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Vibratory manipulation of elastically unconstrained part on a horizontal plane

B. Bakšys*, K. Ramanauskytė, A. B. Povilionis*****
**Kaunas University of Technology, Kęstučio 27, 44312 Kaunas, Lithuania, E-mail: Bronius.Baksys@ktu.lt*
***Kaunas University of Technology, Kęstučio 27, 44312 Kaunas, Lithuania, E-mail: Kristina.Ramanauskyte@ktu.lt*
****Kaunas University of Technology, Kęstučio 27, 44312 Kaunas, Lithuania, E-mail: Algis.Povilionis@ktu.lt*

1. Introduction

Assembling is a final stage of manufacturing instruments, machines and other more complex products, which takes from 25 to 50% of product manufacturing time, and its expenditure is up to 50% of the product price. With an increase in the batches of small products and in their variety, the scope of assembly processes and manipulating operations related to them also increase. For this reason more flexible manufacture and assembly systems are needed. For smooth matching, the parts have to be oriented and their connective surfaces have to be matched, thus determining the effectiveness of automated mechanisms. Different scientific papers analyze the processes of the parts manipulation by means of various manipulators and robots whose operation is mostly based on the active methods of matching connective surfaces of the parts. These methods include the use of various sensors, vision systems, control algorithms, feedback systems. All of these devices are expensive and complex.

For assembly automation it is necessary to accomplish operations of parts feeding into assembly position, orienting and positioning, matching, joining and removing assembled units from the working area. Matching of connective surfaces is the main stage of automated assembly during which the parts are matched in the way they can be assembled without difficulties. In recent years the manipulation of the parts and assembly automation are performed using vibrations. The method of vibratory search can be used for connective surfaces matching of the parts. During the search one part must move along certain trajectories in respect to the other part by the plane, which is perpendicular to the connection axis. The centre of the connective part must fall into the zone of allowable error, which is defined by the clearance between the assembled parts, the size of the chamfers and the axial tilt angle.

The horizontal motion of parts on the horizontally excited plane was experimentally investigated by D.S. Reznik [1,2], W.Y.Du [3]. The basic problem is to compute a suitable closed motion of the plane, which creates desirable frictional forces under each part. An important contribution was to show that a sequence of plane rotations about a know set of centres is the desired closed motion. Sensorless manipulations of parts on the vertically vibrating plane were analyzed by Bohringer K.-F. [4], and the parts on the horizontally and vertically excited and segmented plane were examined by Frei P [5,6]. A. Federavičius [7, 8] investigated transportation of a body on a vibrating plane when the effective friction coefficient is controlled. Paper [9] shows, those parts can be manipulated on a plane, witch is excited in two perpendicular directions.

Vibratory transportation of the parts occurs when the plane is excited along the transportation direction. For assembly automation it is necessary to feed the parts into predetermined position and then match the connective surfaces of the parts. During the positioning, the slip motion of the parts should be pointed along the various directions. It can be done by exciting the plane in two directions by equal frequencies under the particular initial phase of the excitation signal. Connective surfaces of the parts are matched when being positioned part accomplishes the search motion near the positioning point. Therefore, it is important to investigate part's motion regimes during the positioning and search, aiming to identify the excitation parameters that ensure the most effective matching of connective surfaces.

In this study the motion of a cylindrical part on a horizontally vibrating plane has been theoretically and experimentally investigated taking into account dynamic processes, and in addition the motion regimes most suitable to manipulation of the parts being automatically assembled have been determined.

2. Equations of part movement

The plane is excited in two perpendicular directions. By changing the excitation amplitude, frequency and phase of the excitation signals, the part can be easily and quickly redirected and provided with search motion of different trajectories, and this way the matching of connective surfaces is possible.

Investigated here is the manipulation of a body on a vibrating plane. Let us assume that $\xi O\eta$ is an immovable coordinate system located in a horizontal plane (Fig. 1). In the same plane a coordinate system xO_1y related to the vibrating plane is located. The coordinate axes x and y are parallel to the axes ξ and η respectively. Let us suppose that the plane motion in $\xi O\eta$ coordinate system is uniform and runs in such a way that every point traces a circle of radius R_e. Thus motion of any point of the plane is determined by the equations:

$$\xi = \xi_0 + R_e cos\omega t; \quad \eta = \eta_0 + R_e sin\omega t \tag{1}$$

where ω is the frequency of harmonic motion, t is time.

The body on a vibrating plane is presented as material particle of mass m, influenced by the inertial force and constant friction force $F=\mu mg$, which has the direction opposite to the relative velocity. Differential equations of the part motion on a vibrating plane are

$$ma_\xi = F_\xi.\ ma_\eta = F_\eta \tag{2}$$

where a_ξ and a_η are the projections of absolute acceleration of the part onto the ξ and η axes.

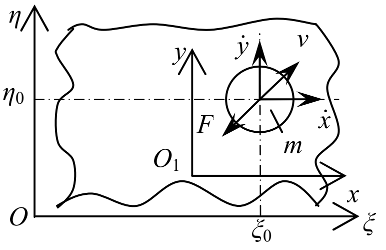


Fig 1 Graphical scheme of the part motion on vibrating plane, where v is relative velocity of the part; F is friction force

When the body slides with respect to the plane $x=x(t)$, $y=y(t)$ and acceleration projections of gravity center of the part are expressed by two components

$$\left.\begin{aligned} a_\xi &= \ddot{x} - \omega^2 R_e \cos\omega t \\ a_\eta &= \ddot{y} - \omega^2 R_e \sin\omega t \end{aligned}\right\} \tag{3}$$

The first component determines relative acceleration of the part, and the second component –acceleration of translation.

Friction force depends on the angle between the relative velocity of the part and projections of the velocity onto the x and y axes. Projections of the relative velocity of the part are denoted by $\dot{x}$ and $\dot{y}$. Then the projections of friction force onto ξ and η axes (also onto the x, y axes) are

$$\left.\begin{aligned} F_\xi &= -\mu mg cos\left(v, \dot{x}\right) = -\mu mg \frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} \\ F_\eta &= -\mu mg sin\left(v, \dot{y}\right) = -\mu mg \frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} \end{aligned}\right\} \tag{4}$$

where μ is coefficient of dry friction between the surfaces of the body and the plane.

Substituting the expressions (3) and (4) into the body motion equations (2) the following is obtained

$$\left.\begin{aligned} \ddot{x} + \mu g \frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} &= \omega^2 R_e cos\, \omega t \\ \ddot{y} + \mu g \frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} &= \omega^2 R_e sin\, \omega t \end{aligned}\right\} \tag{5}$$

These equations are valid as the body slides on the plane, i.e. as $\dot{x}^2 + \dot{y}^2 \neq 0$.

It is necessary to determine such a trajectory of the body motion, which results the connective surfaces search time to be the smallest.

3. Simulation of the part motion on the circularly vibrating plane

To solve Equations of motion (5), MATLAB software was applied and the calculation code was written. By the results of mathematical simulation it was determined that the part moving from the initial point towards the positioning point has characteristic transient and steady regimes of motion (Fig. 2). The character of the trajectory for transient regime depends both on the excitation amplitude and frequency as well as on the coefficient of friction and initial velocity. The part can move to a predetermined position along a circular looping trajectory, or along a curvilinear or linear trajectory. Such a transient motion of the part may be used to accomplish part positioning towards the other mating part. As the plane vibrates by a circular trajectory, the trajectory of steady motion of the part is also circular. The examples of part's center motion trajectories along x and y directions are shown in Fig. 3.

It is possible to control the direction of motion by changing the initial phase of excitation signals [8]. Steady motion of the part may be used in search of the connective surfaces of the parts.

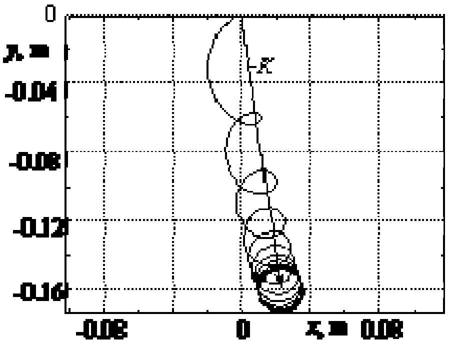


Fig. 2 Trajectory of transient and steady motion regimes of the part on vibrating plane, as μ=0.1, R_e=0.015 m, ω=30 s^{-1}

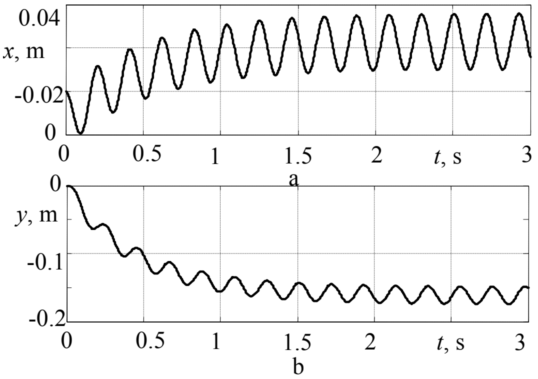


Fig. 3 Characteristic vibrations of the part; a – along x axis direction, b – along y axis direction

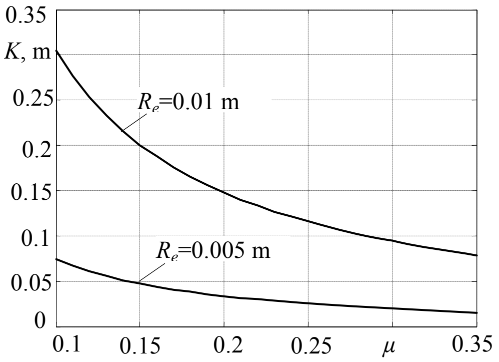


Fig. 4 Dependence of the part displacement K on friction coefficient μ and excitation frequency R_e, as ω=40 s^{-1}

Under a significant positioning error of the connective surfaces, i.e. when axial misalignment between the axes of the peg and the bushing is larger than the diameter of the circular search trajectory, it is necessary to position the surfaces prior to matching. Having known the misalignment between the assembling parts, it is necessary to choose magnitudes of the parameters so, that predefined positioning of the parts to is ensured. As it is seen in Fig. 4, increasing the friction coefficient μ, the displacement K of the part from the initial point to the centre of search motion rapidly decreases, and when friction coefficient exceeds 0.1, the displacement decreases more slowly. Increasing both the excitation frequency ω and amplitude R_e, part's displacement K increases (Fig. 5).

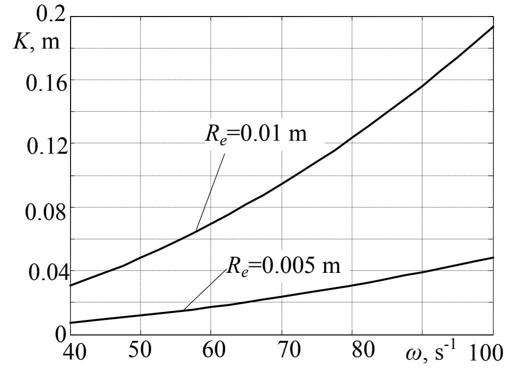


Fig. 5 Dependence of the part displacement K on excitation frequency ω and amplitude R_e, as μ=0.1

4. Simulation of motion of a part on a plane vibrating along elliptical trajectory

As excitation of the plane in one direction has higher amplitude than in the other direction, in the system of coordinates' $\xi O \eta$ every point of the plane traces an ellipse. Then the motion of any point of the plane is determined by the equations

$$\left.\begin{aligned}\xi &= \xi_0 + A_e \cos \omega t\\ \eta &= \eta_0 + B_e \sin \omega t\end{aligned}\right\} \qquad (6)$$

where A_e is the length of major axis of the ellipse, B_e is the length of minor axis of the ellipse

Motion of the part on the plane vibrating along elliptical trajectory is expressed by the equations:

$$\left.\begin{aligned}\ddot{x} + \mu g \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} &= \omega^2 A_e \cos \omega t\\ \ddot{y} + \mu g \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} &= \omega^2 B_e \sin \omega t\end{aligned}\right\} \qquad (7)$$

As the plane is subjected to elliptical vibrations, the trajectory of the steady state motion regime of the part is also elliptical (Fig. 6). The character of the transient motion regime, which describes movement of the part, while the plane moves along elliptical or circular trajectory, practically does not differ.

The performed investigation showed, that under the same friction coefficient and excitation frequency, as the plane is excited by the elliptic trajectory, the displacement of the part from the initial point to the centre of the steady search motion is bigger than that under circular excitation of the plane. As the part is moving along steady elliptic trajectory, the conditions for matching of connective surfaces are more advantageous. If the axis of the part will cross the zone of the allowable error, depends on the position of the trajectory centre relative to the mentioned zone, the radius of the circle or on the length of the ellipsis axes. When the minor axis of the ellipse is smaller than the diameter of allowable error zone, the center of the part will cross the mentioned zone.

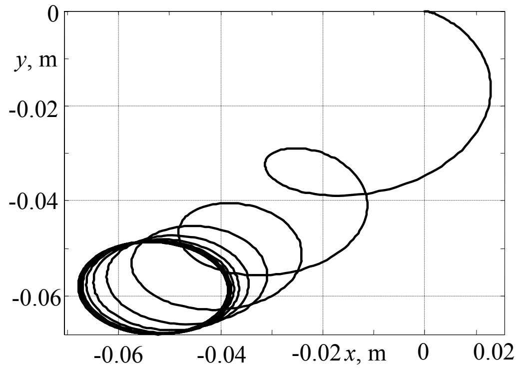
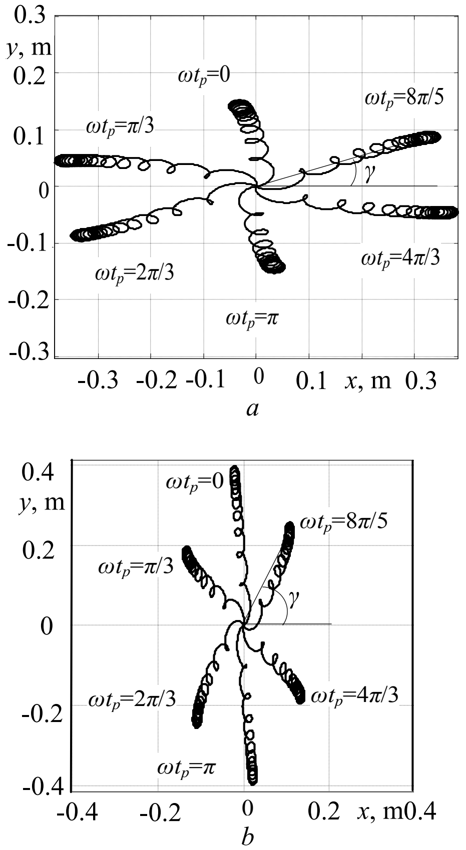


Fig. 6 Motion trajectory of the part on an elliptically moving plane, when ω=26 s^{-1}, μ=0.08, A_e=0.015 m, B_e=0.01 m


a

b

Fig. 7 Motion direction of the part versus the initial phase of the excitation signals, as μ=0.2, ω=35 s^{-1}: a - A_e=0.01 m, B_e=0.015 m, b - A_e=0.015 m, B_e=0.01 m

It is possible to control movement direction of the part the same way as if the plane is excided circularly [8], i.e. by varying the initial phase of the excitation signals (Fig. 7). Under the same initial phase of the excitation signal, but with different excitation amplitudes along the x

and y directions, the results change both in direction angle of the part motion and in covered distance towards the centre of the steady trajectory.

The direction angle γ of the part movement on the plane follows a complex law, which is represented as a sum of the sine curve and linear function (dashed line) (Fig. 8). The larger is the difference between the amplitudes A_e and B_e. of the plane excitation, the larger is amplitude of the sine. The sine graph of the direction angle change, when the amplitude of excitation along the x direction is smaller than that along the y direction ($A_e<B_e$), it is shifted by a half period of the sine.

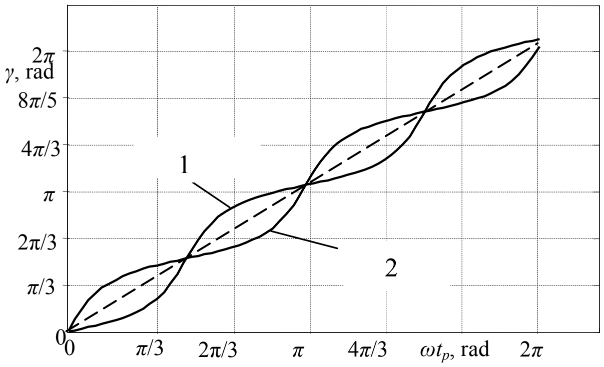


Fig. 8 Direction angle γ of the part motion versus the initial phases ωt_p of the excitation signals: 1 - A_e=0.015 m, B_e=0.01 m, 2 - A_e=0.01 m, B_e=0.015 m

5. Experimental investigation of unconstrained part motion on the vibrating plane

Experimental setup and the method investigation. To verify the results of theoretical the experiments of the unconstrained part motion on vibrating plate analysis were carried out. The experimental setup was designed and made (Fig. 9). Vibrating plate *1* is mounted on four elastic rods *2*. To the bottom surface of the plate DC motor *3* with eccentric mass *4* is attached. The base *5* of the setup is immovably fixed to the floor. DC motor *3* is connected to the voltage adjuster *6*, which provides the possibility to change the frequency of rotation. Eccentric mass *4* was used to control vibration amplitude of the plate *1*. Elastic rods *2* were used to ensure horizontal motion of the plate relative to the base. Part *7* was placed on the vibrating plate *1*. Immovably fixed to the plate reflective spheral marker *8* was used to capture motion of the plate, while motion of the part on the vibrating plate was tracked by means of the reflective marker *9*, attached to the part. Four ProReflex MCU 500Hz cameras were used to track motion of the part. 3D Qualisys Track Manager (QTM) system was used for motion analysis.

Infrared (IR) light diodes, mounted around the cameras' lens, emit the IR light, pointed towards the markers. IR light hits the reflective markers and goes back to the camera lens, and so motion trajectories of the markers are fixed. The center point of the spherical marker is calculated in real time by sub-pixel interpolation algorithm. The obtained data is momentarily transferred to computer, where QTM system processes the data from all four cameras and performs automated recognition of the markers. Then computer monitor reproduces 3D real time trace of each marker. Besides, it is possible to make the 3D or motion along the x, y and z axes graphs of each marker. Obtained by QTM system data is stored in text file. By means of Microsoft Excel software motion trajectories of the plate and the part along the x-y plane were made.

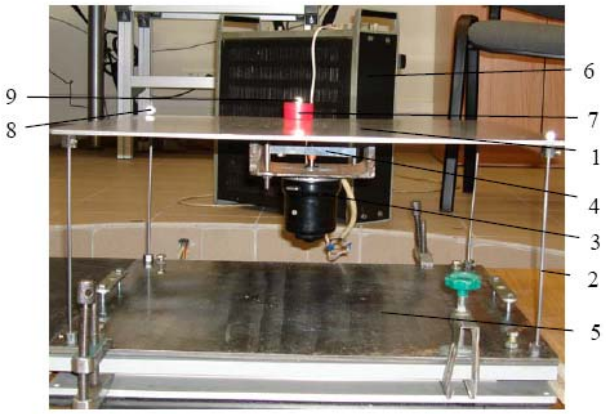
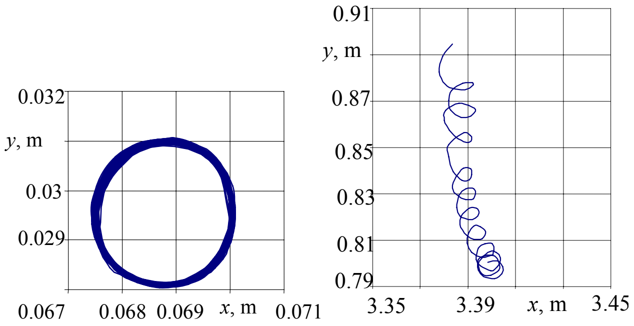


Fig. 9 Experimental setup: *1* - vibrating plate, *2* - rods, *3* - electrical motor, *4* - eccentric mass, *5* - base, *6* - voltage adjuster, *7* - part, *8*, *9* - markers

Results of the experiments. It was determined during the experiment, that providing small amplitude A_e=0.012 m and frequency ω=30 s^{-1} excitation to the plane, it moves along the circular trajectory (Fig. 10, a). The part placed on the aluminium alloy plate, starts transient motion, in the direction, which depends on the part placement moment in respect of the vibration period of the plane, later motion trajectory gets steady (Fig. 10, b). Obtained during the experiments, motion trajectories of the part positioning and search are very similar to those obtained during the simulation (Fig. 3). It was determined, that because of irregular roughness of different parts of the plate, the loops of the transitional motion of the parts are of different magnitudes and steady motion is not quite exactly circular. The performed experiment showed, that under existing surface contact (except of the point contact) of the part and the plate, because of acting friction forces, the part also performs rotation along its own axis, which was neglected during the simulation.



a b

Fig. 10 Trajectories of motion: a - plate trajectory, as A_e=0.015 m, ω=30 s^{-1}; b - trajectory of the part, when $\mu\approx0.08$

Exciting the plane by higher amplitude A_e=0.012 m and frequency ω=30 s^{-1}, due to unequal rigidity of the rods along the x and y directions and varying centrifugal inertia force, the elliptic trajectory of the plane is obtained (Fig. 11, a), as at each rotation an ellipse is drawn not exactly around the same point. As the plate moves along the elliptic trajectory, the obtained steady motion trajectory of the part is also elliptic (Fig. 11, b), which was also defined during the simulation (Fig. 6). Furthermore, steady motion of the part is not quite stable, i.e. the centre of the steady motion is slightly varying. This is because of non-ideal conditions of the plate excitation during the experiment and due to irregular roughness of the plate surface.

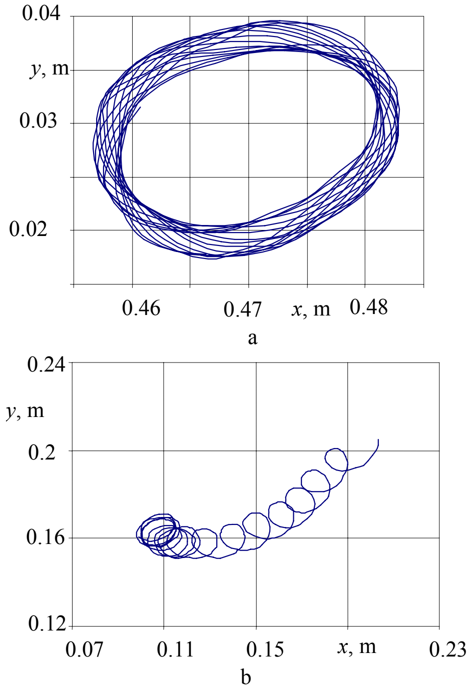


Fig. 11 Motion trajectories: a - the trajectory of the plate, when A_e=0.015 m, B_e=0.01 m, ω=26 s^{-1}; b - the trajectory of the part, when $\mu\approx0.08$

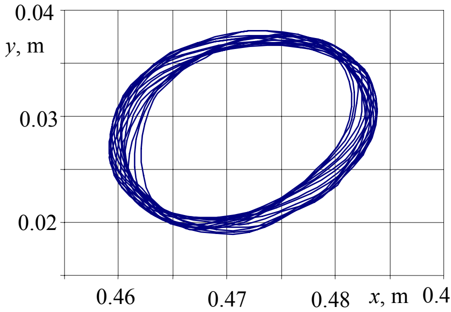


Fig. 12 Motion trajectory of the plate, as A_e=0.125 m, B_e=0.09 m, ω=52 s^{-1}

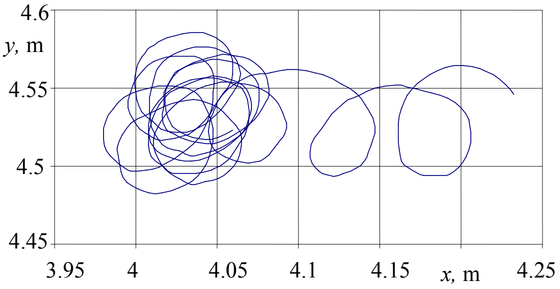


Fig. 13 Motion trajectory of the plastic part, as A_e= = 0.0125 m, B_e=0.009 m, ω=52s^{-1}, $\mu\approx0.08$

Motion trajectories of the parts, made of the different materials, consequently under different friction coefficients between the plane and the part, are written. One of the parts is made of plastic, the other – of steel. The parts were placed on the plane vibrating by the elliptical trajectory (Fig. 12). The obtained motion trajectories of the parts are shown in Figs. 13-14. It was determined during the experiments that as friction coefficient between the steel part and the plane is bigger, the displacement of the part from initial point is smaller. This verifies the correctness of the simulation results.

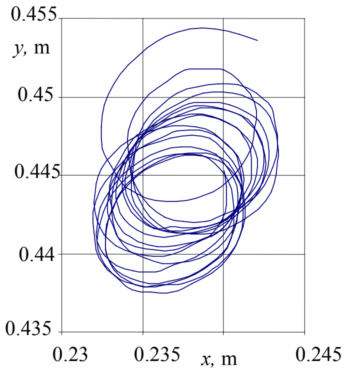


Fig. 14 Motion trajectory of the steel part, as A_e=0.0125 m, B_e=0.009 m, ω=52s^{-1}, $\mu\approx0.14$

Performing the experiments, it was observed, that close to the edge of the plane small amplitude vibrations along the vertical direction are excited. Therefore, close to the edge of the plane, the part changes motion direction. To avoid such change in direction, it is necessary to increase the rigidity of the plane.

6. Conclusions

1. Research was carried out considering the unconstraint part motion on a plane, which is horizontally excited in two perpendicular directions. It was determined that the part on the plane can be positioned to a predefined point and provided with a search motion along circular and elliptic trajectories.

2. Displacement of the part from the initial position to the centre of search trajectory mainly depends on friction coefficient μ and decreases increasing the friction, whereas increasing the excitation frequency ω and the amplitude R_e, displacement increases.

3. Experimental results showed that motion trajectories of the part positioning and search are very similar to simulation results. It was determined, that because of irregular roughness of the different parts of the plate, part's transitional motion loops are of different magnitude and steady-motion is not quite exactly circular or elliptic.

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B. Bakšys, K. Ramanauskytė, A. B. Povilionis

NESUVARŽYTOS DETALĖS VIBRACINIS MANIPULIAVIMAS ANT HORIZONTALIOSIOS PLOKŠTUMOS

Reziumė

Straipsnyje teoriškai ir eksperimentiškai nagrinėjamas detalės judėjimas ant horizontaliai dviem statmenomis kryptimis žadinamos plokštumos. Sudaryti vibracinio slinkimo matematiniai modeliai, kai plokštuma juda apskritimu ir elipse. Modeliuojant ištirtos detalės judesio trajektorijos pozicionavimo ir paieškos metu, nustatytos detalės poslinkio nuo pradinės padėties iki nusistovėjusios trajektorijos centro priklausomybės nuo sistemos ir žadinimo parametrų. Detalė pozicionuojama esant pereinamajam judesio režimui, o paieškai būdingas nusistovėjęs judesio režimas. Eksperimentiškai patvirtinta, kad ant apskritimu ar elipse judančios plokštumos detalę galima nukreipti į nustatytą tašką, o paskui apie šį tašką ji gali atlikti apskritiminės ar elipsinės trajektorijos paieškos judesį.

B. Bakšys, K. Ramanauskytė, A. B. Povilionis

VIBRATORY MANIPULATION OF ELASTICALLY UNCONSTRAINED PART ON A HORIZONTAL PLANE

Summary

Theoretical and experimental investigation of the part motion on an excited in two perpendicular directions horizontal plane is analyzed in presented paper. The mathematical models of vibratory displacement, as plane moves along circular or elliptical trajectories, were made. By the simulation and experiments, motion trajectories of the parts during positioning and search were investigated. The dependencies of the part displacement, from the initial point towards the centre of steady search, on the parameters of the systems and excitation were defined. The part is positioned during the transient motion regime, while the search occurs during the steady motion regime. It was verified experimentally, that the part on the circularly or elliptically vibrating plane may be directed to predefined point and later is able to perform search motion by circular or elliptical trajectory.

Б. Бакшис, К. Раманауските, А. Б. Повилионис

ВИБРАЦИОННОЕ МАНИПУЛИРОВАНИЕ УПРУГИМИ СВЯЗЯМИ НЕОГРАНИЧЕННОЙ ДЕТАЛИ НА ГОРИЗОНТАЛЬНОЙ ПЛОСКОСТИ

Резюме

В статье теоретически и экспериментальным путем рассматривается движение детали на горизонтальной плоскости, возбуждаемой в двух перпендикулярных направлениях. Составлены математические модели вибрационного скольжения детали при движении плоскости по круговой и эллиптической траектории. Моделированием изучены траектории перемещения детали и во время позиционирования и абтопоиска, определены зависимости перемещения детали от начального положения до центра установившейся траектории автопоиска от параметров системы и возбуждения. Позиционирование осуществляется при переходном режиме движения, а автопоиску характерен установившийся режим. Экспериментально подтверждено, что на вибрирующей по круговой и эллиптической траектории плоскости деталь можно направить в определенную точку и потом около этой точки она может осуществлять поисковое движение по круговой или эллиптической траектории.

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