i) \end{cases} \tag{2}$$

and distribution formulas [2-4]:

$$\begin{cases} \ddot{\Phi}_i(\xi_i) = F_i(1-\xi_i), \Phi_i(\eta_i) = G_i(1-\eta_i), \\ \dot{\Phi}_i(\kappa_i) = H_i(1-\kappa_i). \end{cases} \tag{3}$$

Densities [3]:

$$\begin{cases} h_1(\tau_1) = \dfrac{8(1-\tau_1)}{(1+3\tau_1)^3}, \\ h_2(\tau_2) = \dfrac{4}{(1+3\tau_2)^2}, \\ h(\tau) = \dfrac{96(1-\tau)}{(7+9\tau)^3}\left\{\dfrac{41-9\tau}{7+9\tau}\ln\dfrac{(1-3\tau)^2}{1-\tau} - \right. \\ \left. -\left[\dfrac{8(7+9\tau)}{9(1-\tau)(1+3\tau)} - 3\tau + 11\right]\dfrac{1}{1+3\tau} + \dfrac{128}{9(1-\tau)} + 3 \right\}. \end{cases} \tag{4}$$

Equal levels of defects

Defect levels of separate parameters θ_i are distributed evenly: $\theta_i \sim be(1, b_i)$, i.e. averages $\mu_i = \mu_{i+1} = (b_i + 1)^{-1}$, when parameters of distribution shape $a_i = a_{i+1} = 1$, $b_i = b_{i+1} \geq 1$ (integer number) and $b_i = 1 - \dfrac{1}{\mu_i}$. Then $g_i(\theta_i) = b_i(1-\theta_i)^{b_i - 1}$, $g_i(0) = b_i$, $g_i(1) = 0$, $g_i(\theta_i^*) = 1$, when $\theta_i^* = 1 - \sqrt[b_i - 1]{\dfrac{1}{b_i}}$; $i = 1 - l$. If [3] $b_1 = b_2 = \ldots = b_e$, $a_i = 1$, then

$$g(\theta) = \frac{b_1^l}{(l-1)!}(1-\theta)^{b_1 - 1}\left(\ln\frac{1}{1-\theta}\right)^{l-1}; \tag{5}$$

mode

$$\theta_M = 1 - e^{-\frac{l-1}{b_1 - 1}}. \tag{6}$$

Approximation of the density $h(\tau)$ is [7]

$$h_\Sigma(\tau) = \frac{b_1^l (1-\tau)^{b_1 - 1}}{\tilde{\beta}(l-1)!(1+c\tau)^{b_1+1}}\left(\ln\frac{1+c\tau}{1-\tau}\right)^{l-1}, \tag{7}$$

here $\tilde{\beta}$ is the solution of the equation and is equal

$$\frac{\tilde{\beta} b_1^l}{(l-1)!}\sum_{S=0}^{\infty}\tilde{\gamma}^S \int_0^1 \theta^{S+1}(1-\theta)^{b_1 - 1}\left(\ln\frac{1}{1-\theta}\right)^{l-1} d\theta =$$

$$= \tilde{\beta} b_1^l \sum_{S=0}^{\infty}\tilde{\gamma}^S \sum_{k=0}^{S+1} C_{S+1}^k \frac{(-1)^k}{(b_1 + k)^l} = \mu_\tau \equiv 1 - \bar{\mu}_{\tau_1}^l, \tag{8}$$

here $i = 1 - l$.

According to [7]

$$h_i(\tau_i) = \frac{b_i(1-\tau_i)^{b_{i-1}}}{\tilde{\beta}_i(1+c_i\tau_i)^{b_{i+1}}}, \ h_i(0) = \frac{b_i}{\tilde{\beta}_i}, \ h_i(1) = 0, \tag{9}$$

$$f_i(\omega_i) = \frac{\tilde{\beta}_i b_i (1-\omega_i)^{b_{i-1}}}{(1-\tilde{\gamma}_i\omega_i)^{b_{i+1}}}, \ f_i(0) = \tilde{\beta}_i b_i, \ f_i(1) = 0, \tag{10}$$

$$\omega_{iM} = \frac{1}{2}\left[(b_i + 1) - \frac{1}{\tilde{\gamma}_i}(b_i - 1)\right]. \tag{11}$$

In a separate case, when $a_i = b_i = 1$, we have

$$h_i(\tau_i) = \frac{1}{\tilde{\beta}_i(1+c_i\tau_i)^2}, \ h_i(0) = \frac{1}{\tilde{\beta}_i}, \ h_i(1) = \tilde{\beta}_i,$$

$$h_i(\tau_i^*) = 1 \text{ when } \tau_i^* = \frac{1}{c_i}\left(\sqrt{\frac{1}{\tilde{\beta}_i - 1}}\right); \tag{12}$$

$$f_i(\omega_i) = \frac{\tilde{\beta}}{(1-\tilde{\gamma}_i\omega_i)^2}, \ f_i(0) = \tilde{\beta}_i, \ f_i(1) = \frac{1}{\tilde{\beta}_i},$$

$$f_i(\omega_i^*) = 1 \text{ when } \omega_i^* = \frac{1}{\tilde{\gamma}_i}\left(1 - \sqrt{\tilde{\beta}_i}\right). \tag{13}$$

If the $\tilde{\beta}$ in the model is substituted by $\dfrac{1}{\tilde{\beta}}$, τ is substituted by ω, then

$$\frac{\tilde{\beta} b_1^l}{(l-1)!}\sum_{S=0}^{\infty}\tilde{\gamma}^S \int_0^1 \eta^{b_1 + S}\left(\ln\frac{1}{\eta}\right)^{l-1} d\eta =$$

$$= \widetilde{\beta} b_1^l \sum_{S=0}^{\infty} \frac{\widetilde{\gamma}^S}{(b_1 + S + 1)^l} = \overline{\mu}_{\omega} = \overline{\mu}_{\omega_1}^l , \quad (14)$$

here $\eta = 1 - \theta$.

Level of defect varies evenly. Averages μ_i increase gradually: $\mu_1 < \mu_2 < \ldots < \mu_{l-1} < \mu_l$, when $a_i \geq 1$ and $b_i \geq 1$; $b_i = b_{i+1} + a_{i+1}$ or $b_{i+1} = b_i - a_{i+1}$, then $\theta \sim be(a, b)$, when $a = \sum_{i=1}^{l} a_i$, $b = b_l$.

When $a_i = a_{i+1}$, we have $b_1 < b_2 < \ldots < b_{l-1} < b_l$. The following a_i values are recommended for the modeling: $a_i = 1,2,3$. Main value is $a_i = 1$, and $a_i = 2$ and $a_i = 3$ are reasonable to use for parameters of parameter groups with higher level of defects.

When $a_i = 2$ we have

$$\begin{cases} g_i(\theta_i) = b_i (b_i + 1) \theta_i (1 - \theta_i)^{b_i - 1}, \\ g_i(0) = g_i(1) = 0; \end{cases} \quad (15)$$

mode $\theta_{iM} = \frac{1}{b_i}$. Maximum density

$$g_i(\theta_{iM}) = (b_i + 1)\left(1 - \frac{1}{b_i}\right)^{b_i - 1}. \quad (16)$$

When $a_i = 3$:

$$\begin{cases} g_i(\theta_i) = \frac{b_i}{2}(b_i + 1)(b_i + 2)\theta_i^2 (1 - \theta_i)^{b_i - 1}, \\ g_i(0) = g_i(1) = 0; \end{cases} \quad (17)$$

mode $\theta_{iM} = \frac{2}{b_i + 1}$, $g_i(\theta_{iM}) = 2 b_i (b_i + 2)\left(\frac{b_i - 1}{b_i + 1}\right)^{b_i - 1}$.

Various cases of defect levels

In this case [1–4] the averages $\mu_i = \mu_{i+1}$, $\mu_i \neq \mu_{i+1}$, parameters $a_i > 0$, $b_i > 0$, then

$$\begin{cases} g_i(\theta_i) = \frac{\Gamma(a_i + b_i)}{\Gamma(a_i \mid \Gamma(b_i))} \theta_i^{a_i - 1} (1 - \theta_i)^{b_i - 1}, \\ \text{mode}\, \theta_{iM} = \frac{a_i - 1}{a_i + b_i - 2}, \\ g_i(\theta_{iM}) = \frac{\Gamma(a_i + b_i)}{\Gamma(a_i \mid \Gamma(b_i))} \cdot \frac{(a_i - 1)^{a_i - 1} (b_i - 1)^{b_i - 1}}{(a_i + b_i - 2)^{a_i + b_i - 2}}. \end{cases} \quad (18)$$

Instead of the precise model of $g(\theta)$ we apply the approximation $g_\sigma(\theta)$, i.e. $\theta \sim be(a^*, b^*)$, when $a^* = \mu\left(\frac{\mu\overline{\mu}}{\sigma^2} - 1\right)$; $b^* = a^* \frac{\overline{\mu}}{\mu}$, mode $\theta_M = \frac{a^* - 1}{a^* + b^* - 2}$, By using the material from [7], we obtain $\mu_i = \frac{a_i}{a_i + b_i}$, $\overline{\mu}_i = 1 - \mu_i = \frac{b_i}{a_i + b_i}$, $\overline{\mu} = \prod_{i=1}^{l} \overline{\mu}_i$, $\mu = 1 - \overline{\mu}$, $\sigma_i^2 = \frac{\mu_i \overline{\mu}_i}{a_i + b_i + 1}$; $\sigma_{12}^2 = \sigma_1^2 \overline{\mu}_2^2 + \sigma_2^2 \overline{\mu}_1^2 + \sigma_1^2 \sigma_2^2$, etc., up to σ^2.

Random values of the level of defects will be $\tau \sim \widetilde{b}_e(a^*, b^* \mid \widetilde{\beta})$ with the approximated density $h_\sigma(\tau)$, $\omega \sim \widetilde{b}_e(a^*, b^* \mid \widetilde{\beta})$. We substitute $\widetilde{\beta}_i$, ω_i, a_i, b_i with $\widetilde{\beta}$, ω, a^*, b^*. We receive

$$\frac{\widetilde{\beta}}{B^*} \sum_{S=0}^{\infty} \widetilde{\gamma}^S \int_0^1 \eta^{S + b^*} (1 - \eta)^{a^* - 1} d\eta = \widetilde{\beta} \sum_{S=0}^{\infty} \widetilde{\gamma}^S \prod_{r=0}^{S} \frac{b^* + r}{a^* + b^* + r} = \overline{\mu}_\omega, \quad (19)$$

here $\mu_\tau = 1 - \overline{\mu}_\tau$, $\overline{\mu}_\tau = \prod_{i=1}^{l} \overline{\mu}_{\tau i}$, $\overline{\mu}_{\tau i} = 1 - \mu_{\tau i}$; $\mu_\omega = 1 - \overline{\mu}_\omega$, $\overline{\mu}_\omega = \prod_{i=1}^{l} \overline{\mu}_{\omega i}$, $\overline{\mu}_{\omega i} = 1 - \mu_{\omega i}$, $B^* = B(a^*, b^*)$.

Modeling example

Initial data: object of control – product with five parameters $(l = 5)$ with two groups of parameters A and B. Group A: $i = 1,2,3$, group B: $i = 4,5$. For parameters: $\theta_i \sim Be(1, b_i)$, when $a_i = 1$, $b_i = \{6,5,4,3,2\}$, $i = 1 \div 5$, $\mu_i = (b_i + 1)^{-1}$ and obtain: $\mu_i = \left\{ 1/7, 1/6, 1/5, 1/4, 1/3 \right\}$.

A problem: to model levels of defects for random quantities $\theta_A, \theta_B, \theta$, when $\widetilde{\beta}_i = \frac{1}{3}$ and the nomenclature of the controlled parameters in group B is changed according to the following scheme:

a) $\widetilde{\beta}_5 = 1$, $\widetilde{\beta}_4 = \frac{1}{3}$ - the fifth parameter is not controlled;

b) $\widetilde{\beta}_4 = 1$, $\widetilde{\beta}_5 = \frac{1}{3}$ – the fourth parameter is not controlled;

c) $\widetilde{\beta}_4 = \widetilde{\beta}_5 = \widetilde{\beta}_B = 1$ – the B group is not controlled.

Results: $\theta_A \sim Be(3;4)$, $\theta_B \sim Be(2;2)$, $\theta \sim bE(5;2)$, $\mu_A = \frac{3}{7}$, $\mu_B = \frac{1}{2}$, $\mu = \frac{5}{7}$. $\tau_i \sim \widetilde{B}e\left(1; b_i \mid \frac{1}{3}\right)$;
$\tau_A \sim \widetilde{B}e(a_A, b_A \mid \widetilde{\beta}_A) = \widetilde{B}e(3;4 \mid 0.292)$;
$\tau_B \sim \widetilde{B}e(a_B, b_B \mid \widetilde{\beta}_B) = \widetilde{B}e(2;2 \mid 0.297)$;
$\tau \sim \widetilde{B}e(a, b \mid \widetilde{\beta}) = \widetilde{B}e(5;2 \mid 0.235)$;
$\mu_{\tau i} = \{0.058;0.070;0.088;0.118;0.176\}$, $i = 1 \div 5$;
$\mu_{\tau A} = 0.201$, $\mu_{\tau B} = 0.273$; $\mu_\tau = 0.419$.

Densities $h_i(\tau_i)$ monotonically decrease from $h_i(0) = 3b_i$ to $h_i(1) = 0$. Approximated densities $h_A(\tau)$, $h_B(\tau)$, $h(\tau)$ are of the unimodal type with mode $h_M = h(\tau_M)$. Here

$$
\begin{cases}
\tau_M = Q_\tau - \sqrt{Q_\tau^2 - \dfrac{a-1}{2c}},\\
Q_\tau = \dfrac{1}{4}\left[(b+1) + \dfrac{a+b-2}{c}\right],\\
c = \dfrac{\tilde{\gamma}}{\tilde{\beta}}.
\end{cases} \quad (20)
$$

Values of modes and densities are given in Table 1.

Table 1. Values of τ_M and $h(\tau_M)$, $h(0) = h(1) = 0$

Group	A	B				A b_i (A+B)			
$\tilde{\beta}_i$	1/3	1/3	$\tilde{\beta}_4 = 1$	$\tilde{\beta}_5 = 1$	$\tilde{\beta}_B = 1$	1/3	$\tilde{\beta}_4 = 1$	$\tilde{\beta}_5 = 1$	$\tilde{\beta}_B = 1$
τ_M	0.121	0.121	0.246	0.300	0.5	0.314	0.454	0.496	0.646
$h(\tau_M)$	3.961	2.742	1.768	1.646	1.5	2.020	1.831	1.822	1.923

Conclusions

It is better to use beta densities for different parameters levels description for multiparameter electronics systems defect levels probabilities modeling, because it simplifies further modeling procedure.

Exact multiparameter electronics system product defect level density expressions are very complicated because of multinomial integration procedure. Offered engineering accepted approximated models, that allows describing defect level density transformations fairly exact, for any parameter value.

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This publication is about continuous between-operational control main probabilities characteristics modelling techniques for multilevel ES, when separate independent parameters defect level probabilities distributions are set (known) in chosen control schematics. Made ES defect level probability density direct and reverse transformations models by separate parameter levels densities transformations. Offered to use approximated models instead of exact whole ES defect level because of complicated process of integration and models get more simple expressions. Ill. 1, bibl. 7 (in English; abstracts in English, Russian and Lithuanian).

Д. Эйдукас. Моделирование уровня дефектов электронных систем // Электроника и электротехника. – Каунас: Технология, 2010. – № 3(99). – С. 13–16.

Предложена методика моделирования основных вероятностных характеристик системы сплошного контроля изделий, когда заданы вероятностные распределения уровней дефектности отдельных независимых параметров в каждой точке схемы контроля. Потоки забракованных изделий возвращаются в процесс производства для регенерации, а правила классификации изделий на отдельных ступенях контроля – одинаковы при наличии существенных ошибок классификации по отдельным параметрам (плохие изделия бракуются, а дефектные признаются годными). Представлены расчётные модели основных вероятностных числовых характеристик, когда значения отдельных параметров описываются бета распределением. Ил. 1, библ. 7 (на английском языке; рефераты на английском, русском и литовском яз.).

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Pateikta metodika daugiaparametrių ES tarpoperacinės kontrolės pagrindinėms tikimybinėms charakteristikoms modeliuoti, kai yra pateikti atskirų nepriklausomų parametrų defektingumo lygių tikimybių skirstiniai. Išbrokuotų ES srautai grąžinami į gamybos procesą regeneruoti, o klasifikavimo taisyklės atskirose kontrolės pakopose analogiškos, esant nepaneigtinoms ES klasifikavimo pirmos ir antros rūšies klaidoms pagal atskirus parametrus. Pagal atskirų parametrų defektingumo lygių tankių transformacijas sudaryti ES defektingumo lygio tikimybių tankio tiesioginės ir atvirkštinės transformacijų modeliai. Il. 1, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).