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# Algorithm for Optimal Supplement of Train Traffic Schedule

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## Introduction

Currently high level of informational transport control systems presents possibility to install automatic or even self-acting correction system for disturbed train traffic. Science fiction [1-4] contains similar tasks; however they are solved either by applying the criterion of passenger comfort or by adapting them for railroad infrastructure with semi-automatic blocking without possibility of operative correction for train mobility [2-4]. Such systems are not typical for railroad networks of various countries: semi-automatic blockings are already obsolete and modern traffic control technologies (e.g. the EU system ERTMS) enable to control train traffic at high level of exactness.

This issue presents theoretical principles for optimal correction of train traffic schedule being disturbed in railroad line, having installed self – acting control system.

The issue presents the algorithms, which might be successfully applied for automation of "last minute" (ad hoc) application service (by inserting additional train line in fully prepared traffic schedule). The current situation of "last minute" (ad hoc) application service is left as the option of infrastructure managers and the service procedure itself is such that the driver (applicant) is not aware of how to formulate the application and satisfy maximum of his interests as well as how to find out the detail, which may cancel the application.

## Formulation of the task

The train traffic schedule is comprised of a number of so called "train lines", each of them describes a particular route.

In diagram (Fig. 1) the "train line" on the plane of time ($t$) and distance (s) is represented as broken line, the horizontal segments of which correspond to standing (at the station) and oblique segments correspond to moving at appropriate speed.

Stations on the train schedule are labeled only by their axial line (i.e. length of the station is not assessed). Acceleration and deceleration sections are not shown also: it seems that train stops momentarily and in the same way develops necessary speed.

These parameters are enough to describe the traffic schedule nominally:

departure time matrix $\boldsymbol{t}^i = \left\| t^i_{ji} \right\|$,

speed matrix $\boldsymbol{v} = \left\| v_{ji} \right\|$,

coordinate vector of station axial lines $\boldsymbol{S} = \left\| S_i \right\|$:

$$\mathbf{S}(t) = \{ \boldsymbol{t}^i, \boldsymbol{v}, \boldsymbol{S} \} . \quad (1)$$

The disturbance of traffic schedule is described as difference between planned diagram $\mathbf{S}^p(t)$ and factual diagram $\mathbf{S}^f(t)$, as even for a single "train line" this equation is applied: $s^p_j(t) - s^f_j(t) > \Delta(s)$, here $\Delta(s)$ - permissible delay of the train (measured in terms of losses).

Hereinafter such traffic disturbances are analyzed when only a single train fails behind the schedule impermissibly.

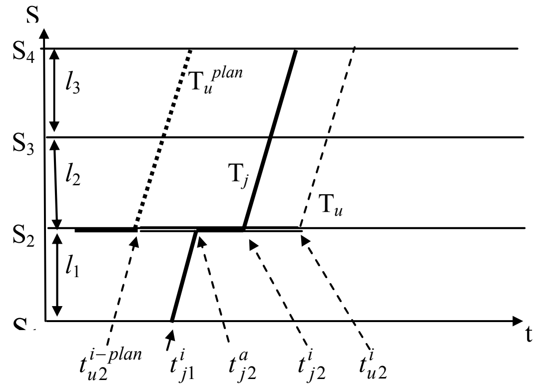


**Fig. 1.** The example of train traffic schedule diagram

The train $T_u$ according to plan had to run in line $T_u^{plan}$ from the station $S_2$ at the time moment $t_{u2}^{i-plan}$, however because of any reason $T_u$ could not leave from the station $S_2$ until the time moment $t_{u2}^{i}$.

Losses incurred because of the train $T_u$ delay (compensations for clients' losses etc.) depend on the amount of delay time ( $\tau_u = t_u^i - t_u^{i-plan}$ ) and the point of the route ($s$) at which the train delays.

Practically the function $N_u(\tau_u, s)$ is discrete: $s$ acquires coordinate values of station axial lines only $s \in \{s_z, ..., s_n\}$,

where $z$ –the station index, where traffic schedule of the train $T_u$ was disturbed;

$n$ - the index of terminal station.

In such case $N_u(\tau_u, s) \to N_u(\tau_{ui})$.

The function $N_u(\tau_{ui})$ assesses losses involved in delay of the train $T_u$ only. This is enough as the task of traffic optimization for delayed train is analyzed, but if completely new train line must be inserted in traffic schedule (i.e. as addition but not correction of traffic schedule ) it is necessary to use the function:

$$W_{ui}^T = W_{Tui}^T + W_{Eui}^T + N_{ui}, \qquad (2)$$

where $W_{Tui}^T$ - component depending on time and involved in exploitation costs of locomotives and carriages of the train $T_u$ as well as calculated for the period, during which the train $T_u$ crosses the line $i$ ($S_i - S_{i+1}$);

$W_{Eui}^T$ - costs for the team of the train $T_u$, calculated for the period, during which the train $T_u$ crosses the line $i$;

$N_{ui}$ - losses, incurred because of ill-timed arrival of the train $T_u$ to the terminal station $S_{i+1}$ of the line $i$ ($S_i - S_{i+1}$).

If it would be possible to neglect other traffic participants, the train $T_u$ should run all remaining distance (from the station $S_z$, where if was forced to delay, up to the terminal station $S_n$ ) at the reasonable speed ( $v_u^{opt}(s) \le v_{ui}^{rib}(s)$ ) resulted in minimal costs on fuel and the losses $W_u$, incurred because of delay

$$W_u = \sum_{i=z}^{n} \left( W_{ui}^E + N_u(\tau_{ui}) \right) \to min. \qquad (3)$$

Methodology of calculating the expenditures $W_{ui}^E$, $W_{ui}^T$ and $W_{ui}^S$ is known [8].

Unfortunately the assumption (made before the formula (3) has been written) that other traffic participants may be neglected (equal to the assumption that other traffic participants do not intervene) is often valid for road transport facilities, but not for trains.

## Algorithm for optimal correction of traffic schedule

In railroads with modern traffic control system being implemented, the train $T_u$, performing manipulations of its speed is able to:

1. to catch up to oncoming slower train,
2. to be caught up by faster train running after.

In the first case two events are possible again

1.1 The ongoing train $T_j$ stops in the primary station of the line, in which it will be caught up and lets the train $T_u$ to pass.

1.2 The ongoing train does not stop until its panned station.

These cases are illustrated by Fig. 2.

In the second case the train $T_u$ has always to stop and let the train, which is running after, to pass, because faster trains are usually of higher category.

The analysis of variations of possible traffic situations shows that the situations below should be applied for analysis of each line $k$ (with its terminal station $S_{k+1}$):

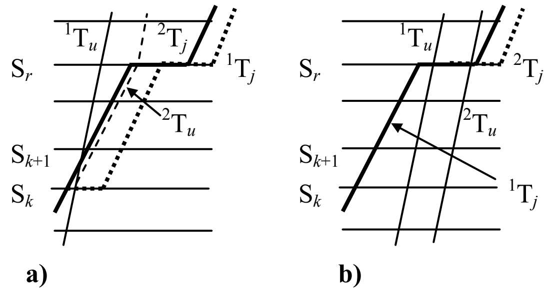


**Fig. 2**. Traffic variations:
a) The slower train $T_j$ makes way (the situation $^1T_u$–$^1T_j$) or does not make way (the situation $^2T_u$–$^2T_j$) for the train $T_u$;
b) The train $T_u$ does not disturb (the situation $^1T_u$–$^1T_j$) or disturbs (the situation $^2T_u$–$^2T_j$) the train $T_j$. to leave timely.

**1.** The train $T_u$ neither in line $k$ nor in its terminal station ($S_{k+1}$) does not catch up to ongoing train $T_j$ and can not be caught up by the train $T_{j+1}$, which is running after.

The other variation of this case: the train $T_j$ can not be caught up in line $k$ , but according to schedule stops in the terminal station $S_{k+1}$ , belonging to this line. In this case the train $T_u$ in the station $S_{k+1}$ is able to overtake the train $T_j$, without disturbance of its traffic schedule (Right side of Fig. 2. the situation $^1T_u$–$^1T_j$).

**2.** The train $T_j$ can not be caught up in line $k$ but according to schedule stops in the terminal station $S_{k+1}$,of the line, where it is overtaken by the train $T_u$ , disturbing the train $T_j$ to leave timely from the station $S_{k+1}$ (Right side of Fig. 2. the situation $^2T_u$–$^2T_j$).

**3.** The train $T_u$ in line $k$ as forecasted catches up to ongoing train $T_j$

$$\left( l_k - \left( t_{j(k+1)}^a - t_{uk}^i \right) v_{uk} < L_{uk}^{st} \right). \qquad (4)$$

Train $T_j$ must do on purpose stopping in the primary station $S_k$, of the line, avoiding traffic disturbance for the train $T_u$. (Left side of Fig. 2. the situation $^1T_u$–$^1T_j$).

**4.** The train $T_u$ in line $k$ catches up to ongoing train $T_j$, which does not let the train $T_u$ to pass. The situation is complicated by the fact that according to the schedule $T_j$ should not stop in terminal station $S_{k+1}$ of the line.

**5.** The train $T_u$ in line $k$ catches up to ongoing train $T_j$, which does not let the train $T_u$ to pass, but according to the schedule it has to stop in the terminal station $S_{k+1}$ of the line.

Possible two variants:

$T_u$ overtake the train $T_j$ standing in the station $S_{k+1}$

$$\left(l_k - \left(t^a_{j(k+1)} - t^i_{uk}\right)v_{uk} > L^{st}_{uk}\right);\ \left(t^i_{j(k+1)} > t^i_{u(k+1)}\right). \quad (5)$$

$T_u$ disturbs the train $T_j$ to leave the station $S_{k+1}$ timely

$$\left(t^i_{j(k+1)} - t^i_{u(k+1)} < \frac{L^{st}_{(j+1)}}{v_{j(k+1)}}\right). \quad (6)$$

**6.** The train $T_u$ in line $k$ is caught up by the train $T_{j+1}$:

$$\left(l_k - \left(t^a_{u(k+1)} - t^i_{(j+1)k}\right)v_{(j+1)k} < L^{st}_{(j+1)k}\right). \quad (7)$$

Train $T_u$ must do on purpose stopping in the primary station $S_k$. of the line.

These entire situations are defined by different formulae of general costs.

**Situation 1.** To run a single line $k$ general costs are

$$W_{uk} = W^E_{uk} + N_u(\tau_{uk}). \quad (8)$$

To run the whole distance from the station $S_z$ to the terminal station $S_n$ of the route general costs are:

$$W_{uy} = \sum_{i=z}^{n}\left(W_{ui}\right) + W^S_{uz}; \quad (9)$$

where

$W_{ui}$ - calculated according to formula (8), as $k \to i$;

$W^S_{uz}$ - the costs of arbitrary fuel for the train $T_u$ to develop speed in primary station $S_z$, are calculated according to the same methods as costs of arbitrary fuel for train's speed development after its stopping.

**Situation 2.**

$$W_{uy} = \sum_{i=z}^{n}\left(W_{ui}\right) + W^S_{uz} + \min_{\forall v_{ji}}\sum_{i=k}^{jg} W_{ji} - \sum_{i=k}^{jg} W^{plan}_{ji}. \quad (10)$$

In this formula $jg$ is the index of the last line of the train $T_j$ route.

The formula (10) provides the optimization procedure for further movement of the train $T_j$ being overtaken. The procedure usually does not produce marked effect thus the formula (9) may be applied instead of formula (10) with a slight error.

**Situation 3.** To run the line $k$ general costs are

$${}^{C}W_{uk} = W^E_{uk} + N_u(\tau_{uk}) + W^S_{jk}. \quad (11)$$

To run the whole distance from the station $S_z$ to the terminal station $S_n$ of the $T_u$ route general costs are

$$W_{uy} = \sum_{i=z}^{n}\left(W_{ui}\right) + W^S_{uz} + W^S_{jk} + \min_{\forall v_{ji}}\sum_{i=k}^{jg} W_{ji} - \sum_{i=k}^{jg} W^{plan}_{ji}. \quad (12)$$

The formula (12) also provides the optimization procedure for further movement of the train $T_j$ being overtaken. The procedure usually does not produce marked effect thus instead of the formula (12) the formula below may be applied with slight error

$$W_{uy} = \sum_{i=z}^{n}\left(W_{ui}\right) + W^S_{uz} + W^S_{jk}, \quad (13)$$

where $W^S_{jk}$ involved in formulae (10) and (13) are the costs of arbitrary fuel necessary to develop the speed for the train $j$ after it was forced to stop.

**4 and 5 situations** analytically are not complicated: additional conditions must be satisfied only

$$v_{ui} \le v_{ji}, \quad i = k, k+1, ..., r, \quad (14)$$

($S_r$ – station of planned stopping for the train $T_j$ ) in case **4** and other condition

$$v_{ui} \le v_{ji}, \quad i = k. \quad (15)$$

in case **5**.

**Situation 6.** To run the line $k$ general costs are

$${}^{F}W_{uk} = W^E_{uk} + N_u(\tau_{uk}) + W^S_{uk}. \quad (16)$$

To run the whole distance from the station $S_z$ to the terminal station $S_n$ of the $T_u$ route general costs are

$$W_{uy} = \sum_{i=z}^{n}\left(W_{ui}\right) + W^S_{uz} + W^S_{uk}. \quad (17)$$

The costs $W_{uy}$ in accordance with the formula (8)-(17) are calculated being aware of the parameters of traffic schedule and traffic participants.

1. Planned traffic schedule **S**($t$) (see (1) formula).
2. Functions of delay losses (penalty) $N_u(\tau_{ui})$.
3. Train parameters, and parameters necessary to calculate $w_{0ji}$ values.
4. Relief parameters $i_e(s)$.
5. Air temperature $t^0$.
6. Index $z$ of the station , from which the line of the train $T_u$ is corrected (inserted).
7. Index $p$ of the first train line, after which the correction line may be inserted.
8. Element values $c_{jk} \in \{1, \infty\}$ of „pass/ do not pass" matrix $\boldsymbol{C} = \left\|c_{jk}\right\|$, which may be given or produced accidentally for each iteration of optimization process.
9. Departure time $t^i_{uz}$ of the train $T_u$ from the station $S_z$, (chosen with the regard to application of regular or stochastic search algorithms , determinate order or at random) and technical speeds of all lines $\boldsymbol{v}_u = \left\|v_{ui}\right\|$, (as $z \le i \le n-1$).

By choosing the mentioned parameters, the restrictions should be satisfied as: $t^i_{(p+m)z} > t^i_{uz} > t^i_{pz}$ and $v^{\max}_{ui} > v_{ui} > v^{\min}_{ui}$.

The particular value of general costs $W_{uy}$ is calculated for each fixed vector of variables.

Correction of the route is performed within permissible field (following the restrictions) seeking such vector of variable value $\{ \ t_{uz}^{i} \ , \ \boldsymbol{v}_u, \ \boldsymbol{C} \ \}_{ger}$ when value of general costs $W_{uy}$ is minimal.

This problem may be solved by applying algorithms of stochastic search (Monte Carlo method, genetic search algorithm etc.), method of variations reselection (in respect of discrete variables) and other.

## Conclusions

1. The tasks of train traffic correction and addition could be solved after installing modern information control technologies.
2. It is reasonable to solve problems of traffic schedule correction and optimal addition by applying the criterion of general costs. The solution is the optimal route line, which has minimal value of general costs.
3. The algorithms of traffic schedule correction and addition are subject to high "branch" level: there are a lot of conditions, which affect later actions and solutions.
4. The problems of traffic schedule correction and optimal addition could be solved with the help of digital methods (by reselecting all combinations of discrete values of variable or by applying methods of stochastic search).

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**I. Mišauskaitė. Algorithm for Optimal Supplement of Train Traffic Schedule // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – Nr.7(71). – P. 43 – 46.**

Article analyses a problem of train traffic schedule optimal addition according to minimum overall expenditure criteria. Task is to analyze optimal insertion of an additional train track into already made schedule. The methodology is all-right applicable for ad hoc requests submission. Handling importance of ad hoc requests is emphasized in EU directive 2001/14/EU. Task can be solved applying Monte-Carlo, genetic algorithms, variants reselection methods or methods of the shortest (according to waste) way in graphs. All these methods are equally exact and can be implemented with modern computers with short enough time consumption. Ill. 2, bibl.8 (in English; summaries in English, Russian and Lithuanian).

**И. Мишаускайте. Алгоритм для оптимального дополнения графика движения поездов // Электроника и электротехика. – Каунас: Технология, 2006. – №7(71). – С. 43 – 46.**

Статья посвящена проблеме оптимального изменения или дополнения графика движения поездов в случаях их отставания по каким-либо причинам от планового расписания или при необходимости дополнить график новой поездной линией. Проблема дополнения особенно остро возникает в связи с акцентируемой в директиве 2001/14/EU необходимостью организации обслуживания так называемых заявок „последней минуты“ (*ad hoc requests*). Для решения задачи в статье предлагаются методы оптимизации с использованием критерия наименьших общих экономических затрат. Приводится формулировка задачи и алгоритмы её решения. Ил. 2, библ. 8 (на английсом языке; рефераты на английском, русском и литовском яз.).

**I. Mišauskaitė. Traukinių eismo grafiko optimalaus papildymo algoritmas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr.7(71). – P. 43 – 46.**

Straipsnis skirtas traukinių eismo grafiko optimalaus (pagal minimalių bendrųjų sąnaudų kriterijų) koregavimo metodikos analizei. Traukiniui dėl kokių nors priežasčių atsilikus nuo grafiko, naudojant šiuolaikines informacines ir valdymo technologijas ieškoma tokios tolesnių veiksmų sekos, kad nuostoliai dėl traukinių vėlavimų bei papildomų energijos sąnaudų būtų minimalūs. Ši metodika taip pat galėtų būti sėkmingai naudojama ir automatizuojant „paskutinės minutės“ (*ad hoc*) paraiškų aptarnavimą (į sudarytą eismo grafiką įterpiant papildomą traukinio liniją vienam ar keletui reisų). Dabar „paskutinės minutės“ paraiškų aptarnavimo procedūra tokia, kad vežėjui (pareiškėjui) neaišku, kaip formuluoti prašymą, kad maksimaliai patenkintų savo interesus ir kartu kad prašymas nebūtų atmestas dėl kokių nors detalių. Il. 2, bibl.8 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).