ISSN 1392 - 1207. MECHANIKA. 2009. Nr.4(78)

On the safety prediction of deteriorating structures

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1. Introduction

The target design life of deteriorating load-carrying structures and their components must be defined in an early design stage of buildings, construction works and technological equipments. The value of this life must serve as a basis for the selections of materials and structures. The target design life is related to destruction modes of materials and structural components and failure consequences. In any case, higher durability requirements are applied to members which routine or preventive maintenance and repairs require great efforts.

Failures and collapses in load-carrying structures can be caused not only by irresponsibility of gross human errors of designers or erectors but also by some conditionalities of recommendations and directions presented in design codes and standards. The Standards EN 1990 [1] in Europe and ASCE/SEI [2] in the USA require that load-carrying structures to be designed with appropriate degrees of reliability. These Standards are based on the limit state concept and, respectively, on the methods of the partial factor design and the strength or allowable stress design. However, the structural design practice shows that it is impossible to verify the safety and economy parameters of deteriorating structures by using deterministic methods and their universal factors for loads and material properties.

The reliability degree of deteriorating structures may be objectively defined only by fully probability-based concepts and models. Only probabilistic approaches may allow us explicitly predict uncertainties of analysis models of these structures. Besides, the probabilistic analysis of deteriorating members is indispensible in order to predict their destructions or failures and to avoid of economic and psychologic losses. However, the mathematical probabilistic formats used in long-term reliability prediction of structures are based on rather complicated considerations [3-6]. Thus, the engineering modeling of survival probabilities of structures subjected to aggressive environmental actions and extreme live and climate loads are still unsolved.

The main task of this paper is to present new methodological formats on probability-based safety predictions of deteriorating members exposed to permanent loads and recurrent single or joint extreme service and climate actions.

2. Resistances and safety margins of deteriorating members

Multicriteria failure modes and safety of structural members (beams, slabs, columns, joints) may be objectively assessed and predicted only knowing survival probabilities of particular members (normal or oblique sections, connections) for which the only possible failure mode exists. Predicted durability parameters for deteriorating structures depend on chemical diagnosis and the acceptable risk of serviceability failure associated with the damage levels and losses. Besides, the predictions of safety of deteriorating members and their systems will account for all extreme action combinations. In any case, it is expedient to divide the life cycle t_n (Fig. 1) of deteriorating structures into the initiation, t_{in}, and propagation, t_{pr}, phases [7]. The length of initiation phase is a random variable depending on a feature of degradation process, an environment aggressiveness and quality of protective covers. The unvulnerability of structures may be characterized by the duration of this phase. When the degradation process of the members is caused by intrinsic properties of materials, the phase $t_{in} = 0$. The propagation phase is delayed for structures protected by coats.

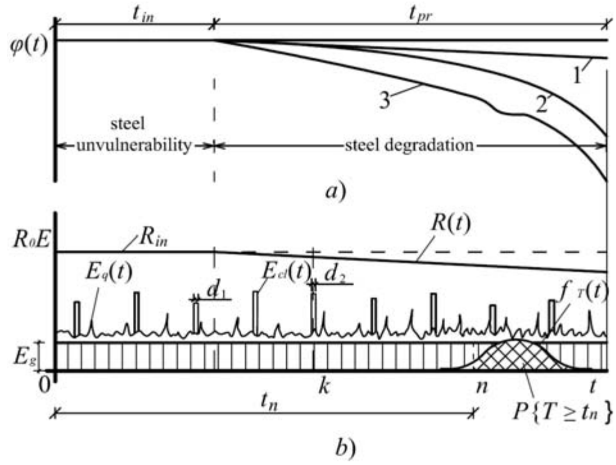

Fig. 1 Degradation function $\varphi(t)(a)$ and dynamic model (b) for time-dependent safety analysis: *1* - unloaded members, *2* - loaded columns, *3* - loaded beams

The resistance of particular members in the propagation phase is treated as a nonstationary random process

$$R(t) = \varphi(t)R_{in} = \varphi(t)R_0 \tag{1}$$

where R_0 is the initial value of member resistance, $\varphi(t)$ denotes the degradation function depending on the rate of a resistance decrease induced by an artificial ageing and degradation of materials. This function for corrosion affected particular members may be presented in the form

$$\varphi(t) = 1 - a(t - t_{in})^b \tag{2}$$

where b defines nonlinearity of the deterioration function and a is degradation intensity factor. A shape of the degradation function is close to linear ($b \approx 1$) and parabolic ($b \approx 2$) when corresponding degradation mechanisms are steel corrosion and aggressive environmental attacks [8, 9]. However, marine corrosion of steel structures is not linear function of time [10].

Action effects of structures are caused by permanent loads g, sustained $q_s(t)$ and extraordinary $q_e(t) = q(t) - q_s(t)$ components of live loads $s(t)$ and wind, surf or seismic actions $w(t)$. The annual extreme sum of sustained and extraordinary live load effects $E_q(t)$ caused by $q_s(t)$ and $q_e(t)$ may be modeled as a rectangular pulse renewal process described by Type I (Gumbel) distribution of extreme values with the coefficient of variation $\delta q = 0.58$ and mean $E_{qm} = 0.47 E_{qk}$, where E_{qk} is its characteristic value [11].

It is proposed to model annual extreme snow and wind action effects by a Gumbel distribution with the mean values equal to $E_{sm} = E_{sk} / (1 + k_{0.98} \delta E_s)$ and $E_{wm} = E_{wk} / (1 + k_{0.98} \delta E_w)$, where E_{sk} and E_{wk} are the characteristic (nominal) values of action effects and $k_{0.98}$ is the characteristic fractile factor of these distributions [7, 12]. The coefficients of variation of snow and wind extreme loads depend on the feature of geographical area and are equal to $\delta s = 0.3 - 0.7$ and $\delta w = 0.2 - 0.5$.

The durations of extreme floor and climate actions are: $d_q = 1 - 14$ days for merchant and 1-3 days for other buildings, $d_s = 14 - 28$ days and $d_w = 8 - 12$ hours. Renewal rates of annual extreme actions are equal to $\lambda = 1 / year$. Therefore, the recurrence number of two joint extreme actions during the design working life of structures, t_n in years, may be calculated by the formulae

$$n_{12} = t_n (d_1 + d_2) \lambda_1 \lambda_2 \tag{3}$$

where $\lambda_1 = \lambda_2 = 1 / t_\lambda$ are the renewal rates of extreme loads. Thus, the recurrence numbers of extreme concurrent live or snow and wind loads during $t_n = 50$ years period are quite actual to $n_{qw} = 0.2 - 2.0$ and $n_{sw} = 2.0 - 4.0$. The bivariate distribution function of two independent extreme action effects may be presented as their conventional joint distribution function with the mean $E_{12m} = E_{1m} + E_{2m}$ and the variance $\sigma^2 E_{12} = \sigma^2 E_1 + \sigma^2 E_2$ [13].

According to probability-based approaches (design level III), the time-dependent safety margin of deteriorating particular members exposed to extreme action effects may be defined as their random performance process and presented as follows

$$\begin{aligned} Z(t) = g[\boldsymbol{X}(t), \boldsymbol{\theta}] &= \theta_R R(t) - \theta_g E_g - \theta_q E_{q_s} - \\ &- \theta_q E_{q_e}(t) - \theta_s E_s(t) - \theta_w E_w(t) = \\ &= \theta_R R(t) - \theta_g E_g(t) - \theta_1 E_1(t) - \theta_2 E_2(t) \end{aligned} \tag{4}$$

where $\boldsymbol{X}(t)$ and $\boldsymbol{\theta}$ are the vectors of basic and additional variables, representing respectively random components (resistances and action effects) and their model uncertainties; $E_1(t) = E_{q_s}(t) + E_{q_e}(t)$ and $E_2(t) = E_s(t)$ or $E_2(t) = E_w(t)$. The mean values and standard deviations of additional variables of the safety margin are: $\theta_{Rm} = 0.99 - 1.04$, $\sigma\theta_R = 0.05 - 0.10$ and $\theta_{Em} \approx 1.00$, $\sigma\theta_E \approx 0.10$ [11, 12].

Gaussian and lognormal distribution laws is to be used for member resistances. The permanent actions can be described by a normal distribution law [13,14]. Therefore, for the sake of simplified but quite exact probabilistic analysis of deteriorating members, it is expedient to present Eq. (4) in the form

$$Z(t) = R_c(t) - E(t) \tag{5}$$

where

$$R_c(t) = \theta_R R(t) - \theta_g E_g \tag{6}$$

is the conventional resistance of members the bivariate probability distribution of which may be modeled by Gaussian distribution

$$E(t) = \theta_1 E_1(t) + \theta_2 E_2(t) \tag{7}$$

is the conventional bivariate distribution process of two stochastically independent annual extreme effects [15].

Inspite of analysis simplifications, the use of continuous stochastic processes of member resistances considerably complicates the durability analysis of deteriorating structures exposed to intermittent extreme gravity and lateral variable actions along with permanent ones. The dangerous cuts of these processes correspond to extreme loading situations of structures. Therefore, in design practice the safety margin process Eq. (5) may be modeled as a random geometric distribution and treated as finite decreasing random sequence

$$Z_k = R_{ck} - E_k, k = 1,2,...,n-1,n \tag{8}$$

where

$$R_{ck} = \varphi_k \theta_R R_{in} - \theta_g E_g \tag{9}$$

is the conventional resistance of deteriorating members at the cut k of this sequence (Fig. 1) and n is the recurrence number of single or coincident extreme action effects, E_k, given by Eq. (7), i.e. $E_k = \theta_1 E_{1k} + \theta_2 E_{2k}$.

When extreme action effects are caused by two stochastically independent variable actions, a failure of members may occur not only in the case of their coincidence but also when the value of one out of two effects is extreme. Therefore, three stochastically dependent safety margins should be considered as follows

$$Z_{1k} = R_{ck} - E_{1k}, k = 1,2,...,n_1 \tag{10}$$

$$Z_{2k} = R_{ck} - E_{2k}, k = 1,2,...,n_2 \tag{11}$$

$$Z_{3k} = R_{ck} - E_{12k} = R_{ck} - E_{1k} - E_{2k}, k = 1,2,...,n_{12} \tag{12}$$

where the number of sequence cuts n_{12} is calculated by Eq. (3).

3. Transformed conditional probability method

For particular and structural members of deteriorating structures subjected to extreme action effects, more than one limit state situations are considered. The number of these situations is equal to recurrence numbers n_1 or n_2 and n_{12} of single and coincident extreme action effects, respectively.

The statistical dependences among failure probabilities of particular members at any time t_k or any cut k of rank sequence and their survival probabilities at previous extreme loading situations exist. Therefore, the instantaneous failure probability of these members at sequence cut k, assuming that they were safe at cuts [$1, k-1$], may be presented in the form:

$$P(Z_k \le 0) = P\{R_{ck} \le E_k \ \exists\ k \in [1,n]\} = P\left(F_k \bigcap_{i=1}^{k-1} S_i\right)$$

where F_k denotes the failure event of members at cut k and S_i denotes the event of their survival at previous cut i of a sequence. Therefore, the instantaneous failure probabilities of particular members at cuts $1, 2, 3, ..., n$ of their safety margin sequences are: $P(Z_1 \le 0) = P(F_1)$

$$P(Z_2 \le 0) = P(F_2 \cap S_1) = P(F_2 \cap 1 - F_1) = P(F_2) - \\ - P(F_2 \cap F_1) \quad (13)$$

$$P(Z_3 \le 0) = P(F_3 \cap S_2 \cap S_1) = P(F_3) - P(F_3 \cap F_1) - \\ - P(F_3 \cap F_2) + P(F_3 \cap F_2 \cap F_1) \quad (14)$$

$$P(Z_n \le 0) = P(F_n \cap S_{n-1} \cap S_{n-2} \cap ... \cap S_2 \cap S_1) = \\ = P(F_n) - \sum_{i=1}^{n-1} P(F_n \cap F_i) + ... \\ ... \mp P(F_n \cap F_{n-1} \cap ... \cap F_2 \cap F_1) \quad (15)$$

The time dependent failure probabilities of deteriorating particular and individual members as autosystems during times $t_1, t_2, t_3, ..., t_n$ may be expressed as

$$P(T < t_1) = P(Z_1 \le 0) = P(F_1) \quad (16)$$

$$P(T < t_2) = P(Z_1 \le 0) + P(Z_2 \le 0) = P\left(\bigcup_{k=1}^{2} F_k\right) = \\ = P(F_1) + P(F_2) - P(F_2 \cap F_1) \quad (17)$$

$$P(T < t_3) = P\left(\bigcup_{k=1}^{3} F_k\right) = P(F_1) + P(F_2) + P(F_3) - \\ - P(F_2 \cap F_1) - P(F_3 \cap F_1) - P(F_3 \cap F_2) + \\ + P(F_3 \cap F_2 \cap F_1) \quad (18)$$

$$P(T < t_n) = P\left(\bigcup_{k=1}^{n} F_k\right) = \sum P(F_k) - \\ - \sum_{l>k}\sum P(F_l \cap F_k) + \sum\sum_{m>l>k}\sum P(F_m \cap F_l \cap F_k) - ... \\ ... \pm P\left(\bigcap_{k=1}^{n} F_k\right) \quad (19)$$

Thus, according to probabilistic approaches, the prediction of time dependent survival probabilities of load-carrying particular members may be based on the analysis of decreasing sequences of random safety margins (Fig. 2), i.e. can be written as

$$P(T \ge t_n) = P\left(\bigcap_{k=1}^{n} S_k\right) = 1 - P\left(\bigcup_{k=1}^{n} F_k\right) \quad (20)$$

When the sequence consists of two dependent cuts, the probability that either or both of two failure events of a series system occur is expressed by Eq. (17). An the evaluation of the probability of a second order intersection of failure events F_2 and F_1, i.e. $P(F_2 \cap F_1)$, may be carried out by rather uncomfortable for structural engineers methods of numerical integrations or Monte Carlo simulation. It is more expedient to use in design practice the unsophisticated method of transformed conditional probabilities (TCTM). According to its approaches, the intersection probability

$$P(F_2 \cap F_1) = P(F_2)P(F_1 | F_2) \quad (21)$$

where the conditional failure probability

$$P(F_1 | F_2) = P(F_1) + \rho_{21}^{x_2}[P(F_1)/P(F_2) - P(F_1)] \quad (22)$$

The indexed correlation factor of two sequence cuts, $\rho_{21}^{x_2}$, characterizes an effect of their statistical dependence on the intersection probability $P(F_2 \cap F_1)$.

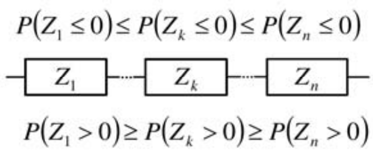

Fig. 2 Safety margin sequences with dependent elements

When sequence cuts are independent, i.e. $\rho_{21}^{x_2} = 0$, the conditional, intersection and failure probabilities of members from Eqs. (22), (21) and (17) are defined as: $P(F_1 | F_2) = P(F_1)$; $P(F_2 \cap F_1) = P(F_2)P(F_1)$; $P(F_2 \cup F_1) = P(F_2) + P(F_1) - P(F_2)P(F_1)$.

When sequence cuts are fully correlated, i.e. $\rho_{21}^{x_2} = 1$, these probabilities are: $P(F_1 | F_2) = P(F_1)/P(F_2)$; $P(F_2 \cap F_1) = P(F_1)$; $P(F_2 \cup F_1) = P(F_2)$.

When the factor $\rho_{21}^{x_2}$ is between 0 and 1, the intersection and failure probabilities by Eqs. (21) and (17) of two cut sequences become as

$$P(F_2 \cap F_1) = P(F_2)P(F_1)\{1 + \rho_{21}^{x_2}[1/P(F_2) - 1]\} \quad (23)$$

$$P\left(F_2 \bigcup F_1\right) = P\left(F_2\right) + P\left(F_1\right) - P\left(F_2\right)P\left(F_1\right) \times \\ \times \left\{1 + \rho_{12}^{x_2}\left[1/P\left(F_2\right) - 1\right]\right\} \tag{24}$$

Analogically to Eq. (23), the probability of an intersection of three failure events may be presented as

$$P\left(F_3 \bigcap F_2 \bigcap F_1\right) = P\left(F_3\right)P\left(F_2\right)P\left(F_1\right) \times \\ \times \left\{1 + \rho_{21}^{x_2}\left[1/P\left(F_2\right) - 1\right]\right\}\left\{1 + \rho_{3,21}^{x_3}\left[1/P\left(F_3\right) - 1\right]\right\} \tag{25}$$

where the correlation factor $\rho_{3,21} \approx 0.5\left(\rho_{32} + \rho_{31}\right)$. The correlation factor and its bounded index are considered in Section 5.

4. Instantaneous survival probability

The instantaneous survival probability of particular members with respect to their single failure mode at sequence cut k, if they were safe at cuts 1-*k*-1 i.e. $P\left(Z_k > 0\right) = P\left(S_k\right) = 1 - P\left(F_k \bigcap_{i=1}^{k-1} S_i\right)$, can be modelled using multidimensional integral as

$$P\left(Z_k > 0\right) = 1 - P\left(Z_k \le 0\right) = \int_{g\left(\mathbf{X}_k|\boldsymbol{\theta}\right) > 0} f_{\mathbf{X}_k|\boldsymbol{\theta}}\left[\mathbf{X}_k|\boldsymbol{\theta}\right] dx$$

For design convenience, the structural safety analysis of deteriorating members may be based on the limit state criteria $R_{ck} - E_k > 0$ or $R_{ck} - \left(E_{q_s k} + E_{2k}\right) > 0$ and, $R_{ck} - \left(E_{1k} + E_{2k}\right) > 0$ where R_{ck} is defined from Eq. (6). Therefore, the instantaneous survival probability may be expressed as

$$P\left(S_k\right) = P\left\{R_{ck} - E_k > 0 \ \exists \ k \in [1, n]\right\} \tag{26}$$

The conventional resistance R_{ck} and single extreme action effect E_k may be treated as statistically independent variables of random safety margins.

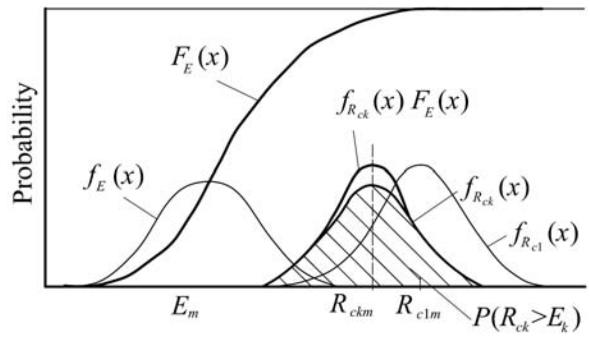

Fig. 3 Schematic representation of an instantaneous survival probability analysis

Therefore, the instantaneous survival probability of deteriorating members can be expressed by convolution integral as

$$P\left(S_k\right) = P\left\{R_{ck} > E_k \ \exists \ k \in [1, n]\right\} = \int_0^\infty f_{R_{ck}}(x) F_{E_k}(x) dx \tag{27}$$

where $f_{R_{ck}}(x)$ is the density function of resistance and $F_{E_k}(x)$ is the cumulative distribution function of their action effect (Fig. 3).

5. Long-term survival probability

Decreasing resistance of particular members must be treated as a nonstationary process. Therefore, it is rather complicated to define the failure probability of multicut sequences in easy perceptible manner. However it is fairly simple to calculate the survival probability of deteriorating members by TCPM. According to Eq. (20), the survival probabilities of these members exposed to two, three and n extreme loading situations may be expressed as follows

$$P\left(T \ge t_2\right) = P\left(S_1 \bigcap S_2\right) = 1 - P\left(F_2 \bigcup F_1\right) = \\ = P\left(S_1\right)P\left(S_2\right)\left\{1 + \rho_{12}^{x_2}\left[\frac{1}{P\left(S_1\right)} - 1\right]\right\} \tag{28}$$

$$P\left(T \ge t_3\right) = P\left(\bigcap_{k=1}^{3} S_k\right) = 1 - P\left(\bigcup_{k=1}^{3} F_k\right) = \\ = P\left(S_1\right)P\left(S_2\right)P\left(S_3\right)\left\{1 + \rho_{21}^{x_2}\left[\frac{1}{P\left(S_1\right)} - 1\right]\right\} \times \\ \times \left\{1 + \rho_{3,21}^{x_3}\left[\frac{1}{P\left(S_1 \bigcap S_2\right)} - 1\right]\right\} \tag{29}$$

$$P\left(T \ge t_n\right) = P\left(\bigcap_{k=1}^{n} S_k\right) = 1 - P\left(\bigcup_{k=1}^{n} F_k\right) = \\ = \prod_{k=1}^{n} P\left(S_k\right)\left\{1 + \rho_{21}^{x_2}\left[\frac{1}{P\left(S_1\right)} - 1\right]\right\} \times \ldots \times \\ \times \left\{1 + \rho_{k,1\ldots k-1}^{x_k}\left[\frac{1}{P\left(S_1 \bigcap \ldots \bigcap S_{k-1}\right)} - 1\right]\right\} \times \ldots \times \\ \times \left\{1 + \rho_{n,1\ldots n-1}^{x_n}\left[\frac{1}{P\left(S_1 \bigcap \ldots \bigcap S_{n-1}\right)} - 1\right]\right\} \tag{30}$$

For the sake of simplified but fairly exact probability-based analysis of deteriorating structures, the conditional survival probability of higher order $P\left(S_k \middle| S_1 \bigcap \ldots \bigcap S_{k-1}\right)$ for particular members may be defined as the probability $P\left(S_k \middle| S_{k-1}\right)$. Therefore, the component $\left\{1 + \rho_{k,1\ldots k-1}^{x_k}\left[\frac{1}{P\left(S_1 \bigcap \ldots \bigcap S_{k-1}\right)} - 1\right]\right\}$ of Eq. (30) may be changed by the factor $\left\{1 + \rho_{k,1\ldots k-1}^{x_k}\left[\frac{1}{P\left(S_{k-1}\right)} - 1\right]\right\}$. Then, Eq. (30) may be rewritten in the form

$$P_i\left(T \ge t_n\right) = P_i\left(\bigcap_{k=1}^{n} S_k\right) \approx \prod_{k=1}^{n} P\left(S_k\right) \times \\ \times \left\{1 + \rho_{21}^{x_2}\left[\frac{1}{P\left(S_1\right)} - 1\right]\right\} \times \ldots \times$$

$$\times\left\{1+\rho_{k,1\ldots k-1}^{x_k}\left[\frac{1}{P(S_{k-1})}-1\right]\right\}\times\ldots\times$$
$$\times\left\{1+\rho_{n,1\ldots n-1}^{x_n}\left[\frac{1}{P(S_{n-1})}-1\right]\right\} \qquad (31)$$

where $P(S_1),\ldots,P(S_{k-1}),\ldots,P(S_{n-1})$ are the instantaneous survival probabilities of members by Eq. (27). The correlation factor of dependent sequence cuts, $\rho_{k,1\ldots k-1}$, is formed from k th row of quadratic matrix of basic coefficients of correlation

$$[P]=\begin{vmatrix} 1 & & & & & & \\ \rho_{21} & 1 & & & & & \\ \cdots & \cdots & \cdots & \cdots & & & \\ \rho_{k1} & \rho_{k2} & \cdots & \rho_{k,k-1} & 1 & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{n,k-1} & \cdots & \rho_{n,n-1} & 1 \end{vmatrix}$$

It may be defined as

$$\rho_{k,1\ldots k-1}\approx(\rho_{k1}+\rho_{k2}+\ldots+\rho_{k,k-1})/(k-1) \qquad (32)$$

The coefficient of correlation of rank safety margin cuts is calculated from the equation

$$\rho_{kl}=Cov(Z_k,Z_l)/(\sigma Z_k\times\sigma Z_l)\approx$$
$$\approx\varphi_k\varphi_l\Big/\left(1+\frac{\sigma^2 S_k}{\sigma^2 R_k}\right) \qquad (33)$$

where $Cov(Z_k,Z_l)$ and $\sigma Z_k,\sigma Z_l$ are an auto covariance and standard deviations of safety margin values.

Then long term survival probabilities of members are calculated by Eq. (31), the bounded index, x_k, of correlation factors of random multicut sequences may be expressed as

$$x_k=P(S_k)[(4.5+4\rho_{k,1\ldots k-1})/(1-0.98\rho_{k,1\ldots k-1})]^{1/2}\approx$$
$$\approx P(S_k)[8.5/(1-0.98\rho_{k,1\ldots k-1})]^{1/2} \qquad (34)$$

For highly reliable load-carrying members, the instantaneous survival probability $P(S_k)\approx 1$ and its effect on bounded indices may be ignored.

The acceptability of this index in design practice is corroborated by Fig. 4, where the position of points for decreasing sequences with two, three and four cuts is calculated by Monte Carlo simulation method. These points belong to the safety margin $Z_k=\varphi_k R_0-M$, where φ_k is its degradation function with reference values 0.97, 0.92, 0.87 and 0.82; R_0 is the initial bending resistance and M is the bending moment. The means and variances of its independent variables are: R_{0m} = 200 kNm, $\sigma^2 R_0$ = 1600 (kNm)^2 and M_m = 60 kNm, $\sigma^2 M$ = 36, 144, 576, 1296 (kNm)^2.

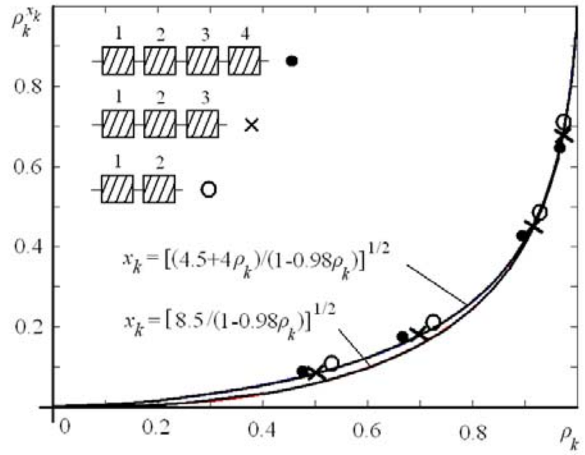

Fig. 4 The indexed correlation factor $\rho_k^{x_k}$ of series systems versus its basic value ρ_k

When extreme action effects are caused by two independent loads and three safety margins (10), (11) and (12) are considered, the long-term survival probability of particular members as series stochastic systems may be expressed as

$$P(T\geq t_n)=P_1P_2P_3\left[1+\rho_{21}^{x_2}\left(\frac{1}{P_1}-1\right)\right]\times$$
$$\times\left[1+\rho_{3,21}^{x_3}\left(\frac{1}{P_2}-1\right)\right] \qquad (35)$$

where the ranked survival probabilities $P_1>P_2>P_3$ are calculated by Eq. (31) and the correlation factor $\rho_{3,21}=0.5(\rho_{31}+\rho_{32})$.

The survival probability of members may be also introduced by the generalized reliability index

$$\beta=\Phi^{-1}\{P(T\geq t_n)\} \qquad (36)$$

where $\Phi^{-1}(\bullet)$ is the cumulative distribution function of the standard normal distribution. The target reliability index β_{min} of the structural members depends on their reliability classes by considering the human life, economic, social and environmental consequences of failure or malfunction [1, 15]. For persistent design situations, the values of β_{min} are equal to 3.3, 3.8 and 4.3 for reliability classes RC1, RC2 and RC3 of structural members. The value of β_{min} for particular members should be not less. However, for members of hyper static structures, it may be decreased to 1.64 [16].

According to TCPM, the total survival probabilities of structural members (beams, columns, plates, trusses) as series, parallel and mixed microsystems may be calculated by the equations

$$P_{ser}=P(S_1\cap S_2)=P_1P_2\left[1+\rho_{21}^{x_2}\left(\frac{1}{P_1}-1\right)\right] \qquad (37)$$

$$P_{par} = P(S_1 \bigcup S_2) = P_1 + P_2 - P_1 P_2 \left[1 + \rho_{21}^{x_2} \left(\frac{1}{P_1} - 1 \right) \right] \quad (38)$$

$$P_{mix} = P(S_1 \bigcap S_2 \bigcap S_3) =$$

$$= P_{par} P_3 \left[1 + \rho_{3,21}^{x_3} \left(\frac{1}{P_{3/par}} - 1 \right) \right] \quad (39)$$

where $P_{3/par}$ is the greater value from the probabilities P_3 and P_{par} by Eq. (38).

6. Numerical example

Consider the long-term survival probability and reliability index of deteriorating roof steel beams of a scrap metal shed exposed to atmosphere corrosion conditions induced by environmental cold, wet and dry actions (Fig. 5). The indicative design working life of beams is 25 year. The initiation degradation phase of beams $t_{in} = 0$ and the degradation function of their bending resistance $\varphi(t) = 1 - 0.00375t$.

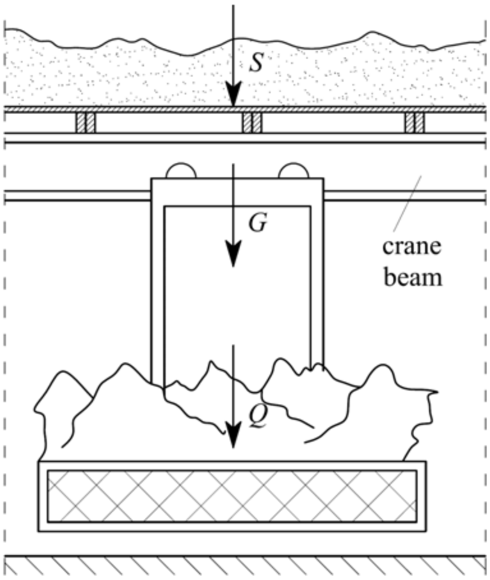

Fig. 5 Traversing underslung crane

The bending moments of beams M_G, M_Q and M_S are caused by permanent load G of steel roof structures and hanging crane crabs, variable loads Q and S of scrap metals and snow depth. The means and coefficients of variation of basic variables of a beam safety margins are: $R_{0m} = 363.5$ kNm, $\delta R_0 = 0.08$; $M_{gm} = 20.0$ kNm, $\delta M_g = 0.10$; $M_{qm} = 49.7$ kNm, $\delta M_q = 0.20$; $M_{sm} = 50.3$ kNm, $\delta M_s = 0.5$. The statistics of additional variables of beam safety margin are: $\theta_{Rm} = \theta_{Mm} = 1.0$, $\sigma^2\theta_R = 0.0025$, $\sigma^2\theta_M = 0$.

The means and variances of the beam parameters are:

$(\theta_R R_0)_m = 363.5$ kNm, $\sigma^2(\theta_R R_0) = (0.08 \times 363.5)^2 + 363.5^2 \times 0.0025 = 1176.2$ (kNm)^2;

$(\theta_M M_g)_m = 20.0$ kNm, $\sigma^2(\theta_M M_g) = (0.10 \times 20.0)^2 = 4.0$ (kNm)^2;

$(\theta_M M_q)_m = 49.7$ kNm, $\sigma^2(\theta_M M_q) = (0.20 \times 49.7)^2 = 98.8$ (kNm)^2;

$(\theta_M M_s)_m = 50.3$ kNm, $\sigma^2(\theta_M M_s) = (0.50 \times 50.3)^2 = 632.5$ (kNm)^2.

These parameters are described by normal (R and M_g), lognormal (M_q) and Gumbel (M_s) probability distributions. The instantaneous and long-term survival probabilities are calculated by Eqs. (26) and (31) and the reliability index is defined by Eq. (36). Their decreases in time are presented in Fig. 6.

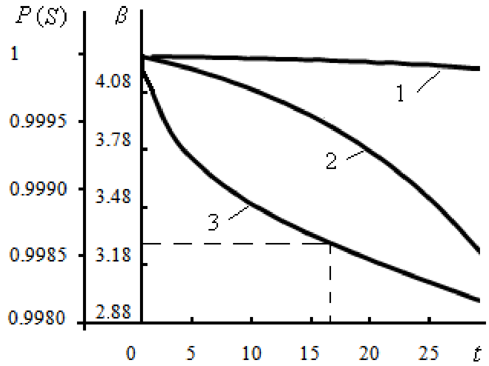

Fig. 6 The decreases of instantaneous (*1*) and long-term (*2*) survival probabilities of beams and their reliability index (*3*)

According to code recommendations [1], the minimum value for reliability index of beams is $\beta_{min} = 3.3$. Therefore, their technical service life is equal to 17 years.

7. Conclusion

The prediction of time-dependent safety of deteriorating structures subjected to aggressive environmental conditions and recurrent extreme service and climate loads can be formulated and solved within unsophisticated probability-based approaches. It is expedient to base the analysis of survival probabilities and reliability indices of deteriorating particular members (sections, bars, connections) on the concept of random decreasing multicut sequences. The position of stochastically dependent cuts of these sequences is matched with extreme loading situations of structures.

The method of transformed conditional probabilities (TCPM) may be successfully introduced into the probability-based design of deteriorating particular and structural members in a simple and easy perceptible manner. This method help us predict the safety parameters of structural members (beams, columns, plate, trusses) as stochastical series, parallel and mixed microsystems.

A closer definition of technical service lives of deteriorating structural members allows us avoid unfounded premature replacements and unexpected damages.

The represented methodological formats on survival probability and technical service life prediction are in force for deteriorating structures subjected both to single and joint extreme loads.

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SILPNĖJANČIŲ KONSTRUKCIJŲ SAUGOS PROGNOZAVIMAS

R e z i u m ė

Nagrinėjamas agresyvios aplinkos ir pavienių ar sutampančių ekstremaliųjų apkrovų stačiakampio atsinaujinančio pulsinio proceso veikiamų konstrukcijų saugos prognozavimo inžinerinis modeliavimas. Silpnėjančių ypačiųjų elementų saugos ribos modeliuojamos atsitiktinėmis mažėjančiomis sekomis. Ilgalaikės konstrukcijų išlikties tikimybės prognozavimas remiasi nesudėtingu transformuotų sąlyginių tikimybių metodu. Tikimybinį silpnėjančio elemento projektavimą iliustruoja skaitinis pavyzdys.

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ON THE SAFETY PREDICTION OF DETERIORATING STRUCTURES

S u m m a r y

Engineering modeling of safety prediction of the structures subjected to aggressive environmental actions and rectangular renewal pulse processes of single and coincident extreme loads is considered. The safety margins of deteriorating particular members are modeled as a random decreasing sequences. The prediction of long-term survival probabilities of structures is based on the unsophisticated method of transformed conditional probabilities. The probability-based design of deteriorating members is illustrated by the numerical example.

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О ПРОГНОЗИРОВАНИИ БЕЗОПАСНОСТИ ОСЛАБЛЯЮЩИХСЯ КОНСТРУКЦИЙ

Р е з ю м е

Рассматривается инженерное моделирование прогнозирования безопасности конструкций, подвергнутых воздействию агрессивной среды и прямоугольных восстановляющихся пульсирующих процессов одиночных и совмещенных экстремальных нагрузок. Запас прочности ослабляющихся частных элементов моделируется случайной снижающейся последовательностью. Прогнозирование вероятностной долговечности конструкций основано на несложном методе трансформированных условных вероятностей. Вероятностное проектирование ослабляющегося элемента иллюстрируется численным примером.

Received June 18, 2009
Accepted August 07, 2009