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# STUDY OF THE SIMULATION AND OPTIMIZATION SYSTEM OF INTERBANK SETTLEMENTS

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**Abstract**. The aim of this paper is a computer study of the system for simulation and optimization of interbank settlements. The system is based on the Poisson-lognormal model of settlement flow and performs iterative simulation and optimization of expected transaction costs using randomly generated samples of transaction flows. The results of study by Monte-Carlo simulation are given, based on data of the payment and settlement system of the Bank of Lithuania.

**Keywords:** Interbank payments, settlements, modelling of interbank settlements, optimisation of settlement costs.

## 1. Introduction

Active introductions of the means of electronic data transfer in banking and concentration of a great part of settlements at the centres of interbank payments were related with the creation of an automated system of clearing. The main purpose of such systems is to warrant a fast and rational turnover of settlements, to balance payments, and to reduce the movement of money supply. These systems should provide the principles of stability, efficiency, and security. Participants of the system must satisfy the requirements of liquidity and capital adequacy measures. The owner, operator, and supervisor of such a system by default are the central bank. It installs a request for the participants of the system, conducts supervision over their performance and takes measures that guarantee a stable system operation.

The target of this paper is computer study of the system for simulation and optimization of interbank settlements. The system is based on the Puasson-lognormal model of settlements flow and performs iterative simulation and optimization of expected transaction costs using randomly generated samples of transaction flows. The main parts of the system are described by Bakšys and Sakalauskas, 2007 [1]. The results of the system study by Monte-Carlo simulation are given based on data of the payment and settlement system of the Bank of Lithuania.

## 2. A description of simulation and optimization system of settlements

The principal scheme of modelling, simulation, and optimization of the settlements system should consist of the following parts:

- the subsystem of analysis and calibration;
- the subsystem of simulation and optimization.

In Figure 1, we present the scheme of modelling, simulation and optimisation of the settlements system.

The data are scanned in the part of statistical analysis. These data are used to calibrate the settlement model and compute the parameters. The calibration procedures are developed on the base of the Poisson-lognormal model [1].

In the part of generating the settlement flow, a fixed number of applications is generated using the generator of random numbers by means of the Poisson-lognormal model [1]. In the part of simulation of the settlement process the time of transactions are simulated considering the address of applications and liquidity characteristics. In the subsystem of analyses of costs and liquidity, the settlement costs and the fixed loss of liquidity are computed. In the part of optimization, different strategies for management of the correspondent account of participants and the central bank are explored. In the simulation process a continuous net settlement system is realized when transactions are booked immediately [3].

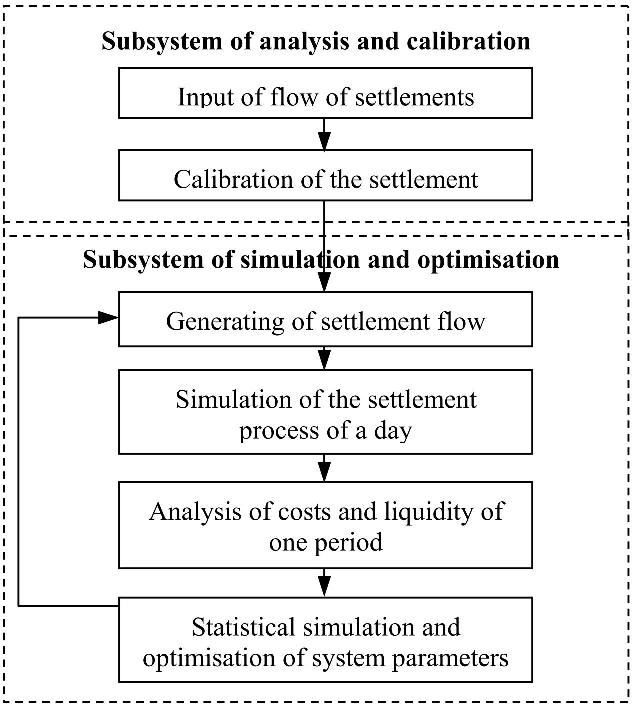


**Figure 1.** The system of modelling, simulation, and optimization of settlements

### 2.1. Modelling and simulation of data

Let us consider the system consisting of $J$ agents, who execute payments between themselves. We call by agents the participants of a system: banks, foreign bank branches, credit unions, and other financial or clearing institution members of the payment and settlement system. The participants send applications to the payment and settlement system. Each application is described in the system by the name of a sender, name of the addressee, moment of delivery of the application, and the volume of transaction.

The receipt of real data is bound up with a problem of confidentiality. Usually the institutions which take part in interbanking operations avoid to reveal the data of transactions. Exceptionally it is possible to receive encoded data.

We consider anonymous data of the interbank settlement session of a typical labour day presented by the Bank of Lithuania. These data consist of 74637 applications of $J=11$ participants of the Payment and Settlement System. The data include the code name (number) of a participant of the payment and settlement system, time of delivery of the applications, volume of applications, and the flow of applications. Further we use the term "payment" instead of "payment order", for simplicity.

According to the transaction model used the system generates flows of moments of bilateral payments by the Poisson distribution and the corresponding flow of payment volumes according to lognormal distribution. The primary data were obtained from these data by an imitative model of generation of payments flow:

- frequency of submission of applications;
- average of the value of one transfer;
- standard deviation of one transfer.

### 2.2. Simulation of settlement costs

A successful performance of the payment system is guaranteed by keeping sufficient sums in the correspondent accounts. The correspondent account of the $l^{\text{th}}$ day consists of the correspondent account of the previous day, net balance, and the deposited (or withdrawn) amount of asset of the participant himself. Thus, the amount on the correspondent account may be computed as follows:

$$K_i^l = \max(0, K_i^{l-1} + \delta_i^l + G_i^l), \tag{1}$$

where, $\delta_i^l$ is the balance, $K_i^l$ is the correspondent account residue, $G_i^l$ is the deposited or withdrawn sum of the bank $i$ for day $l$ [1].

Let us analyze how banks can manage settlement costs by depositing (or withdrawing) assets on the correspondent account. We consider the policy when banks deposit or withdraw certain fixed sums $X_i$. When computing operational costs we have to take into consideration that a bank cannot withdraw a sum larger than that present in the corresponding account. Thus, after simple considerations, we have that the deposited or withdrawn amount is as follows:

$$G_i^l = \max\left(X_i, -\max\left(K_i^{l-1} + \delta_i^{l-1}, 0\right)\right). \tag{2}$$

Insufficient sums of the clearing accounts cannot satisfy the credit obligations, because this fact destabilizes interbank payments and sets gridlocks in the payment and settlement system. The Central bank allows borrowing overnight loans and installs reserve requirements to the settlement system participants in order to prevent the illiquidity in the payment system. Therefore the Central bank establishes reserve requirements $RR_i$ for the participants of the settlement system. The reserve requirements depend on liabilities of a participant.

In order to study the policies of credit and liquidity risk control, we consider operational costs of settlements. The total cost of settlements of the $i^{\text{th}}$ agent during one period consists of several parts:

$$D_i = RE_i + F_i + B_i + TT_i + AC_i \tag{3}$$

where $RE_i$ is the premium for deposit, $F_i$ is the pay of nonconformity of reserve requirements, $B_i$ is the cost of short-term loans, $TT_i$ is the indirect bank losses due to the freeze of the deposited amount of assets (or possible profit of withdrawal) in the correspondent account, and $AC_i$ is the operation cost. The main parts of the total costs of settlements are described by Bakšys and Sakalauskas, 2007 [1]

The payment and settlement system is characterised by total settlement costs

$$D = \sum_{i=1}^{J} D_i , \tag{4}$$

Let us calculate the average costs of service by simulating a few periods of settlements.

Denote the cost of transactions during one period by $D_i = D_i(X_i, \delta_i)$, which is a random function in general, depending on the deposit $X_i$ and the vector of balances of the correspondent account $\delta_i = (\delta_i^1, \delta_i^2, \ldots, \delta_i^T)$. Let us denote the expected cost during one period by

$$L_i(X_i) = ED_i(X_i, \delta_i). \tag{5}$$

Using the formulas of stochastic differentiation [6], we can compute the subgradients $\partial_x D_i(X_i, \delta_i)$. It is easy to make sure that expectation of the subgradient of the cost function yields the gradient of expected costs [4]:

$$\frac{dL_i(X_i)}{dX_i} = E\partial_x D_i(X_i, \delta_i). \tag{6}$$

Let $N$ periods of settlement performance be simulated and random vectors of incomes and outcomes $\delta_{i,n}$, $1 \le n \le N$, $1 \le i \le J$, be generated. Thus, the statistical estimate of settlement costs are the average cost:

$$\tilde{L}_i(X_i) = \frac{1}{N}\sum_{n=1}^{N} D_i\left(X_i, \delta_{i,n}\right). \tag{7}$$

The Monte-Carlo estimator of gradient (6) is obtained by virtue of:

$$Q_i(X_i) = \frac{1}{N}\sum_{n=1}^{N} \partial_x D_i\left(X_i, \delta_{i,n}\right). \tag{8}$$

Denote the vector of agent impact on its correspondent account as $X = (X_1, \ldots, X_J)$. The quality of the settlement system can be defined by the general expected cost

$$\tilde{L}(X) = \sum_{i=1}^{J} \tilde{L}_i(X_i). \tag{9}$$

During the simulation the sampling variance can be computed

$$d_N^2(X) = \frac{1}{N}\sum_{n=1}^{N}\left(D^n - \tilde{L}(X)\right)^2, \tag{10}$$

where $D^n = \sum_{i=1}^{J} D_i\left(X_i, \delta_{i,n}\right)$, $1 \le n \le N$.

## 3. Statistical optimisation of settlement costs

We develop a statistical optimization procedure for minimizing the costs using the approach of stochastic nonlinear programming by the Monte-Carlo estimators [5]. Let some initial vector of agents deposits $X^0 = \left(X_1^0, X_2^0, \ldots, X_J^0\right)$ be given, and a random sample of income and outcome vectors be generated. Let the initial sample size be $N^0$. Now, the Monte-Carlo estimators of the gradient of expected costs are computed according to (9). Next, an iterative stochastic procedure of gradient search could be introduced:

$$X^{t+1} = X^t - \rho \cdot Q_i(X_i), \tag{11}$$

where $\rho > 0$ is a certain step-length multiplier.

We choose the sample size at each next iteration inversely proportional to the square of the gradient estimator from the current iteration:

$$N^{t+1} = \frac{n \cdot Fish(\gamma, n, N^t - n)}{\rho \cdot Q(X^t) \cdot (A(X^t))^{-1} \cdot (Q(X^t))'}, \tag{12}$$

where $Fish(\gamma, n, N^t - n)$ is the $\gamma$-quintile of the Fisher distribution with $(n, N^t - n)$ degrees of freedom.

The step length $\rho$ could be chosen experimentally. We introduce minimal and maximal values $N_{min}$ and $N_{max}$ are to avoid great fluctuations of sample size in iterations. Usually $N_{\min}$ ~500-1000 and $N_{max}$ chosen from the conditions on the permissible confidence interval of estimates of the objective function [1].

It is convenient to test the hypothesis of equality to zero of the gradient by means of the well-known multidimensional Hotelling $T^2$-statistics [2]. Hence, the optimality hypothesis could be accepted for some point $X^t$ with the significance value $1-\mu$, if the following condition is satisfied:

$$(N^t - n)\cdot(Q(X^t))\cdot(A(X^t))^{-1}\cdot(Q(X^t))/n \le Fish(\mu, n, N^t - n)\,. \quad (13)$$

Next, we decide that the objective function is estimated with a permissible accuracy $\varepsilon$, if its confidence bound does not exceed this value:

$$\eta_\beta \cdot d_{N^t}(X^t)/\sqrt{N^t} \le \varepsilon\,, \quad (14)$$

where $\eta_\beta$ is the $\beta$ -quintile of the standard normal distribution, and the standard deviation $d_{N^t}$ is defined by (10). Thus, the procedure (11) is iterated adjusting the sample size according to (12) and testing conditions (13) and (14) at each iteration. If the latter conditions are met at some iteration, then there are no reasons to reject the hypothesis on the optimality of the current solution. Therefore, there is a basis to stop the optimization and make a decision on the optimum finding with a permissible accuracy. If at least one condition in (13), (14) is violated, then the next sample is generated and the optimization is continued. As follows from the previous section, the optimization should stop after generating a finite number of Monte-Carlo samples.

## 4. Results of simulation and optimization

In this section, we present some Monte-Carlo simulation results, which were obtained using the proposed model calibrated with respect to real data. The parameters of the Poisson-lognormal model were taken from Bakšys and Sakalauskas [1]. Figures 2-4 illustrate the dependencies of the average settlement costs on the number of iteration for the 1st, 9th, and 10th participants. Analogous dependences are similar for other agents. In Figure 5, the dependence of the average total settlements costs on the number of iteration is presented. Figure 6 shows dynamics of the Monte-Carlo sample size during the optimization. In Figure 7, we give a histogram of the iteration number needed for algorithm termination.

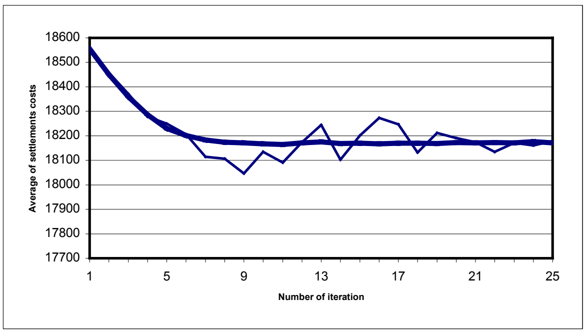


**Figure 2.** Dependence of the average settlement costs of the 1st participant on the number of iteration

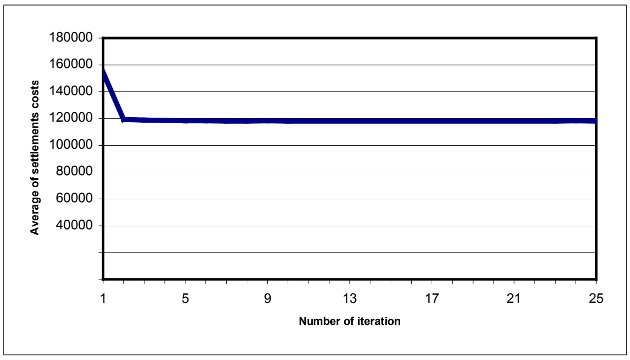


**Figure 3.** Dependence of the average settlement costs of the 9th participant on the number of iteration

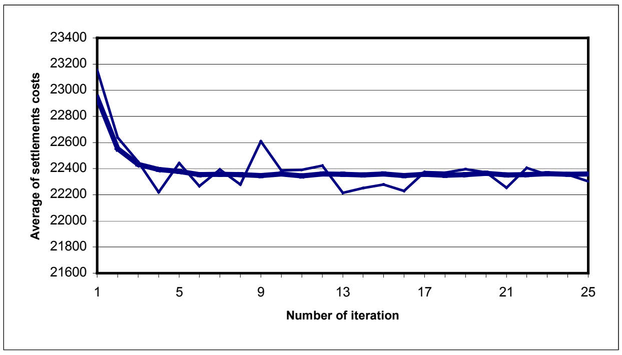


**Figure 4.** Dependence of the average settlement costs of the 10th participant on the number of iteration

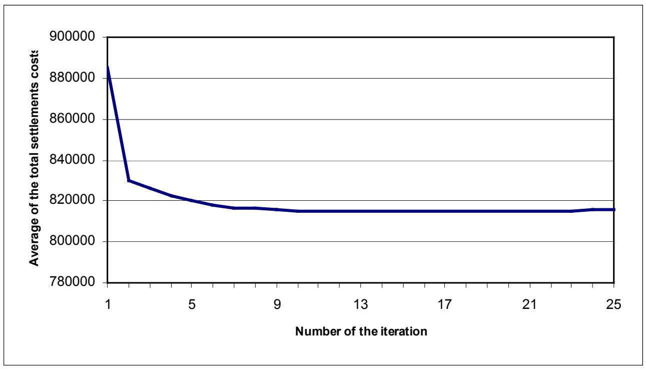


**Figure 5.** Dependence of the average total settlement costs on the number of iteration

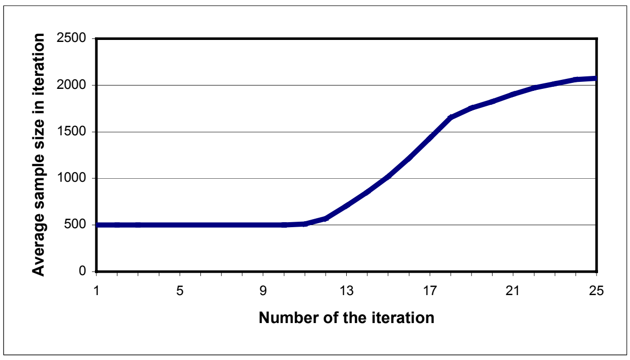


**Figure 6.** Dependence of the average sample size on the number of iteration

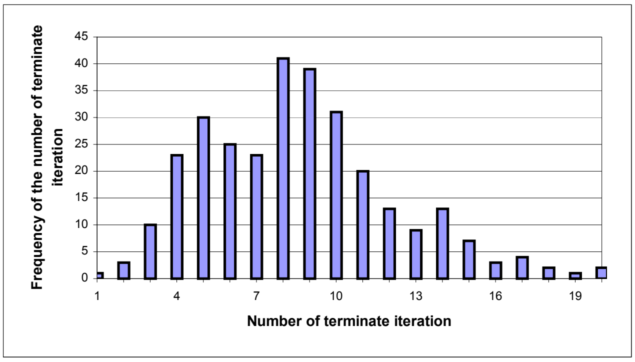


**Figure 7.** Histogram of the iteration numbers for algorithm termination

## 5. Conclusions

The growth of non-cash payments, and the need to execute real-time payments invokes new challenges to electronic systems of interbank clearing.

In this paper, the system of simulation and optimisation of interbank settlements is studied by Monte-Carlo simulation. We analyze how banks can manage the settlement costs by depositing (or withdrawing) assets on the correspondent account. The policy when banks deposit or withdraw certain fixed sums is considered. The stochastic optimisation method to regulate the correspondent agent's account has been developed by Monte-Carlo method and investigated by computer simulation. Since only the methods of the first order work in stochastic optimization, we have confined ourselves to the gradient-descent type methods. The approach surveyed in this paper is grounded by the stopping procedure and the rule for iterative regulation of the size of Monte-Carlo samples, taking into account the stochastic model risk. The stopping procedure proposed allows us to test the optimality hypothesis and to evaluate the confidence intervals of the objective and constraint functions in a statistical way. The regulation of sample size, when this size is taken inversely proportional to the square of the norm of the gradient of the Monte-Carlo estimate, allows us to solve stochastic optimization problems rationally in computational terms and guarantees the convergence a.s. at a linear rate. The numerical study and the practical example corroborate theoretical conclusions and show that the procedures developed enable us to solve optimization problems with a permissible accuracy by using the acceptable volume of computations (14-35 iterations and 20000 total Monte-Carlo trials).

The outcome of the performed simulation shows that, by applying the given model of the income of a Clearinghouse as well as information technologies, it is possible to optimize the parameters for management of risks of the credit, liquidity, and operational costs.

Simulation and optimization of transaction costs illustrate an opportunity for banks to maximize the future profit. In this situation, it is especially important to study the strategies of management by banks of their correspondent accounts in the Clearinghouse.

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