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Insertion simulation cylindrical parts under kinematical excitation of mobile based part

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1. Introduction

The main stages of automated assembly are matching of connective surfaces and insertion of the parts. Successful and competitive manufacture requires to increase productivity, reduce equipment cost and ensure maximal quality. The most effective solution for increasing the productivity of assembly operations is to increase the insertion speed [1]. Quasi-static model of parts insertion, analyzed in the majority of papers [2, 3], is not suitable for research of high speed insertion process. The high speed insertion process is a complex dynamic process, which is under the influence of gravity, inertia, insertion speed, stiffness of basing device, friction, etc. High insertion speed causes high insertion forces, therefore the probability of jamming increases. When the parts to be assembled get jammed, assembly equipment damages, manufacture disconcerts. Jamming can be avoided by exiting vibrations of the parts to be assembled [4].

Insertion process without using vibratory excitation was analyzed in paper [5]. Mathematical model of insertion was presented. Conditions for the shortest duration and the highest reliability of insertion process were determined.

The problem of alignment and matching of the parts is mainly investigated in the literature about vibratory assembly. Paper [6] analyses motion of a part on an excited in two perpendicular directions horizontal plane. Motion of the part was investigated by mathematically modeling analyzing how by changing excitation amplitude, frequency, direction, and coefficient of friction it is possible to perform easy and fast orienting of the parts in order to join it. Paper [7] presents experimental investigation of vibratory alignment and matching of the parts to be assembled automatically, under kinematical excitation of mobile based part. Results obtained performing alignment of circular and rectangular chamferless parts under different excitation, initial pressing force and parts misalignment conditions are given. Excitation parameters which determine the reliability of alignment and matching of the parts were detected.

Paper [8] analyses insertion process of the parts. A device for high speed and precision chamferless assembly, using vibratory technology, is proposed. The device consists of work table, driven by vibratory motion, provided by two pneumatic bellows actuators, piloted by the pseudo-random binary signal and dynamic compliance device. However, design of the proposed device is quite complex. The influence of insertion parameters on insertion process was not determined in the paper. Also the influence of vibration on insertion process and its duration was not determined.

In order the process of vibratory assembly to be reliable and effective, it is necessary to ensure that the parameters, which make an influence on insertion process, were rationally selected.

This paper investigates the insertion of mobile based peg into immobile based bush with guaranteed clearance, under kinematical excitation of the peg in axial direction. With properly selected parameters of kinematical excitation it is possible to avoid insertion process failures due to jamming and to obtain minimal duration of the insertion process.

2. Mathematical model of parts insertion process

Insertion of mobile based peg, which is moved in constant velocity v, into immobile bush is investigated (Fig. 1). The peg is hold by the gripper and only can turn around the centre of compliance C and slip in radial direction. The gripper is excited by vibration of amplitude A_e and frequency ω, in z axis direction. These vibrations are transferred to the peg through elastic elements. This kind of excitation is called kinematical.

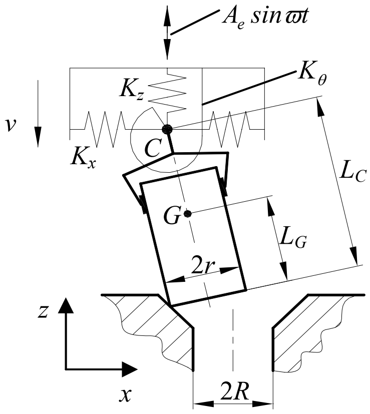


Fig. 1 Scheme of vibratory insertion

In the compliance center of the peg vibratory excitation force F_e acts, elastic component of the insertion force along x axis F_x, along z axis F_z and torsion moment M about point C

$$F_e = K_z A_e \tag{1}$$

$$F_x = K_x \left(x_C - x_{C0} \right) \tag{2}$$

$$F_z = K_z \left(v \cdot t + z_C - z_{C0} \right) \tag{3}$$

$$M = K_\theta \left(\theta - \theta_0 \right) \tag{4}$$

where t is time; x_{C0} and z_{C0} are coordinates of point C at the initial instant of time; θ is tilt angle of the peg; θ_0 is tilt angle of the peg at the initial instant of time; K_x, K_z, K_θ are lateral stiffness, axial stiffness and angular stiffness respectively.

Insertion process can be divided into three stages: chamfer crossing stage, one point contact and two point contact stages.

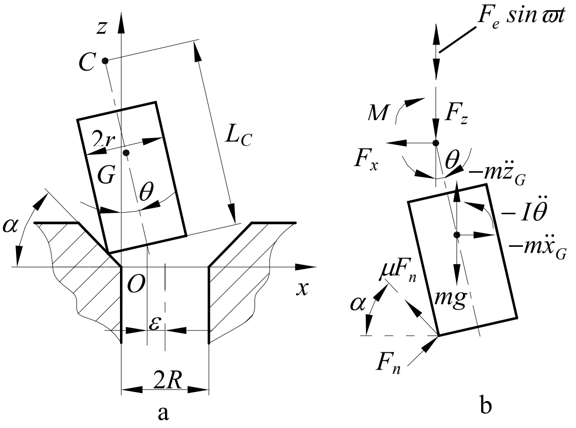


Fig. 2 Peg and chamfer contact

During the assembly process, the peg axis and the hole misalign by distance ε due to positional errors, therefore sliding downward the peg touches the chamfer. Then insertion process starts (Fig. 2, a). During the chamfer crossing stage (Fig. 2, b), the peg is influenced by elastic components of insertion forces and torque (F_x, F_z, M), vibratory excitation force $F_e \sin\omega t$, gravity mg, inertia forces $m\ddot{x}_G$, $m\ddot{z}_G$, inertia torque $I\ddot{\theta}$, reaction force F_n, friction force μF_n; where m is the mass of the peg and the gripper, I is inertia moment of the peg and the gripper about mass centre, g is gravitational constant, μ is dry friction coefficient, x_G, z_G are the coordinates of the mass centre.

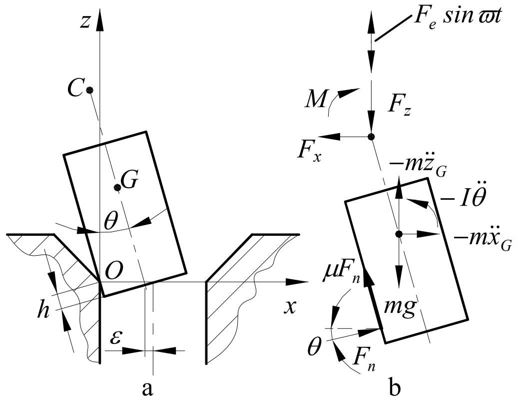


Fig. 3 One point contact stage

Applying D'Alembert principle, movement of the peg during the chamfer crossing stage is determined by the following equations

$$\left.\begin{aligned}
&K_1F_n - F_x - m\ddot{x}_G = 0\\
&K_2F_n - F_z - F_e \sin\omega t - m\ddot{z}_G - mg = 0\\
&\left(F_x \cos\theta + F_z \sin\theta + F_e \sin\omega t \sin\theta\right)\left(L_C - L_G\right) +\\
&+ K_3F_n - M - I\ddot{\theta} = 0
\end{aligned}\right\} \tag{5}$$

where $K_1 = \sin\alpha - \mu\cos\alpha$; $K_2 = \cos\alpha + \mu\sin\alpha$; $K_3 = \left(L_GK_1 - rK_2\right)\cos\theta + \left(L_GK_2 + rK_1\right)\sin\theta$; α is the chamfer angle; L_G is the distance from the lower end surface of the peg to the centre of mass; L_C is the distance to the center of compliance.

The peg slides down the chamfer until cylindrical surface of the peg touches the hole. Insertion process steps into the one point contact stage (Fig. 3, a). After the evaluation of acting forces on the one point contact stage (Fig. 3, b), movement of the peg in the hole is determined by the following equations

$$\left.\begin{aligned}
&K_4F_n - F_x - m\ddot{x}_G = 0\\
&K_5F_n - F_z - F_e \sin\varpi t - m\ddot{z}_G - mg = 0\\
&\left(F_x \cos\theta + F_z \sin\theta + F_e \sin\varpi t \sin\theta\right)\left(L_C - L_G\right) +\\
&+ \left(L_G - h - \mu r\right)F_n - M - I\ddot{\theta} = 0
\end{aligned}\right\} \tag{6}$$

where $K_4 = \cos\theta - \mu\sin\theta$; $K_5 = \sin\theta + \mu\cos\theta$; h is insertion depth.

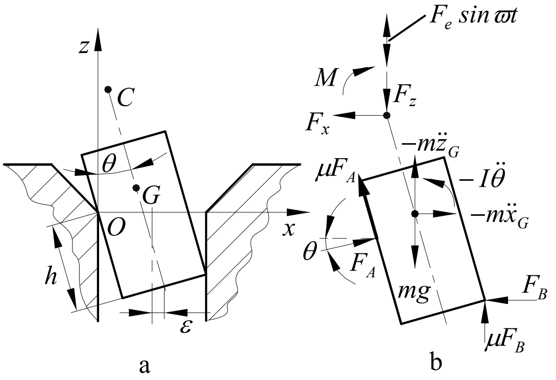


Fig. 4 Two point contact stage

The peg contacts with the hole in one point until the lower edge of the peg reaches internal surface of the hole. Insertion process steps into the two point contact stage (Fig. 4, a). After the evaluation of acting forces in the one point contact stage (Fig. 4, b), movement of the peg in the hole is determined by the following equations

$$\left.\begin{aligned}
&K_4F_A - F_B - F_x - m\ddot{x}_G = 0\\
&K_5F_A + \mu F_B - F_z - F_e \sin\varpi t - m\ddot{z}_G - mg = 0\\
&\left(F_x \cos\theta + F_z \sin\theta + F_e \sin\varpi t \sin\theta\right)\left(L_C - L_G\right) - M -\\
&- I\ddot{\theta} + \left(L_G - h - \mu r\right)F_A - \left(L_GK_4 - rK_5\right)F_B = 0
\end{aligned}\right\} \tag{7}$$

The parts are completely assembled when the required depth of insertion h_i is reached.

The peg can get jammed due to improperly selected parameters of insertion process. Jamming is a condition in which the peg can not move into the hole due to poorly proportioned applied forces and moments. Jamming usually occurs in the two point contact stage. To avoid jamming, it is necessary the applied forces and moment to be greater than or at least be equal to the reaction forces and moment

$$\begin{aligned}
&\left(F_x \cos\theta + F_z \sin\theta + F_e \sin\varpi t \sin\theta\right)\left(L_C - L_G\right) -\\
&- M \ge I\ddot{\theta} - \left(L_G - h - \mu r\right)F_A + \left(L_GK_4 - rK_5\right)F_B
\end{aligned} \tag{8}$$

3. Simulation of insertion process

Programs for the simulation of insertion process were written using MatLab software.

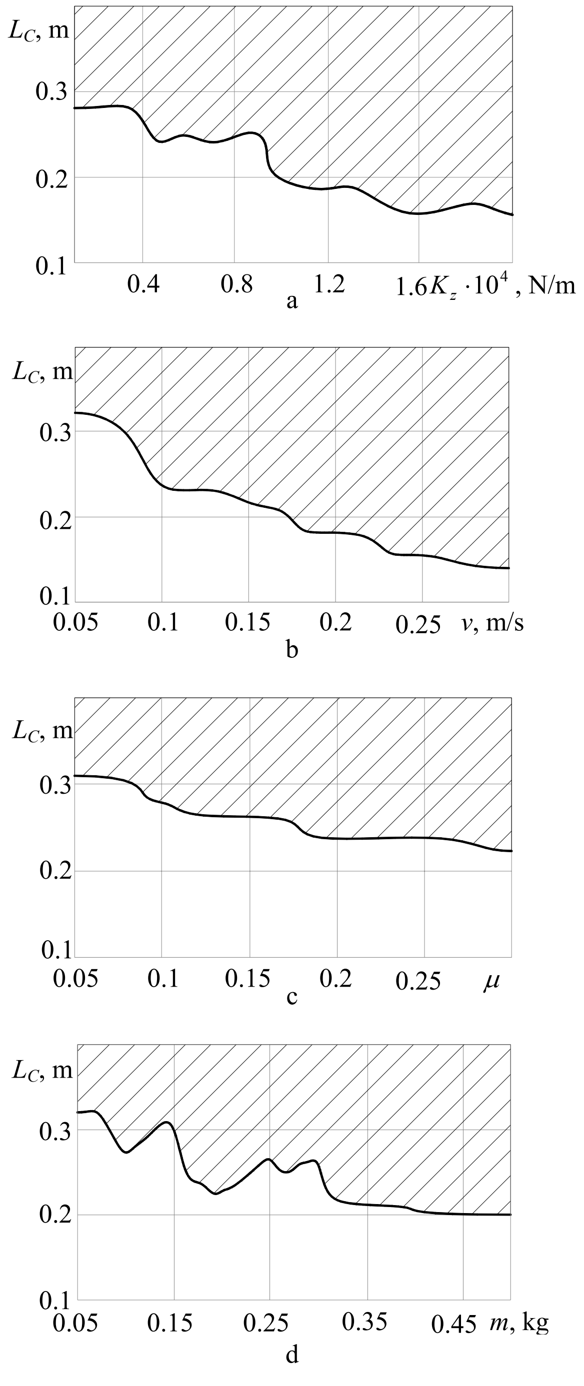


Fig. 5 Area of jamming (hatched) depending on the distance to compliance centre L_C: a – axial stiffness K_z; b – insertion speed v; c – coefficient of friction μ; d – mass m. K_z = 2000 N/m

The following initial values of the parameters of insertion process were used: $m = 0.1$ kg, $r = 0.0099$ m, $R = 0.01$ m, $I = 0.002$ kg m^2, $L_G = 0.05$ m, $L_C = 0.025$ m, $\alpha = \pi / 4$ rad, $\mu = 0.1$, $v = 0.1$ m/s, $\theta_0 = 0.01$ rad, $\dot{\theta}_0 = 0.01$ rad/s, $\varepsilon_0 = -0.001$ m, $\dot{\varepsilon}_0 = 0.001$ m/s, $K_x = 2000$ N/m, $K_z = 10000$ N/m, $K_\theta = 20$ Nm/rad, $h_i = 0.05$ m.

The simulation showed that the highest influence on jamming makes the distance from the lower end surface of the peg to the centre of compliance L_C. Jamming occurs under high values of L_C (Fig. 5, a-d; Fig. 6, b). Also the probability of jamming increases when axial stiffness K_z , peg insertion speed v, coefficient of friction μ, mass of the peg and the gripper m are increasing (Fig. 5, a-d). Assembly clearance also makes an influence on jamming. The probability of jamming is higher under the lower clearance ratio (Fig. 6, a, b). The increase of angular stiffness K_θ has slight influence on jamming (Fig 6, a).

Also it is noticed that the parts get jammed under very high values of lateral K_x and axial K_z stiffness, due to harder compensation of misalignment errors.

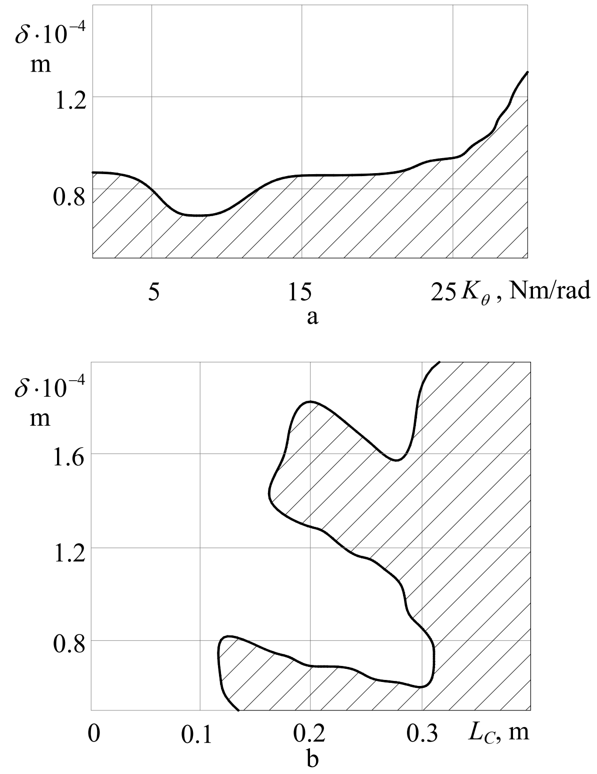


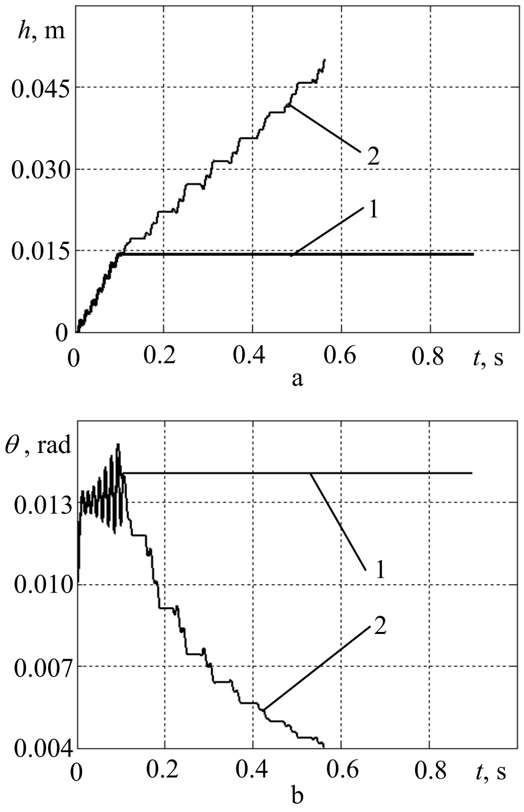
Fig. 6 Area of jamming (hatched) depending on assembly clearance δ and a – angular stiffness K_θ; b – distance L_C. K_z = 2000 N/m

Connection of jammed parts is possible by removing the causes of jamming. For this purpose, axial vibration of the peg can be excited. Due to this, balance of acting forces and moments on the peg changes and insertion process continues.

During the insertion process, when L_C = 0.3 m and $K_z = 10^4$ N/m, when the peg is inserted into the hole at particular depth, jamming occurs. As it seen from curve *1*, insertion process halts, insertion depth (Fig. 7, a) and tilt angle of the peg (Fig. 7, b) do not change more.

Under the same parameters of insertion process, the peg is kinematically excited in axial direction by vibration amplitude $A_e = 0.5 \cdot 10^{-3}$ m and frequency $\omega = 100$ s^{-1}. When the insertion depth is reached wherein jamming occurs, as it seen from curve *2*, the insertion depth keeps increasing (Fig. 7, a), while the tilt angle of the peg is decreasing (Fig. 7, b), until the specified insertion depth is reached and the parts get completely connected.

The effect of vibratory excitation is clearly represented by hatched areas of distance L_C and axial stiffness K_z combinations, under which the peg is not completely inserted into the bush hole. Without using vibrations (Fig. 8, area *1*), the peg gets jammed under high values of distance L_C, therefore insertion process is not finished. The peg gets jammed more often under higher values of axial stiffness also. When mobile based part is kinematically excited in axial direction by vibration amplitude $A_e = 0.5 \cdot 10^{-3}$ m and frequency $\omega = 100$ s^{-1}, the area of not successful insertion distinctly decreases (Fig. 8, area *2*). The parts are not completely connected only in the narrow area, under very high values of distance L_C and low values of axial stiffness K_z.



a

b

Fig. 7 Variation of insertion depth (a) and tilt angle of the peg (b) during insertion process: *1* – without using vibrations; *2* – under kinematical excitation of mobile based part

In order the insertion process of vibratory assembly to run reliably, it is necessary to select properly the values of vibration amplitude and frequency. Areas which represent the combinations of excitation amplitude A_e and frequency ω, under which the parts get jammed, are showed in Fig. 9. Following from an arrangement of the areas, it is advantageous to select lower values of excitation amplitude. The parts do not connect under low excitation frequency, besides under excitation frequency ω = 35–50 s^{-1} and high excitation amplitude, the area of not stable insertion emerges. Consequently, higher vibratory excitation frequency ensures more favorable conditions for the insertion process.

Axial stiffness K_z has high influence on the insertion process, because mobile based part is excited in axial direction. Following from the combinations of axial stiffness and excitation frequency when the parts get jammed, the insertion process is not reliable under too low values of axial stiffness (Fig. 10, a). It is not possible to connect jammed parts when vibration amplitude is too low (Fig. 10, b). Following from the combinations of axial stiffness and excitation amplitude when the parts are not connected, the parts do not connect when axial stiffness is too low. Areas of not reliable insertion emerge under high values of excitation amplitude and axial stiffness. Therefore it is important to select rational value of axial stiffness K_z (Fig. 10, b).

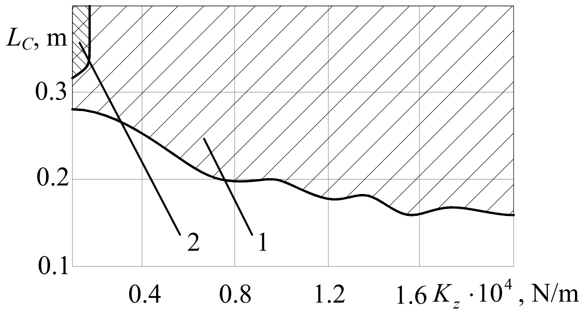


Fig. 8 Areas of distance to compliance centre L_C and axial stiffness K_z combinations when the parts are not connected (hatched): *1* – without using vibrations; *2* – under kinematical excitation of mobile based part

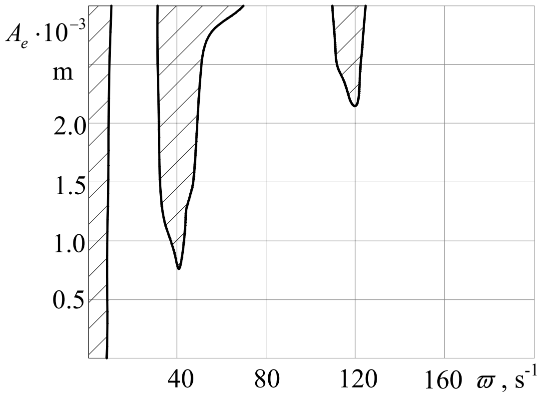


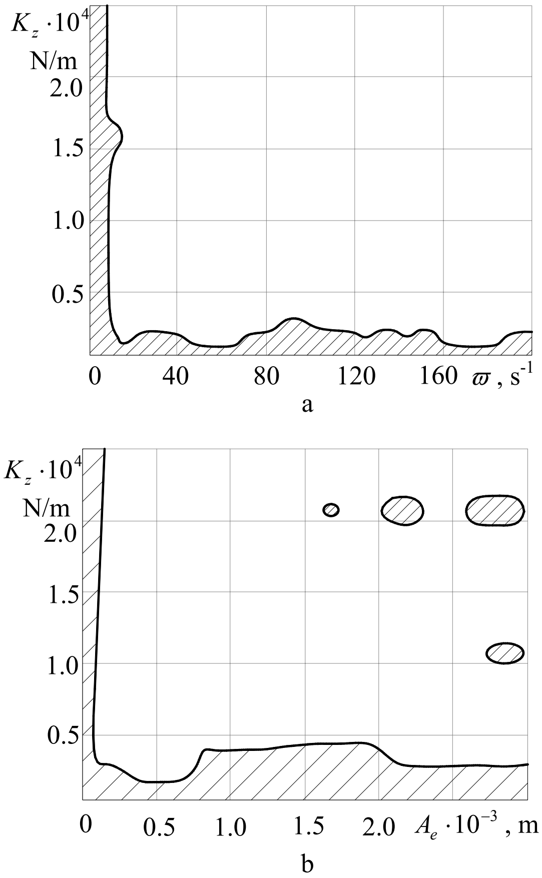
Fig. 9 Areas of excitation frequency ω and amplitude A_e combinations when jammed parts are not connected (hatched): K_z = 2500 N/m, L_C = 0.4 m

Distance L_C to the centre of compliance makes noticeable influence on jamming. Areas of distance L_C and excitation frequency ω and excitation amplitude A_e combinations, when the components are not connected due to jamming, were determined. Under kinematical excitation, mobile based part can not be inserted into the base part only when excitation frequency is too low (Fig. 11, a), the value of excitation amplitude and distance L_C (Fig. 11, b) is high.

Dependencies of insertion stages durations on the system parameters were determined.

Chamfer crossing duration t_1 is the duration from the beginning of insertion process to the occurrence of one point contact. The simulation showed that when excitation frequency ω is increasing, chamfer crossing duration t_1 decreases (Fig. 12, a).

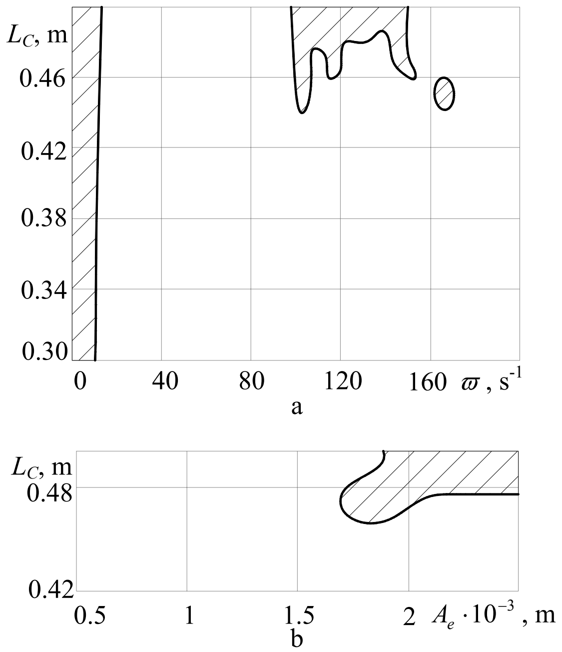
Total insertion duration t_3 is the duration from the beginning of insertion process until the parts are completely inserted. It consists of the chamfer crossing dura-



a

b

Fig. 10 Area, wherein the parts get jammed (hatched), depending on axial stiffness K_z: a – excitation frequency ω; b – excitation amplitude A_e. L_C = 0.4 m



a

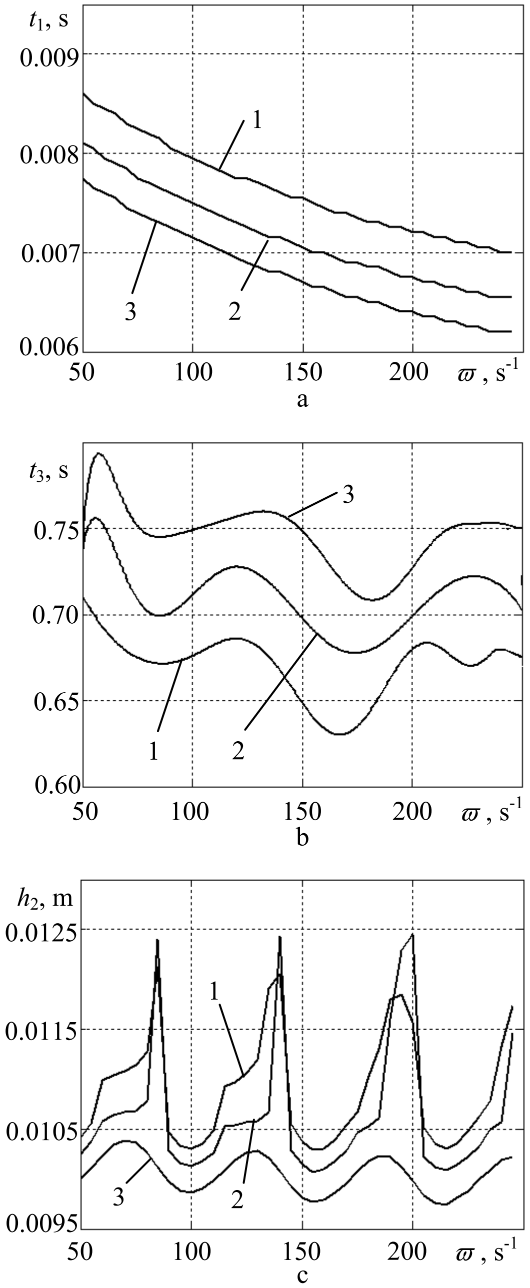
b

Fig. 11 Area, wherein the parts get jammed (hatched), depending on the distance to compliance centre L_C: a – excitation frequency ω; b – excitation amplitude A_e. K_z = 3000 N/m

tion t_1, one point contact duration and two point contact duration.

When excitation frequency is increasing, the insertion process duration unevenly decreases. It decreases more noticeable under excitation frequency ω = 150-200 s^{-1} (Fig. 12, b).

Excitation amplitude has more noticeable influence on the insertion process duration. The duration of insertion process t_3 increases when excitation amplitude A_e is increasing (Fig. 13, a). The duration of insertion process increases because amplitude of uneven movement of the peg increases under the influence of higher excitation amplitude.



a

b

c

Fig. 12 Dependences of: a – chamfer crossing duration t_1; b – insertion process duration t_3; c – depth h_2 wherein the two point contact appears, on excitation frequency under different values of axial stiffness: *1* – $K_z = 2 \cdot 10^4$ N/m; *2* – $K_z = 2.5 \cdot 10^4$ N/m; *3* – $K_z = 3 \cdot 10^4$ N/m. L_C = 0.075 m

Reliability of insertion process depends on the depth h_2, wherein the two point contact appears. It is noticed, that wedging or jamming usually occurs when the two point contact appears in a small depth. Besides, the probability increases that the peg will jump out of the hole when the two point contact appears in a small depth due to its uneven movement. Therefore, it is necessary to select such values of insertion process parameters, which influence the higher value of depth h_2.

When excitation frequency ω is increasing, the depth h_2, wherein the two point contact appears, varies with pulsation (Fig. 12, c). In particular frequency intervals, the depth h_2 more significantly increases, under lower axial stiffness K_z.

When excitation amplitude is decreasing, the two point contact appears in higher depth (Fig. 13, b), therefore the insertion process is more reliable.

After the evaluation of numerical experiment results, the conclusion could be made, that from the viewpoint of reliable and effective insertion, the most acceptable excitation frequency is ω = 80–140 s^{-1}, excitation amplitude $A_e = 0.5\cdot10^{-3}$–$1\cdot10^{-3}$ m.

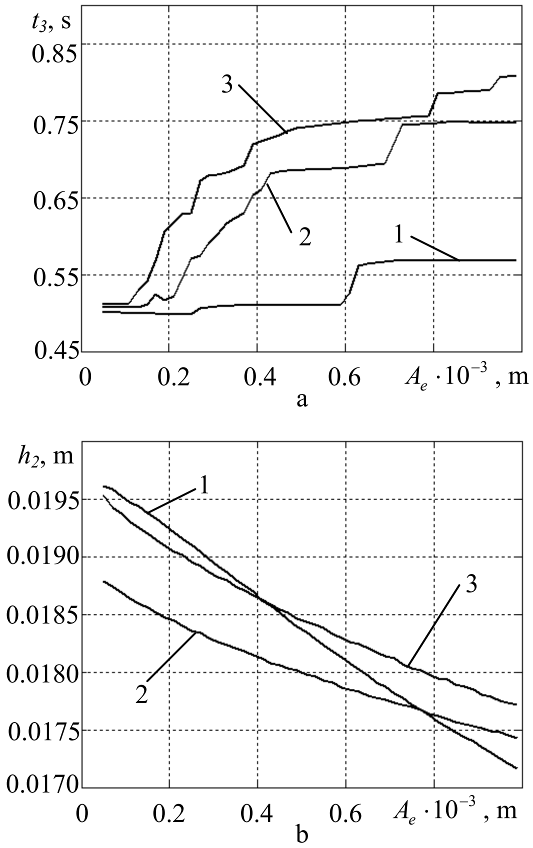


Fig. 13 Dependences of: a – insertion process duration t_3; b – depth h_2 wherein the two point contact appears, on excitation frequency, under different values of axial stiffness: $1 - K_z = 2\cdot10^3$ N/m; $2 - K_z = 5\cdot10^3$ N/m; $3 - K_z = 8\cdot10^3$ N/m

Dependences of insertion process duration on insertion speed, under kinematical excitation of mobile based part by vibration amplitude $A_e = 0.5\cdot10^{-3}$ m and frequency $\omega = 100$ s^{-1}, were determined (Fig. 14, a). The results were compared with the dependences presented in paper [5], which were obtained without using vibrations (Fig. 14, b). It was determined that the insertion process duration is shorter under kinematical excitation of mobile based part and properly selected excitation parameters.

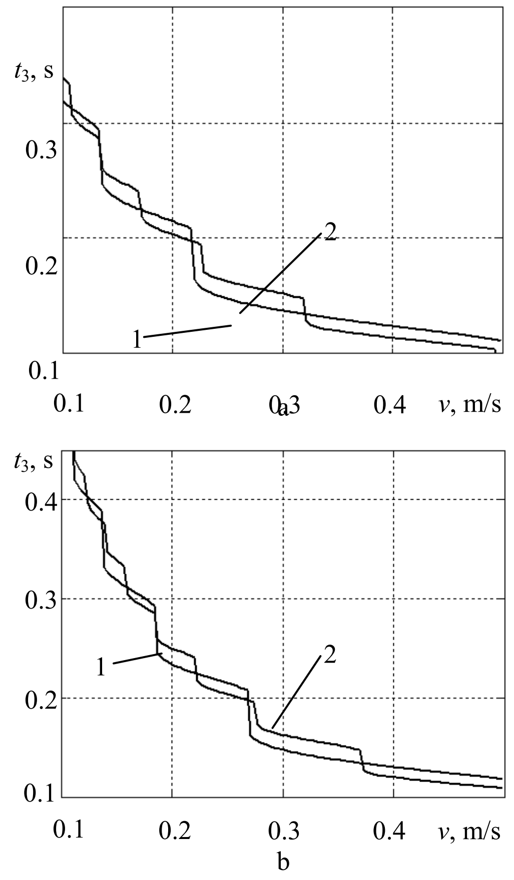


Fig. 14 Dependences of insertion process duration t_3 on insertion speed v: a – under kinematical excitation of mobile based part; b – without using vibrations, under different values of axial stiffness: $1 - K_z = 2\cdot10^3$ N/m; $2 - K_z = 5\cdot10^3$ N/m

4. Conclusions

1. Mathematical model of cylindrical parts insertion, under kinematical excitation of mobile based part in axial direction, was formed. Programs for simulation of the insertion process were written using MatLab software.

2. It was noticed that the highest influence on jamming makes the distance from the lower end surface of the peg to the centre of compliance L_C. Jamming occurs under high values of L_C. Also the probability of jamming increases when axial stiffness K_z, peg insertion speed v, coefficient of friction μ, mass of the peg and the gripper m, is increasing and clearance ratio δ is decreasing.

3. It was determined that connection of jammed parts is possible by exciting vibrations of the peg in axial direction. Reliability and effectiveness of insertion process can be increased by the selection of proper values of excitation amplitude and frequency.

4. Simulation showed that jammed components are not connected when the values of excitation amplitude A_e, frequency ω or axial stiffness K_z are too low. Also jammed components are not connected under too high value of excitation amplitude. Besides, under excitation by very high amplitudes, the reliability of insertion process decreases due to the increase of probability of peg jump out from the hole.

5. Chamfer crossing duration t_1 decreases when excitation frequency ω is increasing. Insertion process du-

ration t_3 decreases when excitation amplitude A_e is decreasing and excitation frequency ω is increasing.

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CILINDRINIŲ DETALIŲ SUJUNGIMO TYRIMAS KINEMATIŠKAI ŽADINANT PASLANKIAI BAZUOJAMĄ DETALĘ

Reziumė

Straipsnyje nagrinėjamas paslankiai bazuojamo strypo sujungimo su nejudama įvore procesas, kai strypas kinematiškai žadinamas ašine kryptimi. Sudarytas vibracinio sujungimo proceso matematinis modelis, aprašantis strypo kontaktą su nuožula, strypo vieno ir dviejų taškų kontaktą su įvorės skyle. Nustatyti parametrai, kuriems esant renkamos detalės įstringa. Įstrigusius komponentus galima sujungti sužadinus strypo virpesius ašine kryptimi. Nustatyti kinematinio žadinimo parametrai, kuriems esant sujungimo procesas trunka trumpiausiai ir yra patikimiausias.

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INSERTION SIMULATION CYLINDRICAL PARTS UNDER KINEMATICAL EXCITATION OF MOBILE BASED PART

Summary

The process of mobile based peg insertion into immobile based bush, while the peg is kinematically excited in axial direction, is analyzed in the paper. Mathematical model of vibratory parts insertion process, which determines peg contact with the chamfer of the bush, peg one point and two point contact with the bush hole, was formed. Parameters which influence jamming of the parts to be assembled were determined. Connection of jammed parts is possible by exciting vibrations of the peg in axial direction. Parameters of kinematical excitation, under which insertion process duration is the shortest and the process is the most reliable, were detected.

Б. Бакшис, С. Киликевичюс

ИССЛЕДОВАНИЕ СОЕДИНЕНИЯ ЦИЛИНДРИЧЕСКИХ ДЕТАЛЕЙ ПРИ КИНЕМАТИЧЕСКОМ ВОЗБУЖДЕНИИ ПОДВИЖНО БАЗИРУЕМОЙ ДЕТАЛИ

Резюме

В статье рассматривается процесс соединения подвижно базируемого стержня с неподвижной втулкой при кинематическом возбуждении стержня в осевом направлении. Составлена математическая модель процесса вибрационного соединения, описывающая контакт стержня с фаской, одноточечный и двухточечный контакт стержня с отверстием втулки. Определены параметры, при которых возникает заклинивание деталей. Показано, что возбуждение колебаний стержня в осевом направлении способствует освобождению заклиненной детали и дальнейшему соединению. Выявлены параметры кинематического возбуждения, обеспечивающие надежность и наименьшую продолжительность процесса соединения.

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