Dynamics of a body vibrating according to the law of harmonics on an air-cushion

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1. Introduction

Automatic assembling is essentially aimed at ensuring precise matching of connective surfaces of the parts prior to their join. One method of solving this problem is to apply pneumatic-vibrational automatic assembly. It is based on directional matching of joining surfaces by a directional self-search method. During self-search process the relative position of joining surfaces is directionally changing within a certain limited area. At a moment when the error of these surfaces does not exceed a permissible one, the surfaces are unhinderedly joined. The perspective pneumatic-vibrational assembling method involves excitation of vibrations of an element based on air-cushion perpendicularly to the joint axis. The surfaces overlap due to aligning effect created by the air current flowing through the clearance between joining elements. In the previous works the vibration regimes of an element on air-cushion and their characteristics were determined for incompressible [1,2] and compressible [3,4] air. Technical assembling possibilities extend when a track is placed on a vibrator platform and forced vibrations are induced in it. The paper analyses this method to be applied for assembling cylindrical and prismatic elements. In order to use the pneumatic-vibrational assembling in practice, the matching characteristics of joining surfaces are to be determined. The vibrations of an element based on air-cushion are due to both air distribution in the air-cushion and air flow parameters. Pressure in the air-cushion in the case of compressible and incompressible air is described by nonstationary Reynold’s equations which are solved by applying the variational formulation of the finite elements method. The air-cushion area is divided by finite elements into linear triangles. In the case of incompressible air, a set of linear algebraic equations is derived by means of the finite elements method which is solved by Gauss elimination. In the case of compressible air, by applying the finite elements method a matrix differential equation is derived which, for obtaining a matrix algebraic equation, is approximated by finite differences according to the central difference scheme. The derived nonlinear matrix algebraic equation is solved by Newton-Raphson method. Simultaneously solving the pressure distribution and part motion equations, the latter is solved by Runge-Kutta’s method, and vibrations of a part on an air-cushion vibrating according to the harmonic law are investigated.

2. Scheme of pneumatic-vibrational assembling

Joining part 2 is placed on pneumatic track 1 having one air blow line (Fig. 1). Other part 3 is rigidly fixed at the rear surface of the track.

The compressed air making an air-cushion between the part and the supporting track surface is blown through aperture 4 from the pneumatic track cell. In the assembly position apertures 4 are formed with pockets 5. At certain blow parameters vibrations the part on air-cushion are excited and they provide the search of the joining surfaces of parts. To increase the vibrations amplitude of the part on air-cushion, track 1 is placed on vibrator platform 6 and harmonic track vibrations are excited. When the surfaces are matched the parts are joined easily.

3. Pressure distribution equation assuming air density to be constant

To determine pressure in the clearance between the part and the supporting pneumatic track surface, nonstationary Reynold’s equation used in the theory of gas lubrication is applied [5,6]. This equation is derived from general equations of viscous liquid laminar flow and that of stream continuity. When solving practical problems it is supposed that air mass forces are small and temperature in the clearance between the surfaces where the air is blown is constant. Then, assuming that air density is constant, dimensionless Reynold’s equation is written in the following form [1]

\[
\widetilde{h}^2 \left( \frac{\partial^2 \tilde{p}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{p}}{\partial \tilde{z}^2} \right) = \sigma \frac{\partial \tilde{h}}{\partial \tau}
\]

where \( \sigma = \frac{12pl}{p_o l_0} \); \( \tilde{h} = \frac{h}{h_s} \); \( \tau = \frac{t}{t_0} \); \( \tilde{p} = \frac{p}{p_o} \); \( \tilde{x} = \frac{x}{l} \); \( \tilde{z} = \frac{z}{l} \); \( \sigma \) is coefficient of compression; \( \eta \) is coefficient...
of dynamic viscosity; \( l \) is length of the part placed on an air-cushion; \( h = h(t) \) is dimensionless thickness of air-cushion; \( \bar{p} = \bar{p}(\bar{x}, \bar{z}, t) \) is dimensionless pressure of air-cushion; \( p \) is pressure of air-cushion; \( h_d \) is thickness of air-cushion when the part is in dynamical equilibrium state; \( t_o \) is vibrations period of the part; \( p_o \) is pressure in the blow cell.

When \( \bar{z} = 0 \), boundary conditions in the blow line are expressed by

\[
-h^3 \frac{\partial \bar{p}}{\partial \bar{z}} |_{\bar{z}=0} = q = 0
\]  

(2)

where \( q = \alpha \nu (\frac{p_b}{p_o}) \) is air mass flow; \( \alpha = \frac{6m^* S_p p_0}{\rho p_0 h_0^3} \) is function describing critical air mass flow to the blow line length unit (dimensionless value); \( m^* \) is maximum permissible flow to discharge area unit; \( S_p \) is discharge area to the blow line length unit; \( \rho \) is air density; \( p_0 \) is pressure in the blow line; \( \nu (p/= p_o) \) is discharge function [5].

Boundary condition at part edges is

\[
p(x, z_b) = p(0, z) = p(1, z) = p_a
\]  

(3)

where \( p_a \) is atmosphere pressure.

To solve Reynold’s equation (1) with boundary conditions (2) and (3) the statement of the variational finite elements method is applied. In the fourth subsection the tildes are not put over the values.

4. Finite elements model for incompressible air

Variationally, the solution of Eq. (1) with boundary conditions (2) and (3) is equivalent to functional minimization [7 - 9]

\[
\chi = \int_0^1 \int_0^1 h \left( \frac{\partial \bar{p}}{\partial \bar{x}} \right)^2 + \left( \frac{\partial \bar{p}}{\partial \bar{z}} \right)^2 \partial x \partial z + \sigma \int_0^1 \int_0^1 \partial \bar{h} \partial p \partial x \partial z - \int_0^1 q p \partial x = 0
\]  

(4)

Having done functional minimization by pressure \( p \) the set of equations is obtained

\[
[K] [\bar{p}] + [C] [\bar{h}] = [F]
\]  

(5)

where \([K]\) is matrix of the system yield; \([C]\) is matrix of the system damping; \([F]\) is vector of the system load; \( h = -1 \) is nodal compression velocity [10].

The air-cushion area is split into triangle finite elements [1]. For the sake of symmetry a half of the air-cushion is taken. The expressions of matrices \([K]^{(v)}\), \([C]^{(v)}\) and vector \([F]^{(v)}\) are given [1].

5. Pressure distribution equation for compressible air

In case of compressible air the pressure in the air clearance between the part and supporting pneumatic track surface can be calculated by means of Reynolds’ equation derived from both viscous liquid laminar flow equations called Navier–Stokes equations and that of stream continuity. Reynolds’ equation is written in a dimensionless form as [3]

\[
2\sigma \frac{\partial (h \bar{p})}{\partial \bar{t}} = h \left( \frac{\partial^2 \bar{p}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{p}}{\partial \bar{z}^2} \right)
\]  

(6)

where \( \bar{p} = \bar{p}(\bar{x}, \bar{z}, t) \) is dimensionless pressure of the air-cushion; \( p \) is air pressure.

When \( \bar{z} = 0 \), boundary condition in the air blow line is expressed by

\[
\sigma V \frac{\partial (\bar{p})}{\partial \bar{t}} = q + 2h^3 \frac{\partial \bar{p}}{\partial \bar{t}} = 0
\]  

(7)

where \( V = V_o + \nu h \) is volume of track blow line pockets; \( V_o \) is volume of the pockets for the blow line length unit; \( \nu \) is dimension of the pockets in \( z \) axis direction [5].

Boundary conditions at the edges of the part are described by Eq. (3). Further on, the tildes will not be written over the values.

6. Finite elements model for compressible air

Having designated \( p^2 = P \), Reynold’s equation can be written as

\[
h \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} \right) = 2\sigma \frac{\partial (h \sqrt{P})}{\partial \bar{t}} = 0
\]  

(8)

Boundary condition in the blow line is

\[
\frac{\sigma V}{2} \frac{\partial \sqrt{P}}{\partial \bar{t}} \frac{q}{2} - h^3 \frac{\partial \sqrt{P}}{\partial \bar{z}} = 0
\]  

(9)

Boundary condition at the part edges is

\[
p(x, z_b) = p(0, z) = p(1, z) = \sqrt{P_a}
\]  

(10)

Eq. (8) and boundary conditions (9) and (10) are not being solved directly, instead the statement of the variational finite elements method is applied. The first functional variation is

\[
\delta X [P] = \int_0^1 \int_0^1 h \left[ \frac{\partial \sqrt{P}}{\partial \bar{x}} \delta \sqrt{P} + \frac{\partial \sqrt{P}}{\partial \bar{z}} \delta \sqrt{P} \right] \partial x \partial z - \int_0^1 \int_0^1 2\sigma \left[ \frac{\partial \sqrt{P}}{\partial \bar{x}} \delta \sqrt{P} + h \frac{\partial \sqrt{P}}{\partial \bar{z}} \delta \sqrt{P} \right] \partial x \partial z - \int_0^1 \frac{1}{2} q \delta P \partial x + \frac{\sigma V}{2} \int_0^1 \frac{\partial \sqrt{P}}{\partial \bar{t}} \delta \sqrt{P} \partial x
\]  

(11)

When pressure \( P \), variation \( \delta P \), compression velocity \( h \) and pressure derivative according to time are expressed by the form functions, and Eq. (11) is rewritten in
a matrix form, and when $\partial P$ is evaluated as independent, then a matrix differential equation is derived for the element

$$\left[K^{(e)}\right]\{p\} - \left[G^{(e)}\right]\{\sqrt{P}\} + \left[C^{(e)}\right]\{\frac{\partial \sqrt{P}}{\partial t}\} = \{F^{(e)}\}$$

where $\left[K^{(e)}\right]$ is matrix of the element yield; $\{F^{(e)}\}$ is vector of the element load; $\left[G^{(e)}\right]$ is matrix of the element compression; $\left[C^{(e)}\right]$ is matrix of the element damping; $\{C^{(e)}\}_1$ is volume matrix of the element damping.

The set of differential Eq. (12) is solved by approximating it with the finite differences method according to the central differential scheme. Having done the approximation, matrix equation is derived

$$\left[A\sqrt{P}\right]_{new} + \left[K\right]_{new} = \left[H\sqrt{P}\right]_{old} + \{F\}$$

(13)

Matrix $[A]$ is a combination of matrices $[C]$, $[C_1]$ and $[G]$ while matrix, $[H]$ is a combination of matrices $[C]$ and $[C_1]$. The expressions of matrices $\left[K^{(e)}\right]$, $\left[G^{(e)}\right]$, $\left[C^{(e)}\right]$, $\{C^{(e)}\}_1$ and vector $\{F^{(e)}\}$ are given in [3].

7. Investigation of a body motion on an air-cushion vibrating at harmonic frequency

A pneumatic track is placed on a vibrator platform and its forced vibrations are excited. The part elevation $h$ is expressed with respect to air-cushion thickness $h_1$

$$h = h_1(t) + A \sin \omega t$$

(14)

Then the equation of part motion acquires the form

$$\ddot{h} + \frac{L \sin \frac{\omega}{v}}{\nu} \dot{h} + 2 \int_0^{\frac{1}{2}} \left(\ddot{p} - \ddot{p}_a\right) \xi d\xi = F$$

(15)

where $\ddot{m} = \frac{m h_1}{l^2 p_0 \gamma_0}$; $F = \frac{mg}{p_0 l^2}$; $b = h / l$; $t_0 = 1 / \nu$; $L = A m \omega^2 / \left(p_0 l^2\right)$; $A$ and $\omega$ are amplitude and frequency of pneumatic track vibrations, respectively; $\nu$ is frequency of vibrations of the part on air-cushion; $m$ is mass of the part on air-cushion; $l$ is length of the part; $b$ is width of a half of the part.

In the case of incompressible air, Eq. (5) is co-solved with Eq. (15). In the case of compressible air, Eq. (13) is co-solved with Eq. (15). In MATLAB the programs for solving algebraic Eqs. (5) and (15) by the finite elements method are formed. Ordinary differential Eq. (15) is also solved in MATLAB system by the Runge-Kutta’s method. At an initial moment air-cushion thickness $h$ is determined and air-cushion pressure is calculated by solving Eq. (5) in the case of incompressible air, and Eq. (13) in the case of compressible air. The new air-cushion thickness is calculated in both cases by solving Eq. (15), and new pressure – Eq. (5) in the case of incompressible air, and Eq. (13) in the case of compressible air.

The effect of various parameters on the part motion on a harmonically vibrating air-cushion is analysed. The vibrations character depends on regime parameter $\alpha_\nu$, compression coefficient $\sigma$, parameter $V$, defining the ratio of track pockets volume to that of lubricant layer (Fig. 1), the frequency ratio $\omega/\nu$. Parameter $\alpha_\nu$ defines the air blow degree. It is included in the load vector expression. Compression coefficient $\sigma$ depends on both the coefficient of dynamic air viscosity in the pressure cell and air-cushion thickness when the part is in dynamic equilibrium. When analysing dynamics of the part on-air-cushion vibrating at harmonic frequency, it has been found out that vibrations character can be diverse. In the case of incompressible air, diverse vibrations are represented in Fig. 2. Vibrations of a constant amplitude are represented in Fig. 2, a, slightly modulated vibrations – in Fig. 2, b, vibrations with more pronounced modulation – in Fig. 2, c, vibrations whose amplitude abruptly changes – in Fig. 2, d, beating – in Fig. 2, e, slightly modulated vibrations with higher relative frequency – in Fig. 2, f and vibrations when the vibration frequency of the part on-air-cushion is the lowest – in Fig. 2, g. The dependences are formed with these values of parameters: $L = 0.01$, $p_a = 0.6$, $\sigma = 3.95$, $\alpha_\nu = 3$, $m = 0.0006$. In the case of compressible air, the character of the part vibrations is analogous, while the reciprocal amplitude is somewhat lower.

The zones of these vibrations for incompressible air are represented in Fig. 3, a, in compressible air – in Fig. 3, b. The zones when air is compressible are formed with these values of parameters: $m = 0.0006$, $V = 0.3$, $L = 0.01$, $\sigma = 3$. The zones marking corresponds to the one of unlike character vibrations presented in vibrations diagrams (Fig. 2).

The character of vibrations is mostly affected by relative frequency $\omega/\nu$.

In the diagrams representing unlike character vibration zones the vibrations proceeding in zone $e$ are called beating. In this case the vibrations amplitude of the part is a periodically varying time function of period $T_{\nu}$. Forced vibrations period is marked as $T_{\nu}$ (Fig. 2, e).

Beating period $T_{\nu}$ depends on the ratio of forced vibrations and vibration frequencies of the part on air-cushion. Its variation in incompressible air is represented in Fig. 4. In both cases the curve acquires a similar shape and the beating period is the longest when $\omega/\nu = 19$. All further dependences are formed with these values of parameters: $m = 0.0006$, $L = 0.01$, $p_a = 0.6$, $V = 0.3$. Only $\alpha_\nu$ and $\sigma$ are varying.

The beating reduces when the relative frequency deviates from this value to one or other side. The maximum and minimum dependences of mutual amplitudes when beating occurs in incompressible air is represented in Fig. 5, a, in compressible air – in Fig. 5, b.

In the case of incompressible air, the maximum mutual amplitude is the highest when $\omega/\nu = 18.5$. With an
Fig. 2 The character of part vibrations on air-cushion: a - constant amplitude vibrations, $\omega/\nu = 5$; b - slightly modulated vibrations, $\omega/\nu = 10$; c - vibrations with pronounced modulation, $\omega/\nu = 13$; d - vibrations whose amplitude abruptly changes $\omega/\nu = 15$; e - vibrations of the part at beating, $\omega/\nu = 20$; f - higher frequency slightly modulated vibrations, $\omega/\nu = 24$; g - low frequency modulated vibrations of the part on air-cushion, $\omega/\nu = 25$. 
Fig. 3 Vibration zones on $\omega/\nu - \alpha_s$ coordinates when:

- a - air is incompressible;
- b - air is compressible

Fig. 4 Dependence of beating period on frequencies ratio, air being incompressible

Increase in relative frequency $2h_{\text{max}}$ decreases, whereas $2h_{\text{min}}$ increases. In the case of compressible air $2h_{\text{max}}$ reaches maximum when $\omega/\nu = 18$ (Fig 5, b). At this value of relative frequency $2h_{\text{min}}$ is minimum. With an increase in frequency, $2h_{\text{min}}$ decreases, whereas $2h_{\text{max}}$ increases.

During beating amplitudinal modulation appears when a part on an air-cushion rises over the supporting pneumatic track surface according to the law of lower frequency vibrations and not according to the forced track vibrations law. For this reason, the self-search zone of the parts joining surfaces increases.

Fig. 5 Dependence of maximum (1) and minimum (2) mutual amplitudes on frequencies ratio when:

- a - air is incompressible $\alpha_s = 3$, $\sigma = 3.95$;
- b - air is compressible, $\alpha_s = 6.2$, $\sigma = 3$

Fig. 6 Dependence of mutual amplitude at beating on $L$ when $\omega/\nu = 20$:

- a - air is incompressible, $l - \alpha_s = 3$, $\sigma = 3.95$; $2 - \alpha_s = 3.5$, $\sigma = 5$;
- b - air is compressible: $l - \alpha_s = 6.2$, $\sigma = 3$; $2 - \alpha_s = 8$, $\sigma = 3$
The maximum and minimum rise values of the part on air-cushion greatly depend on the forced pneumatic track vibrations amplitude with an increase of which the mutual amplitude of the part also increases (Fig. 6, a in the case of incompressible air), Fig. 6, b in the case of compressible air). Moreover, at higher air blow parameter $\alpha$, the mutual amplitude is greater in both cases.

In the analysis of compression coefficient effect on maximum vibrations amplitude at beating it has been found out that in the case of incompressible air $2h_{\text{max}}$ increases with an increase in $\sigma$ (Fig. 7, a), while in compressible air $2h_{\text{max}}$ decreases with an increase in $\sigma$, (Fig. 7, b).

With an increase in pockets volume $V$, $2h_{\text{max}}$ of beating increases in compressible air (Fig. 8). Thus, in compressible air with an increase in parameters $\alpha_s$ and $V$ the self-search zone increases, and herewith the assembling conditions improve.

![Figure 7](image1.png)

**Fig. 7** Dependence of $2h_{\text{max}}$ on $\sigma$ when: a – air is incompressible: $1 - \sigma = 6; 2 - \sigma = 3.95; \alpha_s = 3$; b – air is compressible: $1 - \sigma = 3; 2 - \sigma = 10; \alpha_s = 6.2$

![Figure 8](image2.png)

**Fig. 8** Dependence of $2h_{\text{max}}$ on relative frequency and $V$ at beating, air being compressible when $\alpha_s = 6.2$, $\sigma = 3$: $1 - V = 1.5; 2 - V = 0.3$

![Figure 9](image3.png)

**Fig. 9** Dependence of $2h_{\text{max}}$ on relative frequency and $\alpha_s$: a – at beating, air being compressible, $\sigma = 3$: $1 - \alpha_s = 15; 2 - \alpha_s = 6.2$; b – in the absence of beating, air being incompressible, $\sigma = 3.95$: $1 - \alpha_s = 10; 2 - \alpha_s = 3$

With an increase in blow parameter $\alpha_s$ the maximum mutual beating amplitude increases in compressible air (Fig. 9, a). For incompressible air when blowing parameter $\alpha_s$ increases the mutual amplitude in the absence of beating also increases (Fig. 9, b).

![Figure 10](image4.png)

**Fig. 10** Dependence of $2h_{\text{max}}$ on $\omega/v$ in the absence of beating when: $1$ - air is compressible, $\alpha_s = 8$, $\sigma = 3$; $2$ - air is incompressible, $\alpha_s = 3$, $\sigma = 3.95$
The maximum mutual amplitude dependences on $\omega/\nu$ have been developed in case of the absence of beating. In compressible air, (Fig. 10, graph 1), and in incompressible air (Fig. 10, graph 2), an increase in $2h_{\text{max}}$ has been observed with an increase in $\omega/\nu$.

The algorithm of task solution has made it possible to define the pressure distribution all over the air-cushion. The highest pressure has been found in the blow line. Approaching the part edges the pressure decreases and at the edges it is equal to atmospheric.

Preferable vibrations for assembling are from the zones $b$, $c$, $e$, $f$ and $g$. To guarantee the proper self-search zone the vibrations of the zone $a$ have to be of sufficient amplitude. The parts can be joined when the amplitude varies abruptly, zone $d$, however this vibrations regime is not desirable.

7. Conclusions

1. Diverse character vibrations zones of a part on a harmonically vibrating air-cushion have been determined in compressible and incompressible air. The vibrations in these zones are of constant amplitude, of slighter or stronger amplitudinal modulation, of abruptly changing amplitude, or the vibrations turning into beating. Relative frequency $\omega/\nu$ has the greatest effect on vibrations character.

2. At beating, the maximum mutual amplitude is reached when $\omega/\nu = 18$ in compressible air, and when $\omega/\nu = 18.5$ in incompressible air. With an increase in the amplitude of forced vibrations and blow parameter $\alpha_s$, the maximum mutual amplitude increases when the air is compressible and incompressible. With an increase in compression coefficient $\sigma$, the maximum mutual amplitude increases when the air is compressible and incompressible. With an increase in compression coefficient $2h_{\text{max}}$ increases with an increase in pockets volume $\nu$.

3. In the absence of beating, when relative frequency increases, the maximum mutual amplitude increases in compressible and incompressible air. When air is incompressible, $2h_{\text{max}}$ increases with an increase in parameter $\alpha_s$.

4. The regimes of a part on an air-cushion vibrating according to harmonical law have been determined and their characteristics can be applied to the development of pneumatic-vibrational automatic assembling devices.

References


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DYNAMICS OF A BODY VIBRATING ACCORDING TO THE LAW OF HARMONICS ON AN AIR-CUSHION

Summary

Vibrations of a part on a compressible and incompressible air-cushion vibrating at harmonical frequency are analysed and applied to automatic assembling. The equations of pressure distribution in air-cushion between the part and the supporting track surface are given. To solve them a finite elements model is set up. The pres-
sure distribution equations are co-solved with the equation of motion of the part based on air-cushion vibrating according to harmonic law. The areas of the parameter combinations of air supply and part excitation are determined at constant amplitude vibrations, at slightly and strongly modulated vibrations, at abruptly changing amplitude vibrations and beating. The dependences of part vibrations amplitudes, beating period on frequency and amplitude of track excitation and air blow parameters are determined.

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ДИНАМИКА ТЕЛА НА ВОЗДУШНОЙ ПОДУШКЕ, ВИБРИРУЮЩЕЙ ПО ГАРМОНИЧЕСКОМУ ЗАКОНУ

Резюме

Применительно к автоматической сборке рас-сматриваются вибрации детали на воздушной подушке, вибрирующей по гармоническому закону с учетом и без учета сжимаемости воздуха. Представлены уравнения распределения давления в воздушной подушке между деталью и опорной поверхностью лотка. Составлена модель конечных элементов для решения этих уравнений. Совместно с уравнениями распределения давления решается уравнение движения детали на воздушной подушке, вибрирующей по гармоническому закону. Определены области сочетания параметров поддува и возбуждения детали, когда происходят вибрации детали с постоянной амплитудой, слабо и четко модулированные и вибрации с скачкообразно изменяющейся амплитудой и биение колебаний. Определены зависимости амплитуды вибраций детали, периода бienia от частоты и амплитуды возбуждения лотка и параметров поддува.

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