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Displacement of the body on the oscillatory plane

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1. Introduction

One of stages of the automated assembly is relative orientation of the mating parts. Now there are active and passive methods of relative orientation which allow an effective and reliable application of them in mechanical engineering and robotics.

Active methods are based on the use of a different type of sensors which define relative position of the mating parts in assembly position and form control signals for actuators of orienting mechanisms. These mechanisms compensate errors of relative positioning of the parts and perform matching of their connective surfaces.

Passive relative orientation of the mating parts is ensured by means of devices with remoter centre compliance, elastic compensators, using the method of auto-search [1].

In this article the new method of relative orientation of the parts which is based on vibratory displacement of a body on a oscillatory plane is proposed. The oscillatory plane may be classified as analogue of such systems – the oscillatory conveyors, crank-rod, cam and side scene mechanisms where output links make oscillatory movements [2 - 4].

In article [5] the vibratory displacement of the elastically based body on an inclined surface is investigated. By means of the obtained dependences the necessary conditions for the body motion on a plane were defined. It was determined, that the size of the maximum displacement of the body highly depends on the inclination angle of the plane and in a smaller measure on a preliminary preload.

In work [6] the motion of the body on the horizontal plane, which is excited in two perpendicular directions, is considered. The authors have been analysing the characteristics, obtained under varying amplitude and frequency of excitation and changing the direction and magnitude of the friction coefficient to ensure easy and fast orientation of a part on the plane. The plane moves following the circular, elliptic and complex closed trajectories. Mentioned article states that character of the movement of a body depends on the phase angle between the components of excitation and the ratio of excitation frequencies.

Authors of the article [7] investigated the motion of the part on a horizontal plane taking into account elastic and damping constraints and provided dynamic and mathematical models. By the obtained results it was determined, that pressing the peg by changing linearly the force the bushing moves by helical interwinding trajectory. The zones with existing different motion laws of the body are given; and optimal conditions for mutual search of the parts applying for automated assembly are defined.

In modern engineering investigation of the transportation of a body on a plane is of great interest. Fedaravičius A., Tarasevičius K., Sližys E. [8] have offered a new method of vibratory transportation under controlled dry friction and presented the dependences of the vibrotransportation force and speed on the moments of the dry friction control relative to the period of horizontal oscillations of the platform.

Fedaravičius A., Tarasevičius K., Bačkauskas V. [9] investigated the vibratory transportation of a body on a plane under controlled dry friction. In this case the plane makes circular movements, whereas the body on the plane performs arbitrary motion. The performed analysis showed, that given method has a big potential for practical application in orientation systems of automatic mechanical assembly of the parts in radio-electronics, equipment industry and other fields.

Main goal of the present work is to analyze the dynamics of vibratory displacement of a body on a oscillatory plane applying for automated assembly of the parts.

Body movement is investigated varying the factor of rigidity, the angle of oscillatory and existing zones with various trajectories of the body motion on a oscillatory plane are defined.

2. Dynamic model, the differential equation and motion trajectories of a body

Let us to consider the dynamic model of vibratory displacement of a body under the influence of kinematic excitation on the oscillatory plane (Fig. 1).

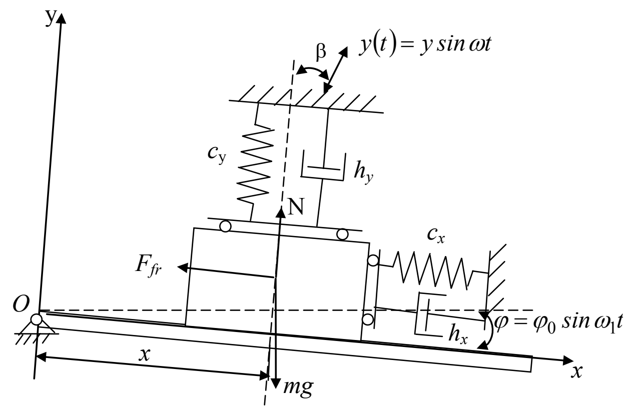

Fig. 1 Movably based body on the oscillatory plane

The mass m represents movably based component which under the influence of vibrations moves in respect of the immovable component, represented by the oscillatory plane.

The motion equations of the body in coordinates xOy of the system rigidly attached to the oscillatory plane are

$$\left.\begin{aligned} &m\ddot{x} + h_x\dot{x} + c_x x = F\sin\omega t \sin\beta + F_{fr} + mg\sin(\varphi_0 \sin\omega_1 t) \\ &m\ddot{y} + h_y\dot{y} + c_y y = F\sin\omega t\cos\beta + N - mg\cos(\varphi_0\sin\omega_1 t) - c_y y_{st} + m\omega_1^2 x\varphi_0\sin\omega_1 t \end{aligned}\right\} \quad (1)$$

where m is mass of the body, ω is excitation frequency, h_x, h_y are damping coefficients along the x and y axis direction respectively, c_x, c_y are rigidities of the spring along the direction of x and y axes respectively, β is inclination angle of the oscillation trajectory relative to the plane (angle of vibration), F is the equivalent excitation force $F(t) = c_y y(t)$, where $y(t) = y\sin\omega t$, y is the amplitude of kinematic excitation, F_{fr} is dry friction force, g is acceleration due to gravity, φ_0 is the angle of oscillation of the plane, ω_1 is vibration frequency of the plane, y_{st} is the preload, N is the force of normal pressing of the body to the plane, t is time.

During motion of the body on vibrating surface $y \equiv 0$ then from the second equation of system (1) the normal reaction $N(t)$ is defined

$$N(t) = mg\cos(\varphi_0\sin\omega_1 t) + c_y y_{st} - \\ - F\sin\omega t\cos\beta - m\omega_1^2 x\varphi_0\sin\omega_1 t \quad (2)$$

The force of dry friction F_{fr} between the surfaces of the body and vibrating plane depends on normal reaction N and changes a sign depending on the direction of body's motion. Mentioned force is expressed by dependences

$$F_{fr} = \begin{cases} -\mu N, & \text{when } \dot{x} > 0, \\ +\mu N, & \text{when } \dot{x} < 0, \\ (-1,+1)\mu N, & \text{when } \dot{x} = 0 \end{cases} \quad (3)$$

here μ is coefficient of sliding friction.

In the first equation of system (1) having substituted the Eqs. (2), (3) we will receive the following equation of the motion of a body on vibrating plane

$$m\ddot{x} + h_x\dot{x} + c_x x = F\sin\omega t\sin\beta \mp \mu\big(mg\cos(\varphi_0\sin\omega_1 t) + \\ + c_y y_{st} - m\omega_1^2 x\varphi_0\sin\omega_1 t - F\sin\omega t\cos\beta\big) + \\ + mg\sin(\varphi_0\sin\omega_1 t) \quad (4)$$

The sign "-" in Eq. (4) corresponds to the motion of the body along the x axis direction and sign "+" represents motion in opposite direction.

Eq. (4) was solved by a numerical method using MatLab software package. To analyze the behavior of the considered system the basic laws of the body's motion on the oscillatory plane under the influence of kinematic excitation have been defined.

The process of vibratory displacement of the body occurs in transient modes of motion. The body moves on the surface from the position of static equilibrium to dynamic equilibrium.

The mode of transient motion ends, when total resistance force, which prevents the displacement, is equal to horizontal component of the excitation force. At further excitation the body continues vibratory motion on the plane near the position of dynamic equilibrium. During automated assembly the given vibratory motion can facilitate matching of the parts.

The analysis of motion diagrams of the body shows, that three types of motion may occur:

1) sliding forward ($\dot{x} > 0$),
2) sliding backward ($\dot{x} < 0$),
3) an instant stop ($\dot{x} = 0$), where $\dot{x}$ is motion velocity of the body.

As starting point of the time scale we take the start point of the excitation with the following initial conditions: $x = 0$, $\dot{x} = 0$.

Examples of the motion trajectories of the body along the x axis direction on the oscillatory plane are shown on the Figs. 2 - 4.

Fig. 2 presents the motion trajectory of the body, which moves from the position of static equilibrium to dynamic equilibrium position where continue vibrations near this position with steady amplitude.

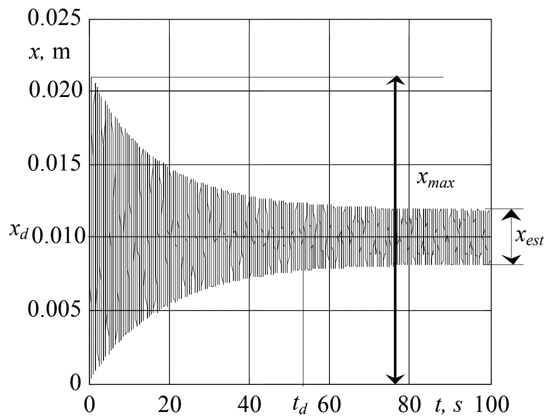

Fig. 2 Motion trajectory of the body, as $\varphi_0 = 0.05$ rad, $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_x = c_y = 50$ N/m, $y = 0.01$ m, $y_{st} = 0.002$ m, $h = 0.1$ N·s/m, $m = 0.5$ kg, $\beta = 0.1$ rad, $F = 4$ N, $\mu = 0.1$

In Fig. 2 x_{max} is maximum displacement of the body from the position of static equilibrium; t_d is time required to reach steady amplitude of vibrations of the body near the position of dynamic equilibrium; x_d is coordinate of dynamic equilibrium; x_{est} is peak-to peak magnitude of the steady vibrations of the body near the coordinate of dynamic equilibrium.

It is seen from Fig. 3, that the amplitude of vibrations of the body on the oscillatory plane initially increases and later undamped periodic vibration near the position of static equilibrium is taking place.

Thus, it is possible to provide a search with constant amplitude vibration.

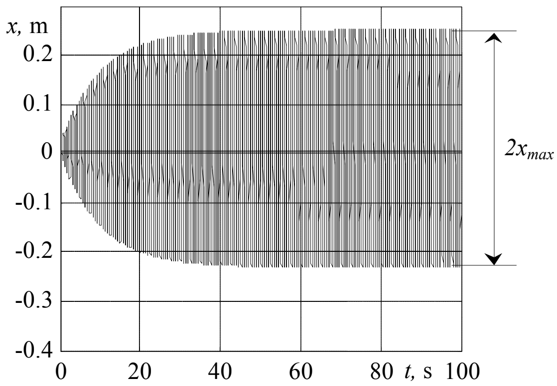

Fig. 3 Motion trajectory of the body, as $\varphi_0 = 0.05$ rad, $\omega = \omega_1 = 10$ s^{-1}, $c_x = c_y = 50$ N/m, $y = 0.01$ m, $y_{st} = 0.002$ m, $h = 0.1$ N·s/m, $m = 0.5$ kg, $\beta = 0.1$ rad, $F = 4$ N, $\mu = 0.1$

In Fig. 4, the body displaces from the position of static equilibrium and continues vibration with constant steady amplitude.

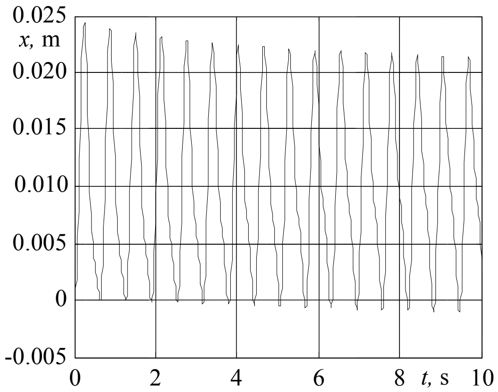

Fig. 4 Motion trajectory of the body, as $\varphi_0 = 0.1$ rad, $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_x = c_y = 50$ N/m, $y = 0.01$ m, $y_{st} = 0.002$ m, $h = 0.1$ N·s/m, $m = 0.5$ kg, $\beta = 0.1$ rad, $F = 4$ N, $\mu = 0.1$

3. Simulation of the body displacement on the oscillatory plane

Investigation of motion characteristics of a body on an oscillatory plane was carried out for the trajectory of motion presented on Fig. 2.

The dependences $x_{max} = f(\varphi_0)$ under particular parameter c_x (Fig. 5) show, that as oscillatory angle $\varphi_0 \le 0.1$ rad, the magnitude of the maximum displacement x_{max} of the body practically does not vary, and at $\varphi_0 > 0.1$ rad, it starts increasing linearly.

Fig. 6 presents dependences $x_{max} = f(c_x)$ under particular values of the parameter φ_0. The increase in elastic resistance force, results decrease in x_{max}.

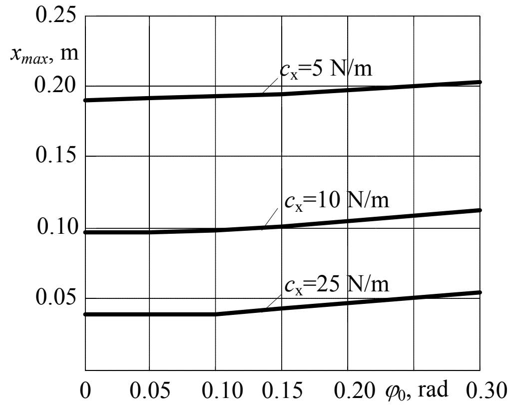

Fig. 5 Graph of $x_{max} = f(\varphi_0)$, as $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_y = 50$ N/m

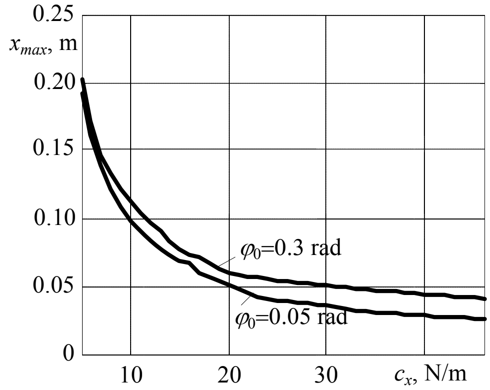

Fig. 6 Graph of $x_{max} = f(c_x)$, as $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_y = 50$ N/m

Increasing the coefficient of rigidity c_y at invariable parameter φ_0 (Fig. 7) the maximum displacement x_{max} increases linearly. Varying the rigidity coefficient of the spring c_y causes both the change in constant component of the friction force and horizontal component of the excitation force of the body.

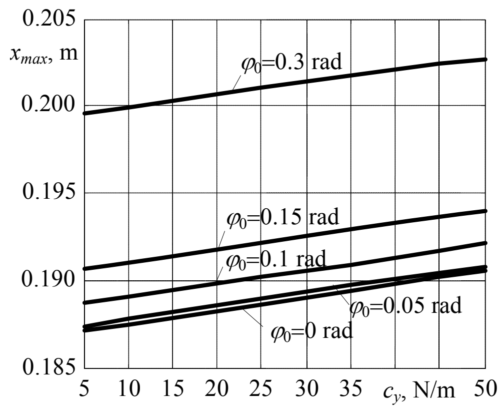

Fig. 7 Graph of $x_{max} = f(c_y)$, as $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_x = 5$ N/m

Increasing the frequency ω_1 of oscillation of the plane and particular oscillatory angle φ_0 (Fig. 8), the magnitude of x_{max} initially decreases, then at

$\omega_1 = 20 - 25\ \text{s}^{-1}$ – nonsignificantly increases, further increase in frequency of oscillations $\omega_1 > 25\ \text{s}^{-1}$, results decrease in x_{max}, whereas further increasing the ω_1, maximum displacement practically does not vary.

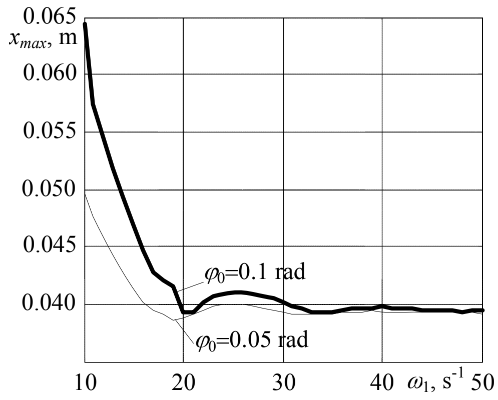

Fig. 8 Graph of $x_{max} = f(\omega_1)$, as $\omega = 10\ \text{s}^{-1}$, $c_x = 25$ N/m, $c_y = 50$ N/m

To define the dependence $x_{max} = f(\alpha)$ under existing phase difference, by the angle α, between the excitation and plane oscillation exists, let us in Eq. (4) to replace the value $sin\,\omega_1 t$ by $sin(\omega_1 + \alpha)$. Obtained dependences under particular parameter c_x are presented in Fig. 9. Increasing the phase difference angle α, the maximum displacement of the body from the position of static equilibrium decreases.

Maximum displacement of the body from the position of static equilibrium basically depends on parameters φ_0, c_x, c_y, ω_1 and is less dependent on ω, y_{st}, y and β.

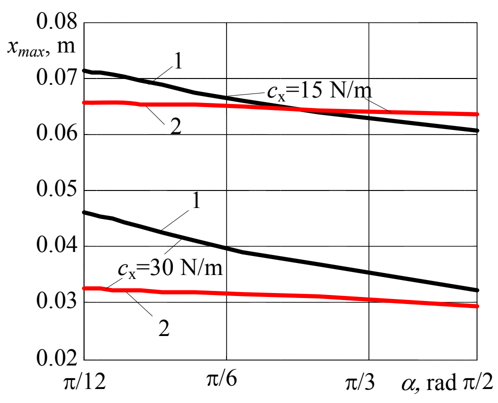

Fig. 9 Graph of $x_{max} = f(\alpha)$, as $c_x = 25$ N/m, $c_y = 50$ N/m and exclusion line: 1 - $\omega = 10\ \text{s}^{-1}$, $\omega_1 = 10\ \text{s}^{-1}$, 2 - $\omega = \omega_1 = 20\ \text{s}^{-1}$

One of the necessary criterions characterizing automated assembly is time necessary to join the parts. It is defined, that time t_d, which is required to obtain constant amplitude oscillation of a body near the position of dynamic equilibrium, is practically marginally dependent on φ_0, c_x and c_y (Fig. 10 - 12).

The value of a coordinate of dynamic equilibrium of the body practically is not dependent on variation of the oscillatory angle φ_0 (Fig. 13).

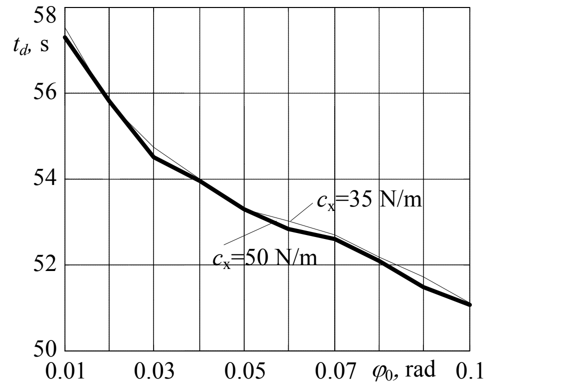

Fig. 10 Graph of $t_d = f(\varphi_0)$, as $\omega = 10\ \text{s}^{-1}$, $\omega_1 = 20\ \text{s}^{-1}$, $c_y = 50$ N/m

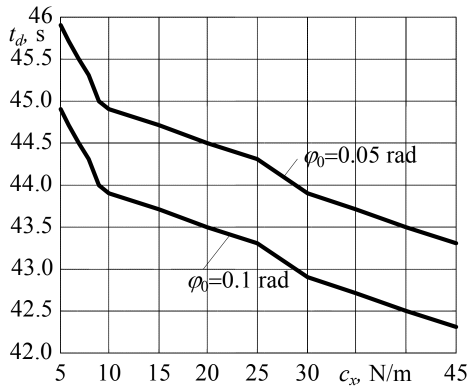

Fig. 11 Graph of $t_d = f(c_x)$, as $\omega = 10\ \text{s}^{-1}$, $\omega_1 = 20\ \text{s}^{-1}$, $c_y = 50$ N/m

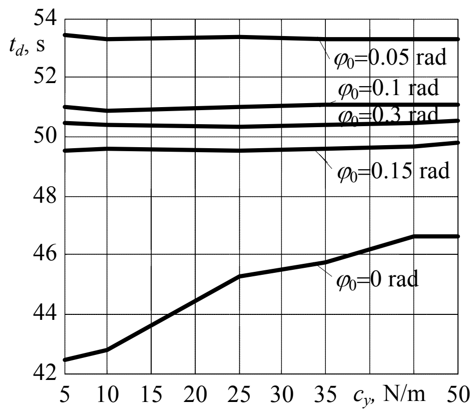

Fig. 12 Graph of $t_d = f(c_y)$, as $\omega = 10\ \text{s}^{-1}$, $\omega_1 = 20\ \text{s}^{-1}$, $c_x = 5$ N/m

Increasing the force of static pressing of the body to the plane and particular parameter φ_0, the magnitude of the coordinate of dynamic equilibrium x_d increases (Fig. 14).

For effective and accurate joining of the of parts at automated assembly the big role is played by the size of scope of the established oscillations of the body near the coordinate of dynamic equilibrium x_{est}. Increasing both the oscillatory angle φ_0 (Fig. 15) and forces of elastic resistance, results in x_{est} increase (Fig. 16).

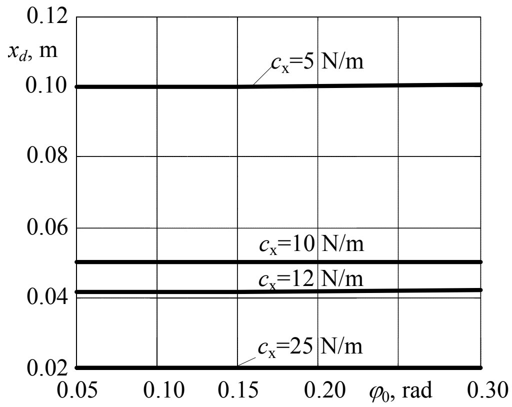

Fig. 13 Graph of $x_d = f(\varphi_0)$, as $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_y = 50$ N/m

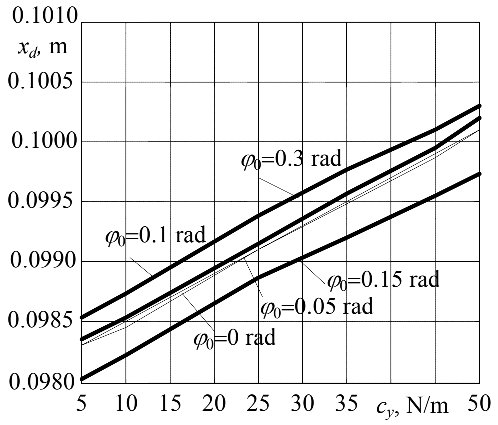

Fig. 14 Graph of $x_d = f(c_y)$, as $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_x = 5$ N/m

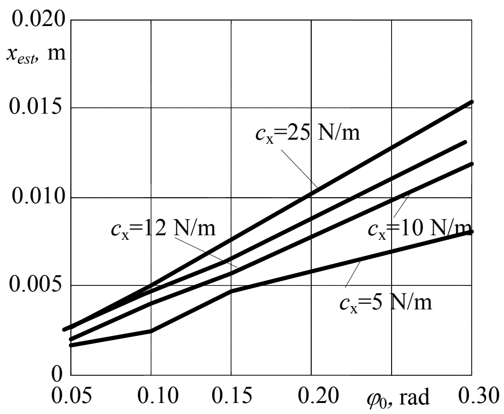

Fig. 15 Graph of $x_{est} = f(\varphi_0)$, as $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_y = 50$ N/m

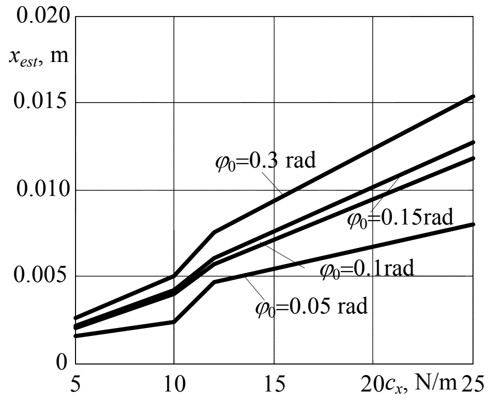

Fig. 16 Graph of $x_{est} = f(c_x)$, as $\omega = 10$ s^{-1}, $\omega_1 = 20$ s^{-1}, $c_y = 50$ N/m

Existing zones of the motion of the body on a oscillatory plane were determined under different combinations of the excitation frequency ω and oscillation frequency ω_1 of the plane (Fig. 17), oscillatory angle φ_0 and oscillation frequency ω_1 of the plane (Fig. 18), coefficient of rigidity c_x and excitation frequency ω (Fig. 19), coefficient of rigidity c_x and oscillation frequency ω_1 of the plane (Fig. 20).

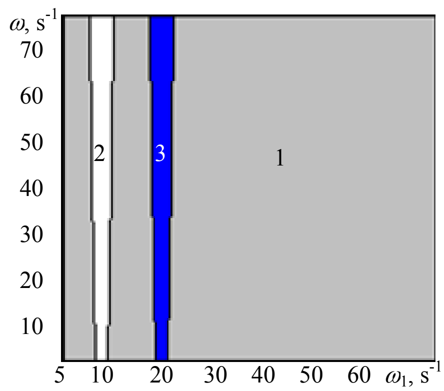

Fig. 17 Zones of motion regimes in coordinates ω and ω_1, as $c_x = c_y = 50$ N/m, $\varphi_0 = 0.1$ rad

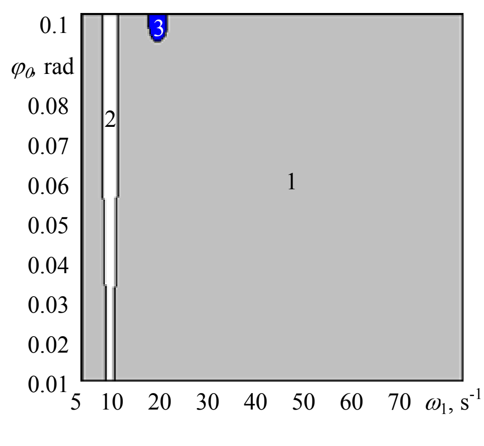

Fig. 18 Zones of motion regimes in coordinates ω_1 and φ_0, as $c_x = c_y = 50$ N/m, $\omega = 10$ s^{-1}

The first zone corresponds to the motion trajectory of the body shown on Fig. 2, the second - motion trajectory of the body, presented on Fig. 3 and the third corresponds to the trajectory presented on Fig. 4.

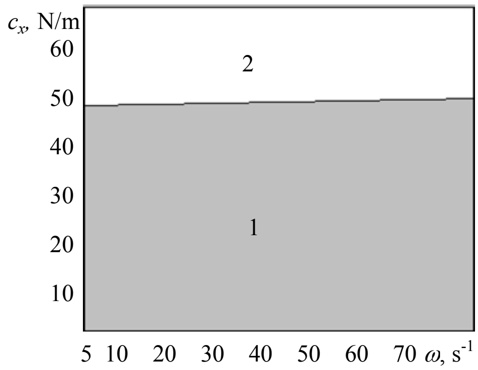

Fig. 19 Zones of motion regimes in coordinates c_x and ω, as $\varphi_0 = 0.05$ rad, $c_y = 50$ N/m, $\omega_1 = 10$ s^{-1}, $y = 0.01$ m, $y_{st} = 0.002$ m, $h = 0.1$ N·s/m, $m = 0.5$ kg, $\beta = 0.1$ rad, $F = 4$ N, $\mu = 0.1$

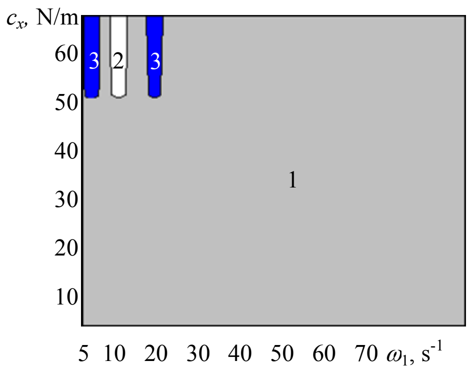

Fig. 20 Zones of motion regimes in coordinates c_x and ω_1, as $\varphi_0 = 0.1$ rad, $\omega = 10$ s^{-1}, $c_y = 50$ N/m, $y = 0.01$ m, $y_{st} = 0.002$ m, $h = 0.1$ N·s/m, $m = 0.5$ kg, $\beta = 0.1$ rad, $F = 4$ N, $\mu = 0.1$

The trajectory of the body motion, presented on Fig. 2, has the maximum zone of existence.

4. Conclusions

1. The analysis of motion trajectories of a body on a oscillatory plane showed, that three types of motion are characteristic. The process of vibratory displacement occurs in transient modes of motion.

2. It was determined that the magnitude of the maximum displacement x_{max} highly depends on parameters φ_0, c_x, c_y, ω_1, and is less dependent on ω, y, y_{st} and β. As oscillatory angle $\varphi_0 < 0.1$ rad, x_{max} practically remains constant, whereas oscillatory angle $\varphi_0 > 0.1$ rad, results linear increase in x_{max}.

3. Increasing the factor of rigidity c_x under particular parameter φ_0, x_{max} decreases, whereas increasing the coefficient of rigidity, c_y decreases.

4. Increasing the frequency ω_1 of oscillations of the plane, x_{max} decreases. At $\omega_1 = 20 \div 25$ rad - increases and as $\omega_1 > 25$ rad – decreases while reaches particular magnitude and further remains practically constant.

5. With the increase in phase difference between the excitation and oscillatory of the plane maximum displacement of the body on plane decreases.

6. The time t_d, necessary to obtain constant amplitude oscillations of the body near the position of dynamic equilibrium, practically is not dependent on parameters φ_0, c_x, c_y.

7. The coordinate of the dynamic equilibrium x_d practically is not dependent on variation of the angle φ_0.

Increasing the force of static pressing of the body to the plane, under particular parameter φ_0, the magnitude of x_d increases.

8. The magnitude of the peak-to-peak amplitude x_{est} of steady-state vibrations of the body near the position of dynamic equilibrium depends on parameters φ_0 and c_x.

Increasing both the oscillatory angle of the plane and forces of elastic resistance, results an increase in x_{est}.

9. Under identical parameters of the system and considering complete sets of parameters both, ω and ω_1, φ_0 and ω_1, c_x and ω, c_x and ω_1, three existing zones with characteristic modes of motion of the body on a oscillatory plane were defined. The obtained trajectories of the body motion on a oscillatory plane can be applied for relative positioning of the mating parts during automated assembly.

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KŪNO POSLINKIS SVYRUOJANČIA PLOKŠTUMA

R e z i u m ė

Straipsnyje pateikti automatiniam surinkimui naudojamo kūno vibracinio poslinkio svyruojančia plokštuma tyrimai. Sudaryti dinaminis ir matematinis vibracinio poslinkio modeliai. Naudojant programų paketą MatLab parašytos lygčių sprendimo programos, nustatyta, kaip kūno judėjimo svyruojančia plokštuma charakteristikos priklauso nuo svyravimo kampo, plokštumos svyravimo dažnio, spyruoklės standumo koeficiento, žadinimo ir plokštumos svyravimo fazių skirtumo. Sudarytos judėjimo režimų sričių priklausomybės nuo žadinimo ir plokštumos svyravimo dažnio, svyravimo kampo ir spyruoklės standumo koeficiento. Nustatyta, kad kūno judėjimo pobūdis priklauso nuo plokštumos svyravimo dažnio ir kampo, spyruoklės standumo koeficiento bei žadinimo ir plokštumos svyravimo fazių skirtumo. Gautos kūno poslinkio svyruojančia plokštuma trajektorijos gali būti taikomos automatiškai surenkamoms detalėms tarpusavyje pozicionuoti.

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DISPLACEMENT OF THE BODY ON THE OSCILLATORY PLANE

S u m m a r y

Presented work analyzes characteristics of vibratory displacement of a body on a oscillatory plane with reference to automated assembly of the parts. Mathematical and dynamic models of vibratory displacement are made. Applying MatLab software, programs to solve the equations of motion were written. Characteristics of a body motion on a oscillatory plane depending on the oscillatory angle and oscillation frequencies of the plane, coefficients of rigidity of the spring and phase difference between the excitation plane oscillations were defined. Existing areas of different modes of motion of the body depending on excitation frequency and oscillation of the plane, on oscillatory angle of the plane and rigidity factor were defined. It was determined, that motion character of the body depends on oscillatory angle and oscillation frequency of the plane, coefficient of rigidity of the spring and also phase difference between the excitation and oscillation of the oscillatory plane. Obtained trajectories of the body motion on a oscillatory plane can be applied for relative positioning of the mating parts during automated assembly.

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ПЕРЕМЕЩЕНИЕ ТЕЛА НА КАЧАЮЩЕЙСЯ ПЛОСКОСТИ

Р е з ю м е

В настоящей работе приведено исследование характеристик вибрационного перемещения тела на качающейся плоскости применительно к автоматической сборке деталей. Составлены динамическая и математическая модели вибрационного перемещения. С помощью пакета MatLab составлены программы для решения уравнений движения, определены характеристики движения тела на качающейся плоскости в зависимости от угла качания, частоты колебаний плоскости, коэффициентов жесткости пружины, разности фаз между возбуждением и колебанием плоскости. Определены области существования режимов движения тела в зависимости от частоты возбуждения и колебания плоскости, угла качания плоскости и коэффициента жесткости. Выявлено, что характер движения тела зависит от угла качания и частоты плоскости, коэффициента жесткости пружины и сдвига фаз между возбуждением и колебанием плоскости. Полученные траектории движения тела на качающейся плоскости могут быть применены для относительного позиционирования сопрягаемых деталей при автоматической сборке.

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