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## **Electronic Payment Systems for Communications Services**

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#### Introduction

Electronic payment systems for communication services (EPSCS) are being developed for the purpose of collecting toll and the payment for usage of technical means attributed to roads. The structure of such system is shown in Fig. 1:



Fig. 1. Horizontally integrated EPSCS

In Fig. 1  $ES_{A1}$  is the electronic vehicle identification system (ES);  $ES_{A2}$  is the ES of the collection of payment for services;  $ES_{A3}$  is the traffic routing ES;  $ES_A$  is the toplevel (centralised control) ES.  $ES_{A1}$  consists of the scanning, vehicle identification and registration systems. ES<sub>A2</sub> consists of the toll collection system, the parking fee collection system, the route fee collection system as well as the system of the collection of fees for the maintenance services. The sets of all systems shall be distributed in various locations of the taxable section of the road and shall be linked to ES<sub>A</sub> information and control instruction transmission measures. The question is: where and what ES should be installed? It is evident that this question can be answered by the specific definition of the selected section of the road and by the analysis of the traffic flow volumes in this section (its segments) [1,2].

#### Analysis of the traffic flow volumes

The traffic flow shall be assumed to be all vehicles  $(N_T)$  passing the analysed road section (i) over the

established period of time  $(t_1 \div t_2)$ . The flow volumes  $(\lambda i(t))$  shall be the number of vehicles passing road section i in time t

$$\lambda_i(t) = \frac{n_T(t,\Delta t)}{\Delta t},\tag{1}$$

where  $\Delta t$  – the observation period. This value shall be correct for the period  $t \pm \frac{\Delta t}{2}$ .

Let us assume that the diagram provided in Fig. 2 describes the selected road section.



Fig. 2. Diagram of the road section

In the diagram, the circles denote road points (1, i, i+1, K, i1 and 1N1), while ij1 and ij2 mark road branches without terminal points.  $G_{1A}$  and  $G_{1B}$  are traffic flows from/to point 1 respectively.  $G_{ijA}$  is the flow from direction ij1, which is directed towards point i+1.

$$\begin{cases} G_{ij1A}^{'} = G_{ijA1} + G_{ijB1}, \\ G_{ij1B}^{'} = G_{ij1A} + G_{ij1B}. \end{cases}$$
(2)

Other flows can be depicted equivalently. The algorithm shown in Fig. 3 can be used for the analysis of flow volumes.



Fig. 3. Algorithm of the analysis of flow volumes

The entire period of the analysis is grouped into intervals (e.g. spring, summer, autumn, winter). These periods are further grouped into smaller intervals (e.g. days) ( $\Delta$ t). Value of  $\lambda_i$  (t) of parameter  $\Lambda_i$  is obtained with the help of formula (1). Each value of  $\lambda_i$  (t) is partially random [3,4]. Therefore, the entirely of these values is characterised by distribution density  $f_{\Delta tj}(\lambda_i(t))$  in Fig. 4. Very often this density differs in byroads ( $f_{\Delta tj}(\lambda_i(t))_s$ ), roads of average importance ( $f_{\Delta tj}(\lambda_i(t))_v$ ) and main roads ( $f_{\Delta tj}(\lambda_i(t))_m$ )



Fig. 4. Distribution density of values  $\lambda_i(t)$  in j big interval of time

The average value of flow volumes during that (j) interval (e.g.  $(t_0 \div t_1)$  is  $\lambda_{ijv}$ . Afterwards, the graphs shown in Fig. 5 are drawn for the analysis of the dynamics of  $\lambda_i$ .



Fig. 5. Dynamics of the distribution density of  $\lambda_i$  values

It is evident that each road section will have different  $\lambda_i$  values as well as different distribution density and different dynamics. The comparison of average volumes in spring ( $\lambda_{ip}$ ), summer ( $\lambda_{iv}$ ), autumn ( $\lambda_{ipr}$ ) and winter ( $\lambda_{iz}$ ) allows us to draw the graph presented in Fig. 6.



Fig. 6. Dynamics of  $\lambda_i$  values

These values will determine the average flow volume per day over each big interval of time.

$$\begin{cases} G_p = \lambda_{ip} \cdot \Delta t, \\ G_{\check{z}} = \lambda_{i\check{z}} \cdot \Delta t, \end{cases}$$
(3)

where  $G_p$ ,  $G_v$ ,  $G_r$  and  $G_z$  will influence the loading of individual components of the EPSCS, the volume of information flows among systems, etc. In a way it will also influence the amount of toll (especially in the section i – (i+1) between ij1, ij2, ... ijN', where N' – the number of branches between points i and i+1 without terminal systems. Ignoring the dynamics provided in Fig. 6 and stating that all big intervals of time are of the same size, it can be assumed that the average flow volumes in the analysed road section is as follows:

$$\overline{\lambda_i} = \frac{\lambda_{ip} + \lambda_{iv} + \lambda_{ir} + \lambda_{i\tilde{z}}}{4}$$
(4)

A more accurate result will be obtained by using the following formula:

$$\overline{\lambda_i} = \frac{\int\limits_{t_0}^{t_z + t^*} \lambda_i(t) dt}{\int\limits_{t_z + t^* - t_0}^{t_z + t^* - t_0}};$$
(5)

where  $t^*$  – one half of the winter period. This analysis provides with the opportunity to establish the reliability of further solutions obtained by using values  $\lambda_{ip}, \dots \lambda_{iz}$ . E.g. if  $f_p$  ( $\lambda_i$  (t)) is the normal distribution density, then  $\lambda_{ip}$  and lesser values will be guaranteed with the probability of  $P_p$ = 0.5. Therefore, EPSCS measures suitable for such volumes may be excessively overloaded in 50% of the cases. If we select EPSCS measures and use value  $\lambda'_{ip}$ , we shall receive another value of this probability, viz.

$$P_{p}^{'} = \int_{0}^{\lambda_{ip}} f_{p}(\lambda_{i}, t) dt \approx 0.8.$$
 (6)

Other probabilities  $(P'_v, P'_r \text{ and } P'_{\check{z}})$  have equivalent dependency. The selection of the rational value  $\lambda_{ip}$  during the development of the EPSCS is a separate objective.

#### Rational dislocation of payment collection systems

In order to select a rational dislocation of EPSCS components, we can employ the algorithm provided in Fig. 7. The realisation of this algorithm starts with the formulation of goals.

Setting of a target function. Let us assume that the selected road section (length from 0 to L) is uneven in certain areas. Therefore, installation price  $C_k$  (*l*) shall be the function of road area (*l*). Operation costs (over time)  $C_E$  (*l*) will change accordingly. Therefore, the total cost of the road shall be:

$$C_{s}\left(l\right) = \frac{C_{K}\left(l\right)}{T\left(l\right)} + C_{E}\left(l\right);\tag{7}$$

where T(l) – the period of operation of the facilities in that area of the road. The intensity of operation of all road sections shall differ. It is evident that the toll  $C_M(l)$  pertaining to the sole use of that area of the road shall be a function of the area. The graphs reflecting the said interdependence are provided in Fig. 8.

The payment collection system should be structured so as to comply with the following provision:

$$C_0(l) = C_M(l) - \left(\frac{C_K(l)}{T(l)} + C_E(l)\right) \ge C_p(l), \quad (8)$$

where  $C_0(l)$  – the corporate profit, and  $C_p(l)$  – the acceptable profit level.



Fig. 7. Algorithm of the selection of the dislocation option for the system



Fig. 8. Interdependence of indicators and areas of the road section

Payment

$$C_{M}(l) = \sum_{i=1}^{Y} C_{Mi}(l) = \sum_{i=1}^{Y} C_{M1i}(l) \cdot \lambda_{i}(l); \qquad (9)$$

where  $C_{Mi}(l)$  – the lump-sum payment of toll for using the 1<sup>st</sup> road section by all vehicles of type i, Y – the number of types of vehicles,  $C_{M1i}(l)$  – the lump-sum payment of toll for using the 1<sup>st</sup> road section by one vehicle of type i,  $\lambda_i(l)$  – the type i vehicle volumes on the 1<sup>st</sup> road section. Then the target function of the selection of rational areas of EPSCS components shall be as follows:

$$\max_{l} \int_{0}^{L} \left[ \sum_{i=1}^{Y} C_{Mli}(l) \cdot \lambda_{l}(l) - \left( \frac{C_{E}(l)}{T(l)} + C_{E}(l) \right) \right] dl \Big| C_{0}(l) \ge C_{p}(l),$$
(10)

where l = 0 | L, L – the length of the road section. Or

$$\max_{l} \int_{0}^{L} \left[ \sum_{l=1}^{Y} C_{Mll}(l) \cdot \lambda_{l}(l) - \left( \frac{C_{K}(l)}{T(l)} + C_{E}(l) \right) \right] dl \Big| C_{0\Sigma} \ge C_{p\Sigma}, \quad (11)$$

where

$$C_{0\Sigma} = \int_{0}^{L} C_0(l) dl, \qquad (12)$$

where  $C_{P\Sigma}$  – the acceptable amount of profit when the entire road section is used. However, (11) target function is maybe rather too theoretical. It has to be simplified in practice as a result of large volumes of necessary information.

**Description of the road section.** In order to emphasise the key peculiarities of the road section, let us analyse the part of the main road that is not related to any particular area. This part is shown in Fig. 9.



Fig. 9. Dislocation of EPSCS components in the road section

In the illustrative diagram of the road section: S<sub>p</sub> and S<sub>g</sub> are the initial and the final electronic payment collection systems; STS<sub>i</sub> is electronic vehicle distribution system i; KS<sub>ii</sub> is the electronic traffic flow control system j attributed to system  $STS_i$ ;  $STS_{11}$  and  $S_{1N1}$  are in the jurisdiction of the  $1^{st}$  STS: the first STS and the  $N_1^{th}$  S;  $DS_1$  and  $AS_1$  is the first electronic system of the fuel station and the first electronic system of the parking lot; PS<sub>i</sub> is the electronic system of the rest area j;  $PK_{ji}$  is road i exiting from rest area j;  $K_{ij}$  is road j in the segment between  $STS_i$  and  $STS_{i+1}$ (M<sub>Ki</sub> is the number of roads in this segment); G'<sub>Aij</sub> and G"<sub>Aij</sub> are the flows of vehicles in K<sub>ij</sub> branch in both directions;  $G_{Aiji}$  and  $G_{Aij(i+1)}$  are flows of vehicles arriving from the direction of  $STS_i$  and  $STS_{i+1}$  and turning to branch  $K_{ij}$ ;  $G^*_{Aiji}$  and  $G^*_{Aij(i+1)}$  are flows that turn from  $K_{ij}$  in the direction of STS<sub>i</sub> and STS<sub>i+1</sub>; CVS is the central control system; S<sub>NS</sub> is service system N<sub>S</sub>; Letter I marks flows of information and control instructions between the systems. It is evident that

$$\begin{cases} G'_{Aij}(t) = G_{Aiji}(t) + G_{Aij(i+1)}(t), \\ G^*_{Aij}(t) = G^*_{Aiji}(t) + G^*_{Aij(i+1)}(t). \end{cases}$$
(13)

For the purpose of saving the means of communications, shortening the length of lines and integration of controls in each control level of the EPSCS, it is well worth selecting the jurisdiction diagram shown in Fig. 9. All  $\{S_i\}$ ,  $\{STS_i\}$ ,  $\{DS_i\}$ ,  $\{AS_i\}$ ,  $\{SS_i\}$  and  $\{PS_j\}$  could be in the jurisdiction of the central control system. All  $\{S_{ij}\}$ ,  $\{S_{ijv}\}$  and  $\{KS_{ij}\}$  located in the area of any STS<sub>i</sub> could be subordinated to this system. In this way it would be much easier to co-ordinate the functions of  $S_{ij}$  and  $STS_i$ , and the time period required to convey information and control instructions as well as the number of functions of the CVS would be reduced.

**Dislocation of**  $\{S_i\}$ ,  $\{STS_i\}$  and  $\{KS_i\}$ . If the traffic flow on the main roads are the greatest, e.g.

$$\begin{cases} G'_{p} >> G'_{Aij}, \\ G''_{p} >> G'_{Aij} \dots, \end{cases}$$
(14)

then  $S_p$  and  $S_g$  must be attributed to  $\{S_i\}$  set. Other  $\{S_i\}$ may be installed only in branches of the main road (e.g. A<sub>11</sub>, ... A<sub>1N1</sub>) or in separate branches of the said branches (e.g.  $A_{111}$ , ...  $A_{11N11}$ ). The results of the analysis of the flows of vehicles will determine whether any S<sub>ij</sub> is installed. When such installation is inexpedient, we shall have a branch (e.g.  $K_{ij}$ ) without  $S_{ij}$ . It is evident that if we fail to consider  $G'_{Aij}$  during the establishment of the necessity of S<sub>ii</sub>, we shall partially loose information about the dislocation of vehicles (once they leave the road section); however, we will not incur any significant economic loss. Therefore, the expedience of S<sub>ij</sub> will be determined by  $G_{Aij}^*$  flow. As STS<sub>i</sub> and STS<sub>i+1</sub> control and manage flows, vehicles of this flow will not be able to reach further than them on the main road; they could only move towards the direction of K<sub>i1</sub>, ... K<sub>iMKi</sub>. However, even in these sections it can fall under the control of {KS<sub>ii</sub>}. Even though it will not be subject to payment at KS<sub>ii</sub>, it still has to pay upon exiting the main road. It is more relevant for the monitoring of transit (main road) vehicles. Subscription (local) vehicles will use subscriber service systems.  $S_{1N1}$  will be installed on road  $A_{1N1}$  when the following condition is met:

$$\sum_{i=1}^{Y} C_{M1i} \cdot \lambda_{i1N1} \ge \frac{C_{K1N1}}{T_{1N1}} + C_{E1N1} - C_{p1N1}; \qquad (15)$$

where  $C_{M1i}$  and  $\lambda_{i1N1}$  is the toll paid in relation to one vehicle of type i for using the road and the flow volumes of that type of vehicles on road  $A_{1N1}$ ;  $C_{K1N1}$  is the cost for

equipping road  $A_{1N1}$ ;  $T_{1N1}$  is the duration of the operation of road facilities;  $C_{E1N1}$  is the expenditure for the operation of road  $A_{1N1}$  per time unit;  $C_{p1N1}$  is the acceptable profit rate during the operation of this road. The sites for the installation of other  $\{S_i\}$  and  $\{S_{ij}\}$  shall be established accordingly.  $\{STS_i\}$  shall be installed in the principal branching sites of the road. They can collect payments, control flows (individual vehicles) and control their movement (position them). The expedience of the installation of  $\{STS_i\}$  is determined by their efficiency (profit). Therefore, it is well-worth installing them in sites meeting the following condition:

$$\sum_{i=1}^{Y} C'_{M1i} \cdot \lambda_{iSTSj} \ge \frac{C_{KSTSj}}{T_{STSj}} + C_{ESTSj} - C_{pSTSj} ; \qquad (16)$$

where  $\lambda_{iSTSj}$  – the traffic volumes of vehicles driving though  $j^{th}$  STS;  $C_{KSTSj},\ T_{STSj}$  and  $C_{ESTSj}$  – the expenses related to the installation of their STS, the duration of operation and maintenance costs per time unit; C<sub>pSTSi</sub> equivalent to C<sub>p1N1</sub>; C'<sub>M1i</sub> - the additional increase of payments received after the installation of STS<sub>i</sub>. It must be noted that the majority of these solutions are interrelated. Therefore, the optimal solution may be obtained by performing the systematic assessment of the efficiency of the EPSCS and by analysing all potential combinations for the dislocation of  $\{S_i\}$ ,  $\{STS_i\}$  and  $\{KS_i\}$ . However, in this case the systematic module of the efficiency of the EPSCS is a must.  $\{KS_i\}$  may be installed only in road segments with the sufficient number of roads (branches) of type  $\{K_{ii}\}$ . The diagram provided in Fig. 10 may be used in order to solve this problem.



Fig. 10. Diagram of flows in branches

In order to simplify the marking, let us temporarily mark all flows in between  $STS_i$  and  $STS_{i+1}$  by numbers, and sites of the branching of the roads from the main road - by letters (L<sub>0</sub>, L<sub>1</sub>, L<sub>2</sub>, L<sub>3</sub> and L<sub>4</sub>). Then the overall volumes of uncontrolled vehicle flows in the main road segment L<sub>0</sub> to L<sub>4</sub> will be depicted by the chart in Fig. 11.

In Fig. 11 the heights of the rectangles show the flow volumes. If we assume that the volumes of all flows are more or less the same, Fig. 11 demonstrates that the accumulated flow volumes in segment  $L_2 - L_3$  are higher than in any other segment.



Fig. 11. Flow volumes in main road sections

Therefore, we can already predict that the site of  $KS_{ij}$ on the main road most probably will be B-B. In order to make sure of the expedience, the driving distances of vehicles of these flows on the main road must be considered first. Flow  $G_3$  will only drive the segment from  $L_1$  to  $L_0$ , while flow  $G_1$  – the segment from  $L_3$  to  $L_0$ . If one vehicle of type i pays  $C_{1li}$  per road length unit, then the income generated by flow  $G_1$  per time unit should be as follows:

$$C_{1} = \sum_{i=1}^{Y} C_{1ii} \cdot \lambda_{1i} \cdot (L_{3} - L_{0}); \quad (17)$$

where  $\lambda_{1i}$  – the vehicle volumes of flow i. Total income generated by all uncontrolled flows shall be

$$C_{GB} = C_{G1} + C_{G6} + C_{G7} + C_{G10} + C_{G11} + C_{G13} + C_{G14} + C_{G15} + C_{G18} + C_{G20};$$
(18)

where  $C_{Gi}$  – the income generated by uncontrolled vehicle flow i. Then the loss (in the absence of  $KS_{ij}$ ) will be

$$\Delta C_{\rm GB} = C_{\rm GB} - C_{i} - C_{i+1}; \eqno(19)$$

where  $C_i + C_{i+1}$  – the amounts paid by the owners of vehicles of these flows upon arriving at  $STS_i$  and  $STS_{i+1}$ .

If

$$\Delta C_{GB} - C_p^* > \frac{C_{KS}}{T_{KS}} + C_{EKS}; \qquad (20)$$

where  $C_p^*$  – the acceptable profit level during the development of KS;  $T_{KS}$  – the installation price and the duration of operation of KS;  $C_{EKS}$  – the KS maintenance expenditure per time unit, then it is expedient to install KS<sub>ij</sub> in site B-B of the main road. This selected option of {KS<sub>ij</sub>} only considers flows created by permanent subscribers of the EPSCS, as KS<sub>ij</sub> will register vehicles and convey

information to  $STS_i$  system, which in its turn - to CVS, and respective amounts will be deducted from accounts of these subscribers. Transit and other non-subscription vehicles not subject to the control of  $STS_i$  and  $STS_{i+1}$  shall remain indebted to this system. If the methods of the selection of the site for the installation of  $\{KS_{ij}\}$  are employed and the number of these systems is increased, the loss can be reduced.

#### Conclusions

In order to ensure higher efficiency of the EPSCS, the dislocation of its ES must be optimal and they must be integrated by forming the integrated territorial network.

The rational dislocation of the ES and their components can be obtained by simulating traffic flows and by selecting the economic target function.

When selecting rational parameters of individual ES, due consideration must be given to the dynamics and distribution of traffic flow volumes

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The structure of the electronic payment systems for communications services is presented. The analysis of traffic flow volumes and the algorithm of flow volumes. The distribution density of values in the big interval of time as well as the dynamics of densities and the dynamics of values are provided. The rational dislocation of payment collection systems is presented. The target function is developed, and the road section is described. Ill. 11, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

# П. Балайшис, Д. Эйдукас, Л. Гочелькене. Электронная система налогообложения услуг транспортных связей // Электроника и электротехника. – Каунас: Технология, 2009. – № 3(91). – С. 3–8.

Представлена структура электронной системы налогообложения транспортных услуг. Предложены принципы и алгоритмы исследования интенсивности потоков транспортаю При выборе рационального решения применяются экономические критериию Система охватывает не только сбор налогов за поезд, но и монитоинг расчетов за другие услуги на данном участке дороги. Ил. 11, библ. 5 (на английском языке; рефераты на английском, русском и литовском яз.).

# P. Balaišis, D. Eidukas, L. Gočelkienė. Elektroninė susisiekimo paslaugų apmokestinimo sistema // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 3(91). – P. 3–8.

Pateikta elektroninės susisiekimo paslaugų apmokestinimo sistemos struktūra. Pasiūlyti: transporto srautų intensyvumų tyrimo principai ir algoritmas. Parenkant racionalų sprendimą, naudojami ekonominiai kriterijai. Sistema apima ne tik mokesčių už pravažiavimą surinkimą, bet ir atsiskaitymų už kitas paslaugas tame kelio ruože monitoringą. Il. 11, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).