361. Investigation of dynamics of fluid in the elements of a pipe robot

L. Ragulskis[1], K. Ragulskis[2], A. Bubulis[3], B. Spruogis[4], V. Mištinas[5], A. Matuliauskas[6]
[1]Vytautas Magnus University,
Vileikos 8-702, LT-44404 Kaunas, Lithuania,
[2, 3] Kaunas University of Technology,
Studentų 50-222, LT-51368 Kaunas, Lithuania
Phone: +37069822456; **Fax:** +37037330446
[4, 5, 6] Vilnius Gediminas Technical University,
Plytines 27, LT-10223 Vilnius, Lithuania
E-mail: [1] *l.ragulskis@if.vdu.lt* [2] *kazimieras3@hotmail.com,* [3] *algimantas.bubulis@ktu.lt*
[4] *bs@vgtu.lt,* [5] *vygantas@ti.vgtu.lt,* [6] *arvydas@ti.vgtu.lt*

(Received 19 May 2008; accepted 13 June 2008)

Abstract. The dynamics of fluid in the elements of a pipe robot is investigated in this paper. Plane vibrations of ideal compressible fluid are analyzed by taking account the constraint of non-rotation by means of the penalty method. Reduced order numerical integration of the penalty term is performed. Graphical representation of rotations at the points of reduced integration is proposed for validation of the calculated eigenmodes.

Fluid flow through different cross sections of nano-holes in metal sheets separating different compartments of a pipe robot is important in the design of a pipe robot. Calculation of the mass flow rate is necessary when performing the comparison of different cross-sections. A two-dimensional model performing the discretization of the cross-section is used for the solution of this problem.

The obtained results are used in the process of design of a pipe robot.

Keywords: pipe robot, ideal compressible fluid, eigenmode, penalty method, reduced integration, finite elements, fluid flow, cross section, mass flow rate.

Introduction

In this paper the dynamics of fluid in the elements of a pipe robot is investigated.

The plane vibrations of ideal compressible fluid are analyzed by using the displacement formulation [1, 2] and taking into account the constraint of non-rotation by means of the penalty method [3, 4]. Reduced order numerical integration of the penalty term is performed [3, 4].

Graphical representation of rotations at the points of reduced order numerical integration is proposed for validation of the calculated eigenmodes.

Fluid flow through different cross sections of nano-holes in metal sheets separating different compartments of a pipe robot is important in the design of a pipe robot. Calculation of the mass flow rate is necessary when performing the comparison of different cross-sections. Related problems of analysis of fluid flow are presented in [5, 6, 7]. A two-dimensional model performing the discretization of the cross-section is used for the solution of this problem [5].

The pipe robots of various types and their dynamical motions as well as the motions of their elements are investigated and described in [8] and other papers. This paper presents the continuation of investigations presented in the previous papers.

The obtained results are used in the process of design of the elements of a pipe robot.

Finite element model of the element of a pipe robot

The nodal variables for the analyzed two dimensional problem of vibrations of the fluid are the displacements u and v in the directions x and y of the orthogonal Cartesian system of coordinates.

It is assumed that the angular frequency of excitation coincides with the eigenfrequency of the appropriate eigenmode and thus the eigenproblem is analyzed. For the calculation of the eigenmodes the expressions for the mass and stiffness matrices are necessary.

The mass matrix of the fluid is:

$$[M] = \int [N]^T \rho [N] dxdy, \qquad (1)$$

where ρ is the density of the fluid in the status of equilibrium, $[N]$ is the matrix of the shape functions determined by the following relationship:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N]\{\delta\}, \tag{2}$$

where $\{\delta\}$ is the vector of nodal displacements. Explicitly:

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & \cdots \\ 0 & N_1 & 0 & N_2 & \cdots \end{bmatrix}, \tag{3}$$

where N_i are the shape functions of the analyzed finite element.

The stiffness matrix of the fluid is:

$$[K] = \\ = \int [B]^T \rho c^2 [B] dxdy + \int [\tilde{B}]^T \lambda [\tilde{B}] dxdy, \tag{4}$$

where c is the speed of sound in the fluid, λ is the penalty parameter for the constraint of non-rotation. The matrix $[B]$ relating the volumetric strain with the displacements is defined from:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = [B]\{\delta\}. \tag{5}$$

Explicitly:

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_2}{\partial y} & \cdots \end{bmatrix}. \tag{6}$$

The matrix [$\tilde{B}$] is used to characterize the rotation and is defined from:

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = [\tilde{B}]\{\delta\}. \tag{7}$$

Thus:

$$[\tilde{B}] = \\ = \begin{bmatrix} \frac{\partial N_1}{\partial y} & -\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & -\frac{\partial N_2}{\partial x} & \cdots \end{bmatrix}. \tag{8}$$

The calculation of the second integral of the stiffness matrix is performed using numerical integration rule of reduced order.

The rotations for the eigenmode are calculated at the points of reduced integration order of the second integral of the stiffness matrix:

$$\omega = -\frac{1}{2}[\tilde{B}]\{\delta\}. \tag{9}$$

Results of analysis of vibrations of the fluid in the element of a pipe robot

The problem in the rectangular domain is analyzed. The length of the analyzed structure is equal to twice of its dimension in the direction of the y axis. The displacements normal to the boundaries are assumed equal to zero.

As it is assumed that the angular frequency of excitation coincides with the eigenfrequency of the appropriate eigenmode and the excitation is not orthogonal to the eigenmode, the excitation is not specified explicitly and vibrations according to the eigenmode are analyzed.

The rotations for the eigenmode are calculated at the points of reduced integration order and represented in a circle around this point as a black angle from the positive direction of the x axis. This angle is equal to the rotation multiplied by a large constant. This constant is chosen for each eigenmode in order to achieve visible representation of the results.

The first eigenmode is presented in Fig. 1, the second eigenmode in Fig. 2, … , the tenth eigenmode in Fig. 10. The second and the third eigenmodes are multiple and also the eighth and the ninth eigenmodes are multiple.

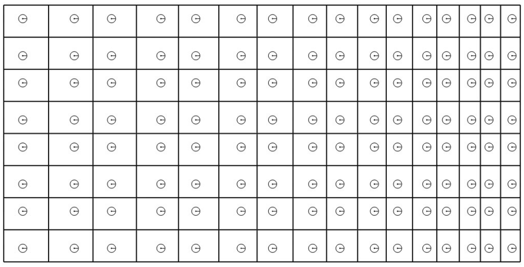

Fig. 1. The first eigenmode

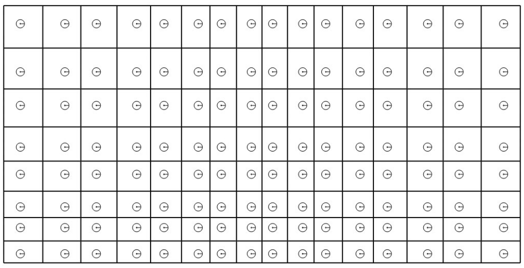

Fig. 2. The second eigenmode

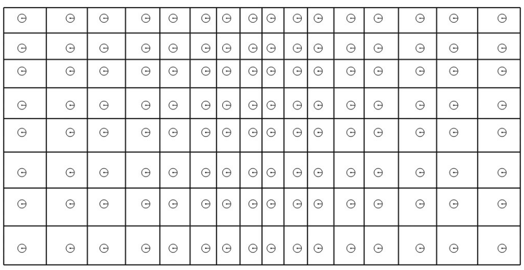

Fig. 3. The third eigenmode

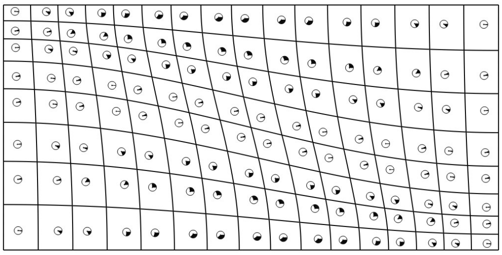

Fig. 4. The fourth eigenmode

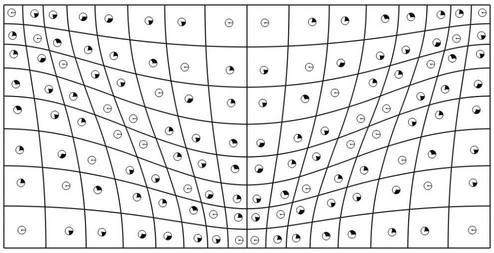

Fig. 5. The fifth eigenmode

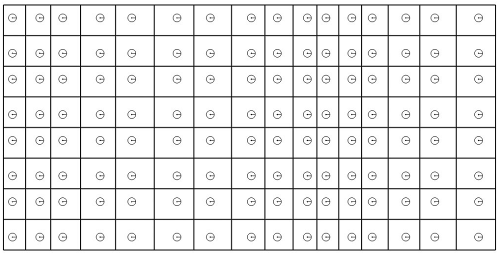

Fig. 6. The sixth eigenmode

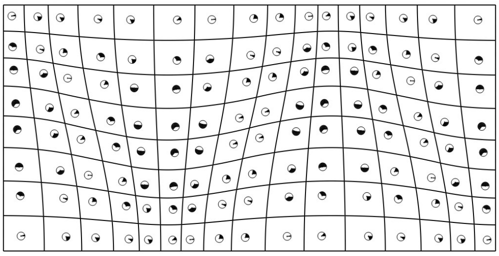

Fig. 7. The seventh eigenmode

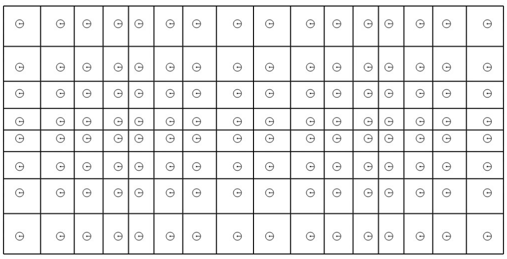

Fig. 8. The eighth eigenmode

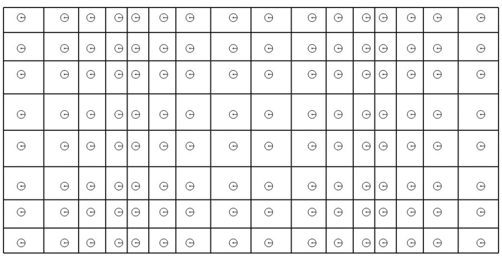

Fig. 9. The ninth eigenmode

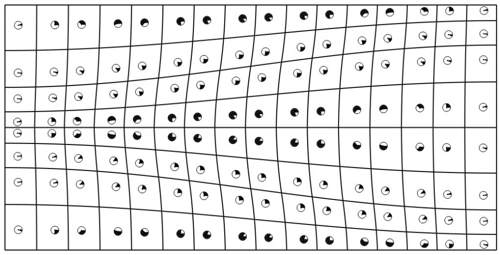

Figure 10. The tenth eigenmode

This graphical representation indicates the quality of satisfaction of the constraint of non-rotation for the eigenmode over the whole structure and thus serves for the validation of the results of calculations. It should be noted that for this problem in several eigenmodes there is no rotational motion because of the displacements parallel to the axes of coordinates.

Analysis of mass flow rate between different elements of a pipe robot

Cartesian coordinate system is used with the z axis parallel to the axis of the tube. The cross-section of the tube does not vary with the z coordinate. The velocity of fluid flow in the direction of the z axis is the function of the coordinates of the cross-section: $w = w(x, y)$. The components of velocity in the plane of the cross-section are equal to zero.

The stresses are: $\sigma_x = \sigma_y = \sigma_z = -p,$ $\sigma_{xy} = 0,\ \sigma_{yz} = \mu \frac{\partial w}{\partial y},\ \sigma_{zx} = \mu \frac{\partial w}{\partial x},$ where p is the pressure and μ is the viscosity of the fluid.

The equilibrium equation in the direction of the z axis has the form:

$$\frac{\partial}{\partial x}\left(\mu \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial w}{\partial y}\right) - \frac{\partial p}{\partial z} + \rho g = 0, \qquad (10)$$

where ρ is the density of the fluid, g is the acceleration of gravity, $\frac{\partial p}{\partial z}$ is the gradient of pressure in the direction of the z axis and it is assumed constant.

The boundary condition at the wall has the form:

$$-\mu \frac{\partial w}{\partial n} = \alpha w, \qquad (11)$$

where n is the outward normal to the boundary of the cross section of the flow, α is the coefficient of slippage between the fluid and the surface of the tube.

The stiffness matrix has the form:

$$[K] = \int [B]^T \mu [B] dxdy + \int [M]^T \alpha [M] ds, \qquad (12)$$

where s is the boundary line of the cross-section of the flow, $[B]$ is the matrix of the derivatives of the shape functions (the first row with respect to x and the second row with respect to y), $[M]$ is the row vector of the shape functions on the boundary of the cross-section.

The loading vector has the form:

$$\{F\} = \int [N]^T \left(\rho g - \frac{\partial p}{\partial z} \right) dxdy, \qquad (13)$$

where $[N]$ is the row vector of the shape functions in the cross-section of the flow.

The mass flow rate is found:

$$Q = \int \rho w(x, y) dxdy, \qquad (14)$$

where $w(x, y)$ are calculated from the obtained solution of the system of linear algebraic equations by using the shape functions of the appropriate finite elements.

Cross-section is assumed to be a circle and one fourth of it is analyzed. The characteristics of the fluid:

$$\mu = .004 \frac{g}{mms} = 4cP, \quad \alpha = .04 \frac{g}{mm^2 s},$$

$$\rho = .001 \frac{g}{mm^3}, \qquad -\frac{\partial p}{\partial z} + \rho g = 1 \frac{g}{mm^2 s^2}.$$

Radius of the cross section is 10 mm. The obtained mass flow rate $\frac{Q}{4} = 254.86 \, \frac{\mathrm{g}}{\mathrm{s}}$.

Conclusions

The dynamics of a fluid in the element of a pipe robot is investigated.

Vibrations of ideal compressible fluid are analyzed by taking into account the constraint of non-rotation by the penalty method using reduced order numerical integration of the penalty term.

Graphical representation of rotations at the points of reduced order numerical integration is proposed for validation of the calculated eigenmodes. This graphical representation shows the quality of satisfaction of the constraint of non-rotation over the whole structure.

The mass flow rate of the fluid through various cross-sections is determined. This is important in the design of separating elements between the different compartments of a pipe robot.

The obtained results are used in the process of design of the elements of a pipe robot.

References

[1] **Iershov N. F., Shahvierdi G. G.** The Method of Finite Elements in the Problems of Hydrodynamics and Hydroelasticity. Leningrad: Sudostroienie, 1984.

[2] **Eibl J., Stempniewski L.** Dynamic analysis of liquid filled tanks including plasticity and fluid interaction - earthquake effects. Shell and Spatial Structures: Computational Aspects, Proceedings of the International Symposium. July 1986. Leuven, Belgium, Berlin: Springer-Verlag, 1987, p. 261-269.

[3] **Reddy J. N.** Applied Functional Analysis and Variational Methods in Engineering. New York: McGraw-Hill, 1986.

[4] **Bathe K. J.** Finite Element Procedures in Engineering Analysis. New Jersey: Prentice-Hall, 1982.

[5] **Rao S. S.** The Finite Element Method in Engineering. Oxford: Pergamon Press, 1982.

[6] **Connor J. J., Brebbia C. A.** Finite Element Techniques for Fluid Flow. Leningrad: Sudostroienie, 1979.

[7] **Zienkiewicz O. C.** The Finite Element Method in Engineering Science. Moscow: Mir, 1975.

[8] **Matuliauskas A., Mištinas V., Spruogis B., Ragulskis K.** Pipe crawling robots with vibratory drives. Journal of Vibroengineering 4(5). Vilnius, 2000. P. 9-14.