

## “Droplet” problem in the aspect of modern thermal technologies

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### Nomenclature

$a$  - thermal diffusivity,  $m^2/s$ ;  $B$  - Spalding transfer number;  $c_p$  - mass specific heat,  $kJ/(kg\ K)$ ;  $C_l$  - droplet drag coefficient;  $D$  - mass diffusivity,  $m^2/s$ ;  $Fo$  - Fourier number;  $I$  - control time index;  $Kn$  - Knudsen number;  $l$  - Knudsen layer thickness,  $m$ ;  $L$  - latent heat of evaporation,  $J/kg$ ;  $m$  - vapour mass flux density,  $kg/(s\ m^2)$ ;  $M$  - molecular mass,  $kg/mol$ ;  $n$  - number of the term in infinite sum;  $Nu$  - Nusselt number;  $p$  - pressure,  $Pa$ ;  $Pr$  - Prandtl number;  $q$  - heat flux density,  $W/m^2$ ;  $r$  - coordinate of a droplet,  $m$ ;  $R$  - radius of a droplet,  $m$ ;  $Re$  - Reynolds number;  $R_M$  - universal gas constant,  $J/(kmol\ K)$ ;  $T$  - temperature,  $K$ .

### Greek symbols

$\alpha_k$  - condensation coefficients;  $\alpha_v$  - evaporations coefficients;  $\eta$  - dimensionless coordinate;  $\lambda$  - thermal conductivity,  $W/(K\ m)$ ;  $\mu$  - dynamic viscosity,  $kg/m\ s$ ;  $\rho$  - density,  $kg/m^3$ ;  $\tau$  - time,  $s$ .

### Subscripts

$e$  - stabilized evaporation;  $f$  - evaporation;  $g$  - gas;  $i$  - time index in a digital scheme;  $I$  - index of control time;  $j$  - index of droplet cross-section;  $J$  - index of droplet surface;  $k$  - iteration index in a digital scheme;  $Kn$  - Knudsen layer limit;  $l$  - conductive;  $L$  - liquid;  $R$  - droplet surface;  $v$  - vapour;  $vg$  - vapour-gas mixture;  $0$  - initial state;  $\infty$  - far from a droplet.

### Superscripts

+ - external side of a surface; - - internal side of a surface.

## 1. Introduction

Liquid injection is used in liquid fuel combustion, gas purification, evaporation of liquid solutions, protection of various surfaces from direct energetic influence of high temperatures and many other thermal technologies. Reliability of these thermal technologies depends on the capability to control optimally the intensity of liquid phase transformations. Right amount of liquid, evaporating in the right place, leads to effective control of combustion process, optimal reduction of pollutants' formation, increase of thermal equipment efficiency and reliable protection of their surfaces.

Water spray - related technologies have been developing recently. Highly developed interphase contact area in water flows, effective energetic interaction between gas and liquid phases, significant energy consumption for droplet heating and evaporation ensure reliable protection of surfaces [1]. That makes assumptions to use today - available materials for the development of plasmochemical or even nuclear synthesis technologies. Experiments prove that higher efficiency of gas turbines is reached by spraying water into air compressor [2], thus making air compression process close to isothermal and consequently reducing energy required for external work. Water, injected into air flow, which enters the cylinders of internal com-

bustion engines, allows significant reducing of NO<sub>x</sub> amount. Rapid development of water spray technologies, used to increase humidity of flue gas, thus cleaning it and utilizing condensation energy, is observed.

It is necessary to ensure rapid process of water droplet's evaporation. The rate of phase transformations in two-phase flows is traditionally controlled by changing dispersity of sprayed liquid. If a two-phase flow consists of small droplets, the rate of transfer processes in such flow is very high. It is very complicated to produce the flow, which contains one micron-size droplets, using conventional water dispersion methods, because nozzles have wide dispersivity characteristics. Present stage of technological development is able to offer quite unexpected solutions, which allow approaching conventional thermal technologies to nano technologies. Experiments show, that water droplets suddenly shift to an explosive evaporating mode, as their mass mean temperature exceeds saturation temperature, corresponding to medium parameters by 18 degrees [3]. An explosive evaporation of water droplet is evoked by intensive energetic influence of a laser beam or by sudden change of pressure in droplet surroundings. If pressure difference is selected correctly, explosion transforms polydispersal water flow into a flow of small droplets, close to monodispersal, in which evaporation rate is uniform and high. The flow, produced in such way, contains hot water droplets, temperature of which is close to saturation under conditions of low-pressure surroundings. Phase transformations of hot water evaporating droplets, comparing to well-investigated case of cold water droplets, are peculiar [4].

The change of phase transformation intensity according to dispersity of sprayed liquid is insufficient, when it is necessary to control the intensity of liquid evaporation, as equipment operates in slightly variable modes. Multifactorial control opens wider regulation possibilities, as droplet dispersity is combined with correctly selected thermal state of dispersed liquid. In order to ensure such regulation, it is necessary to know thoroughly the regularities of heat and mass transfer in a wide range of boundary heat and mass transfer conditions. It is necessary to take into account unsteadiness and interaction of heat and mass transfer processes in droplets and their surroundings, which occur under the influence of such factors as radiant selectivity, Stefan hydrodynamic flow, temperature and pressure drops in the Knudsen layer, droplet slip in gas, turbulence of two-phase flow, variable physical properties, etc. These aspects of research of the “droplet” problem are important for the application of modern technologies. All these aspects together make the “droplet” problem “hard - to - solve” even evaluating the level of modern thermodynamical sciences and wide possibilities of numerical research of transfer processes. Therefore the methodology

of a continuous research of the “droplet” problem is being developed, as the limitations in the solutions are gradually approaching to real conditions.

The study [5] generalizes the results of the first studies regarding the “droplet” problem, as steady heat and mass transfer is researched in the vicinity of a droplet, as it evaporates in a low – temperature medium. Introduction of the Spalding heat transfer number  $B$  [6] allowed evaluating the influence of phase transformations on droplet heating intensity using the principles of similarity theory. Thorough overview of “droplet” research in the case of convective heat transfer in the medium, surrounding droplets, is presented in [7]. Recently the methodology of a combined analytical and numerical research is more and more widely applied in “droplet” researches [8-10]. The peculiarities of the interaction of unsteady radiant and conductive heat transfer processes in water droplets are investigated in [8]. The peculiarities of the application of a combined analytical, asymptotic and numerical method for modelling of a liquid fuel droplet heating and evaporation are discussed in [9]. A significant influence of initial nonisothermality of water droplets on their thermal state change during unsteady evaporation is described in [10]. Note that the method of iterative research of unsteady heat and mass transfer in large droplets is developed in [8,10]. Modelling phase transformations in small droplets, the problem of temperature and pressure drops in the Knudsen layer arises, the solution of which requires the application of kinetic theory methods and solution of complicated Boltzman equations. Temperature and pressure drops for a spherical droplet with isothermal surface are described using the system of algebraic equations, presented in the studies [11, 12], making assumptions, that vapor molecules do not collide in the Knudsen layer; their velocities distribute according to the Grad function; the values of the ratio of phase transformation rate sound velocity and the ratio of gas and droplet temperature difference gas temperature are significantly smaller than 1.

This study contains the expanded combined analytical – numerical research method of the “droplet” problem, developed in the studies [8,10], for the modelling of phase transformations in small droplets, using mathematical apparatus of pressure and temperature drops in the Knudsen layer [11,12]. The results of cold water evaporation and the change of thermal state are presented and generalised.

## 2. Analytical – numerical research method

The goal of heat and mass transfer in two – phase flows, consisting of gas and droplets, is to determine the intensity of transfer processes. Heat and mass transfer in modern thermal technologies, dealing with two – phase flows between small droplets and gasses, is intensive, distinctively unsteady and is related to phase transformations. Experimental investigations are very complicated from technical point of view. Investigating theoretically, it is necessary to pay attention to several peculiarities. The influence of mass transfer processes on heating of evaporating droplets is huge, the weight of kinetic processes near droplet surface increases, therefore in parallel to diffusion processes it is necessary to take into consideration the processes of abruption of vapour molecules from liquid surface and simultaneous return to this surface. These fac-

tors make the modelling more complicated. Radiant influence on thermal state of small droplets is negligible; droplet inertness is small, therefore, as droplet and gas flow velocities become equal, conductive heat transfer prevails in the droplets and their vicinity. These are the factors that facilitate modelling.

The vapour flux density, which defines the intensity of the droplet phase transformations, can be expressed by the ratio of difference of heat flux densities and the external and internal sides of the droplet surface on latent heat of evaporation

$$m_v^+(\tau) = \frac{q_l^+(\tau) - q_l^-(\tau)}{L(\tau)} \quad (1)$$

Unsteady and equilibrium liquid evaporation modes can be distinguished according energy supplied to the droplets for their heating. In the first case  $q_l^- = 0$  in the equation (1); heat flux supplied to a droplet, evaporates it. Density of heat flux, supplied to the droplet by conduction, is calculated according to the conditions of a solid spherical particle, using the function of Spalding transfer number and evaluating the influence of Stefan hydrodynamic flow

$$q_l^+(\tau) = [1 + B(\tau)]^{-0.7} \frac{\lambda_{vg}(\tau)}{R(\tau)} [T_g - T_R(\tau)] \quad (2)$$

In the case of unsteady evaporation, heat flux density in a droplet is defined by the temperature field gradient

$$q_l^-(\tau) = -\lambda_L(\tau) \frac{\partial T(\tau, r)}{\partial r} \Big|_{r=R^-} \quad (3)$$

In the case of unsteady evaporation the Spalding transfer number also depends on temperature field gradient in the droplet [9]

$$B(\tau) = \frac{c_{p,vg} [T_g - T_R(\tau)]}{L(\tau)} \left[ 1 - \frac{\partial T(\tau, r)}{\partial r} \Big|_{r=R^-} / q_l^+(\tau) \right] \quad (4)$$

Therefore a link between transfer processes in the droplet and in its vicinity is obvious. All these processes define instant temperature of heating and evaporating on the droplet surface. Energy flux densities in droplets and in gas can be calculated only if the droplet surface temperature is known. Therefore the necessity of iterative numerical research is evident and the unambiguously conditions for the system can be formed in regard to yet unknown temporal function that defines the change of droplet surface temperature

$$T(r,0) = T_0(r), \quad T(R,0) = T_{R,0}, \quad T(R,\tau) = T_R(\tau) \quad (5)$$

Therefore the gradient of unsteady temperature field of thea droplet can be defined using the following integral equation [10]

$$\frac{\partial T(r,\tau)}{\partial r} \Big|_{r=R^-} = \frac{2\pi}{R^2(\tau)} \sum_{n=1}^{\infty} (-1)^n n \int_0^{\tau} f_n(R,\tau') \times \\ \times \exp \left\{ a(\tau') \left[ \frac{n\pi}{R(\tau')} \right]^2 (\tau' - \tau) \right\} d\tau' + \frac{T_R(\tau)}{T_{R,0}} \frac{\partial T_0(r)}{\partial r} \Big|_{r=R^-} \quad (6)$$

in which the function of an infinite sum term, time and coordinate evaluates the peculiarities of initial temperature field of the droplet and the rate of the droplet surface temperature change

$$f_n(r, \tau') = \frac{(-1)^n T_0(r) dT_R(\tau')}{n\pi T_{R,0} d\tau'} - \int_0^R \left( \frac{n\pi r'}{R} \cos \frac{n\pi r'}{R} - \sin \frac{n\pi r'}{R} \right) \left[ \frac{a(r') dT_0(r') T_R(\tau')}{R^2 dr' T_{R,0}} \right] dr \quad (7)$$

The vapour flux density on the droplet surface is influenced by the nature of liquid and its surrounding medium, diffusive mass transfer rate, which is caused by the gradient of vapour concentration, and convective mass transfer rate, caused by the influence of Stefan hydrodynamic flow. As the hydrodynamic flow's influence is evaluated by the Stefan logarithm, the vapour flux density on the droplet surface is defined by the expression [13]

$$m_v^+(\tau) = \frac{D_{vg}(\tau)}{T_{vg,R}(\tau)} \frac{M_v}{R_M R(\tau)} p \ln \frac{p - p_{v,\infty}(\tau)}{p - p_{v,R}(\tau)} \quad (8)$$

The direction of vapour flow is unambiguously described by a logarithmic function, called Stefan logarithm. When liquid vapour pressure near the droplet is higher than vapour partial pressure in gas that surrounds the droplet, the vapour flow is positive; the droplet decreases, as liquid evaporates. In the opposite case the vapour flow is negative and the droplet grows, as liquid vapour condenses on its surface. The droplet dimension changes, caused by heating and mass transfer processes, are described by the equation of radius dynamics:

$$\frac{\partial}{\partial \tau} [\rho_L(\tau) R^3(\tau)] = -3R^2(\tau) m_v^+(\tau) \quad (9)$$

When the influence of Knudsen layer on heat and mass transfer processes can be neglected, temperature of the vapour and gas mixture near the droplet surface in expression (8) is considered to be the droplet surface temperature; vapour partial pressure near the droplet surface is considered to be saturated vapour pressure according to the droplet surface temperature:  $T_{vg,R}(\tau) \equiv T_R(\tau)$ ,  $p_{v,R}(\tau) \equiv p_{s,R}(\tau)$ . In the case of small droplets it is necessary to evaluate temperature drop  $T_{vg,R}(\tau) \equiv T_{Kn}(\tau) = T_R(\tau) + \Delta T_{Kn}(\tau)$  and pressure drop  $p_{v,R}(\tau) \equiv p_{v,Kn}(\tau) = p_{v,s}(T_R, \tau) + \Delta p_{Kn}(\tau)$  in the Knudsen layer. Following the method, presented in [12,13], temperature and pressure drops can be described by the system of algebraic equations

$$\left( 1 - \frac{\alpha_k R^2}{2R_{Kn}^2} \right) \frac{\sqrt{(2\pi R_{\mu,v} T_\infty)}}{p_{v,\infty}} m_v^+ = \alpha_v \left( 1 - \frac{T_R - T_\infty}{2T_\infty} + \frac{p_{s,R} - p_{v,\infty}}{p_{v,\infty}} \right) \alpha_k \left( 1 + \frac{p_{v,Kn} - p_{v,\infty}}{p_{v,\infty}} - \frac{T_{Kn} - T_\infty}{2T_\infty} \right) \quad (10)$$

$$\begin{aligned} & \left( \frac{R}{R_{Kn}} + \frac{8k_{vg}}{1 - \frac{R^2}{2R_{Kn}^2}} \frac{Kn}{k_{vg} + 1 Pr_m} \right) \frac{T_R - T_{Kn}}{T_\infty} = \\ & = \frac{8k_{vg}}{k_{vg} + 1 Pr_m} \left( \frac{T_R - T_\infty}{T_\infty} \right) + \\ & + \frac{R}{R_{Kn}} \frac{k_{vg} - 1}{k_{vg} + 1} \frac{R_{\mu,v}}{R_{\mu,vg}} \frac{\sqrt{2\pi R_{\mu,vg} T_{Kn}}}{p} m_v^+ \end{aligned} \quad (11)$$

External radius of Knudsen layer is proportional to the droplet radius:  $R_{Kn} = R + \beta l$ . The coefficient  $\beta = 0.75$  [11]; the free path of molecules is calculated in regard to the parameters of gas mixture far from the droplet [14]. In equation (11) the modified Prandtl number and adiabatic coefficient of the vapour and gas mixture are calculated according to the method presented in [12]. In the case of determined gas flow, the presented system of nonlinear equations allows unambiguous calculating of temperature of an evaporating droplet surface according to the iterative numerical research scheme:

$$\begin{aligned} & (1 + B_{I,k-1})^{-0.7} \frac{\lambda_{vg,I,k-1}}{R} (T_{\infty,I,k-1} - T_{R,I,k}) = \\ & = \lambda_{I,k} \frac{\partial T_L}{\partial r} \Big|_{I,J} + \frac{D_{vg,I,k-1}}{T_{Kn,I,k}} \frac{L_{I,k} \mu_v}{R_\mu R} p \ln \frac{p - p_{v,\infty,I,k-1}}{p - p_{Kn,I,k}} = 0 \end{aligned} \quad (12)$$

which is constructed in regard to the premise, that all energy fluxes coming to the droplet surface and going out of it are equal. For that dimensionless radius is divided into

$$J-1 \text{ number of parts } \Delta \eta_j: \quad \eta_j R \equiv \sum_{j=2}^J \Delta \eta_j R = R \sum_{j=2}^J (\eta_j - \eta_{j-1}). \text{ Control time interval is predicted:}$$

$\Delta \tau_I \equiv 0 \div \tau_I$ . Its division  $\tau_{i=1} \equiv 0$  determines initial thermal state of the droplet; the division  $\tau_{i=I} \equiv \tau_I$  defines time instant, for which a new droplet's thermal state, which satisfies the condition  $\tau_I \equiv \sum_{i=2}^I \Delta \tau_i = \sum_{i=2}^I (\tau_i - \tau_{i-1})$  is calculated.

Energy fluxes are set to be positive, if their vector is directed towards droplet surface. Thermal conductive flux density  $\bar{q}_I^+$  is unambiguously defined by the difference between gas and droplet surface temperatures; thermal convective heat flux density  $\bar{q}_I^-$  is unambiguously defined by the droplet's temperature field gradient; energy flux density  $\bar{q}_f^+ = m_v^+ L$  of phase transformations is unambiguously defined by Stefan logarithm in equation (8). The calculation of unsteady temperature field gradient of the droplet for time instant  $\tau_I$  requires evaluation of the change of the droplet's thermal state in the interval  $0 \div \tau_I$ . Therefore a scheme, similar to the scheme (12), should be applied to every intermediate time instant  $\tau_i$ . Control time is selected in such a way that the influence of the droplet dimension change could be neglected in the time interval  $0 \div \tau_I$ .

During intensive liquid heating period the change of mean temperature of the droplet mass by one degree is being controlled. During equilibrium evaporation of liquid the change of the droplet radius by one tenth of micron is being controlled. As the function that defines the change of surface temperature of warming and evaporating droplet is determined, functions that define the change in time of other heat and mass transfer parameters are calculated simultaneously. At the initial thermal state of the droplet, for every newly selected control time, temperature field of the previous time  $\tau_l$  is being assigned:  $T_0(\eta) \equiv T_L(\eta, \tau_l)$  and the droplet dimension is being specified according to the equation (9).

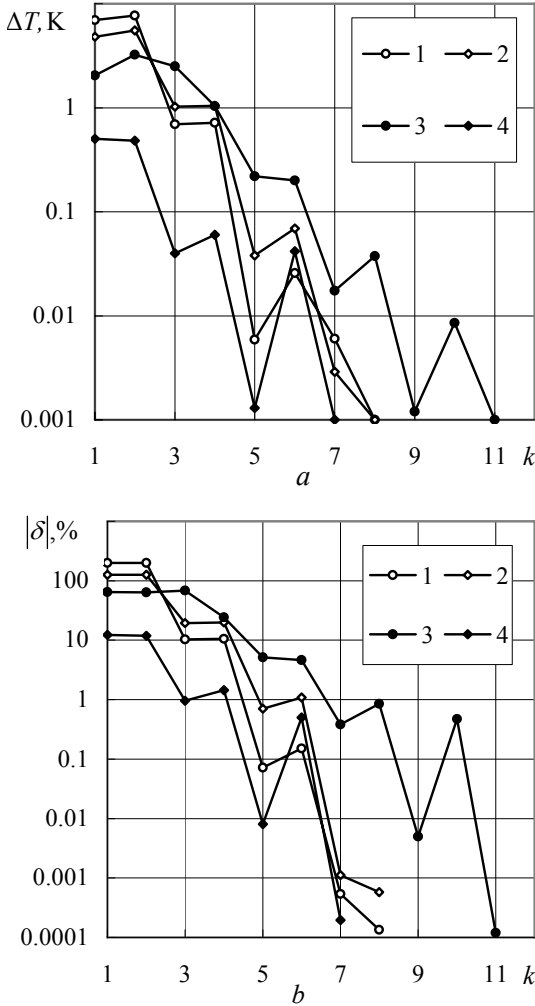


Fig. 1 Selection (a) of evaporating droplet temperature by iteration computations using the method of the fastest descent and the influence of this selection on disbalance of energy fluxes on the droplet surface (b). Duration of conductive heating  $Fo$ : 1 – 0.11; 2 – 0.439; 3 – 0.878; 4 – 4.39.  $T_g = 573 \text{ K}$ .  $T_{R,0} = 275 \text{ K}$ .  $\Delta T = |T_{R,k} - T_{R,k}| + 0.001, \text{ K}$ .  $R_0 = 5 \cdot 10^{-6} \text{ m}$ .  $p = \left( 1 - \frac{q_f^+ + q_c^-}{q_c^+} \right) 100\%$ .  $p_{v,\infty} / p = 0$

In the case of single droplet evaporation, the changes of gas flow parameters are not taken into account. Indeterminacy, related to an infinite sum, exists in the numerical scheme (12), as temperature field gradient of the droplet is being calculated [10]. Numerical experiment

proves that the number of terms in the infinite sum should be not less than 100. Such condition assures unambiguous values of the parameters of combined heat transfer in the droplet in all modelled cases. During iterative scheme optimisation the analysis of the parameters that define droplet heating and evaporation processes pointed up a huge influence of liquid surface temperature on satisfying the condition of energy flux balance on the droplet surface (Fig. 1). Instant value of the droplet surface temperature is selected by iteration computations according to the scheme (12), using the method of the fastest descent in such a way, that it has to follow the condition of energy flux balance on the droplet surface with the accuracy of one hundredth percent. When calculating the instant value of the droplet surface temperature, it is necessary to evaluate the change of this temperature, which is smaller than one thousandth degree (Fig. 1, a).

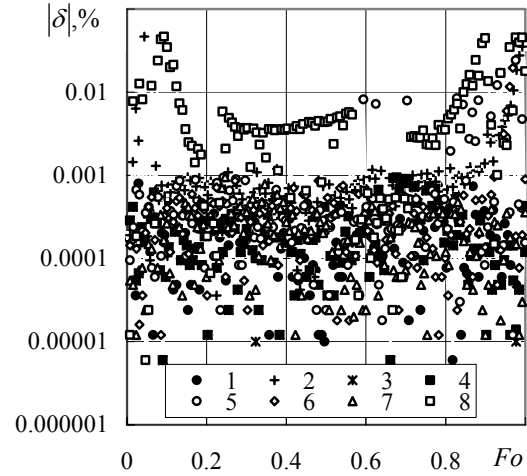


Fig. 2 Control of energy flux disbalance during the numerical experiment of heat and mass transfer in a droplet.  $T_g, \text{ K}$ : 1,2 – 573; 3-6 – 473; 7,8 – 1073.  $T_{R,0}, \text{ K}$ : 1-2,5-8 – 275; 3,4 – 303.  $R_0 \cdot 10^{-6} \text{ m}$ : 1-4,7,8 – 5; 5,6 – 2.  $p_{v,\infty} / p$ : 1,2 – 0.4; 3,4 – 0; 5-6 – 0.1

High reliability of the research results was ensured during all cases of numerical experiment (Fig. 2). Calculated heat fluxes ensured the condition of energy balance on the droplet surface with the accuracy of  $\pm 0.001$  percent during the entire lifetime of the droplet in all cases of liquid evaporation in dry medium, neglecting the influence of Knudsen layer.

As liquid evaporates in humid air, relative error, which defines the disbalance of energy fluxes, suddenly increases, as phase transformation mode on the liquid surface changes from vapour condensation to liquid evaporation. This jump is more significant, if Knudsen layer's influence is taken into account. During final liquid evaporation stage the increase of relative error is observed as well. This jump is related to highly increasing influence of Knudsen layer during rapid decrease of the droplet dimension. In all modelled cases during the mode of phase transformation change, as well as during the final liquid evaporation stage, the accuracy of  $\pm 0.1$  percent was ensured in the energy balance condition.

During major part of the droplet heating and evaporation process the condition of energy balance on its surface was ensured with the accuracy of  $\pm 0.1$  percent, evaluating the influence of Knudsen layer (Fig. 2).

**3. Research results**

Phase transformation of single cold water droplet is modelled in various boundary conditions of heat and mass transfer. Sprayed liquid is set to be cold, if its initial temperature is lower than the temperature of equilibrium evaporation beginning  $T_{R,e,l}(\tau_e)$ , as the droplets are heated conductively. When the influence of gravitational forces on droplets movement in gas can be neglected, conductive heating is observed, as the droplets are carried by gas flow without slipping. This is the simplest droplet heating case. Pointing up of the peculiarities of heat and mass transfer processes in the case of conductive heating of droplets is very important for the evaluation of liquid thermal state change and phase transformation in more complicated droplet heating cases.

**3.1. Phase transformation of cold water droplets during conductive heating**

Unsteady and equilibrium mass transfer modes of a droplet can be distinguished according to the distribution of heat supplied to the droplet for its heating and evaporating.

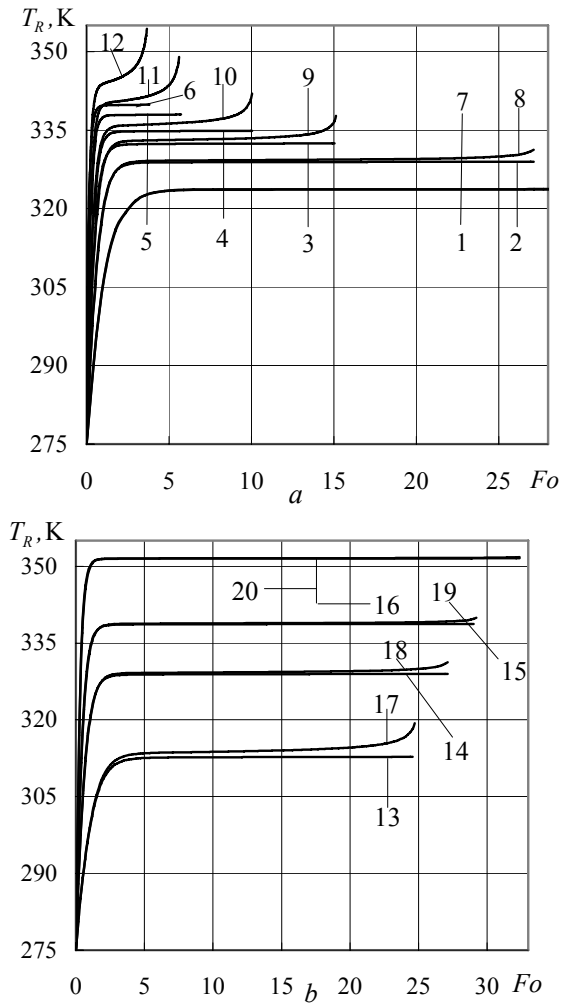


Fig. 3 Influence of gas temperature (a) and liquid vapour pressure (b) on water droplet surface temperature in the case of conductive heating. 1-6,13-16 – Knudsen layer’s influence is neglected.  $T_g$ , K: 1,2 – 373; 3,4,13-20 – 473; 5,6 – 573; 7,8 – 673; 9,10 – 873; 11,12 – 1073.  $p_{v,\infty} / p$ : 1-12,14,18 – 0.1; 13,17 – 0; 15,19 – 0.2; 16,20 – 0.4.  $R_0=5 \cdot 10^{-6}$ m.  $T_{R,0}=275$ K

The change of the droplet thermal state depends on the peculiarities of phase transformations on liquid surface. If liquid vapour is present in gas and the temperature of sprayed liquid is lower than dew point, then the liquid vapour condenses on the droplet surface for some time. In this case the liquid is heated by condensation energy, accumulated in the vapour and heat delivered from gas by conduction. That influences rapid warm of the liquid during the initial stage of its thermal state change (Fig. 3). Vapour condensation process can occur only in the case of unsteady phase transformation, until droplet surface warms up to dew point. During this mode phase transformation intensity gradually decreases (Fig. 4) until vapour flux density passes through its zero value and changes direction. Then the vapour condensation on the droplet surface stops and an unsteady evaporation mode begins - the rate of phase transformation gradually increases.

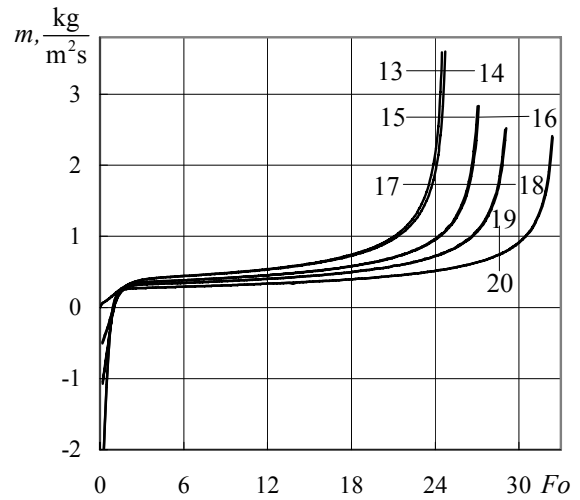


Fig. 4 Change of vapour flux density on a droplet surface. Boundary conditions the same as in Fig. 1

Equilibrium phase transformation mode is being reached, as all supplied heat begins to evaporate liquid (Fig. 5). The approach of liquid unsteady evaporation

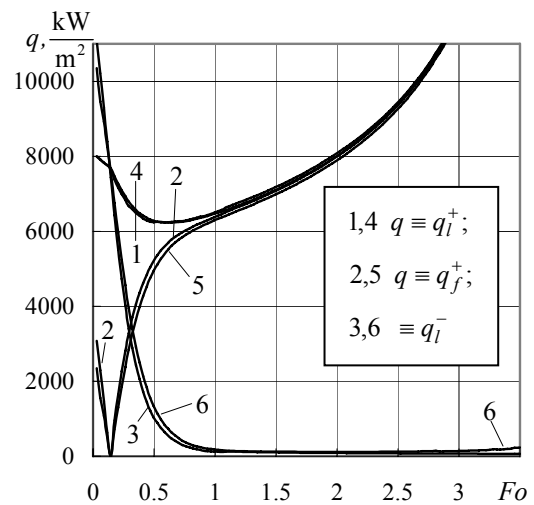


Fig. 5 Energy flux densities on a droplet surface. 4-6 – The influence of Knudsen layer is neglected.  $T_{R,0}=275$ K.  $R_0=5 \cdot 10^{-6}$ m.  $T_g=1073$ K.  $p_{v,\infty} / p = 0.1$

mode towards equilibrium evaporation mode is indicated by: (a) approaching of the curve 5 that defines heat flux density of phase transformations towards the curve 1 that defines conductive heat flux in gas and thus defines external droplet heating intensity, and approaching of the curve 3, which defines conductive heating flux density in liquid and defines warming rate of the droplet, towards its zero value.

When the influence of Knudsen layer is insignificant, the liquid approaches to equilibrium evaporation mode and still continues to warm (Fig. 3), therefore fully equilibrium evaporation is not reached.

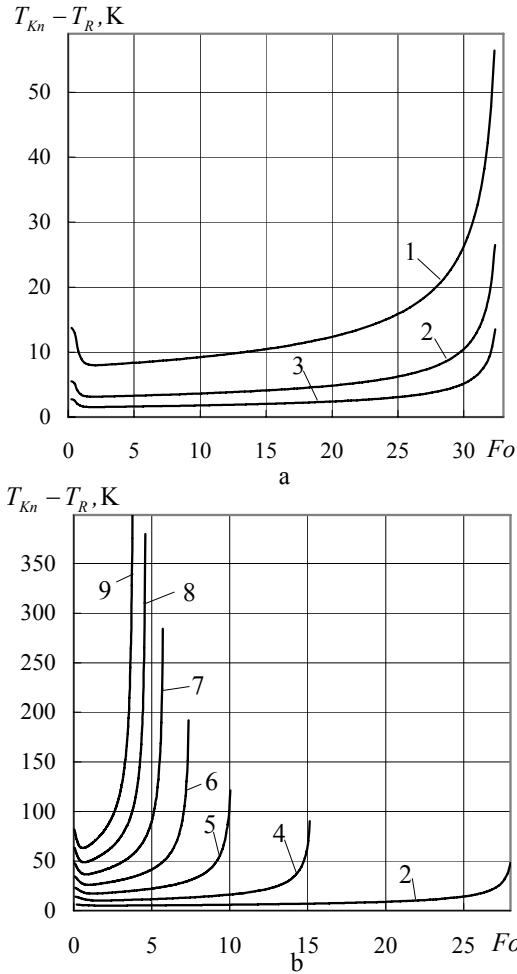


Fig. 6 Dependence of temperature drop in Knudsen layer on the size of droplets (a) and gas temperature (b).  $T_g$ , K: 1-3 – 473; 4 – 573; 5 – 673; 6 – 773; 7 – 873; 8 – 873; 9 – 1073.  $R_0 \cdot 10^{-6}$ , m: 1 – 2; 2,4-9 – 5; 3 – 10.  $p_{v,\infty} / p = 0.4$ .  $T_{R,0} = 275K$ .

That is confirmed by deviation of the curve 6, which represents conductive heat flux density in the liquid, from its zero value during the final stage of liquid evaporation (Fig. 5).

During unsteady evaporation the temperature of droplet surface warming  $T_R(\tau)$  increases up to the temperature  $T_{R,e}(\tau_e)$ , which is characteristic to the beginning moment of unsteady evaporation. This temperature depends only on gas flow parameters - on temperature of gas mixture and liquid vapour partial pressure in it and on the way of droplet heating [10]. Conductively heated liquid droplet, having the lowest temperature, will reach equilib-

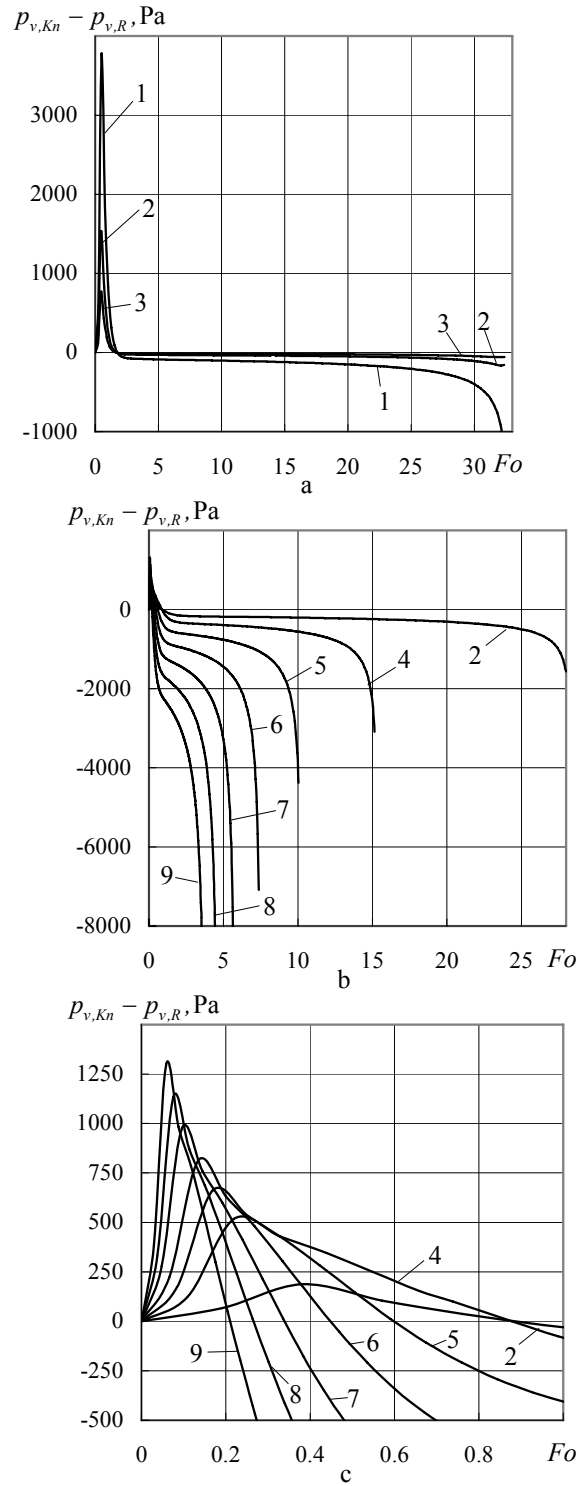


Fig. 7 Dependence of vapour pressure drop in Knudsen layer on the size of droplets (a) and gas temperature (b,c). Boundary conditions are the same as in Fig. 6

rium evaporation for the given gas parameters. This temperature  $T_{R,e}(\tau_e)$  will remain almost constant during equilibrium evaporation mode only if the influence of Knudsen layer's is insignificant (Fig. 3). Hence, the temperature of liquid equilibrium evaporation in the case of conductive droplet heating is very important parameter for the evaluation of the change of dispersed liquid thermal state. During phase transformation mode, close to equilibrium evaporation, the curve, that represents surface temperature  $T_{R,e}(\tau)$  of a single conductively heated droplet,

can deviate from temperature  $T_{R,e,l}(\tau_e)$  only due to temperature and vapour pressure drops in the Knudsen layer. The influence of the latter factors is higher for smaller droplets (Fig. 6,7, a). Temperature and pressure drops for similar size droplets become more significant, as gas temperature and liquid vapour partial pressure in gas increase (Fig. 6,7, a). In these cases more significant influence of the latter parameters on the change of droplet surface temperature is being observed as well (Fig. 3).

The change of phase transformation character makes essential influence on dynamics of liquid vapour pressure in Knudsen layer (Fig. 7, c). At the beginning pressure drop increases under the influence of intensive condensation. After that, weakening of condensation decreases pressure drop down to zero. At this mode of unsteady phase transformation liquid vapour pressure on the boundary of Knudsen layer exceeds saturated liquid vapour pressure near the droplet surface. As the liquid begins evaporating, vapour pressure drop in Knudsen layer starts to grow, but remains lower than saturated vapour pressure near the droplet surface. The amount of vapour in gas determines the duration of distinctly unsteady condensation – based phase transformation mode. Gas temperature, on which actually depends the rate of droplet heating, also makes certain influence (Fig. 7, c).

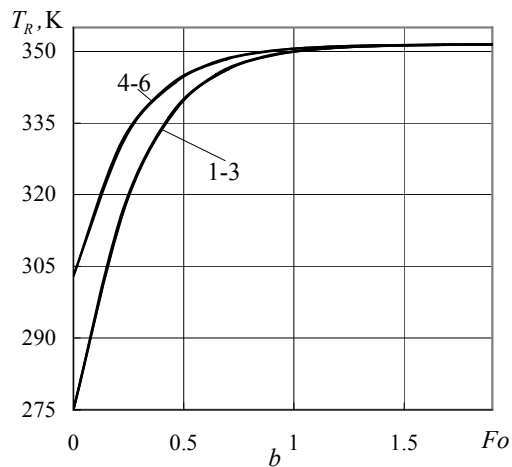
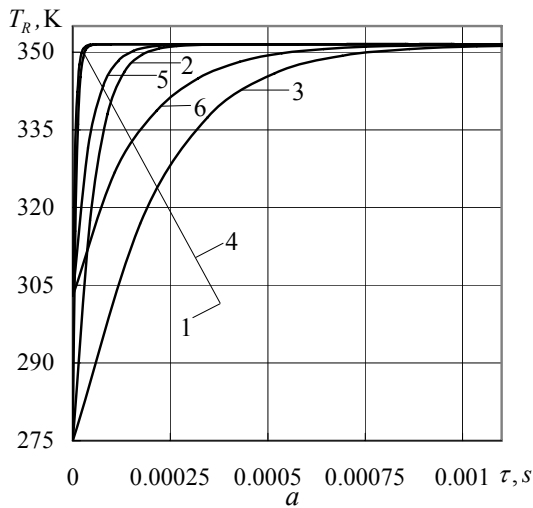


Fig. 8 Heating dynamics of droplets in time (a) and Fourier number (b) scales. The Knudsen layer's influence is cancelled.  $T_{R,0}, K$ : 1-3 – 275; 4-6 – 303.  $R_0 \cdot 10^{-6}, m$ : 1,4 – 2; 2,5 – 5; 3,6 – 10.  $p_{v,\infty} / p = 0.4$ .  $T_g = 473 K$

### 3.2. Discussion on results

Intensity of thermal energy, supplied to the droplets, which are carried without slipping by nonradiating gas flow, directly depends on their size. One can detect that vapour flux density on the surface of spherically symmetrical droplets also linearly depends on their dimension. That makes an assumption for temperature of liquid droplet during its equilibrium evaporation to stay constant, as temperature and vapour pressure drops in Knudsen layer are insignificant. The following fact is important:

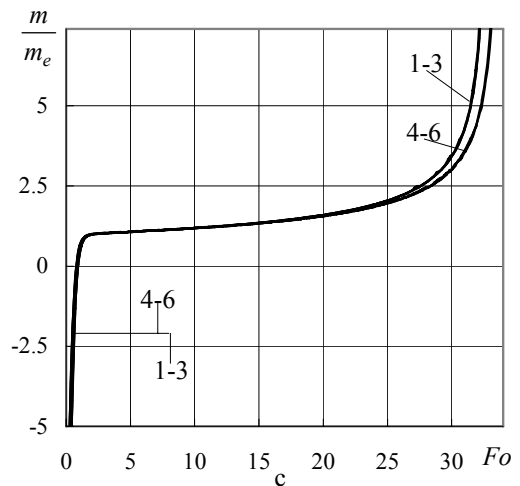
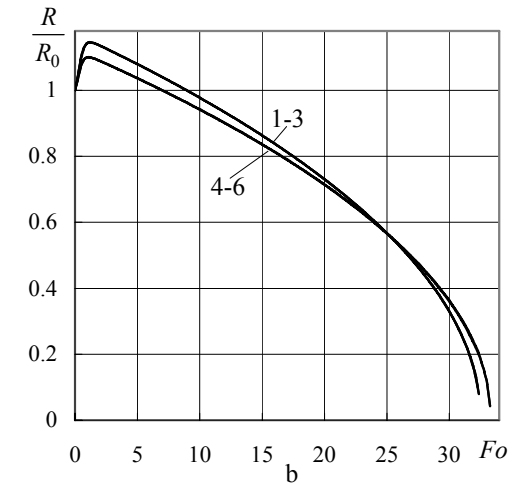
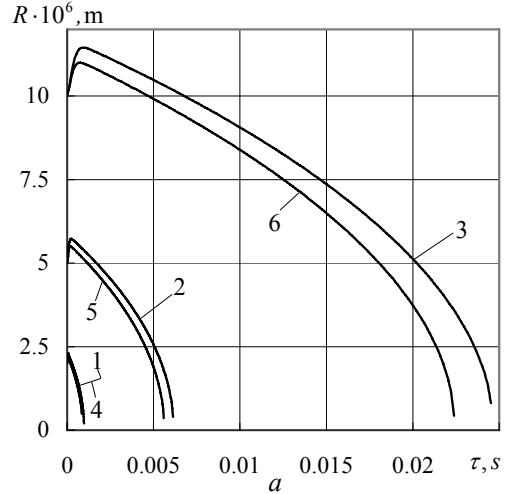


Fig. 9 Evaporation dynamics of droplets in time (a) and Fourier number (b,c) scales. The influence of Knudsen layer is cancelled. Boundary conditions are the same as in Fig. 8

the curves, which represent surface temperatures of evaporating droplets with the same initial surfaces temperature, coincide without reference to the droplet size in the scale of nondimensional time, defined by Fourier number, as the influence of Knudsen layer on transfer processes is insignificant (Fig. 8).

Therefore the curves 1-6, 13-16 (Fig. 3) and the curves 1,4 (Fig. 8, b) represent heating dynamics of infinite set of various initial dimension droplets in corresponding boundary conditions, as the influence of Knudsen layer is insignificant. As the droplets are heated by conduction, one can detect universality of the curves, which represent the change of droplet dimension variation (Fig. 9, a) in relative form of the droplet dimension in nondimensional time scale (Fig. 9, b), as the influence of Knudsen layer on transfer processes can be neglected.

Universality of the curve that represents dynamics of relative droplet dimension is possible only if the quality of similar dynamics of vapour flux density on the droplet surface is present. In this case it must completely coincide in relative form  $m(\tau)/m(\tau_e)$  (Fig. 9, c).

As the influence of Knudsen layer grows, the curves of vapour flux density deviate from the universal scale; this deviation is directly proportional to the sprayed liquid (Fig. 10).

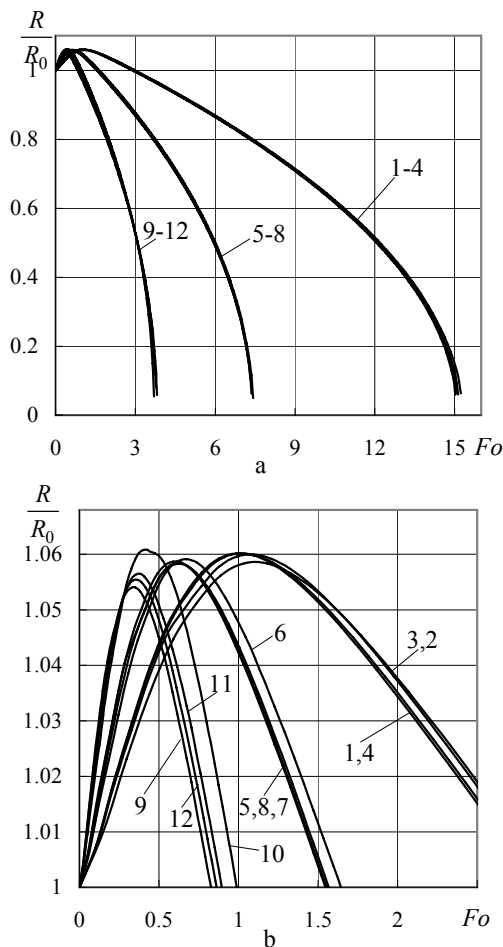


Fig. 10 Influence of the Knudsen's layer on the change of a droplet dimension: a – general view; b – initial stage of phase transformation.  $T_g, K$ : 1-4 – 573; 5-8 – 773; 9-12 – 1073.  $R_0 \cdot 10^{-6}$ , m: 2,6,10 – 2; 3,7,11 – 5; 4,8,12 – 10.  $p_{v,\infty} / p = 0.1$ .  $T_{R,0} = 275K$

Stratification of the curves, which represent dynamics of vapour flux density on the droplet surface, according to the droplet size, is especially distinctive at the final stage of droplet evaporation (Fig. 10, a) and is specific at the initial phase transformation stage (Fig. 10, b). The processes of water heating and phase transformation condition the change of droplet volume in different way: liquid evaporation causes the droplet volume to decrease; condensation and expansion of warming liquid cause the droplet volume to increase. As cold water is injected, the rate of droplet heating decreases during unsteady evaporation, though the rate and character of phase transformation grows. During the initial condensation stage the droplet increases for a while due to intensive warming. As unsteady evaporation begins and droplet warming rate gradually decreases, the droplet begins to decrease. That explains an extreme point in the curve that indicates the variation of the droplet volume (Fig. 10, b). This extreme point represents energy equilibrium between heating and evaporation processes.

#### 4. Conclusions

1. The peculiarities of phase transformation on droplet surface have essential influence on the change of droplet thermal state, especially the change of phase transformation character.

2. The change of water droplet state and dynamics of phase transformations in Fourier number - based time interval can be defined by universal curves. They represent temperature variation of infinite set of various size droplets, having the same initial temperature in the case of conductive heating, as the influence of Knudsen layer on transfer processes can be neglected.

3. Universal curves of the change of liquid thermal state and phase transformations in Fourier number - based time interval are very important for the evaluation of heat and mass transfer peculiarities and the influence of Knudsen layer, as the droplets evaporate in complicated boundary conditions.

4. The difference between droplet equilibrium evaporation temperatures in the cases of purely conductive and combined heating allows evaluating the influence of droplet slip velocity and radiation, absorbed in droplets on liquid thermal state and phase transformation rate.

5. The influence of temperature and pressure drops on liquid thermal state and phase transformation rate can be evaluated according to the change of droplet equilibrium evaporation temperature, calculated evaluating the influence of Knudsen layer.

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#### „LAŠO“ UŽDAVINYS ŠIUOLAIKINIŲ TERMINIŲ TECHNOLOGIJŲ RETROSPEKTYVOJE

#### R e z i ū m ė

Pateikta išpurkšto skysčio fazinių virsmų kombinuoto analitinio ir skaitinio tyrimo metodika, kurią taikant lašelio didumas neribojamas. Ištyrus laidumu šildomų šalto vandens lašelių fazinius virsmus, nustatytos skysčio terminės būsenos kitimą ir fazinių virsmų dinamiką per Furjė kriterijumi išreikštą trukmę aprašančios universaliosios kreivės. Jos aprašo tos pačios pradinės skysčio temperatūros begalinės aibės skirtingo didumo lašelių paviršiaus temperatūros kitimą, kai lašeliai šildomi laidumu ir Knudseno sluoksnio įtakos pernašos procesams galima nepaisyti. Pagal nukrypimą nuo šių kreivių, lašeliams garuojant sudėtingomis ribinėmis sąlygomis, galima vertinti lašelių slydimo greičio dujose, spinduliuotės sugėrimo pusskaidriame skystyje ir Knudseno sluoksnio įtaką lašelių šilimui ir garavimui.

G. Miliauskas, V. Garmus

#### “DROPLET” PROBLEM IN THE ASPECT OF MODERN THERMAL TECHNOLOGIES

#### S u m m a r y

The study presents the combined analytical and numerical method, which allows researching phase transformations of dispersed liquid without reference to droplet size limitation. Phase transformations of conductively heated cold water droplets were examined; the universal curves of the change of liquid thermal state and phase transformations in Fourier number - based time interval were determined. These curves represent temperature variation of infinite set of various size droplets, having the same initial temperature in the case of conductive heating, as the influence of Knudsen layer on transfer processes can be neglected. Deviation from these curves, as droplets evaporate in complicated boundary conditions, allows evaluating the influence of droplet slip velocity, radiation, absorbed in semitransparent liquid and Knudsen layer on droplet heating and evaporation processes.

Г. Миляускас, В. Гармус

#### ЗАДАЧА „КАПЛИ“ В СВЕТЕ СОВРЕМЕННЫХ ТЕРМИЧЕСКИХ ТЕХНОЛОГИЙ

#### Р е з ю м е

Предложена свободная от ограничения величины капли методика комбинированного аналитически-численного исследования фазовых превращений диспергированной жидкости. Изучив испарение капель холодной воды при кондуктивном нагреве, установлены универсальные кривые динамики изменения термического состояния и фазовых превращений жидкости через критерий Фурье. Эти кривые описывают изменение термического состояния и динамики фазовых превращений жидкости бесконечного числа разной величины капель, одинаковой начальной температуры жидкости, при кондуктивном нагреве капель в условиях малого влияния слоя Кнудсена. По отклонению от универсальных кривых при сложном нагреве капель можно оценить влияние скорости скольжения капель и процесса поглощения теплового излучения полупрозрачной жидкостью, а также влияние слоя Кнудсена на процесс нагрева и испарения капель.

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