363. Dynamics and reliability of gear driver with antifriction bearings

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Abstract. The heavy duty gear driver with antifriction bearings have been modeled, simulated and tested in situ. The experimental data of rotors shafts vibration displacements were used in physical model of gear teeth meshing dynamics simulation. The modeling and simulation of rotating system and gears teeth meshing was provided by finite element method. The aims of this paper are: to protect unexpected failures of rotating system with gear power transmission running on antifriction bearings, to simulate contact stresses of gears teeth meshing versus rotor rotation axis position changes that caused by damaged bearings vibration displacements. The experimental and simulation results were implemented in industry.

Keywords: gear driver transmission, antifriction bearings, modeling and simulation, vibration, contact stresses

Introduction

The condition monitoring and failure diagnostics technology of the heavy duty gear drivers with antifriction bearings enables to avoid unexpected failures of the machines running in long term continuous operation mode. In paper [1] a computer simulation for static and dynamic behavior of mesh gear teeth is presented including aspects of parametric modeling of involutes' spur gears pairs in meshing process. In paper [2] the use of vibroacoustic method for analyzing of gear drives vibration is proposed. The method is based on the idea that early gear faults or some other irregularities of working gear drives can be determined by the analysis of emitted vibration parameters. The paper [3] concerns the dynamics following the operation of gear power transmission. The experimentally confirmed dependences are proposed for determination of the dynamic parameters of gear mesh loading. Sources [4, 5] introduce the problems of diagnostics of modern machines comprising gear drivers with antifriction bearings. Wide range vibration frequency interval caused by machines and bearing elements is divided into characteristic frequency intervals.

The aims of this paper are: to design physical and dynamic models of rotating system with gearings and to simulate dynamics of the meshing teeth versus rotor position changes and to identify the causality of involutes gears teeth damages; to monitor the gear driver rotor shaft displacement relative to the bearing housing and to evaluate indirectly the quality of teeth meshing and to prevent failures of gears.

The Object of the Research

The heavy duty gear drivers are used in sugar production diffusion machine. This machine comprises two identical (right Fig.1 and left) transmissions mechanically coupled by two driven low-speed gears z_4 providing high torque (up to 440 000 Nm). DC electric motor EE has nominal rotation speed 850-1100 rpm, gear box with speed ration 50 and heavy duty gear driver with speed ratio 26.2482. Rotation speed of rotor II is 4.474-5.67 rpm. Rotors I and II rotating in radial double row spherical roller bearings with cylindrical bore SKF 22230 CC/W33. The rotation speed of III rotor is 0.660-0.838 rpm. During the continuous long term run operation of the machine the contact surfaces of the gears teeth were damaged, as pointed in Fig. 2.

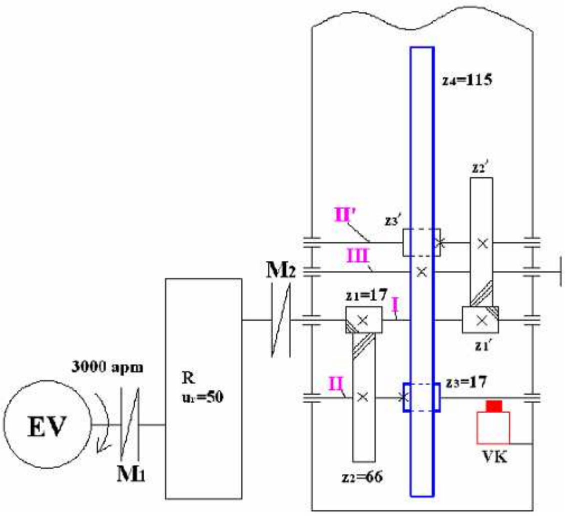


Fig. 1. The gear power transmission: *EE* – electric motor; M_1 and M_2 – elastic couplings; *VK* – shaft vibration displacement measurement sensors; *I, II*, and *III* rotors with gears

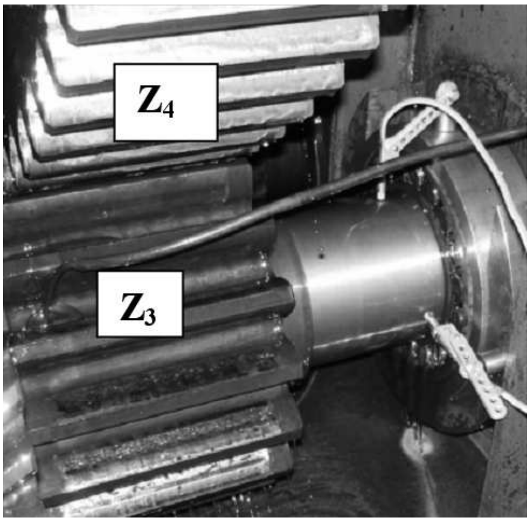


Fig. 2. The damaged teeth and the *II* rotor shaft displacement and vibration displacement measurements sensors

Testing of the Gear Driver Rotor with Roller Bearings

Experimental testing of gear driver in situ of *II* rotor shaft with pinion z_3 (Fig.1), were carried out under variable load from 80% to 110% of rated load values.

The aim of this test was to determine the radial displacement and vibration displacement of rotors *II* shaft point located near the antifriction bearing, Fig.2. Experimental research was carried out with two contact less sensors (*VK*, Fig. 1) and dynamic machine analyzer DMA 04 (Epro, Germany). The left side transmissions vibration displacement values of the driving rotor shaft *II* show that antifriction bearing support has low radial and axial stiffness's and large mechanical looseness. The vibration displacement $s_{p-p}(t)$ of II rotor of right side transmission is shown in Fig.3 and has twice low values in comparison with left side transmission rotor which was serious damaged.

The left side transmissions gear's z_4 rotor III eccentricity plot is shown in Fig 4. The radial displacement reached 133 µm values. In comparison – the right side transmissions gear's z_4 rotor III radial displacement is 60 µm values.

These tests results determined that during the operation of a pinions z_3, z_3' and gears z_4, the teeth base pitch permanently changes its position on the involute teeth surfaces.

The left side transmission rotor III with driven gear z_4 provide large eccentricity values that negatively influences pinions z_3, z_3' and gears z_4 teeth meshing; increased dynamic forces acting on teeth and finally increased contact stresses reaching inadmissible values. The experimental results were applied in the modeling to simulate contact stresses in the gears. The contact stresses versus radial displacement of driving rotor *II* with gear z_3 were simulated in static and dynamic mode.

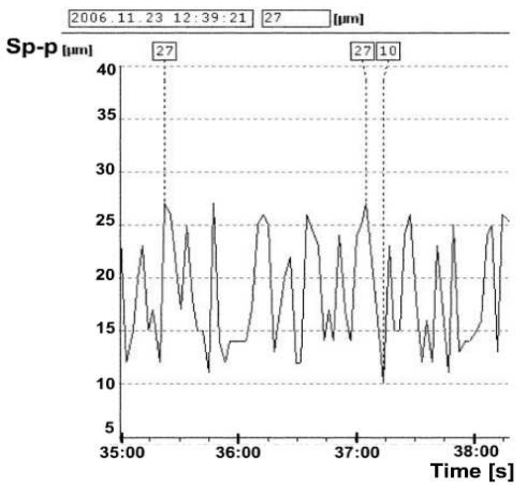


Fig. 3. The rotor II vibration displacement $s_{p-p}(t)$ plot versus time; teeth z_3-z_4 meshing frequency 1.32 Hz (4.665 rpm)

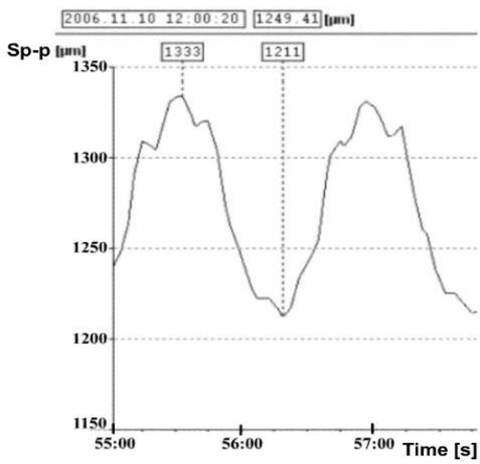


Fig. 4. The rotor III eccentricity (radial displacement) plot versus time , full rotation at 87 s

Modeling and Simulation of Rotating system

The physical and dynamic model of whole rotating system is shown in Fig. 5. Model consists of a five rotors supported by bearings, couplings and electrical engine EE. The general assumptions made are: the material of the rotors and coupling is elastic; shear forces are evaluated; the deflection of the rotor is produced by the displacement of points of the center line; the axial motion of the rotors is neglected; the semi couplings are treated as rigid, teeth of gear are deformable.

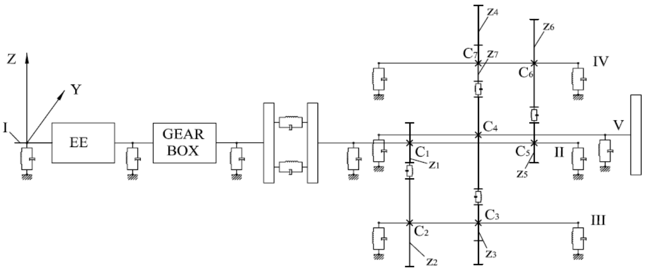


Fig. 5. The physical and dynamic model of the rotating system

The rotor dynamic is simulated by finite element method when finite element consists of two nodes and five degrees of freedom (DOF) at each node. The first and second DOF are displacements along Y and Z axes and last three DOF are rotation angles about *X*, *Y* and *Z* axes.

The $\{q_R\}$ - the nodal element displacement vector, v_R, w_R translation displacement of rotor finite element by Y and Z axes, α_R, β_R, γ_R Cardin's angles.

$$\{q_R\}=[v_{R1} \quad w_{R1} \quad \alpha_{R1} \quad \beta_{R1} \quad \gamma_{R1} \quad v_{R2} \quad w_{R2} \quad \alpha_{R2} \quad \beta_{R2} \quad \gamma_{R2}]^T \quad (1)$$

The equations of motion of the rotor finite element are derived by applying Lagrange equation of second order and written as

$$\begin{aligned}&[M_R]\{\ddot{q}_R\}+([C_R]+[G_R])\{\dot{q}_R\}+\\&+[K_R]\{q_R\}=\{F_R(t,q_{R,}\dot{q}_R q,q)\}\end{aligned} \quad (2)$$

where $[M_R]$ is the composite mass matrix of rotor finite element, $[C_R]$ is the finite element damping matrix; $[G_R]$is the finite element gyroscopic matrix; $[K_R]$ is stiffness matrix of rotor finite element; $\{F_R(t,q_{R,}\dot{q}_R q,q)\}$ is vector of external load and contact forces; $\{q\}$ is vector of gear tooth displacement in the local gear coordinate system.

The system of equations of the tooth that is concentrated in the j-th node of rotor is equal

$$\begin{aligned}&\begin{bmatrix} M_{RR} & M_{Rq} \\ M_{qR} & M_{qq} \end{bmatrix}\begin{Bmatrix} \ddot{q}_R \\ \ddot{q} \end{Bmatrix}+\begin{bmatrix} C_{RR} & 0 \\ 0 & C_{qq} \end{bmatrix}\begin{Bmatrix} \dot{q}_R \\ \dot{q} \end{Bmatrix}+\\&+\begin{bmatrix} K_{RR} & 0 \\ 0 & K_{qq} \end{bmatrix}\begin{Bmatrix} q_R \\ q \end{Bmatrix}=\begin{Bmatrix} F_{Rj}+F_{RP} \\ F_q+F_{qP} \end{Bmatrix}\end{aligned} \quad (3)$$

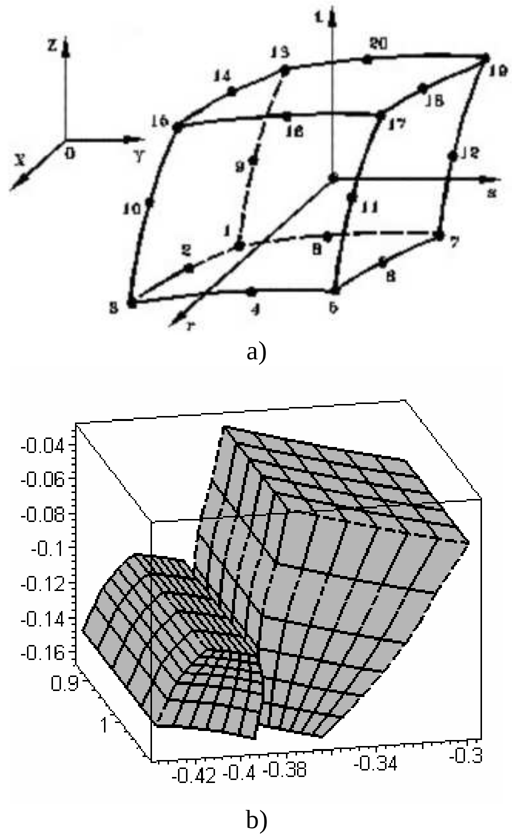
where $M_{RR}, M_{Rq}, M_{qR}, M_{qq}$ are blocks of mass matrices of rotor and tooth; C_{RR}, C_{qq} are blocks of damping matrices of rotor and tooth; K_{RR}, K_{qq} are blocks of stiffness matrices of rotor and tooth; F_{Rj}, F_q are vectors of forces of rotor and tooth; F_{RjP}, F_{qP} are vectors of contact forces of rotor and tooth. Relation between displacement of j-th node and total displacement of rotor is $\{q_{Rj}\}=[B_R]\{q_R\}$, where $[B_R]$ is relation matrix.

The gear tooth dynamic is simulated by FEM when finite element consists of twenty nodes and three degrees of freedom (DOF) at each node, Fig.6 [7].

The total system of equations of finite element of gear tooth is equal:

$$\begin{aligned}&[M_a]\{\ddot{Q}\}+[C_a]\{\dot{Q}\}+[K_a]\{Q\}=\\&=\{F(t)\}-\{F_a(Q,\dot{Q})\}+\{F_{contact}\}\end{aligned} \quad (4)$$

where $\{Q\}$ is total displacement vector of gear, $\{Q\}^T=[\{q_R\}^T,\{q\}^T]$; $[M_a],[C_a]$, $[K_a]$ are mass, damping and stiffness matrices; $\{F(t)\}$ is vector of external load (weight force); $\{F_a(Q,\dot{Q})\}$ is vector of inertia forces; $\{F_{contact}\}$ is vector of contact forces.



a)

b)

Fig. 6. Finite element of twenty nodes (a) and tooth discrete model (b)

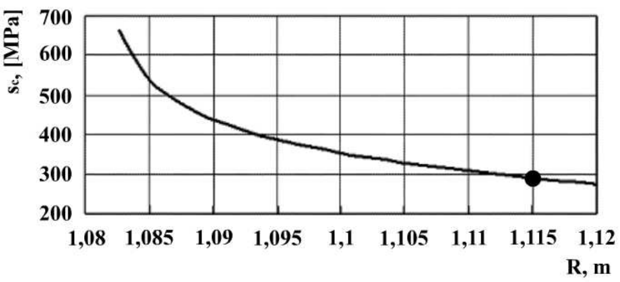


Fig. 7. The simulated maximum contact stressess s_c plot versus teeth meshing contact position (half pitch diameter R=1.115 m)

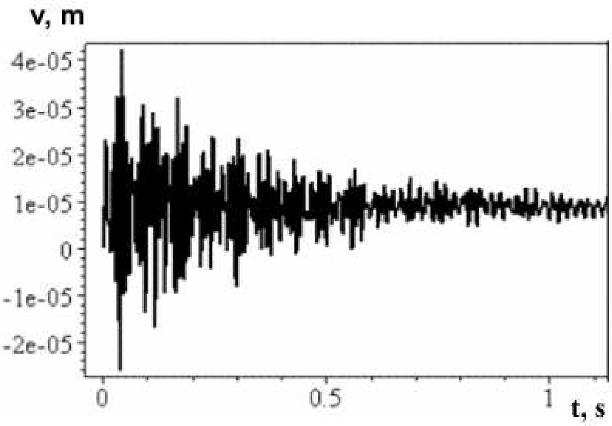


Fig. 8. The gear‘s z_4 teeth vibration displacement $s_{p\text{-}p}$ plot versus time

The simulated maximum contact stress numbers s_c plotted versus teeth meshing contact position reference to the gear z_4 rotating axis is shown in Fig. 7. In the dynamic process the maximum contact stress number at pitch point is s_c<300 MPa (half pitch diameter 1.115 m). The contact stress numbers s_c increased sufficiently when meshing point reaches dedendum distance in z_4., e.g. reaches final point of contact in z_4.

The gear‘s z_4 simulated maximum tangential displacement value at initial meshing moment reaches ~0.1 mm (Fig. 8) and generate large dynamic forces acting on meshing teeth.

Modeling of Gears Teeth Meshing and Simulation Results

Static modeling and simulation of a gear driver was carried out applying the FEM according to designed physical model and experimental testing results. The contact stresses s_c of meshing teeth surfaces was calculated. The simulation was provided applying ANSYS. After accepting the most unfavorable conditions for teeth meshing of gear z_3, z_3' and z_4, mathematical model were developed - with one pair of teeth is in contact, but changing rotors II displacement value from 25 µm up to 250 µm, as measured during experimental test (see Fig. 2).

Geometric model of a gear driver was developed using SolidWorks graphical software and ANSYS scripts, assigning for drive gear z_3 and driven gear z_4 identical real performances of materials: elasticity module $E=2{\cdot}10^{11}$ Pa and Poisson’s ratio $\mu=0,3$. Geometric model of gears teeth was divided into hexahedral 3D elements of SOLID45 as presented in Fig. 9.

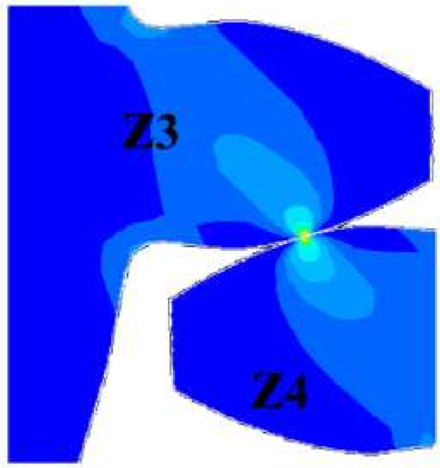


Fig. 9. The teeth meshing in FEM

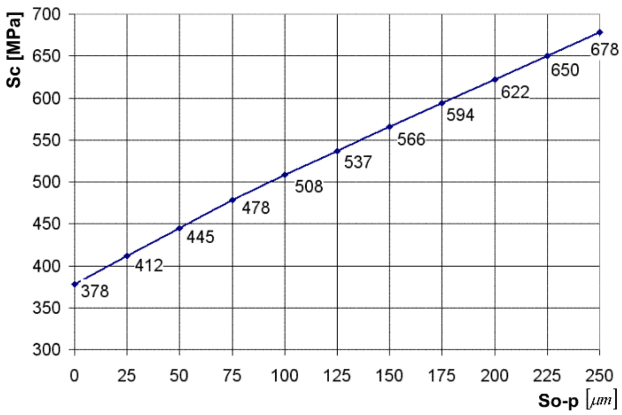


Fig. 10. The contact stresses s_c plot versus radial displacement of the II rotor

Modeling and simulation results of meshing teeth of z_3 and z_4 are shown in Fig. 10. Under ideal tooth meshing condition, when vibration displacement amplitude of the rotor *II* equals 0 µm, the maximum contact stresses at the surface of teeth pitch point is low – in pinion teeth 378 MPa. When rotor’s II radial displacement reaches 200 µm, the maximum s_c at teeth pitch point increases valuable – up to 622 MPa.

On the base of gearing design results by making several adjustments, it was accepted that allowable contact stress numbers s_{ac}=*450-500* MPa. The static model simulation results indicated - when rotor II radial displacement exceeds 100 µm, the contact stresses of the meshing teeth approached s_{ac} value. This amplitude of vibration displacement is the main reason causing damage of involute teeth.

Conclusions

1. Dynamic and mathematical model of the whole rotating system of heavy duty gear power transmission was designed and simulated maximum contact stress numbers s_c as the static pinion and gear teeth meshing model too.

2. The simulation results indicated that the maximum contact stress numbers s_c sufficiently exceeds allowable contact stress numbers. The static model of teeth meshing has limited area of application in comparison with dynamic model, but it is simple and acceptable for the first stage simulation.

3. Experimental testing and theoretical simulation results have approved idea that low frequency vibration displacement and dynamic eccentricity of pinion rotor *II* is the main parameters that describe condition of involutes teeth meshing quality and was suggested as the parameters for gearings applied diagnostics.

4. Damages of working surfaces of involutes teeth in pinion z_3 and gear z_4 were caused by the high radial displacement of rotor *II* caused by roller bearing damages or insufficient radial stiffness of bearing hausing.

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