912. Dynamic study of transportation containers with packages

Kazimieras Ragulskis[1], Laura Gegeckienė[2], Edmundas Kibirkštis[3], Valdas Miliūnas[4], Lina Zubrickaitė[5], Arvydas Pauliukas[6], Liutauras Ragulskis[7]
[1]Kaunas University of Technology, K. Donelaičio str. 73, LT-44029 Kaunas, Lithuania
[2, 3, 4, 5]Kaunas University of Technology, Studentų str. 56-350, LT-51424 Kaunas, Lithuania
[6]Aleksandras Stulginskis University, Studentų str. 11, LT-53361 Akademija, Kaunas District, Lithuania
[7]Vytautas Magnus University, Vileikos str. 8, LT-44404 Kaunas, Lithuania
E-mail: [1]*kazimieras3@hotmail.com*, [2]*laura.siuipyte@stud.ktu.lt*, [3]*edmundas.kibirkstis@ktu.lt*, [4]*valdas.miliunas@ktu.lt*, [5]*lina.zubrickaite@ktu.lt*, [6]*arvydas.pauliukas@asu.lt*, [7]*l.ragulskis@if.vdu.lt*

(Received 30 August 2012; accepted 4 December 2012)

Abstract. The paper studies the motion of transportation machine with packages on uneven road. Conditions of transportation that reduce the influence of harmful vibrations to the packages are determined. For this purpose the region was established where the slippage of packages with respect to the transported box of packages does not occur. The conditions for location of eigenfrequencies and forced frequencies are determined with the purpose to ensure that the system operates far from resonances. In order to avoid large overloads it is recommended to choose the velocity of motion in such a way, that the difference between the eigenfrequencies and the forced frequencies would be maxmin.

Keywords: transportation, road, non-uniformity, packages, protection, harmful vibrations.

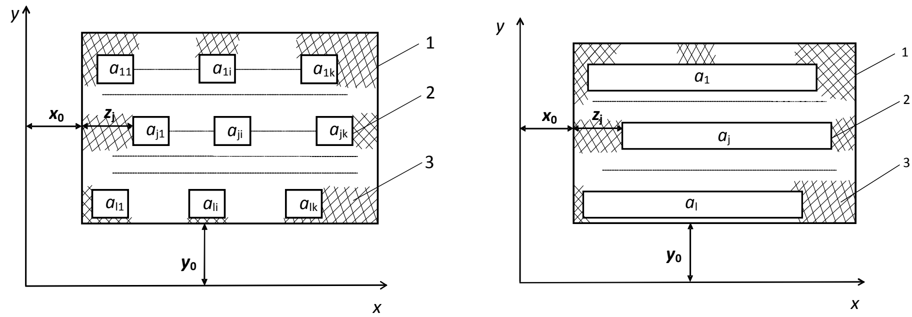
1. Introduction

The one-dimensional system of transportation of the same type as described in [1] is analyzed in this paper, but here a higher number of degrees of freedom is taken into account, while control is not investigated. In practical applications it is important to ensure that the transported packages are not damaged by harmful vibrations. This may be achieved by choosing appropriate velocity of the transportation machine. For this purpose the conditions that provide non-slippage of packages with respect to the box of packages are determined. Also it is proposed to choose the eigenfrequencies of the system and the forced frequencies in such a way that the system operates far from resonances. The forced frequencies excited by the non-uniformities of the road are determined. In order to avoid high overloads of packages it is recommended to choose the velocity of motion in such a way that the difference between the eigenfrequencies and the forced frequencies would be maxmin. Eigenmodes of ten rows and ten columns of packages in a box are analyzed. A package is represented as a single plane strain element. Interconnection of packages is assumed by special elements ensuring approximate continuity of the normal displacement between the packages by the penalty method. Eigenmodes of ten long packages in a box are analyzed. A long package is represented as consisting from ten plane strain elements.

Problems of transportation are described by Foley with coauthors [2], Ostrem and Rumerman [3, 4], and Ostrem and Godshall [5]. They analyze the influence of vibrations to transported objects, which are excited by the non-evenness of the road in the transport means such as trucks. The influence of various defects of the road (Fig. 1) to the transported objects is analyzed here.

Fig. 1. Possible defects of the road: a) step up; b) step down; c) step down and up; d) bending down [6]

Hoppe and Gerok [7] performed a survey on the vibration on the platform of the vehicles used in transportation. In their study, the frequency range of the signals encountered in the transportation was obtained and the levels of bumps and shocks as well as their durations for different vehicles were measured. The authors of references [8, 9] collected and analyzed the data of excited frequencies of vibrations from various transportation devices and compared them with existing models. Rouillard and Sek [10] claimed that tracking the resonance by feedback control of the excitation frequency during resonance dwell test is essential. In a number of references [11, 12] the increase of probability of buckling of the lower row of packages in the process of transportation because of non-evenness of the road is described. It is assumed that the packages are loaded one over the other. The two most typical schematic representations of the box with packages are presented in Fig. 2.



a) b)

Fig. 2. a) Box with packages: 1 – case; 2 – packages; 3 – isolating material;
b) simplified representation of the box with packages: 1 – case; 2 – packages; 3 – isolating material

Simplified model for investigating transportation of a box of packages is presented in Fig. 3. The model of the system consists of the case of the transportation machine 1 and amortized part of the transportation machine 2 with non-deformable box of packages. The transportation machine 1 is in contact with the non-deformable profile of the surface of the road $\xi(x)$. Connecting elements between the bodies 1 and 2 include dissipative and elastic elements.

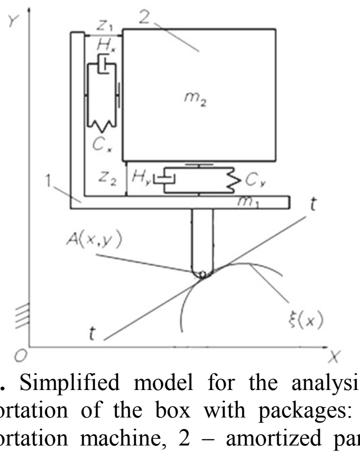


Fig. 3. Simplified model for the analysis of transportation of the box with packages: 1 – transportation machine, 2 – amortized part of the transportation machine with the box of packages, $\xi(x)$ denotes the profile of the unevenness of the road

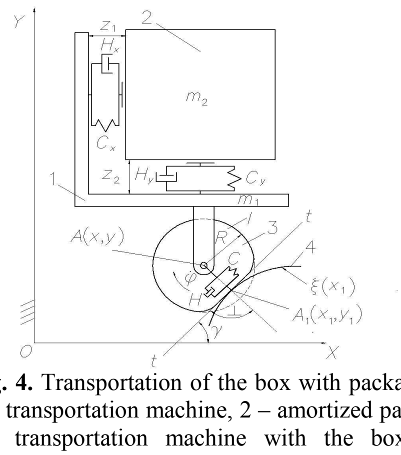


Fig. 4. Transportation of the box with packages: 1 – transportation machine, 2 – amortized part of the transportation machine with the box of packages, 3 – deformable wheel of the transportation machine, 4 – profile of the unevenness of the road $\xi(x_1)$

The obtained results are used in the process of package design.

2. The analyzed system

A more general model which is analyzed further is presented in Fig. 4. The model of the system consists of the case of the transportation machine 1, amortized part of the transportation machine 2 with non-deformable box of packages and with a deformable wheel 3. The wheel 3 is in contact with the non-deformable profile of the surface of the road 4 $\xi(x)$. Connecting elements between bodies 1 and 2 consist from dissipative and elastic elements.

The position of body 1 is determined by the points $A(x, y)$ and $A_1(x_1, y_1)$, while the position of body 2 with respect to body 1 according to the axis Ox is z_1 and according to the axis Oy is z_2 and also the wheel has the angle of rotation φ. In the case of contact between bodies 3 and 4 the position of the system is determined by x_1, z_1, z_2, $r = \overline{AA_1}$, φ.

The point $A_1(x_1, y_1)$ is determined by:

$$y_1 = \xi\left(x_1\right),$$
$$\tan\gamma = \frac{d\xi}{dx_1} = \xi', \tag{1}$$
$$\dot{y}_1 = \xi'\dot{x}_1.$$

Velocities and accelerations of the point $A(x, y)$ are determined by the following equations:

$$\dot{x} = a\dot{x}_1 - \delta\xi'\dot{r} + \delta r\dot{\varphi},$$
$$\dot{y} = a\xi'\dot{x}_1 + \delta\dot{r} + \delta\xi' r\dot{\varphi}, \tag{2}$$

$$\ddot{x} = a\ddot{x}_1 - \delta\xi'\ddot{r} + \delta r\ddot{\varphi} + a'\dot{x}_1^2 - \left(\delta^2\xi'' + \delta'\xi' + \delta\xi''\right)\dot{x}_1\dot{r} + \delta'\dot{x}_1 r\dot{\varphi} + \delta\dot{r}\dot{\varphi},$$
$$\ddot{y} = \xi' a\ddot{x}_1 + \delta\ddot{r} + \delta\xi' r\ddot{\varphi} + \left(a\xi'' + a'\xi'\right)\dot{x}_1^2 - \left(\delta^3\xi'' - \delta'\right)\xi'\dot{x}_1\dot{r} + \left(\delta\xi'' + \delta'\xi'\right)\dot{x}_1 r\dot{\varphi} + \delta\xi'\dot{r}\dot{\varphi}, \tag{3}$$

where:

$$\delta = \frac{1}{\sqrt{1+\xi'^2}},\ a = 1 - \delta^3\xi'' r,\ \ ' = \partial/\partial x_1,\ \ \dot{} = d/dt,\ r = \overline{AA_1}. \tag{4}$$

The change of variables which is convenient for the investigation of steady state regimes is introduced:

$$x_1 = \bar{\dot{x}}_1 t + u,\ \ \varphi = \bar{\dot{\varphi}} t + \psi, \tag{5}$$

where $\bar{\dot{x}}_1$ and $\bar{\dot{\varphi}}$ are average velocities in the steady state regimes, u and ψ are the new variables.

If ξ is a periodic function of x_1, then by taking into account the equations (5):

$$\xi = \xi\left(2\pi\frac{x_1}{\lambda}\right) = \xi\left(\omega t + \frac{u}{\lambda}\right), \tag{6}$$

where:

$$\omega = 2\pi\frac{\bar{\dot{x}}_1}{\lambda}, \tag{7}$$

and λ is the wavelength of the unevenness of the road profile.

Further the differential equations of motion for separate cases are obtained.

Case 1: general. Here the following unknowns are considered: x_1, r, z_1, z_2, φ.

$$m_1\left(\ddot{x} + g\sin\alpha\right) + \delta H_0 N \operatorname{sgn}\dot{x}_1 + H_1\dot{x}_1 + \delta\xi' N - F_{xs} = 0,$$
$$m_1\left(\ddot{y} + g\cos\alpha\right) + \delta\xi' H_0 N \operatorname{sgn}\dot{x}_1 + \xi' H_1\dot{x}_1 - \delta N - F_{ys} = 0, \tag{8}$$

$$m_2\left(\ddot{x} + g\sin\alpha + \ddot{z}_1\right) + F_{xs} = 0,$$
$$m_2\left(\ddot{y} + g\cos\alpha + \ddot{z}_2\right) + F_{ys} = 0, \tag{9}$$

$$J\ddot{\varphi}+\left(H_0 N \operatorname{sgn}\dot{x}_1+H_1\delta^{-2}\dot{x}_1\right)r=M\left(\dot{\varphi}\right), \tag{10}$$

where F_f is the force of sliding friction acting onto wheel 3 at point A_1:

$$F_f=H_0 N\operatorname{sgn}\dot{x}_1+H_1\dot{x}_1,\ N=C_r\left(R-r\right)+H_\rho\dot{r},\ F_{xs}=C_x z_1+H_x\dot{z}_1,\ F_{ys}=C_y z_2+H_y\dot{z}_2, \tag{11}$$

H_0 and H_1 are the coefficients of the forces of dry and viscous friction at point A_1 between the wheel 3 and the road profile 4, C_x, C_y, C_r are the coefficients of stiffness of elastic forces, H_x, H_y, H_ρ are the coefficients of forces of viscous friction, $M\left(\dot{\varphi}\right)$ is the driving moment reduced according to the coordinate $\dot{\varphi}$, g is acceleration of gravity of the earth, α is the angle of rotation of the axes Ox and Oy in the counterclockwise direction, $\dot{s}=\delta^{-1}\dot{x}_1$ is the velocity of slippage of the wheel with respect to the road profile at the point A_1.

In the equations (8, 9) the values of $\ddot{x}$ and $\ddot{y}$ are determined by the equations (3).

Case 2: $\dot{r}=0$, that is $r=R$, $C_r=\infty$. Here the following unknowns are considered: x_1, z_1, z_2, φ. (12)

In this case according to the equations (8) the force of normal pressure to the wheel 3 is:

$$N=-\frac{F_x+H_1\dot{x}_1}{\delta\left(H_0\operatorname{sgn}\dot{x}_1+\xi'\right)}=-\frac{F_y-\xi' H_1\dot{x}_1}{\delta\left(\xi' H_0\operatorname{sgn}\dot{x}_1-1\right)}, \tag{13}$$

where:

$$F_x=m_1\left(\ddot{x}+g\sin\alpha\right)-F_{xs},\ F_y=m_1\left(\ddot{y}+g\cos\alpha\right)-F_{ys}. \tag{14}$$

In this case instead of two equations (8) there is only one differential equation given as:

$$F_x+\xi' F_y+H_1\left(1+\xi'^2\right)\dot{x}_1+\left(-\xi' F_x+F_y\right)H_0\operatorname{sgn}\dot{x}_1=0, \tag{15}$$

and the other equations (9, 10, 11) remain valid, only here we take into account (13, 14).

Case 3: $r=R=0$, the unknowns are: $x=x_1$, z_1, z_2. (16)

Here the equations (9, 15) remain valid, it is only necessary to assume instead of F_x according to equation (14) the following:

$$F_x=m_1\left(\ddot{x}+g\sin\alpha\right)-F_{xs}-P\left(\dot{x}\right), \tag{17}$$

where $P\left(\dot{x}\right)$ is the driving force.

If $H_0=0$, after introduction of the notations:

$$n_x=\sqrt{C_x/m_1},\ n_y=\sqrt{C_y/m_1},\ h_x=H_x/m_1,\ h_y=H_y/m_1,\ \mu=m_2/m_1, \tag{18}$$

the differential equations of motion of the system (15, 9) after taking into account (17, 18) acquires the following form:

$$\begin{aligned}&\left(1+\xi'^2\right)\ddot{u}+\xi'\xi''\left(\bar{\dot{x}}+\dot{u}\right)+g\left(\sin\alpha+\xi'\cos\alpha\right)-\left(n_x^2 z_1+h_x\dot{z}_1\right)-\\&-\xi'\left(n_y^2 z_2+h_y\dot{z}_2\right)+h_1\left(1+\xi'^2\right)\left(\bar{\dot{x}}+\dot{u}\right)=\frac{1}{m_1}P\left(\bar{\dot{x}}+\dot{u}\right),\end{aligned} \tag{19}$$

$$\begin{aligned}&\mu\left(\ddot{u}+\ddot{z}_1\right)+n_x^2 z_1+h_x\dot{z}_1=0,\\&\mu\left(\ddot{u}+\ddot{z}_2\right)+n_y^2 z_2+h_y\dot{z}_2=0.\end{aligned} \tag{20}$$

When the profile of the road is harmonic, then according to the equations (6, 7):

$$\begin{aligned}&\xi=B\cos\left(2\pi\frac{\bar{\dot{x}}t}{\lambda}+\frac{u}{\lambda}\right)=B\cos\left(\omega t+\frac{u}{\lambda}\right),\\&\xi'=-\frac{2\pi}{\lambda}B\sin 2\pi\frac{x}{\lambda},\ \xi''=-\left(\frac{2\pi}{\lambda}\right)^2 B\cos 2\pi\frac{x}{\lambda}.\end{aligned} \tag{21}$$

For the case of zero approximation, when $\overline{x} = const$ is given and the dissipative forces are neglected, the steady state regime according to the equations (19, 20) by taking into account the equation (21) is obtained as:

$$u = \frac{n_x^2 - 4\mu\omega^2}{4\omega^2} K\left(-\frac{h_1}{\omega}\cos 2\omega t + \sin 2\omega t\right) - \frac{n_x^2 - \mu\omega^2}{\omega^2} L \sin\omega t,$$

$$z_1 = \mu K\left(-\frac{h_1}{\omega}\cos 2\omega t + \sin 2\omega t\right) - \mu L \sin\omega t,$$

$$z_2 = \frac{n_x^2 - 4\mu\omega^2}{n_y^2 - 4\mu\omega^2}\mu K\left(-\frac{h_1}{\omega}\cos 2\omega t + \sin 2\omega t\right) - \frac{n_x^2 - \mu\omega^2}{n_y^2 - \mu\omega^2}\mu L \sin\omega t,$$

where:

$$K = \frac{\pi\omega^2 B^2}{\lambda}\cdot\frac{1}{(1+\mu)n_x^2 - 4\mu\omega^2},\ L = \frac{2\pi B g}{\lambda}\cdot\frac{1}{(1+\mu)n_x^2 - \mu\omega^2}.$$

Case 4: the package is located on the bottom of the non-deformable box for packages and its motion with respect to the box is limited by the forces of friction between the package and the box for packages and also by the elastic member.

In this case when x, y, z_1 and z_2 are known variables as functions of time t, the differential equation of motion of the package is as follows:

$$m(\ddot{x} + \ddot{z}_1 + \ddot{\rho}) + H_0 m(g\cos\alpha + \ddot{y} + \ddot{z}_2)\operatorname{sgn}\dot{\rho} + H_1\dot{\rho} + mg\sin\alpha + C_\rho \rho = 0, \qquad (22)$$

where m is the mass of the package, ρ is the displacement of the package with respect to the bottom of the box according to the axis Ox, H_0 and H_1 are the coefficients of dry and viscous friction between the package and the bottom of the box, C_ρ is the coefficient of stiffness of the amortizing element between the package and the case of the box according to the axis Ox.

3. Choice of system parameters

In order to protect the packages and the objects inside from harmful vibrations it is necessary to choose the corresponding parameters of the system.

3.1. Limitation of the package displacement with respect to the case of the box

On the basis of the equation (22), for the package not to move with respect to the box during the motion of the transportation machine the following conditions must be satisfied:

$$g\cos\alpha + \ddot{y} + \ddot{z}_2 > 0,\ \ddot{\rho} = 0. \qquad (23)$$

For these conditions to be met, the following inequality must be valid:

$$\left|\frac{\ddot{x} + \ddot{z}_1 + g\sin\alpha + \frac{C_\rho}{m}\rho}{H_0(g\cos\alpha + \ddot{y} + \ddot{z}_1)}\right| < 1. \qquad (24)$$

In the case when $\alpha = \rho = 0$:

$$x + z_1 = A\cos\omega t,\ y + z_2 = B\sin\omega t,$$

inequality (24) becomes the following one:

$$\left|\frac{A}{H_0 B}\cdot\frac{-\cos\omega t}{\frac{z}{\omega^2 B} - \sin\omega t}\right| < 1, \qquad (25)$$

© VIBROENGINEERING. JOURNAL OF VIBROENGINEERING. DECEMBER 2012. VOLUME 14, ISSUE 4. ISSN 1392-8716

which is shown in Fig. 5.

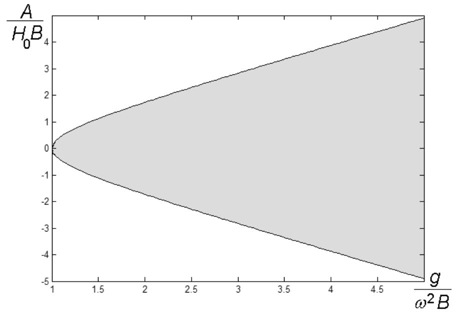
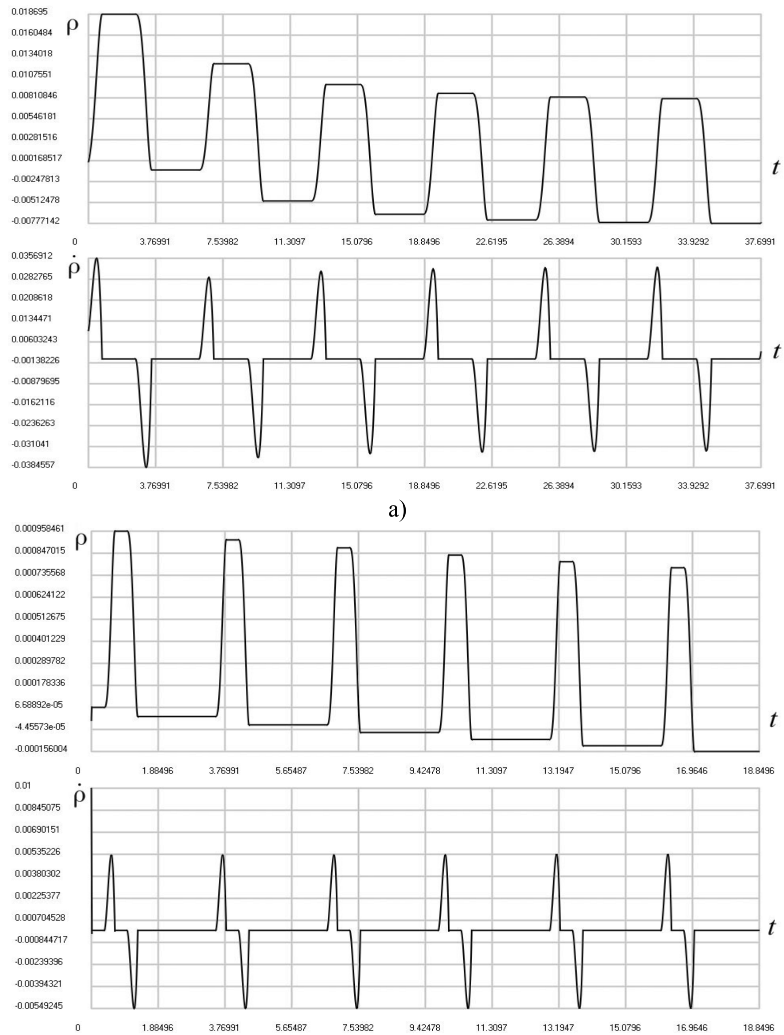
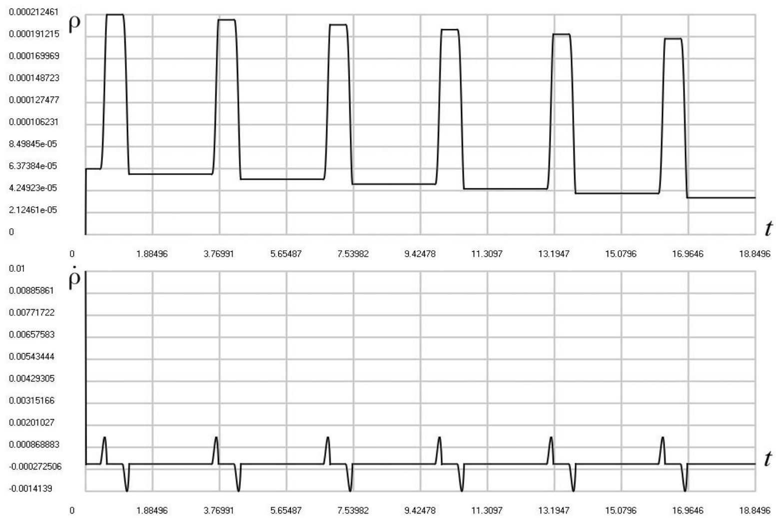


Fig. 5. The region of non-sliding motion is shown in grey color

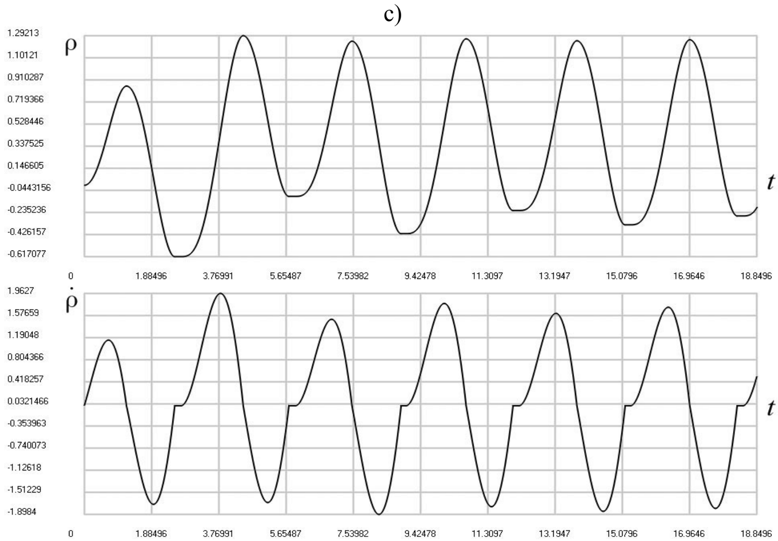
Fig. 6 illustrates package motions $\rho = \rho(t)$.



b)



c)



d)

Fig. 6. Motions of the package at $p = \sqrt{C_\rho / m} = 1$; $g = 9.8\ \mathrm{m}/\mathrm{s}^2$; $H_0 = 0.2$; $H_1 = 0$; $\alpha = 0$:

a) $\omega^2 A = 0.3$; b) $\omega^2 A = 0.5$; c) $\omega^2 A = 0.7$; d) $\omega^2 A = 1.0$

3.2. Maxmin of the differences of the forced frequencies and the eigenfrequencies

The influence of harmful vibrations may be reduced by choosing the parameters of the system in such a way that several first frequencies would be further from the resonances. Here the minmax problem is solved:

$$\max \min \left(i \cdot 2\pi \frac{\bar{\dot{x}}}{\lambda} - p_j \right), \tag{26}$$

where $i = 1, 2, \ldots,$ $\bar{\dot{x}} = \bar{\dot{x}}_i$ in the steady state regime, p_j are the eigenfrequencies of the box with packages.

In general, non-uniformities of the road profile are represented in the following way:

$$\xi = \xi(x) = \sum_{i=1}^{s} A_i \cos\left(2\pi i \frac{\bar{\dot{x}}}{\lambda_i} + \alpha_i\right), \tag{27}$$

where A_i are the amplitudes (i = 1, 2, …, s), x is the longitudinal coordinate, λ_i are the wavelengths and α_i are the phases.

In case when $x = \bar{\dot{x}}t$, the following notation is introduced:

$$\omega_i = 2\pi i \frac{\bar{\dot{x}}}{\lambda_i}, \tag{28}$$

$\bar{\dot{x}} = \text{const}$ – average velocity.

In order to avoid resonance vibrations and minimize the level of vibrations it is recommended to choose the velocity of box transportation in such a way that the minimum distance between the forced vibrations and the eigenfrequencies would be maximum, i.e.:

$$\max \min\left(\omega_i - p_j\right), \tag{29}$$

where p_j denotes the eigenfrequency with number i.

Length of the structure and width of the structure are equal to 2 m. On all the boundaries of the structure all the nodal displacements are assumed to be zero. The following parameters of the packages are assumed: modulus of elasticity E = 6000 Pa, Poisson's ratio ν = 0.3, density of the material ρ = 785 kg/m^3, while $\lambda = 10^9$ N/m^3. The first eigenvalues are listed in Table 1.

Table 1. The first eigenvalues

Number	$p_1^2, \left(\frac{\text{rad}}{\text{s}}\right)^2$	$p_2^2, \left(\frac{\text{rad}}{\text{s}}\right)^2$	$p_3^2, \left(\frac{\text{rad}}{\text{s}}\right)^2$
Value	14.6263	19.801	19.801

Eigenmodes of ten long packages in a box are analyzed. A long package is represented as consisting from ten plane strain elements. Other geometrical and physical parameters are assumed the same as in the previous problem. The first eigenvalues are listed in Table 2.

Table 2. The first eigenvalues

Number	$p_1^2, \left(\frac{\text{rad}}{\text{s}}\right)^2$	$p_2^2, \left(\frac{\text{rad}}{\text{s}}\right)^2$	$p_3^2, \left(\frac{\text{rad}}{\text{s}}\right)^2$
Value	14.7755	19.8687	20.5246

The obtained results are used in equation (29).

4. Conclusions

The motion of the transportation machine on an uneven road was considered in this paper. The conditions for avoiding damages of the transported package were determined. The conditions preventing the package from moving with respect to the transported box were identified. Minmax location of forced frequencies and eigenfrequencies was performed to ensure that the system is located further away from the resonances. All those problems were solved by choosing the velocity of motion of the transport system.

The proposed model is an idealized one, but the adopted assumptions enable easy determination of the eigenmodes of packages.

In this paper the influence of vibrations in the process of transportation of packages was analyzed by taking into account the non-uniformities of the road profile. The main structure of the performed investigation is presented here, while the detailed optimal synthesis and results of experimental investigations are the objects of further research work.

References

[1] **Vahidi A., Eskandarian A.** Predictive time-delay control of vehicle suspensions. Journal of Vibration and Control, November 2001, Volume 7, No. 8, p. 1195-1211.

[2] **Foley J. T., Gens M. B., Magnuson C. F.** Current predictive models of the dynamic environment of transportation. Environmental Sciences Technology, 1973, Vol. 26(1), p. 18-28.

[3] **Ostrem F. E., Rumerman M. L.** Shock and Vibration Transportation Environment Criteria. NASA, Final Report, 1965, NAS-8-11451.

[4] **Ostrem F. E., Rumerman M. L.** Transportation and Handling Shock and Vibration Environment Criteria. NASA, Final Report, 1967, NAS-8-11451.

[5] **Ostrem F. E., Godshall W. D.** An Assessment of the Common Carrier Shipping Environment. US Madison, Wisconsin, General Tech. Report FPL, 1979, No. 22, p. 1-54.

[6] **Zaabar I., Chatti K.** Impact of Road Roughness Features on Damage to Transported Goods. http://hvttconference.com/wp-content/uploads/2012/09/Ses_G_4_Zaabar-et-al.pdf

[7] **Hoppe W., Gerok J.** Vibration on the platforms of various utility transport vehicles and on the containers during loading and unloading in port. Journal of Society of Environmental Engineers, 1976, Vol. 15, p. 24-30.

[8] A Technical Summary of the Intermodal Environment Study. Association of American Railroads, Operations and Maintenance Department, Casualty Prevention Division, Damage Prevention, Freight Claims, 1991.

[9] **Rouillard V., Richmond R.** A novel approach to analyzing and simulating railcar shock and vibration. J. Packing Technol. Sci., 2007, Vol. 20, p. 17-26.

[10] **Rouillard V., Sek M. A.** Monitoring and simulating non-stationary vibrations for package optimization. Journal of Packing Technol. Sci., 2000, Vol. 13(4), p. 149-156.

[11] **Batt G., Marcondes J.** Dynamic Compression: Development of a New Method for Lab Simulation. Worldpak, 2002.

[12] Standard Practice for Performance Testing of Shipping Containers and Systems. ASTM D 4169-05.