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Probability Model Quality of Informations Systems

R. Kalnius, D. Eidukas
Department of Electronics Engineering, Kaunas University of Technology,
Studentų str. 50, LT-51368 Kaunas, Lithuania, phone:+370 37 351389, + 370 37 773529,
e-mail: danielius.eidukas@ktu.lt

Introduction

In companies which utilize modern technologies databases of manufactured production and technological processes are constantly maintained and complemented. Data is often read and input automatically. By using computer networks and database control and analysis systems, information can be transmitted in real-time and can be used in decision making processes at each intermediate or final stage of manufacture. Thus there is a possibility to use information not only from the current but also from the previous stages of manufacture. That can increase the quality of information systems (IS) [1–4].

Modern IS control system involves control of raw materials, components, transitional control of elements or nodes, technological process and final product. In this way IS control system plays an important role by influencing cost and the final result.

Continuous inter-operational quality control is commonly applied in manufacture process of IS products. Selective control peculiarities of such IS are analyzed in publications [1–6] on the grounds of color picture tube manufacture specifics. In this paper we will describe the performance of multistage continuous inter-operational control with the help of stochastic models, when IS classification errors of the first and second kind are present. Main attention is paid to the transformation of production defectivity level probability distributions, which in turn allows to estimate the efficiency of inter-operational control in the way of modeling, and to select the required number of control stages and their characteristics [5–11].

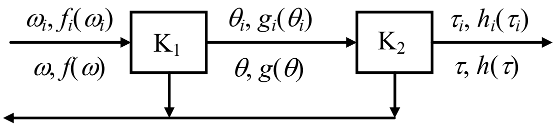

Fig. 1. Multi-parameter IS double level control and its probabilities characteristics

Analyzing double-level control schematics shown in fig. 1, when IS is described as l-dimensional parameter vector [1 – 4], $i = 1 - \ell$.

Here $\omega_i, \theta_i, \tau_i$ – defective IS probabilities by i-th parameter before first level K_1, before and after second level K_2. These are accidental values with densities $f_i(\omega_i), g_i(\theta_i), h_i(\tau_i)$ and distribution functions $F_i(\omega_i), G_i(\theta_i), H_i(\tau_i)$. Respectively ω, θ, τ – analogical characteristics for product by all controlled ℓ parameters.

For further analysis good IS probabilities by *i*-th parameter are needed: $\xi_i = 1 - \omega_i$, $\eta_i = 1 - \theta_i$, $\zeta_i = 1 - \tau_i$ – accidental values with densities:

$$\ddot{\varphi}_i(\xi_i) = f_i(1-\xi_i),\ \varphi_i(\eta_i) = g_i(1-\eta_i),\\ \dot{\varphi}_i(\zeta_i) = 1 - h_i(1-\zeta_i). \quad (1)$$

And distribution functions

$$\ddot{\Phi}_i(\xi_i) = 1 - F_i(1-\xi_i),\ \Phi_i(\eta_i) = 1 - G_i(1-\eta_i),\\ \dot{\Phi}_i(\zeta_i) = 1 - H_i(1-\zeta_i). \quad (2)$$

Accidental values: $\xi = 1-\omega$, $\eta = 1-\theta$, $\zeta = 1-\tau$ – are good IS probabilities by all ℓ parameters.

Analyzing situations, when all accidental values θ_i, also accidental values η_i densities $g_i(\theta_i), \varphi_i(\eta_i)$ are known and written in beta law: $\theta_i \sim \mathrm{Be}(a_i, b_i), \eta_i \sim \mathrm{Be}(b_i, a_i)$, here a_i, b_i – beta law forms parameters [1 – 4]. Then τ_i is directly transformed (T), o ω_i – reverse transformed (A) accidental value θ_i with transformation parameter $\tilde{\beta}_i$:

$$\dot{\beta}_i = \frac{\beta_i}{1-\alpha_i},\quad \alpha_i + \beta_i < 1; \quad (3)$$

where α_i=const, β_i=const – first and second kind errors probabilities [1] by *i*-th parameter, $i = 1 - \ell$.

Densities $h_i(\tau_i)$ ir $f_i(\omega_i)$ by analogy with [1] models are:

$$\begin{cases} h_i(\tau_i) = \dfrac{B_i^{-1}\tau_i^{a_i-1}(1-\tau_i)^{b_i-1}}{\widetilde{\beta}_i^{a_i}(1+c_i\tau_i)^{a_i+b_i}}, \\ f_i(\omega_i) = \dfrac{\widetilde{\beta}_i^{a_i}\omega_i^{a_i-1}(1-\omega_i)^{b_i-1}}{B_i(1-\widetilde{\gamma}_i\omega_i)^{a_i+b_i}}; \end{cases} \quad (4)$$

where $B_i = B(a_i, b_i) = \dfrac{\Gamma(a_i)\Gamma(b_i)}{\Gamma(a_i+b_i)}$ – beta function; $\Gamma(z)$ – gamma function; $\widetilde{\gamma}_i = \dfrac{\gamma_i}{1-\alpha_i} = 1-\widetilde{\beta}_i$, $\gamma_i = 1-\alpha_i-\beta_i$, $c_i = \widetilde{\gamma}_i / \widetilde{\beta}_i \equiv \gamma_i/\beta_i$, $\Gamma(1) = 0! = 1$.

Accidental values θ_i, η_i densities and main numerical characteristics are shown in [1–2]. Distribution function $G_i(\theta_i)$ in common case is calculated by [6–9], for a_i, b_i are whole numbers, we get $G_i(\theta_i)$ and $H_i(\tau_i)$:

$$G_i(\theta_i) = \int_0^{\theta_i} g_i(\theta_i)d\theta_i = B_i^{-1}\sum_{j=0}^{b_i-1} C_{b_{i-1}}^j (-1)^j \frac{\theta_i^{a_i+j}}{a_i+j}, \quad (5)$$

$$H_i(\tau_i) = \frac{B_i^{-1}}{\widetilde{\gamma}_i^{a_i}} \sum_{j=0}^{b_i-1} C_{b_i-1}^j \frac{(-1)^j}{c_i^j} \sum_{k=0}^{a_i+j-1} C_{a_i+j-1}^k \times$$
$$\times \frac{(-1)^{a_i+j-k-1}}{a_i+b_i-k-1}\left[1-\frac{(1+c_i\tau_i)^{k+1}}{(1+c_i\tau_i)^{a_i+b_i}}\right], \quad (6)$$

when $k=a_i+b_i-1$, then in the sum $\sum_k$ setting in member

$$C_{a_i+j-1}^{a_i+b_i-1}\frac{(-1)^{b_i-j}}{c_i^{b_i-j+1}}\ln(1+c_i\tau_i);$$

where C_n^m – combinations from n by m: $\dfrac{n!}{m!(n-m)!}$.

Accidental values τ_i, ω_i averages μ_{τ_i}, μ_{ω_i} and dispersions $\sigma_{\tau_i}^2$, $\sigma_{\omega_i}^2$ calculating in initial moment $\mathrm{E}\omega_i^k, \mathrm{E}\tau_i^k, k=1,2$ help [7 – 10]

$$\begin{cases} \mathrm{E}\tau_i^k = \int_0^1 [\tau_i(\theta_i)]^k g_i(\theta_i)d\theta_i, \\ \mathrm{E}\omega_i^k = \int_0^1 [\omega_i(\theta_i)]^k g_i(\theta_i)d\theta_i. \end{cases} \quad (7)$$

We get, that $\tau_i(\theta_i) = \dfrac{\widetilde{\beta}_i\theta_i}{1-\widetilde{\gamma}_i\theta_i}$, $\omega_i(\theta_i) = \dfrac{\theta_i}{\widetilde{\beta}_i(1+c_i\theta_i)}$ [1] and

$$\begin{cases} \mu_{\tau_i} = \mathrm{E}\tau_i, \\ \mu_{\omega_i} = \mathrm{E}\omega_i, \\ \sigma_{\tau_i}^2 = v\tau_i = \mathrm{E}\tau_i^2 - (\mathrm{E}\tau_i)^2, \\ \sigma_{\omega_i}^2 = v\omega_i = \mathrm{E}\omega_i^2 - (\mathrm{E}\omega_i)^2. \end{cases} \quad (8)$$

In common case, when $a_i>0$, $b_i>0$ applying Taylor series:

$$\mathrm{E}\tau_i = \frac{\widetilde{\beta}_i}{B_i}\int_0^1 \frac{\theta_i^{a_i}(1-\theta_i)^{b_i-1}}{1-\widetilde{\gamma}_i\theta_i}d\theta_i = \widetilde{\beta}_i\sum_{s=0}^{\infty}\widetilde{\gamma}_i^s \prod_{r=0}^{s}\frac{a_i+r}{a_i+b_i+r}, \quad (9)$$

$$\mathrm{E}\tau_i^2 = \frac{\widetilde{\beta}_i^2}{B_i}\int_0^1 \frac{\theta_i^{a_i+1}(1-\theta_i)^{b_i-1}}{(1-\widetilde{\gamma}_i\theta_i)^2}d\theta_i =$$
$$= \widetilde{\beta}_i^2 \sum_{s=1}^{\infty} s\widetilde{\gamma}_i^{s-1}\prod_{r=0}^{s}\frac{a_i+r}{a_i+b_i+r}. \quad (10)$$

If a_i, b_i are whole numbers ≥1, we get

$$\mathrm{E}\tau_i = \frac{B_i^{-1}}{c_i}\sum_{j=0}^{b_i-1} C_{b_{i-1}}^j (-1)^j\left[\frac{1}{\widetilde{\gamma}_i^{a_i+j}}\ln\frac{1}{\widetilde{\beta}_i} - \right.$$
$$\left. - \sum_{k=0}^{a_i+j-1}\frac{1}{(a_i+j-k)\widetilde{\gamma}_i^k}\right], \quad (11)$$

$$\mathrm{E}\tau_i^2 = \frac{B_i^{-1}}{c_i^2}\sum_{j=0}^{b_i-1} C_{b_{i-1}}^j (-1)^j \left\{\frac{1}{\widetilde{\gamma}_i^{a_i+j}}\left[c_i - (a_i+j+1)\times\right.\right.$$
$$\left.\left.\times\ln\frac{1}{\widetilde{\beta}_i}\right] - \sum_{k=1}^{a_i+j}\frac{k}{(a_i+j-k)\widetilde{\gamma}_i^{k-1}}\right\}. \quad (12)$$

Reversed A transformation models immediately got from T transformation models, to direct T transformation expressions instead of parameter $\widetilde{\beta}_i$ we set-in reversed value $1/\widetilde{\beta}_i$ (analogically instead of $-\widetilde{\gamma}_i$ we set-in c_i or vice-versa):

$$\begin{cases} f_i(\omega_i,\widetilde{\beta}_i) = h_i(\omega_i, 1/\widetilde{\beta}_i),\ F_i(\omega_i,\widetilde{\beta}_i) = H_i(\omega_i, 1/\widetilde{\beta}_i), \\ \mathrm{E}(\omega_i,\beta_i)^k = \mathrm{E}_i(\tau_i, 1/\widetilde{\beta}_i)^k,\ \ln\widetilde{\beta}_i = -\ln 1/\widetilde{\beta}_i. \end{cases} \quad (13)$$

Then if a_i, b_i are whole numbers ≥1, we get (when k=1; 2):

$$\mathrm{E}\omega_i = \frac{B_i^{-1}}{\widetilde{\gamma}_i}\sum_{j=0}^{b_i-1} C_{b_{i-1}}^j (-1)^j\left[\frac{(-1)^{a_i+j}}{c_i^{a_i+j}}\ln\frac{1}{\widetilde{\beta}_i} + \right.$$
$$\left. + \sum_{k=0}^{a_i+j-1}\frac{(-1)^k}{(a_i+j-k)c_i^k}\right], \quad (14)$$

$$\mathrm{E}\omega_i^2 = \frac{B_i^{-1}}{\widetilde{\gamma}_i^2}\sum_{j=0}^{b_i-1} C_{b_{i-1}}^j (-1)^j\left\{\frac{(-1)^{a_i+j}}{c_i^{a_i+j}}\left[(a_i+j+1)\times\right.\right.$$
$$\left.\left.\times\ln 1/\widetilde{\beta}_i - \widetilde{\gamma}_i\right] + \sum_{k=1}^{a_i+j}\frac{(-1)^{k-1}k}{(a_i+j-k+1)c_i^{k-1}}\right\}. \quad (15)$$

Double-parameter IS ($\ell = 2$), i=1; 2

Analogically with [1–4] models we get:

$$\Phi(\zeta) = 1 - \int_\zeta^1 \varphi_1(\zeta_1)d\zeta_1 \int_{\zeta/\zeta_1}^1 \varphi_2(\zeta_2)d\zeta_2 = 1 - \frac{\widetilde{\beta}_1^{b_1}\widetilde{\beta}_2^{b_2}}{B_1B_2}\times$$

$$\times\int_{\zeta}^{1}\int_{\zeta/\zeta_1}^{1}\frac{\zeta_1^{b_1-1}\zeta_2^{b_2-1}(1-\zeta_1)^{a_1-1}(1-\zeta_2)^{a_2-1}}{(1-\tilde{\gamma}_1\zeta_1)^{a_1+b_1}(1-\tilde{\gamma}_2\zeta_2)^{a_2+b_2}}d\zeta_1 d\zeta_2, \quad (16)$$

$$\dot{\varphi}(\zeta)=\frac{d\dot{\Phi}(\zeta)}{d\zeta}=\int_{\zeta}^{1}\frac{1}{\zeta_1}\dot{\varphi}_1(\zeta_1)\dot{\varphi}_2\left(\frac{\zeta}{\zeta_1}\right)d\zeta_1=$$

$$=\frac{\tilde{\beta}_1^{b_1}\tilde{\beta}_2^{b_2}\zeta^{b_2-1}}{B_1B_2}\int_{\zeta}^{1}\frac{\zeta_1^{b_1}(1-\zeta_1)^{a_1-1}(1-\zeta_2)^{a_2-1}}{(1-\tilde{\gamma}_1\zeta_1)^{a_1+b_1}(\zeta_1-\tilde{\gamma}_2\zeta)^{a_2+b_2}}d\zeta_1. \quad (17)$$

If $\tilde{\beta}_1=\tilde{\beta}_2=1(\tilde{\gamma}_1=\tilde{\gamma}_2=0)$, i.e. both parameters K_2 level (T transformation) not measured, we get $\zeta=\eta$ and $\dot{\Phi}(\zeta)=\Phi(\eta)$, $\dot{\varphi}(\zeta)=\varphi(\eta)$.

Let IS describe as: $a_i=b_i=1$, $g_i(\theta_i)=\varphi_i(\eta_i)=1$, $\varphi(\eta)=-\ln\eta$, then we get:

$$\dot{\Phi}(\zeta)=1-\frac{1}{1-\tilde{\gamma}_1\tilde{\gamma}_2\zeta}\left[(1-\zeta)-\frac{\tilde{\beta}_1\tilde{\beta}_2\zeta}{1-\tilde{\gamma}_1\tilde{\gamma}_2\zeta}\times\right.$$
$$\left.\times\ln\frac{(1-\tilde{\gamma}_1\zeta)(1-\tilde{\gamma}_2\zeta)}{\tilde{\beta}_1\tilde{\beta}_2\zeta}\right], \quad (18)$$

$$\dot{\varphi}(\zeta)=\frac{\tilde{\beta}_1\tilde{\beta}_2}{(1-\tilde{\gamma}_1\tilde{\gamma}_2\zeta)^2}\left[\frac{1+\tilde{\gamma}_1\tilde{\gamma}_2\zeta}{1-\tilde{\gamma}_1\tilde{\gamma}_2\zeta}\ln\frac{(1-\tilde{\gamma}_1\zeta)(1-\tilde{\gamma}_2\zeta)}{\tilde{\beta}_1\tilde{\beta}_2\zeta}-\right.$$
$$\left.-\frac{1}{\tilde{\gamma}_1\zeta}-\frac{1}{\tilde{\gamma}_2\zeta}+\frac{1}{\tilde{\beta}_1}+\frac{1}{\tilde{\beta}_2}\right], \quad (19)$$

$$H(\tau)=\frac{1}{1-\tilde{\gamma}_1\tilde{\gamma}_2(1-\tau)}\left[\tau-\frac{\tilde{\beta}_1\tilde{\beta}_2(1-\tau)}{1-\tilde{\beta}_1\tilde{\beta}_2(1-\tau)}\times\right.$$
$$\left.\times\ln\frac{(1+c_1\tau)(1+c_2\tau)}{1-\tau}\right], \quad (20)$$

$$h(\tau)=\frac{\tilde{\beta}_1\tilde{\beta}_2}{[1-\tilde{\gamma}_1\tilde{\gamma}_2(1-\tau)]^2}\left[\frac{1+\tilde{\gamma}_1\tilde{\gamma}_2(1-\tau)}{1-\tilde{\gamma}_1\tilde{\gamma}_2(1-\tau)}\ln\frac{(1+c_1\tau)(1+c_2\tau)}{1-\tau}-\right.$$
$$\left.-\frac{1}{\tilde{\beta}_1+\tilde{\gamma}_1\tau}-\frac{1}{\tilde{\beta}_2+\tilde{\gamma}_2\tau}+\frac{1}{\tilde{\beta}_1}+\frac{1}{\tilde{\beta}_2}\right], \quad (21)$$

$$\ddot{\varphi}(\xi)=\frac{1}{\tilde{\beta}_1\tilde{\beta}_2(1-c_1c_2\xi)^2}\left[\frac{1+c_1c_2\xi}{1-c_1c_2\xi}\ln\frac{(1+c_1\xi)(1+c_2\xi)}{(1+c_1)(1+c_2)\xi}-\right.$$
$$\left.-\frac{1}{1+c_1\xi}-\frac{1}{1+c_2\xi}+\tilde{\beta}_1+\tilde{\beta}_2\right], \quad (22)$$

$$f(\omega)=\frac{1}{\tilde{\beta}_1\tilde{\beta}_2[1-c_1c_2(1-\omega)]^2}\left[\frac{1+c_1c_2(1-\omega)}{1-c_1c_2(1-\omega)}\times\right.$$
$$\left.\times\ln\frac{(1-\tilde{\gamma}_1\omega)(1-\tilde{\gamma}_2\omega)}{(1-\omega)}-\frac{\tilde{\beta}_1}{1-\tilde{\gamma}_1\omega}-\frac{\tilde{\beta}_2}{1-\tilde{\gamma}_2\omega}+\tilde{\beta}_1+\tilde{\beta}_2\right], \quad (23)$$

where $h(0)=f(0)=0$, $h(1)=f(1)=\infty$.

When $\mu_i=\bar{\mu}_i=1/2$, $\sigma_i^2=1/12$, we get $\bar{\mu}=\bar{\mu}_1\bar{\mu}_2==1/4$, $\mu=1-\bar{\mu}=3/4$ and when $\sigma_1^2=\sigma_2^2$, $\bar{\mu}_1^2=\bar{\mu}_2^2$, $\sigma^2=\sigma_1^2\bar{\mu}_2^2+\sigma_2^2\bar{\mu}_1^2+\sigma_1^2\sigma_2^2=\sigma_1^2\left(2\bar{\mu}_1^2+\sigma_1^2\right)=7/144$.

In partial case, when $\tilde{\beta}_2=1$ (unmeasured second IS parameter), we get

$$\begin{cases}\dot{\varphi}\left(\zeta_{|\tilde{\beta}_2=1}\right)=\tilde{\beta}_1\left(\ln\frac{1-\tilde{\gamma}_1\zeta}{\tilde{\beta}_1\zeta}-\frac{1}{1-\tilde{\gamma}_1\zeta}\right)+1;\\ \ddot{\varphi}\left(\xi_{|\tilde{\beta}_2=1}\right)=\frac{1}{\tilde{\beta}_1}\left(\ln\frac{1+c_1\xi}{(1+c_1)\xi}-\frac{1}{1-c_1\xi}\right)+1;\end{cases} \quad (24)$$

where $1+c_1\equiv1/\tilde{\beta}_1$.

It is obvious, that analogical expressions are when $\tilde{\beta}_1=1$ (unmeasureable first parameter).

In partial case, when $\tilde{\beta}_2=1$, we get: $\tau_2=\omega_2=\theta_2$ and $\mu_{\tau_2}=\mu_{\omega_2}=\mu_2$; $\bar{\mu}_{\tau_2}=\bar{\mu}_{\omega_2}=\bar{\mu}_2$; $\sigma_{\tau_2}^2=\sigma_{\omega_2}^2=\sigma_2^2$; $h_2(\tau_2)=f_2(\omega_2)=g_2(\theta_2)$; $\dot{\varphi}_2(\eta_2)=\ddot{\varphi}_2(\xi_2)=\varphi_2(\eta_2)$.

Take that IS is characterized with parameters: a_i=1, b_1=2, b_2=1; θ_1~Be(1,2), θ_2~Be(1,1), θ~Be(2,1), then

$$\dot{\varphi}(\zeta)=\frac{2\tilde{\beta}_1^2\tilde{\beta}_2\tilde{\gamma}_2\zeta}{(1-\tilde{\gamma}_1\tilde{\gamma}_2\zeta)^3}\left\{\frac{2+\tilde{\gamma}_1\tilde{\gamma}_2\zeta}{1-\tilde{\gamma}_1\tilde{\gamma}_2\zeta}\ln\frac{(1-\tilde{\gamma}_1\zeta)(1-\tilde{\gamma}_2\zeta)}{\tilde{\beta}_1\tilde{\beta}_2\zeta}+\right.$$
$$+\frac{2}{\tilde{\beta}_1}\left(1-\frac{1}{\tilde{\beta}_1}\right)+\frac{1}{\tilde{\beta}_2}+\frac{1}{2\tilde{\gamma}_1\tilde{\gamma}_2\zeta}\left[\frac{1}{\tilde{\beta}_1^2}-\frac{1-\tilde{\gamma}_1\tilde{\gamma}_2\zeta}{(1-\tilde{\gamma}_1\zeta)^2}\right]-$$
$$\left.-\frac{1}{1-\tilde{\gamma}_2\zeta}\right\}, \quad (25)$$

$$\dot{\varphi}(1)=h(0)=0, \quad (26)$$

$$\dot{\varphi}(0)=h(1)=\tilde{\beta}_2\left(1+\tilde{\beta}_1\right). \quad (27)$$

In this case: $\mu_1=1/3$, $\mu_2=1/2$, $\bar{\mu}_1=2/3$, $\bar{\mu}_2=1/2$, $\bar{\mu}=1/3$, $\mu=2/3$, $\sigma_1^2=1/18$, $\sigma_2^2=1/12$, $\sigma^2=1/18$.

Partial cases ($\bar{\beta}_2=1$ or $\bar{\beta}_1=1$):

$$\dot{\varphi}\left(\zeta_{|\tilde{\beta}_2=1}\right)=\frac{1}{\tilde{\gamma}_1}\left[1-\frac{\tilde{\beta}_1^2}{(1-\tilde{\gamma}_1\zeta)^2}\right], \quad (28)$$

$$\dot{\varphi}\left(\zeta_{|\tilde{\beta}_1=1}\right)=2\tilde{\beta}_2\tilde{\gamma}_2\left(2\ln\frac{1-\tilde{\gamma}_2\zeta}{\tilde{\beta}_2\zeta}-\frac{1}{1-\tilde{\gamma}_2\zeta}+\frac{1}{\tilde{\beta}_2}-\frac{1}{2}\right). \quad (29)$$

Transformed densities approximations

It is obvious, if values a_i, b_i grows, $\dot{\varphi}(\zeta)$ (17) expression after integrating becomes more and more complicated, so it becomes disadvantageous for modeling, also integrating process becomes more complicated. In that case, for density $g(\theta)$ [1–4] and for densities $h(\tau)$ and $f(\omega)$ we apply approximations.

In density $g(\theta)$ regard there are three cases:

1. Known exact density $g(\theta)$ model (not beta density).

Applying $h(\tau)$ approximation to density $h_\Sigma(\tau)$:

$$h_\Sigma(\tau)=\theta'(\tau)g[\theta(\tau)]=\frac{g[\theta(\tau)]}{\widetilde{\beta}(1+c\tau)^2}; \qquad (30)$$

where $\theta(\tau)=\dfrac{\tau}{\widetilde{\beta}(1+c\tau)}$, $\theta'(\tau)=\dfrac{\partial\theta(\tau)}{\partial\tau}=\dfrac{1}{\widetilde{\beta}(1+c\tau)^2}$.

2. One-parameter IS ($\ell=1$) either fixed θ_i value we have

$$\widetilde{\beta}_i=\frac{\beta_i}{1-\alpha_i}=\frac{\eta_i\tau_i}{\theta_i\zeta_i}=\text{const}. \qquad (31)$$

3. Double-parameter IS resultant normalized error probability $\widetilde{\beta}$ (direct transformation parameter for all IS) depends on parameters $\widetilde{\beta}_1,\widetilde{\beta}_2,\theta_1,\theta_2$.

Applying [7–10] models and get

$$\widetilde{\beta}=\frac{\beta}{1-\alpha}=\frac{\eta\tau}{\theta\xi}=\frac{1}{\theta}\left[(1-\widetilde{\gamma}_1\theta_1)(1-\widetilde{\gamma}_2\theta_2)-\mu\right]=$$
$$=\widetilde{\beta}(\theta)\neq\text{const}; \qquad (32)$$

where $1-\alpha=(1-\alpha_1)(1-\alpha_2)$; $\beta=\dfrac{p\tau}{\theta}$; $p=p_1p_2$ – IS acceptance as good probability, $p_i=1-\alpha_i-\gamma_i\theta_i=(1-\alpha_1)\times(1-\widetilde{\gamma}_i\theta_i)$, i=1, 2 – IS acceptance as good by i-th parameter probability.

To limited θ values θ=0 and θ=1 we get

$$\widetilde{\beta}(0)=\frac{1}{2}\left(\widetilde{\beta}_1+\widetilde{\beta}_2\right),\widetilde{\beta}(1)=\widetilde{\beta}_1\widetilde{\beta}_2. \qquad (33)$$

Then parameter $\widetilde{\beta}$ is accidental value with its average value $\mathrm{E}\widetilde{\beta}$:

$$\mathrm{E}\widetilde{\beta}=\int_0^1\widetilde{\beta}(\theta)g(\theta)d\theta; \qquad (34)$$

where $\widetilde{\beta}(\theta)$ – by (20).

To (30) model we set-in fixed parameter $\widetilde{\beta}$ value, which is equal to (34)

$$\mu_\tau\left(\widetilde{\beta}\right)=\mathrm{E}\tau,\ \sigma_\tau^2\left(\widetilde{\beta}\right)=\nu\tau. \qquad (35)$$

Then $\widetilde{\beta}$ is (35) or (36) equations solution

$$\widetilde{\beta}\int_0^1\frac{\theta g(\theta)}{1-\widetilde{\gamma}\theta}d\theta=\widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\gamma}s\int_0^1\theta^{s+1}g(\theta)d\theta=\mu_\tau, \qquad (36)$$

$$\widetilde{\beta}\int_0^1\frac{\eta\varphi(\eta)}{1-\widetilde{\gamma}\eta}d\eta=\widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\gamma}s\int_0^1\eta^{s+1}\varphi(\eta)d\eta=\overline{\mu}_\omega. \qquad (37)$$

Analogically for A transformation we apply density $f(\omega)$ approximation with density $f_\Sigma(\omega)$:

$$f_\Sigma(\omega)=\theta'(\omega)\cdot g[\theta(\omega)]=\frac{\widetilde{\beta}\cdot g[\theta(\omega)]}{(1-\widetilde{\gamma}\omega)^2}; \qquad (38)$$

where $\theta(\omega)=\dfrac{\widetilde{\beta}\omega}{1-\widetilde{\gamma}\omega}$, $\theta'(\omega)=\dfrac{\widetilde{\beta}}{(1-\widetilde{\gamma}\omega)^2}$.

Parameter $\widetilde{\beta}$ is (25) equation solution.

For A transformation we get:

$$\widetilde{\beta}_A=\frac{\theta\xi}{\eta\omega}=\frac{1}{\omega}\left[(1-\widetilde{\gamma}_1\omega_1)(1-\widetilde{\gamma}_2\omega_2)-\xi\right]=\widetilde{\beta}(\omega). \qquad (39)$$

Realizations

When ω=0 and ω=1, $\widetilde{\beta}(\omega)$ gets same values as (32).

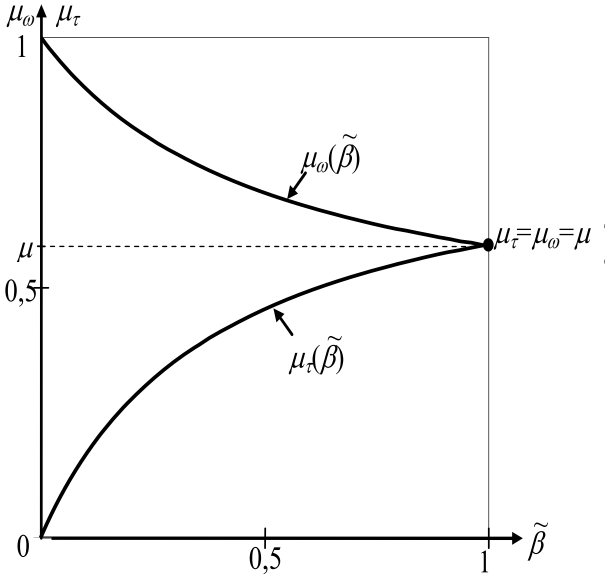

Fig. 2. Functions $\mu_\tau\left(\widetilde{\beta}\right)$ and $\mu_\omega\left(\widetilde{\beta}\right)$

Typical functions $\mu_\tau\left(\widetilde{\beta}\right)$ and $\mu_\omega\left(\widetilde{\beta}\right)$ dependencies curves are shown in fig. 2, if α_1,β_i values are fixed. Numerical IS realizations are shown in fig. 3 – 6.

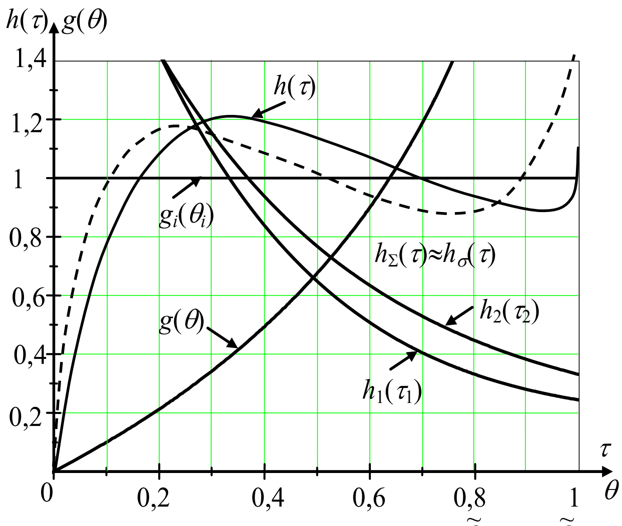

Fig. 3. Densities, ℓ=0; 1 var.: a_i=b_i=1, i=1,2; $\widetilde{\beta}_1$=1/4, $\widetilde{\beta}_2$=1/3; T, - - - - - approximated density

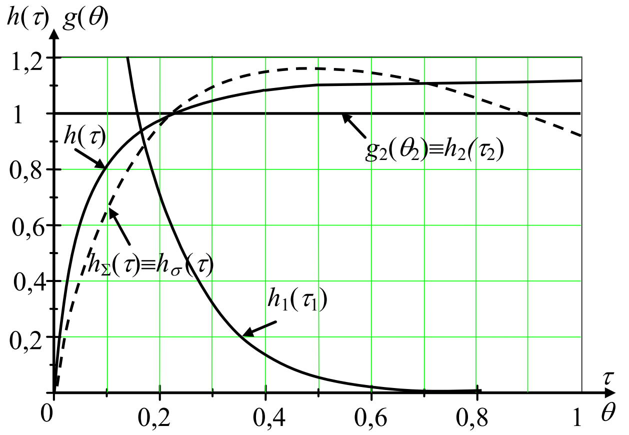

Fig. 4. Densities, ℓ=2; 2 var.: a_i=1, b_1=2, b_2=1; $\tilde{\beta}_1$ =0,1, $\tilde{\beta}_2$ =1; T

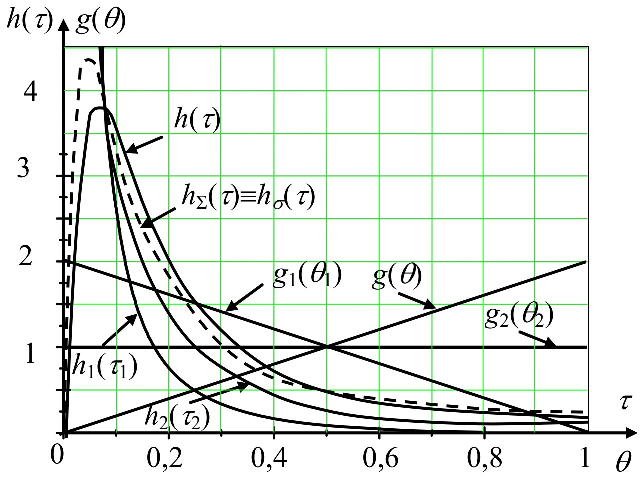

Fig. 5. Densities, ℓ=2; 2 var.: a_i=1, b_1=2, b_2=1; $\tilde{\beta}_1 = \tilde{\beta}_2$ =0,1, T

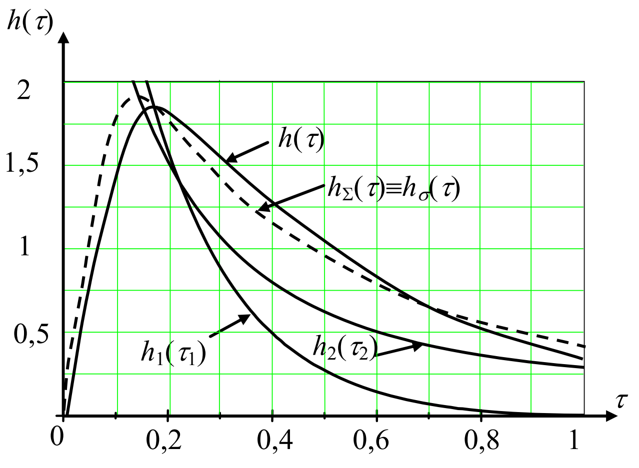

Fig. 6. Densities, ℓ=2; 2 var.: a_i=1, b_1=2, b_2=1; $\tilde{\beta}_1 = \tilde{\beta}_2$ =1/4; T

Conclusions

1. It is better to use beta densities for different parameters levels description for multiparameter measuring systems defect levels probabilities modeling, because it simplifies further modeling procedure.
2. Obtained expressions lets modeling of wanted situations in between operational-control schematics visually by defect level densities transformations (on computer display), giving densities parameters in desirable schematic place and choosing real separate parameters classification probabilities in addition with controlled parameters nomenclature.
3. Exact multiparameter expressions of IS defect levels expressions are very complicated because of multinomial integration procedure. Offered modeling variants should serve as useful instrument for control system design.

References

1. **Eidukas D., Kalnius R.** Stochastic Models of Quality Level of Mechatronic Products // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 3(83). – P. 43–48.
2. **Eidukas D., Kalnius R.** Stochastic Models of Quality in Continuous Information Systems // Proceedings of 30 International Conference on Information Technology Interfaces ITI 2008, June 23 – 26, Cavtat/Dubrovnik, Croatia. – P. 703–708.
3. **Eidukas D.** Stochastic Models Quality Electronics Systems // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 5(85). – P. 41–44.
4. **Eidukas D., Kalnius R., Vaišvila A.** Probability Distribution Transformation in Continuous Information Systems Control // ITI 2007: proceedings of the 29th international conference on Information Technology Interfaces, June 25-28, 2007, Dubrovnik, Croatia / University of Zagreb. University Computing Centre. Zagreb: University of Zagreb. – 2007. – P. 609–614.
5. **Eidukas D., Baraišis P., Valinevičius A., Žilys M.** Reliability and Efficiency of Electronics Systems. – Kaunas: Technologija, 2006. – 316 p.
6. **Kalnius R., Eidukas D.** Applications of Generalized Beta-distribution in Quality Control Models // Electronics and Electrical Engineering. – Kaunas: Technologija. – 2007. – No. 1(73). – P. 5–12.
7. **Kruopis J., Vaišvila A., Kalnius R.** Mechatronikos gaminių kokybė. Vilnius: Vilniaus universiteto leidykla. – 2005. – P. 518.
8. **Caramia M., Dell'Olmo P.** Effective Resource Management in Manufacturing Systems Optimization Algorithms for Production Planning. Springer. – 2006. – P. 216.
9. **Conte G., Moog C. H., Perdon A. M.** Algebraic Methods for Nonlinear Control Systems 2nd ed. Springer. – 2006. – P. 178.
10. **Levitin G.** The Universal Generating Function in Reliability Analysis and Optimization. Springer. – 2005. – P. 442.
11. **Kalnius R., Vaišvila A., Eidukas D.** Probability Distribution Transformation in Continuous Production Control // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 4(68). – P. 29 – 34.

Received 2009 01 13

R. Kalnius, D. Eidukas. Probability Model Quality of Informations Systems // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 4(92). – P. 13–18.

This publication is about continuous between-operational control main probabilities characteristics modeling techniques for multilevel IS, when separate independent parameters defect level probabilities distributions are set (known) in chosen control schematics place. Denied IS streams go back to production process for regeneration and classification rules, in different control levels, are similar when IS classification first and second type errors are not denied by different parameters (good is denied or bad is accepted as good). Made IS defect level probability density direct and reverse transformations models by separate parameter levels densities transformations. Offered to use approximated models instead of exact whole IS defect level probabilities density transformed models because of complicated process of integration and models get more simple expressions. Ill. 6, bibl. 11 (in English; abstracts in English, Russian and Lithuanian).

Р. Кальнюс, Д. Эйдукас. Вероятностные модели контроля качества многопараметрических изделий // Электроника и электротехника. – Каунас: Технология, 2009. – № 4(92). – С. 13–18.

Предложена методика моделирования основных вероятностных характеристик системы сплошного контроля мехатронных изделий, когда заданы (известны) вероятностные распределения уровней дефектности отдельных независимых параметров в каждой точке схемы контроля. Потоки забракованных изделий возвращаются в процесс производства для регенерации, а правила классификации изделий на отдельных ступенях контроля – одинаковы при наличии существенных ошибок классификации по отдельным параметрам (плохие изделия бракуются, а дефектные признаются годными). Получены модели прямой и обратной трансформации плотностей вероятностей уровня дефектности всего изделия на основе моделей трансформации плотностей вероятностей уровня дефектности по отдельным параметрам. Предложено взамен точных моделей трансформированных плотностей вероятностей уровня дефектности многопараметрического изделия применять достаточно точные аппроксимированные модели, для создания которых не нужна сложная процедура интегрирования, а сами модели приобретают гораздо более простые и удобные выражения. Представлены расчётные модели основных вероятностных числовых характеристик (математического ожидания и дисперсии), когда значения отдельных параметров описываются бета распределением. Ил. 6, библ. 11 (на английском языке; рефераты на английском, русском и литовском яз.).

R. Kalnius, D. Eidukas. Tikimybiniai daugiaparametrių gaminių kokybės kontrolės modeliai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 4(92). – P. 13–18.

Pateikta metodika daugiaparametrių IS ištisinės tarpoperacinės kontrolės pagrindinėms tikimybinėms charakteristikoms modeliuoti, kai atskirų nepriklausomų parametrų defektingumo lygių tikimybių skirstiniai yra pateikti (žinomi) pasirinktoje kontrolės schemos vietoje. Išbrokuotų IS srautai grąžinami į gamybos procesą regeneruoti, o klasifikavimo taisyklės atskirose kontrolės pakopose analogiškos, esant nepaneigtinoms IS klasifikavimo pirmos ir antros rūšies klaidoms pagal atskirus parametrus (geras brokuojamas arba blogas pripažįstamas geru). Pagal atskirų parametrų defektingumo lygių tankių transformacijas sudaryti IS defektingumo lygio tikimybių tankio tiesioginės ir atvirkštinės transformacijų modeliai. Pasiūlyta vietoj tikslių visos IS defektingumo lygio tikimybių tankio transformuotų modelių taikyti aproksimuotus modelius, kuriems sudaryti nereikia sudėtingų integravimo procedūrų, o patys modeliai įgauna gerokai paprastesnę išraišką. Il. 6, bibl. 11 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).