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ORIGINAL RESEARCH

Exploring determinants of network stochastic dominance ratios: a causal approach using explainable AI

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Abstract

Various financial ratios are recognised as elements that determine investment decisions, making it essential to identify what factors influence these ratios. The calculation of a ratio is often depicted as a relationship, often in the form of a fraction or percentage, and demonstrates the frequency with which one item is included inside another. The limitations of causal relationships that are derived from observational data are, however, frequently disregarded. We employ structural causal modelling to ascertain the inherent relationship between performance and risk metrics and the network stochastic dominance ratio, as well as how this causal framework influences investment product selection. The network stochastic dominance ratio is an attractive tool for ranking assets with respect to basic stochastic dominance principles. The findings indicate that the extreme Gradient Boosting (XGBoost) technique outperforms the quantile regression method in predicting the network stochastic dominance ratio. To interpret the significance of features, the Shapley Additive Explanations (SHAP) method is employed. The results substantiate the causal importance of network stochastic dominance ratio elements and show the significance of distributional characteristics (Kurtosis) and risk metrics (Max Drawdown and Expected Shortfall) in determining the stochastic dominance ratio. Our research is essential for linking stochastic dominance theories with empirical validation beyond mere correlations.

Keywords Investment portfolio · Uncertainty · Scenario modelling · Stochastic dominance · Explainable AI (XAI)

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1 Introduction

Before making any investment, it is essential for individuals to comprehend their objectives and the constraints that influence them, as these are frequently centred around the most important trade-off between risk and return. Returns signify the financial profits that contribute to the accumulation of long-term wealth; however, they are intrinsically linked to the degree of risk an investor is prepared to undertake (Kamarudin et al., 2021). Stochastic dominance (SD) allows the evaluation of investment possibilities under uncertainty by recognising varying investor utility functions. Despite constraints like computational complexity and interpretability, stochastic dominance's ability to provide a thorough, adaptable, and preference-sensitive assessment for uncertain outcomes makes it essential in decision-making under uncertainty (Levy, 2015; Post & Kopa, 2013).

The concept of dominance between investment options is a situation in which one option is universally preferable over another by all investors within a specific category. This preference establishes stochastic dominance relationships, which are categorised into the first, second, and third orders (Post, 2003). An asset is considered to be dominant over another if its cumulative return distribution is higher across all outcomes for a specific order. Nevertheless, the process of conducting pairwise SD comparisons can be time-consuming when dealing with lots of assets. Kabašinskas et al. (2020) introduced the stochastic dominance ratio (SDR) as a summary measure for ranking a group of funds with the same attributes, thus enabling multi-risk and multi-country comparisons. This SDR technique is fundamentally static, encapsulating dominance relationships for a specific timeframe or distributional assumption. Asset return distributions and dominance relationships may fluctuate over time due to evolving market conditions. This underscores the necessity of a network SDR (NetSDR) that can monitor the evolution of fund rankings across various periods and track the time-varying dominance of certain funds (Kabašinskas et al., 2025). It aims to enhance management of intermediate and ultimate risks while addressing the multi-stage portfolio selection challenge. The NetSDR is constructed in a comparatively straightforward manner, enabling the comparison of specific assets that are not dominated by any other asset.

By defining the essential performance indicators that influence the NetSDR, we may gain a deeper insight into the elements that dictate investment dominance. The use of a ratio to assess the superiority of one financial product over another does not reveal the elements that influenced the ratio's result. The limitations of the causal relationships that can be derived from observational data are often disregarded. We therefore employ structural causal modelling in this paper to examine the intrinsic relationship between performance and risk indicators, as well as the stochastic dominance relation, and the impact of this causal framework on the selection of investment products.

XAI evolved into a recognized domain with a diverse range of techniques and strategies designed to improve the interpretability of machine learning models. Although SHAP (Shapley Additive Explanations) is a robust and often employed strategy for feature attribution, it is but one of the viable methodologies. Alternative methods include model-agnostic techniques such as LIME (Local Interpretable Model-agnostic Explanations), counterfactual explanations, Anchors, Individual Conditional Expectation (ICE), and concept-based interpretability, alongside inherently interpretable models including decision trees, regression, and generalized additive models. A current and thorough review of XAI approaches, with significant information on alternative strategies and their relevance to various problem domains, is available in Bennetot et al. (2024). In this study, we utilize SHAP because of its capacity to provide consistent explanations across various model types and its unified framework for feature attribution. This makes it particularly well-suited for comprehending the intricate relationships between performance indicators and stochastic dominance in our causality-driven approach.

Following the ML predictions of SD relationships, two critical questions arise: How reliably does the predictive model perform across different time periods, and does it accurately represent the underlying relationships, or would an interpretable alternative be preferable to the black-box approach? We implement quantile regression (QR) as an adjunctive method to mitigate these concerns and verify our ML findings. QR, introduced by Koenker and Bassett (1978), enhances standard least squares regression by reducing weighted absolute errors instead of squared errors, so enabling the analysis of correlations over the full conditional distribution rather than only at the mean. Our analysis is significantly enhanced by this approach, which is resilient to deviations from normality (Mata & Machado, 1996), less susceptible to outliers and skewed tails that are frequently encountered in financial data (Floros et al., 2024; Kizhakethalackal et al., 2013) and effectively resolves statistical challenges such as non-Gaussian error distributions (Barnes & Hughes, 2002). In contrast to standard linear regression, which enforces strict assumptions frequently breached by financial datasets displaying heteroscedasticity or non-normal residuals, quantile regression offers a distribution-free framework that ensures dependability across various segments of the distribution. In financial research, this methodological strength has been particularly beneficial for identifying nonlinear relationships. Anton and Afloarei Nucu (2024), for example, demonstrated via quantile regression that financial inclusion has threshold effects on bank stability, a nuanced relationship that traditional methods would have missed. Utilizing quantile regression to analyse the correlation between the NetSDR and asset attributes enhances our understanding of how performance and risk metrics affect dominance rankings across various quantiles, thereby supporting and expanding the patterns discerned through ML while offering a comprehensible alternative that mitigates the shortcomings of opaque methodologies.

Although these approaches have been utilised independently in financial models, the combination of Stochastic Dominance, Machine Learning, and Explainable Artificial Intelligence (XAI) into a causality-driven methodology for investment product selection is novel (Černevičienė and Kabašinskas, 2024). The novel framework offers a more comprehensive and interpretable tool for investors by utilising the strengths of each component—stochastic dominance for evaluating alternatives, machine learning for pattern recognition, and XAI for transparent decision-making. The idea might improve the accuracy and dependability of investing techniques, distinguishing it from conventional approaches.

This paper aims to integrate stochastic dominance, machine learning, and XAI by providing a multi-step interpretable strategy for asset selection. Following the filtration and analysis of stock, commodity, and cryptocurrency returns using daily return data, the resulting data is then processed for further examination. This examination includes selecting a specific time period, evaluating the availability of liquid assets, assessing performance effectiveness, analysing interconnected risks, and incorporating XAI approaches to understand the underlying reasoning behind filtering decisions. The application of a causality-based technique in developing the choosing process of investment product is both novel and offers an alternative perspective on constructing portfolios. From a practical perspective, understanding these causal relationships allows investors to minimize computational time in asset selection by prioritizing the metrics that causally determine the stochastic dominance ratio rather than processing all available financial indicators.

The subsequent section of the paper is organised in the following way. A comprehensive literature review summarizing previous studies related to the questions discussed in the study is provided in Sect. 1. Section 2 presents the approach, including the selection of methodologies and research strategy. Following that, in Sect. 3, the analysis outcomes of the selected assets is provided. Conclusions serve as the final section of the work, summarising and bringing closure to the main points discussed. The appendix at the end of the paper provides additional methodological details, including the search spaces for optimal hyperparameters.

2 Literature review

Structural causal modelling is employed in the literature to quantify and comprehend causal relationships between a variety of factors in complex systems. This modelling can elucidate the links among performance metrics, risk indicators, and a certain ratio, so facilitating the selection of investment products. For example, the study by Ahmed et al. (2022) shows how risk perception influences the link between blue-chip companies and investment choices, emphasizing how crucial it is to comprehend risk while making decisions related to investments. The causal relationships among credit ratings, spread, and return in structured finance products have been examined by structural equation modelling, indicating that ratings can affect investment decisions during the issuance phase but exert diminished influence in the secondary market (Moreira & Zhao, 2018). Integrating structural causal models with complementary techniques, such as fuzzy cognitive maps, allows for an evaluation of performance risk in large-scale projects by demonstrating key determinants and their directional dependencies. These insights also enhance investment analysis by clarifying how variable configurations influence outcomes (Chen et al., 2020).

Understanding the causal relationships behind investment decisions and translating these insights into practical ratings requires a systematic approach that allows for comparison of assets. Asset ranking predominantly relies on performance ratios, with the Sharpe ratio being the most widely used ratio for establishing priority order among investment portfolios, though tailored alternatives such as the Sortino–Satchell, Farinelli–Tibiletti and Rachev ratios can yield substantially different rankings depending on investment style (Chahuán-Jiménez et al., 2022; Eling et al., 2011; Mousavi et al., 2024). Performance ratios including the Sharpe ratio, Sortino ratio, and Ulcer Performance Index are employed to compare trading strategies and establish rankings based on risk-return trade-offs, demonstrating their utility in evaluating whether technical trading rules generate superior risk-adjusted returns compared to benchmark buy-and-hold policies (Nor & Zawawi, 2022). Beyond return-based ratios, asset managers incorporate drawdown metrics such as Maximum Drawdown and Maximum Drawdown at Risk for fund allocation and redemption decisions, as these measures capture long-lasting accumulated losses and provide more accurate risk control for portfolio exposure management (de Melo Mendes and Lavrado, 2017; Van Hemert et al., 2020). While a multitude of performance ratios exist for ranking assets under various risk-return frameworks, these measures fundamentally describe outcomes rather than explain their determinants, creating a need to identify which factors causally influence financial performance and investment decisions.

Stochastic dominance principles offer an alternative framework for asset ranking that extends beyond risk-return ratios by comparing entire return distributions. Stochastic dominance has been used in portfolio selection models as it provides a natural interpretation of risk-averse investor behaviour without requiring specification of the utility function (Kopa & Tichý, 2014; Roman et al., 2013). These models have demonstrated consistent outperformance of traditional index trackers while naturally selecting smaller numbers of stocks without imposing cardinality constraints and exhibiting robustness with respect to changes in scenario sets (Roman et al., 2013). SD rules can be related by systemic risk measured by the Gini coefficient and Lorenz curves, making them more understandable to practitioners (Shalit & Yitzhaki, 2010). The choice of reference distribution is very important in determining portfolio solutions, as different benchmarks lead to substantially different asset allocations. Recent research has explored reshaping reference distributions by adjusting their skewness and variance to obtain portfolios with enhanced return distributions, reduced downside risk, and improved left-tail characteristics, with applications extending from long-only to long-short strategies (Cesarone et al., 2022; Valle et al., 2017). Despite these advances, existing SD applications remain static in nature, relying on fixed distributions that do not capture the time-varying dynamics of market conditions and asset behaviour.

Complementing these assets ranking methodologies, recent studies go beyond simply establishing a causal relationship and quantitatively assess the extent of the impact, allowing for the creation of a ranking of factors affecting financial ratios. Tudose et al. (2022) evaluate the impact of the most critical determinants of financial performance, as measured by four generations of indicators, on outcomes and provide empirical quantification of the interdependencies among profit margin, profit growth rate, return on assets, return on equity, and economic value added. Robust financial performance, indicated by the current ratio (CR), debt-to-equity ratio (DER), return on assets (ROA), earnings per share (EPS), and price-to-earnings ratio (P/E), has varied correlations with stock returns (Endri et al., 2019). Similarly, data from the Amman Stock Exchange demonstrates that ratio analysis is valuable for investment selection: profitability ratios are significantly positive at the 5% level, while credit and asset-utilization ratios are significantly negative at the 10% level (Kaddumi, 2017).

Based on evidence that financial indicators are not uniformly related to financial performance, recent studies have moved from hypothesis-based regressions to data-driven models that quantitatively decompose the factors behind these indicators. Rather than limiting themselves to statistical significance, these studies assess the magnitude of each factor's influence and rank the factors that shape the indicators. While the studies mentioned above quantify which factors matter for ratios and return, most employ static analytical frameworks. ML methods such as Random Forest and LSTM algorithms examine financial ratios over specific time periods to forecast whether stocks will experience gains or losses, enabling more informed and accurate financial decisions (Shende et al., 2022). Similarly, fuzzy chance constrained least squares twin Support Vector Machines (SVM) have been applied to predict business performance through financial ratios across different industries (Song et al., 2018). Advanced ML techniques including XGBoost, SVM, Deep Neural Networks, and Random Forest have demonstrated robust performance in predicting bankruptcy and financial distress, achieving accuracies of 82–83% by analysing financial ratios such as return on assets, current ratio, and solvency ratio, with feature importance analysis revealing that indicators like interest coverage ratio and operating margin ratio are crucial for predictive capabilities (Kristanti et al., 2024; Shetty et al., 2022). Explainability methods such as SHAP values attribute ML predictions to individual input features, proving particularly useful in financial applications for understanding the contribution of specific financial ratios (Agarwal et al., 2022; Lin et al., 2025; Sheela & Girisha, 2024). For instance, by transforming financial data into images and using Convolutional Neural Networks (CNN), researchers have developed bankruptcy prediction models that leverage SHAP and LIME to identify which financial ratios are critical for predictions (Lin et al., 2025). In stock market analysis, XAI techniques such as SHAP and LIME are used to explain the predictions of AI models, making it easier for investors to understand the rationale behind investment advice (Agarwal et al., 2022; Tanveer et al., 2024). Such approaches provide the transparency necessary for reasoned financial decision-making.

Looking beyond static explanation, Kaadoud et al. (2022) research in explainable reinforcement learning extends explanation to the temporal logic of decisions, extracting a theoretical mathematical model that describes a system's behaviour through states and transitions based on inputs from action sequences to reveal how an agent's policy emerges, stabilizes, and generalizes.

While the existing literature demonstrates the effectiveness of ML in predicting financial outcomes and XAI methods in explaining these predictions, a critical gap remains: the limitations of causal relationships derived from observational data are frequently disregarded. Most studies focus on correlations and statistical associations rather than establishing true causal mechanisms that determine financial ratios and investment decisions. Moreover, despite the extensive research on financial ratios analysis and asset ranking, to our knowledge the network stochastic dominance ratio has not been examined in any prior research as a tool for investment decision-making. This study addresses these gaps by employing structural causal modelling to ascertain the inherent causal relationships between performance and risk metrics and NetSDR, moving beyond mere predictive accuracy to understand how this causal framework influences investment product selection.

3 Methodology

3.1 Research schema

The methodological framework for evaluating and predicting the network stochastic dominance ratio using returns data from multiple asset classes, such as stocks, cryptocurrencies, and commodities, is presented in the research schema depicted in Fig. 1.

Data is initially gathered and subsequently divided into two blocks: annual and semi-annual. The selection of historical return periods (semi-annual or annual) influences optimal portfolio allocations and investment results. The research conducted by Waggle and Moon (2006) indicates that the utilization of annual data results in more precise variability measures and superior portfolio performance when contrasted with shorter intervals. In terms of average return, annual-based portfolios, particularly those optimized using advanced strategies, tend to outperform monthly based portfolios (Talebi et al., 2010). The frequency of data collection significantly influences SD prediction, as asset rankings and efficient sets may vary when returns are collected annually or semi-annually. Empirical data regarding OECD equity indexes demonstrates that the composition of the efficient set is contingent on frequency, and that pairwise standard deviation differentials diminish as the sampling interval extends, thereby decreasing cross-asset separability (Uğurlu et al., 2018). Applications of SD

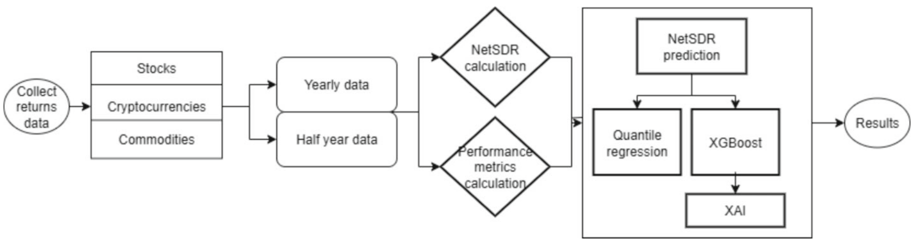


Fig. 1 Research schema

to income distributions indicate that frequency selections influence both the robustness and performance of SD measures, suggesting that results on welfare or inequality may be sensitive to frequency (Gasbarro et al., 2007). Using shorter intervals, such as quarterly periods, may result in higher portfolio allocations and potentially distorted risk assessments by underestimating the variability of specific assets, such as real estate investment trusts (REITs) (Waggle and Moon, 2006). The underappreciation of variability may lead to a flawed depiction of the asset's risk profile. These findings emphasize that temporal aggregation and horizon selection is essential choice in SD testing.

Metrics are computed from data across several timeframes and utilised for subsequent research. Following the determination of various timeframes, NetSDR is computed as a metric for the comparative performance of the assets. Then, in order to assess NetSDR non-directly, a variety of performance metrics are calculated. These calculations serve as the basis for subsequent predictive modelling.

Quantile regression and the XGBoost algorithm are employed to forecast NetSDR. XAI techniques, specifically SHAP, have been included in the system to guarantee transparency of the prediction models. This integration provides a deeper understanding of the factors that influence the possibility to obtain measures of stochastic dominance. These exhaustive framework's findings are combined to provide insights into asset performance and dominance.

3.2 Network stochastic dominance ratio for asset comparison

Three distinct orders (degrees) of stochastic dominance relations are taken into account, based on the hypotheses regarding the utility function $u(x)$ of the portfolio manager:

First-order stochastic dominance (FSD) The only assumption we make about these utility functions is non-satiation, meaning all investors prefer more wealth to less: $u'(x) \geq 0$;

Second-order stochastic dominance (SSD) adds the dimension of risk aversion, aligning investment choices with more common preference for lower risk: $u'(x) \geq 0 \text{and} u''(x) \leq 0$;

Third-order stochastic dominance (TSD) considers investors who are not only risk-averse but also prefer positive skewness. Skewness measures the asymmetry of the return distribution. Positive skewness indicates that the distribution has a longer right tail, meaning there's a greater chance of experiencing extreme positive returns: $u'(x) \geq 0$, $u''(x) \leq 0 \text{and} u'''(x) \geq 0$.

Stochastic dominance criteria were validated by pairwise comparisons, enabling the comparison of two assets. This article only examines the fundamental concept of SD: given conventional assumptions regarding investors utility functions, once a dominating asset pair is recognized, every investor either favours the dominant random return over the dominated one or remains indifferent between the two. The asset i with cumulative distribution function of returns $F_i(x)$ dominates the asset j with cumulative distribution function of returns $F_j(x)$ by First-order stochastic dominance when and only when

$$\left[F_j(x) - F_i(x)\right] \geq 0 \tag{1}$$

for any real number x, with at least one strong inequality. This means that, as long as the investor prefers having more wealth rather than less, the asset i is preferred to asset j.

The asset i dominates asset j with respect to the Second order stochastic dominance (SSD) if

$$\int_{-\infty}^{x} \left[F_j(t) - F_i(t)\right] dt \geq 0 \tag{2}$$

for any real number x, with at least one strong inequality.

The asset i dominates asset j with respect to the Third order stochastic dominance (TSD) if

$$\int_{-\infty}^{x}\int_{-\infty}^{t}\left[F_j(z)-F_i(z)\right]dz\,dt \geq 0 \tag{3}$$

for any real number x, with at least one strong inequality.

We assess stochastic dominance on a period-by-period basis t (annual or semi-annual): for each t, we estimate the return distributions of each asset for that period, construct cumulative distribution functions (empirical or parametric α-stable, NIG, or Student-t), apply the criteria outlined in Eqs. (1)–(3) to all asset pairs utilizing the period-t distributions, and calculate $DySDR_i(t)$ based on the corresponding dominance relations (Levy, 2015).

In general, SD may be verified using either empirical or parametric methods, and it is commonly used as a pairwise measure to compare two assets. In this paper we use non-parametric and parametric approaches with four distributions (empirical, alpha-stable, Normal Inverse Gaussian (NIG) and three parameter Student t). Given that the returns on assets are regarded as random variables, we also recall some theoretical heavy-tailed probability distributions. Modelling heavy-tailed data is a challenge in empirical finance research. Distributions characterized by heavy tails, skewness, and a tendency to outliers commonly occur in asset returns, requiring adaptable and resilient statistical models. The alpha-stable, NIG, and three-parameter Student's t distributions are among the most notable, each providing distinct theoretical attributes and practical benefits for representing non-Gaussian characteristics in empirical data (Kabašinskas et al., 2020; Kopa et al., 2022; Oigard and Hanssen, 2002). If dominance patterns persist across empirical and multiple parametric specifications, this distribution ensures that all stochastically dominant assets are identified.

Next, we provide a short description of NetSDR as presented in Kabašinskas et al. (2025). To calculate the NetSDR, we first need to construct 0–1 matrices $C_{ij}^{ks}(t)$, which represent pairwise SD comparisons in period t between assets i and j under distribution $k \in \{1, \ldots, K_{\text{dist}}\}$ and order $s \in \{1, 2, 3\}$. That is, $C_{ij}^{ks}(t) = 1$ if asset i dominates asset j with respect to s-th order stochastic dominance in t-th period when assuming that returns have probability distribution k *and* $C_{ij}^{ks}(t) = 0$ otherwise. Once these matrices are established, the matrices $D_i^{+}(t)$ and $D_i^{-}(t)$ are defined as the sums of $C_{ij}^{ks}(t)$ over all k distributions and s SD rules (see formula (2) in Kabašinskas et al., 2020). NetSDR is defined by:

$$NetSDR_i(t) = \frac{D_i^{+}(t) - D_i^{-}(t)}{\max\limits_{j\in\{1,\ldots,A\}}\left\{D_j^{+}(t) + D_j^{-}(t)\right\}}, \forall i = 1, \ldots, A,\ t = 1, \ldots, T \tag{4}$$

Values of NetSDR are from the interval $[-1; 1]$. A higher ratio indicates that the asset is more preferable based on stochastic dominance relations.

In the empirical study, we examine $A = 19$ assets and $K_{\text{dist}} = 4$, as empirical, alpha-stable, NIG, and three-parameter Student t distributions were used, with $s = 3$ due to the application of three SD rules (FSD, SSD, and TSD).

3.3 XGBoost

The XGBoost method is employed for regression analysis to ascertain the relationship between the NetSDR and certain performance and risk metrics of assets. Each observation m corresponds to a single asset–period combination. The feature vector x_m encompasses the metrics calculated for the corresponding period, including mean return, volatility, Sharpe ratio, maximum drawdown, skewness, and kurtosis, among others (the precise set utilized in our studies is detailed in Sect. 3.6). The total dataset contains N observations spanning all assets across all analysed time periods.

XGBoost is an ensemble model of high-performing decision trees, demonstrating superior predictive accuracy compared to individual decision trees (Jabeur et al., 2024). XGBoost builds an additive ensemble of decision trees to approximate the unknown function $(\cdot)$ that maps features to NetSDR. After K trees, the prediction for observation n is (Mo et al., 2019):

$$\widehat{y_n^K} = \sum_{k=1}^{K} f_k(x_n) = \widehat{y_n^{K-1}} + f_K(x_n), \tag{5}$$

where f_k- the k-th regression tree. Each new tree f_K is fit to the negative gradients of a chosen regression loss—measuring the discrepancy between y_n and $\widehat{y_n^{K-1}}$—with learning-rate shrinkage and regularization to control complexity. This procedure captures potentially non-linear and interaction effects among performance and risk metrics that drive variation in NetSDR.

This trained model subsequently serves as the basis for our explainability analysis using SHAP values (Sect. 3.6), which quantifies the contribution of each input feature to the predicted NetSDR values.

3.4 Quantile regression

To complement the XGBoost results and address non-Gaussian errors and heteroscedasticity, we estimate quantile regressions tailored to our setting. Each observation n denotes an asset with feature vector x_n and outcome $y_n = \text{NetSDR}_n$. For a quantile level $\theta \in (0, 1)$, the conditional θ-quantile model is

$$y_n = x_n{\prime}\beta_\theta + e_n \tag{6}$$

where β_θ represents the vector of unknown parameters corresponding to the θth quantile and e_n prediction error. The quantile regression formula, which minimizes an asymmetrically weighted sum of absolute errors is (Buchinsky, 1998):

$$\beta_\theta = arg\min_\beta \left[\sum_{n:y_n \geq x_n\beta} \theta|y_n - x_n\beta| + \sum_{n:y_n < x_n\beta} (1-\theta)|y_n - x_n\beta| \right], \tag{7}$$

where θ is the quantile level (e.g., $\theta = 0.5$ for median regression), y_n is the dependent variable (NetSD ratio), x_n is the vector of independent variables (features). If $\theta = 0.5$ the weights for both positive and negative errors are equal, corresponding to median regression. For other values of θ, the weights become asymmetric, focusing more on one tail of the distribution.

3.4.1 Variance inflation factor

The Variance Inflation Factor (VIF) method was implemented in this research to mitigate multicollinearity among 18 features. Ultimately, eight features were determined to be uncorrelated. VIF is a typical method for measuring multicollinearity among predictor variables (Kasraei et al., 2024). The procedure entails regressing each feature against all other variables and calculating the multiple correlation coefficient R_n^2 for each regression. This coefficient denotes the extent of variation in the n-th predictor elucidated by the other predictors.

The VIF for each predictor is then calculated using the following formula (Belsley et al., 2005):

$$VIF_n = \frac{1}{1 - R_n^2} \tag{8}$$

where R_n^2 is the coefficient of determination for predictor n in the regression against all other predictors.

A VIF score of 1 signifies the absence of multicollinearity, values ranging from 1 to 5 imply moderate correlation, while values over 10 denote strong multicollinearity. A stepwise procedure was implemented to address multicollinearity. The highest VIF values were sequentially eliminated, and VIF calculations were repeated until all remaining variables achieved $VIF_n \leq 10$, as advised by previous studies (Akinwande et al., 2015; Craney & Surles, 2002).

3.5 Shapley additive explanation (TreeSHAP)

To interpret the XGBoost predictions and determine the performance and risk metrics that have the most significant impact on NetSDR, we implement SHAP values. We specifically used the shap.Explainer function from the SHAP Python package (Lundberg & Lee, 2017; Lundberg et al., 2020), which automatically employs TreeSHAP, an algorithm developed for tree-based ensemble algorithms such as XGBoost.

Each observation n is an asset with feature vector $x_n = \left(x_{n,1}, \ldots, x_{n,d}\right)$, where d denotes the number of features. Let $f(\cdot)$ denote the fitted XGBoost ensemble (sum of K regression trees from Eq. 5). Define the feature index set $\mathcal{F} = \{1, \ldots, d\}$. In SHAP, $S \subseteq \mathcal{F}$ denotes a subset of features. For any x_n, TreeSHAP returns attributions $\phi_{n,0}, \phi_{n,1}, \ldots, \phi_{n,d}$ satisfying the additivity property:

$$f(x_n) = \phi_{n,0} + \sum_{r \in \mathcal{F}} \phi_{n,r},\ \phi_{n,0} = \mathbb{E}[f(X)]$$

here $\phi_{n,0}$ is the baseline (expected model output under the reference data distribution) and each $\phi_{n,r}$ is the signed contribution of feature r for observation n.

For feature $r \in \mathcal{F}$ is

$$\phi_{n,r} = \sum_{S \subseteq \mathcal{F} \setminus \{r\}} \frac{|S|!\,(d-|S|-1)!}{d!} \left(f_{S \cup \{r\}}(x_n) - f_S(x_n)\right), \tag{9}$$

TreeSHAP leverages the structure of decision trees to compute the Shapley values exactly and efficiently: it follows the decision paths that x_n can take in each tree while implicitly averaging over all possible feature orderings, replaces splits on features treated as unknown with data-weighted averages inside the tree to form the required conditional expectations, and then aggregates the resulting per-tree contributions across the K trees to obtain $\phi_{n,r}$.

$\phi_{n,r}$ measures how much feature r shifts the predicted NetSDR for observation n away from the baseline (the model's expected prediction). A positive $\phi_{n,r}$ raises the predicted NetSDR—indicating stronger stochastic dominance for that asset—whereas a negative $\phi_{n,r}$ lowers it. The absolute value $\left|\phi_{n,r}\right|$ captures the strength of that feature's influence on the prediction.

3.6 Performance and risk metrics

Performance metrics, including volatility, and risk-adjusted ratios, are critical predictors in empirical models, as they capture the relationships between past performance and future risk-adjusted performance (Whittington, 1980). Therefore, to comprehend the causality of the NetSDR, we examine the intrinsic relationship between performance and risk indicators in conjunction with this ratio. To forecast the NetSDR, we implemented a wide variety of performance and risk metrics. These indicators cover traditional metrics including the Sharpe ratio, volatility, and average return, as well as more intricate indicators such as the ulcer performance index, recovery factor, expected shortfall, and maximum drawdown. Furthermore, we analyse higher moments like skewness and kurtosis and conduct the Jarque–Bera test to evaluate the normality of return distributions, along with additional sophisticated metrics such as semi-variance, gain-to-pain ratio, tail ratio, and certainty equivalence. Additionally, we evaluate the correlation dynamics using the minimum, maximum, and average correlation coefficients.

Table 1 presents all selected performance and risk indicators that are used as predictors. Further details regarding performance and risk measures can be found in Bacon's (2021) book.

These metrics provide a comprehensive characterization of asset behaviour, enabling the XGBoost model to learn complex relationships between performance, risk profiles and SD rankings.

3.7 Experimental design

This section outlines the comprehensive process used in our research. The following experimental design is suggested to explore the study topics related to the interaction between performance and risk metrics and NetSDR, as well as the evolution of these relationships over time:

1. Collect daily price data for $A = 19$ assets across three asset classes: stocks, commodities, and cryptocurrencies from Yahoo Finance API for the period 8 November 2017 to 29 May 2024.
2. For each asset i and trading day τ, compute the daily $return_{i,\tau} = (P_{i,\tau} - P_{i,\tau-1})/P_{i,\tau-1}$.
3. Divide the time series into $t = 1 \ldots T$ non-overlapping time intervals: annually windows (each calendar year) and semi-annual windows (two six-month periods each year).
4. Test all asset pairs (i, j) for FSD/SSD/TSD using the empirical distribution; this yields three 19×19 matrices $C^{1s}(t)$ (one for each order $s = 1, 2, 3$), with $C^{1s}_{ij}(t) = 1$ if asset i dominates asset j at order s, else 0. After that sum up these three matrices and create $C^{emp}(t) = \sum_{s=1}^{3} C^{1s}(t)$ (entries 0–3: 0 = none, 1 = TSD only, 2 = TSD + SSD, 3 = TSD + SSD + FSD).
5. Repeat Step 4 for Student-t ($k = 2$), α-stable ($k = 3$), and NIG ($k = 4$), producing nine matrices $C^{ks}(t)$ (three per distribution), each 19×19.
6. By summing up all the matrices of different distributions, create a new matrix $C(t)$ (example provided in Fig. 4x), which reveals all the identified cases of SD.
7. Calculate the NetSDR based on Eq. (4), where $D_i^+(t) = \sum_{j=1}^{A} C_{ij}(t)$ counts how often i dominates other assets, and $D_i^-(t) = \sum_{j=1}^{A} C_{ji}(t)$ counts how often i is dominated (for details see Eq. (2) in Kabašinskas et al., 2020).

Table 1 Performance and risk indicators used as input features

Features	Meaning
Ulcer performance index	The concept of drawdown deviation has a corresponding measure that places additional emphasis on the duration of drawdowns. The purpose of this measure is to account for negative returns experienced during each period when the investment falls below the previous peak
Recovery_factor	The Recovery Factor measures how efficiently an investment recovers from its largest losses (drawdowns). A higher Recovery Factor indicates better resilience and recovery capability
Sharpe ratio	The Sharpe Ratio is a risk-adjusted performance measure that evaluates the excess return of an investment per unit of risk
Kurtosis	Kurtosis measures how much sharpness or smoothness of a certain distribution
Jarque bera	The Jarque–Bera test is a statistical test designed to determine the degree to which the skewness and kurtosis of a data align with the anticipated values for a normal distribution
Semi_Variance	Risk metric focused solely on negative deviations beneath the mean value
Tail ratio	Risk analysis metric that assesses the balance between an asset's extreme positive and negative returns. It is especially beneficial for comprehending the "tail behaviour" of a return distribution, which signifies the danger of significant losses or profits
Min, max and averaged correlation	Min Correlation, Max Correlation, and Mean Correlation denote the minimum, maximum, and average values, respectively, of pairwise correlation coefficients (excluding self-correlations) for a specified asset within the dataset
Max_Drawdown	Max drawdowns represent a significant decline in an investment's value and refer to the most severe and prolonged periods of losses from its peak. It highlights the largest percentage decline experienced during a specific period
Expected shortfall (ES)	Risk management metric used to quantify the potential losses that can occur at the tail (extreme) end of a probability distribution of financial returns
Skewness	Skewness denotes the asymmetry of a distribution relative to its mean
Standard deviation	Standard deviation quantifies the dispersion or variability of data, offering a practical means to express the unpredictability of returns in their original, non-squared units by calculating the square root of the variance
Gain to pain ratio	A performance metric that assesses the efficacy of an investment by contrasting the cumulative net gain with the cumulative losses encountered to achieve that gain, emphasising the relation between reward and risk
Certainty equivalent	Certainty equivalence quantifies rational decision-making preferences. The risk tolerance of the investor is demonstrated by the guaranteed amount of money they prefer to a risky asset in exchange for an uncertain return

8. For each asset i in each period t, compute 18 performance and risk indicators from the return series. This creates a feature vector $x_{i,n}(t)$.
9. Set the target $y_i(t) = NetSDR_i(t)$ and assemble the feature vector $x_{i,n}$ from indicators (Sect. 2.6) and train XGBoost.
10. Benchmark XGBoost with quantile regression. Estimate $y(t) = x_n^T \beta_\theta + \varepsilon_n$ to check how feature effects NetSDR.
11. Explain XGBoost predictions with TreeSHAP.

In accordance with this proposed strategy, it is possible to identify non-dominated assets, quantify their dominance using NetSDR, and associate the results with interpretable variables. This allows decision makers to assess efficient assets and subsequently select those that most closely align with their risk profile.

4 Results

4.1 Data description and statistics

Our data set includes daily closing prices of ten stocks (TSLA-Tesla Inc., MSFT-Microsoft Corporation, AAPL-Apple Inc., VWAGY-Volkswagen AG, NVDA-NVIDIA Corporation, JPM-JPMorgan Chase & Co., AMZN-Amazon.com Inc., MVIS-MicroVision Inc., NOW-ServiceNow Inc., GOOGL-Alphabet Inc. Class A), four cryptocurrencies (Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Dogecoin (DOGE)), and five commodities (Gold Futures (GC = F), Crude Oil Futures (CL = F), Copper Futures (HG = F), Natural Gas Futures (NG = F), Corn Futures (ZC = F)), collected from Yahoo Finance, for the time period between 8 November 2017 and 29 May 2024. The daily prices of cryptocurrencies are reduced to five observations per week to maintain compatibility with standard asset prices.

The variable's minimum, 25th percentile (Q1), 75th percentile (Q3), and maximum values are presented in Table 2, along with the averaged return, annual return, skewness, kurtosis, and values for each of the analysed assets.

The use of cumulative daily returns allowed for a clear and understandable representation of the performance of selected assets over time. This demonstrates the cumulative effect of daily returns, which helps investors understand the overall efficiency of the asset. Figure 2 displays the cumulative daily returns of the selected assets. NVIDIA Corporation has a consistent upward trend in cumulative returns during the specified time, whereas Tesla Inc. displays more fluctuations in its returns. The returns of Dogecoin had a significant and sudden increase, followed by substantial declines towards the end of 2021. It is important to note that this conduct closely resembles that of Microsoft Corporation's stocks. The cumulative returns for commodities exhibited a distinct pattern of growth between the years 2022 and 2023, in contrast to stocks and cryptocurrencies. This behaviour demonstrated the distinctive interplay between supply and demand, geopolitical occurrences, and economic circumstances that influence commodities markets. This was in contrast to the more sentiment-driven and market responsive returns observed in stocks and cryptocurrencies.

The Pearson correlation coefficient measures the degree of linear connection between the chosen assets. Figure 3 displays the Pearson correlations between asset returns across the whole analysis period. Typically, the correlations are rather weak and reach 0.2. There are two exceptions to note: Google stocks have a connection with Apple that is greater than the average, and Ethereum has a correlation value of 0.78 with Bitcoin.

Table 2 Summary statistics of daily returns

	Avg return	Kurtosis	Annualized return	Standard deviation	Skewness	Min	Q1	Q3	Max
TSLA	0.002098	6.546625	0.256313	0.0328	0.317435	− 0.21063	− 0.00792	0.01115	0.198949
MSFT	0.001165	11.07356	0.187344	0.015344	0.076961	− 0.14739	− 0.00296	0.006181	0.142169
AAPL	0.001083	8.696032	0.166892	0.016175	0.054663	− 0.12865	− 0.00369	0.006341	0.119808
VWAGY	0.000174	23.81006	− 0.02285	0.020602	1.019292	− 0.15072	− 0.00609	0.005797	0.292519
NVDA	0.002413	8.379072	0.387359	0.026767	0.283229	− 0.18756	− 0.00653	0.010818	0.243696
JPM	0.000607	21.44022	0.0774	0.015523	0.442025	− 0.14965	− 0.00373	0.004849	0.180125
AMZN	0.00095	7.275747	0.131256	0.018032	0.132781	− 0.14049	− 0.00421	0.006496	0.135359
MVIS	0.002711	122.3438	− 0.03688	0.065845	5.996625	− 0.53448	− 0.01887	0.012605	1.5
NOW	0.001406	5.222917	0.203959	0.021497	0.20291	− 0.12737	− 0.00429	0.008513	0.13444
GOOGL	0.000926	7.221645	0.136109	0.016136	− 0.01754	− 0.11634	− 0.00311	0.006081	0.102244
BTC-USD	0.00164	7.746764	0.267101	0.037253	− 0.11768	− 0.3717	− 0.01408	0.017075	0.252472
ETH-USD	0.002136	6.197465	0.296112	0.046692	− 0.18908	− 0.42347	− 0.01859	0.023776	0.264581
XRP-USD	0.002175	34.62016	0.096901	0.062734	3.143993	− 0.42334	− 0.02098	0.019758	0.834708
DOGE-USD	0.005048	657.0625	0.649563	0.100555	19.48128	− 0.40257	− 0.0228	0.020377	3.555466
GC = F	0.00041	7.814743	0.065145	0.007728	− 0.08243	− 0.04979	− 0.0017	0.002946	0.059477
CL = F	− 0.00159	1363.358	0.034977	0.072496	− 33.5153	− 3.05966	− 0.00475	0.008333	0.376623
HG = F	0.000373	3.882685	0.047979	0.011751	− 0.0004	− 0.06694	− 0.00398	0.004311	0.074642
NG = F	0.000762	17.58977	− 0.02596	0.035642	1.025146	− 0.25954	− 0.00994	0.011114	0.464812
ZC = F	0.000329	23.62484	0.030752	0.014176	− 1.5755	− 0.17387	− 0.00401	0.004785	0.079582

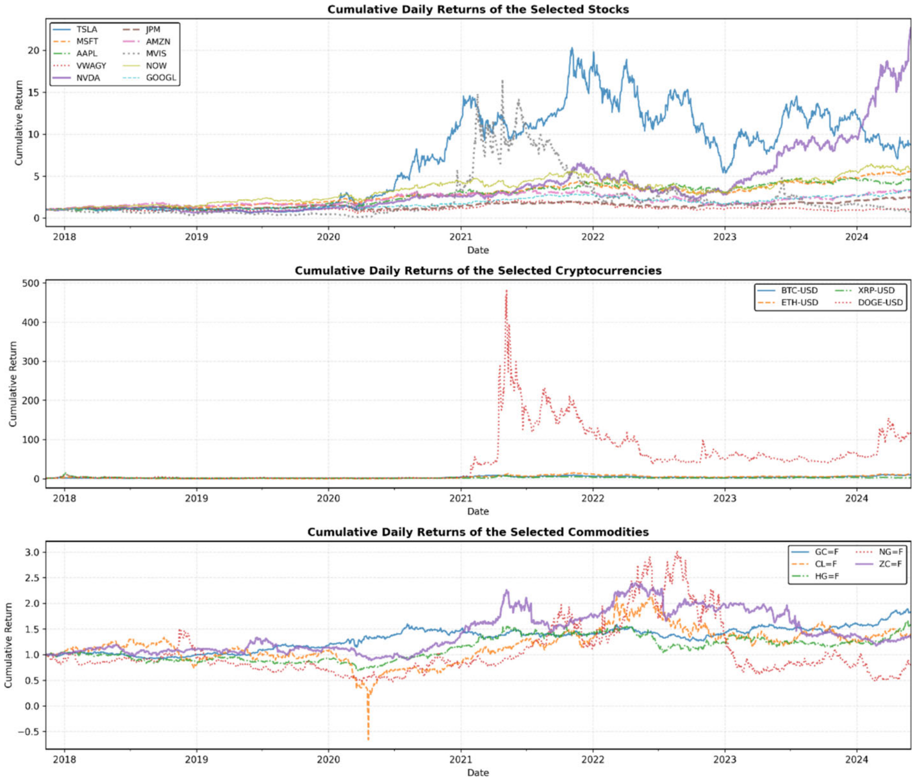


Fig. 2 Cumulative daily returns of the picked investments across three major asset types from 2017 November to 2024 May. The top panel shows stocks, the centre panel shows cryptocurrency, and the bottom panel represents the future of commodities

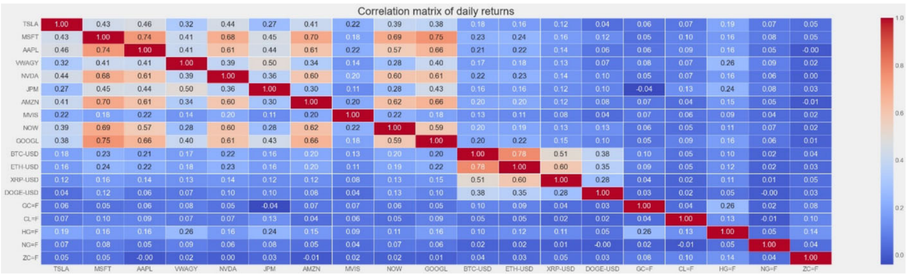


Fig. 3 Correlations between the returns of assets throughout the analysed time period

4.2 Risk and performance analysis of assets

A stochastic dominance analysis was conducted following the process of asset selection. The stochastic dominance is typically pairwise measure and allow to compare two assets. As SD can be calculated in empirical or parametrical way, therefore we get 12 matrices with pairwise results. The elements of such comparison matrix are either zeros or units, see (Kabašinskas

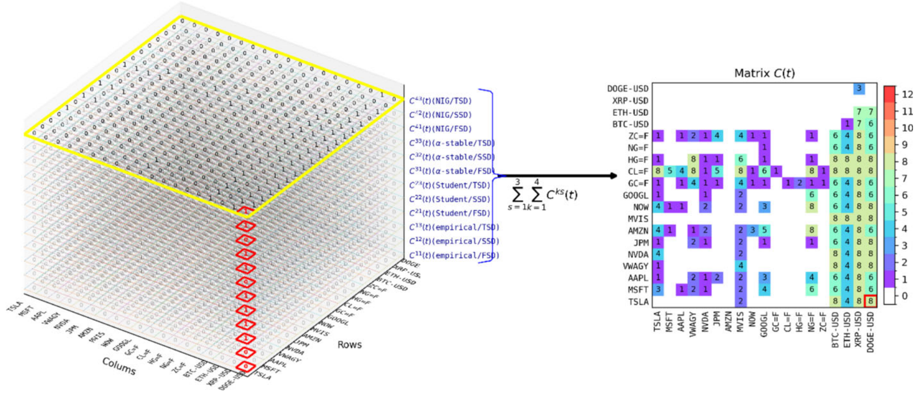


Fig. 4 The process of obtaining matrix $C(t)$. (Color figure online)

et al., 2024) for details. If we sumup all such matrices for all distributions and all orders of SD we get so called aggregated pairwise SD matrix $C(t)$ of stochastic dominance relations. In this paper we use non-parametric and parametric approaches with four distributions (empirical, alpha-stable, NIG and three parameter Student t). See Fig. 4 for detailed visualization of this process for semiannual data from first rolling window in 2018.

In Fig. 4 is highlighted (red) case of stochastic dominance for TSLA and DOGE-USD obtained from semiannual data in first rolling window of 2018. It is possible to see (in the left part) that FSD was not found for this pair in case of empirical, alpha-stable and NIG distributions, however it was found in case of Student t distribution (TSLA dominates DOGE-USD). SSD was not found in empirical case; however, it was found in cases of all parametrical distributions. TSD was observed for all distributions. Therefore, there are 8 units and 4 zeros in the highlighted cells. Finaly the $C(t)$ for this pair is equal to 8 as shown in the right part of Fig. 4. Using aggregated pairwise SD matrix we can calculate the stochastic dominance ratio for every asset.

Now to obtain $D_i^+(t)$ for i = DOGE-USD we need to sumup first row, i.e., $D_i^+(t) = \sum_{j=1}^{19} C_{ij}(t) = 3$ and we sumup last column to obtain $D_i^-(t) = \sum_{j=1}^{19} C_{ji}(t) = 119$. Hence numerator in Eq. (4) becomes equal to -116 and denominator becomes equal to 145 (is the same for all assets). Finaly the $NetSDR_i(t) = \frac{-116}{145} = -0.8$. Once this is done for all assets we can proceed to next rolling window. The network representation of aggregated pairwise matrix $C(t)$ for entire period is given in Fig. 5.

If the asset i more often dominates other assets than is dominated by them (marked green) then the ratio is positive, otherwise it is negative. (marked red). The direction of the pairwise dominance is indicated by the directed arrows, which indicate that the arrows are pointing from the dominating asset to the dominated one. Based on the chart, it is evident that the majority of the stocks has positive NetSDR, unlike all the cryptocurrencies. The darker the arrows are the stronger the dominance in corresponding pair is. Next, we could explore the asset rankings using the NetSD ratio. Figure 6 presents an annual comparison of the NetSDR for each asset.

The NetSD ratio of established technology firms, such as Apple and Microsoft, is more stable than that of emergent technology firms or cryptocurrencies. This implies that these

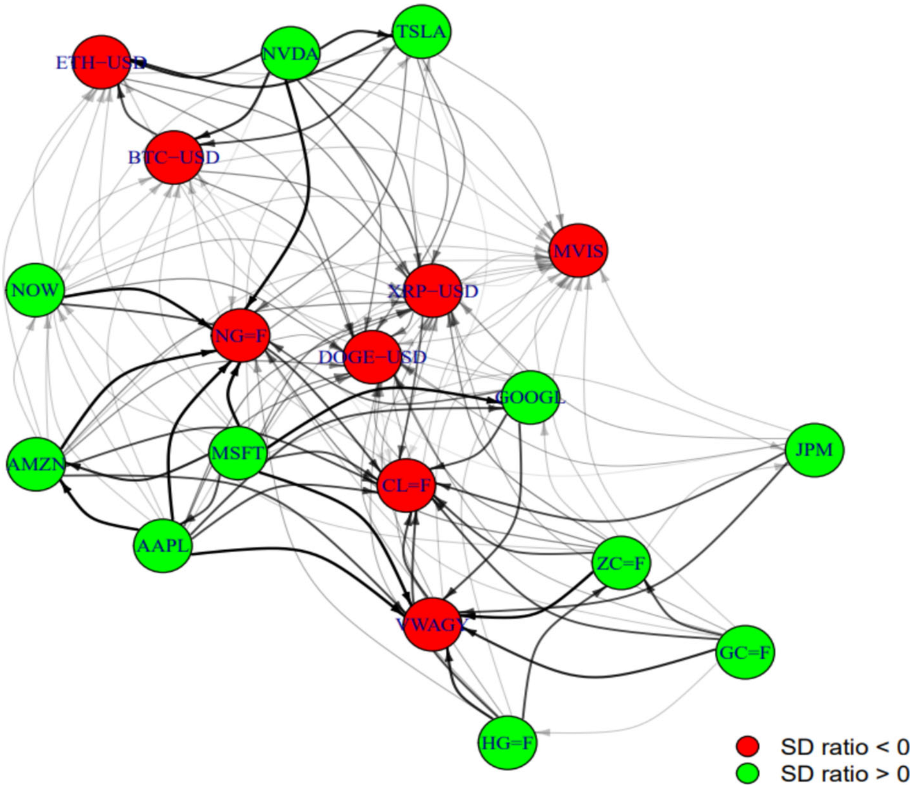
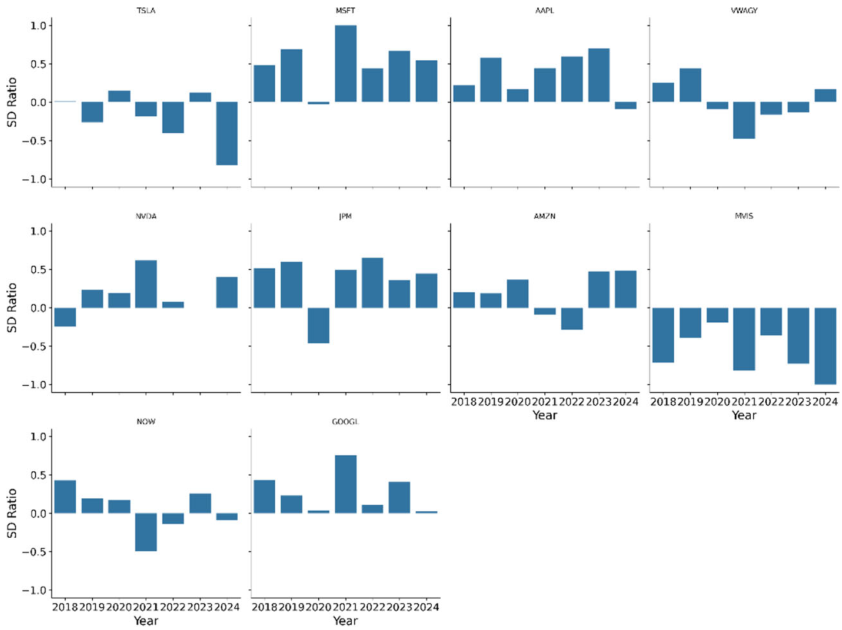


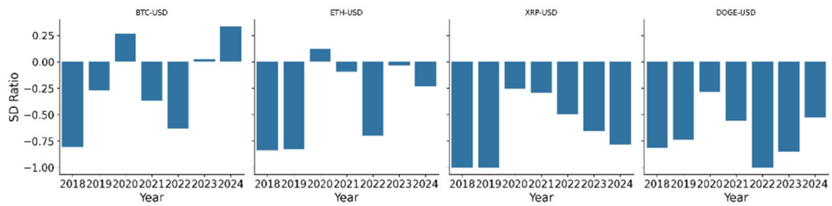
Fig. 5 Network of stochastic dominance according to FSD, SSD, and TSD (for all period). (Color figure online)

well-established companies are less susceptible to large fluctuations in returns, which is likely attributable to their robust market position, consistent revenue streams, and reduced risk exposure. Cryptocurrencies, including some commodities, show the highest volatility in NetSDR, suggesting that they are among the most volatile asset classes. Cryptocurrencies are intrinsically volatile due to their extreme price fluctuations, which renders them higher-risk investments that may accrue both substantial profits and substantial losses.

After identifying assets that demonstrate stochastic dominance over others, the subsequent stage is to comprehend the elements that contribute to this dominance. By identifying the performance indicators that impact the NetSDR, we may obtain a more profound understanding of the factors that determine investment dominance. To do this, the XGBoost algorithm is utilised for regression analysis. The selected stocks were evaluated using various performance indicators, such as the average return, recovery factor, ulcer performance index, expected shortfall, volatility, Sharpe ratio, max drawdown, skewness, kurtosis, annualised return, standard deviation, Jarque Bera, semi variance, gain to pain ratio, and tail ratio. The dependent variable is the NetSDR, whereas the independent variables are the collected performance indicators.



a)



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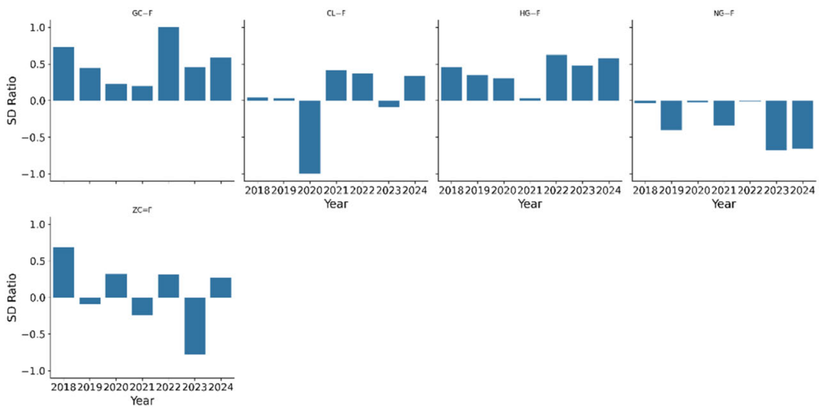

c)

Fig. 6 The NetSDR across various asset classes for each year: **a** Stocks, **b** Cryptocurrency and **c** Commodities

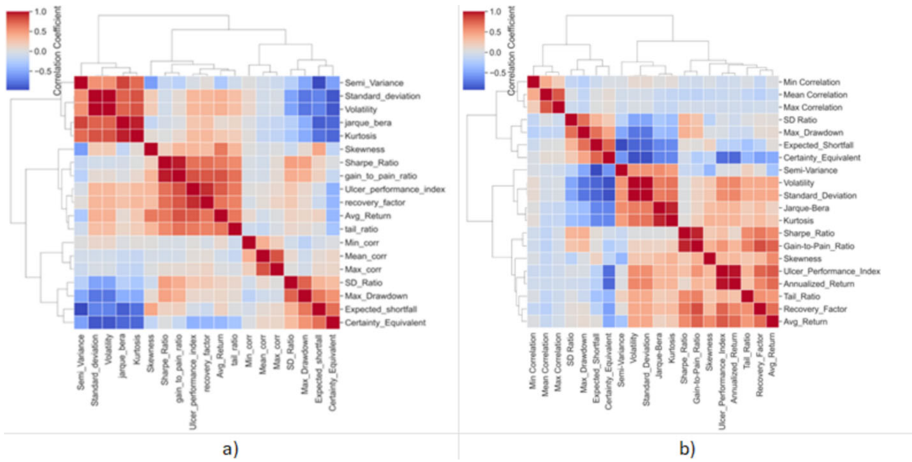

Fig. 7 Correlation estimates between the NetSD ratio and selected features: **a** Calculated for one-year time window, **b** Calculated for half year time window. (Color figure online)

4.3 Interaction of selected metrics and the NetSD ratio

To evaluate the interaction between the chosen performance and risk metrics and the NetSDR, it is essential to study their correlations with each other and with the NetSDR over the specified time period in various time windows. The correlation of the NetSDR remains constant regardless of the time frame. The NetSD ratio has a positive correlation with Max drawdown and Sharpe ratio, while demonstrating a negative correlation with Standard deviation and Volatility, for both yearly and semi-annual performance and risk indicators (Fig. 7).

Figure 7a illustrates the grouping of Semi Variance, Standard Deviation, and Volatility measures, which exhibit a strong positive correlation. The robust positive correlation among these metrics is logical, as all three represent distinct elements of return variability. This cluster indicates that these measurements are fundamentally interchangeable in the analysis of the NetSD ratio causality. Performance metrics like Sharpe Ratio and Gain to Pain Ratio exhibit clustering, signifying a positive correlation. Similar clustering and correlation intensity patterns are observed in b).

4.4 NetSDR prediction

After identifying assets that demonstrate stochastic dominance over others, the subsequent stage is to comprehend the elements that contribute to this dominance. By identifying the most important performance indicators that impact the network stochastic dominance ratio, we may obtain a more profound understanding of the factors that determine investment dominance. The XGBoost algorithm and quantile regression analysis have been utilised for prediction.

4.4.1 Quantile regression analysis

The relationship between NetSDR and 18 explanatory variables was investigated using median quantile regression analysis. The median regression is inherently robust to the violation of homoscedastic and normal assumptions, as it minimises the absolute deviations, in contrast to the ordinary least squares (OLS) or maximum likelihood approaches (Yuan & MacKinnon, 2014). The Variance Inflation Factor values for each predictor were computed to evaluate multicollinearity. Variables exhibiting VIF greater than 10, signifying substantial multicollinearity, were eliminated. Afterward, eight explanatory variables were estimated after the relationship between NetSD ratio and them was excluded. The quantile regression fails to adequately address the prediction task, as evidenced by a pseudo-R-squared value of 0.4129 for the median quantile regression on annual data (Table 3). The Average Return and Ulcer Performance Index are the primary factors that positively influence the annual NetSD ratio. Kurtosis exhibits a little yet notable beneficial influence, whereas Max Correlation demonstrates a considerable negative association, albeit with marginal significance. Other factors, such as Volatility, Tail Ratio, Minimum Correlation, and Average Correlation, demonstrate no statistically significant impact.

Table 4 presents the results of a median quantile regression analysis for the semi-annual NetSDR. Pseudo R-squared 0.3506 suggests that the independent variables in the model with semi-annual windows account for approximately 35 percent of the variability in the NetSDR. The smaller value than the yearly NetSDR indicates that the model is not functioning correctly, despite the presence of a greater number of observations. It could potentially suggest that the NetSDR, which is determined from half-year data, is less informative.

Max_Drawdown, Recovery_Factor, Expected_Shortfall, and Gain-to-Pain Ratio all significantly influence the NetSD ratio calculated for each half year. The negative coefficient of Expected Shortfall (− 0.7107) indicates that increased downside risks are associated with lower NetSD ratios. The positive Maximum Drawdown coefficient suggests that assets with greater drawdowns exhibit significantly higher NetSD ratios, which is indicative of a significantly greater degree of performance variability.

Table 3 Median Quantile Regression Results for yearly NetSD ratio

	Coef	P >|t|
const	0.3055(0.246)	0.216
Avg return	35.9346(14.00)**	0.011
Ulcer performance index	0.0181(0.009)***	0.005
Volatility	− 1.0129(0.6877)	0.988
Kurtosis	0.0083(0.003)***	0.008
Tail ratio	0.0941(0.220)	0.669
Min correlation	− 0.1541(0.474)	0.745
Max correlation	− 0.4426(0.255)***	0.085
Avg correlation	0.8874(0.534)***	0.099
Pseudo R^2	0.4129	

(1) ***, **, and * signify statistical significance at the 1%, 5%, and 10% thresholds, respectively; (2) Values in parenthesis are standard errors

Table 4 Median Quantile Regression Results for semi-annual NetSD ratio

	Coef	P >|t|
const	0.5170(0.143)	0.000
Min correlation	− 0.2292(0.275)	0.405
Mean correlation	0.6222(0.333)*	0.063
Avg_Return	− 1.2829(6.457)	0.843
Expected shortfall	− 0.7107(0.254)***	0.006
Gain-to-Pain_Ratio	0.4692(0.188)**	0.013
Max_Drawdown	1.6149(0.199)***	0.000
Recovery_Factor	− 0.0710(0.031)**	0.022
Tail_Ratio	− 0.0547(0.120)	0.648
Pseudo R^2	0.3506	

(1) ***, **, and * signify statistical significance at the 1%, 5%, and 10% thresholds, respectively; (2) Values in parenthesis are standard errors

Weak explanatory power and poor predictive accuracy are underscored by the quantile regression analysis in predicting the NetSDR. Consequently, we employ XGBoost, which provides enhanced predictive accuracy and profound insights into the NetSD ratio.

4.4.2 Explanation of XGBoost predictions using the SHAP values

After observing the limitations of quantile regression, the XGBoost algorithm is utilised for regression analysis. The dependent variable is the network stochastic dominance ratio, whereas the independent variables are the collected performance indicators, and calculated for each year and for every half year. The NetSD ratio variable was predicted via the XGBoost algorithm, applied to a dataset partitioned into training (70%) and validation (30%) subsets. Evaluations of the validation set predictions were conducted using the Mean Squared Error (MSE), which measures the average squared prediction error, and the R^2 Score, which reflects the proportion of variance in the target variable accounted by the model. We used the integrated cross-validation function of XGBoost to systematically determine the appropriate hyperparameters for the regression task, hence enhancing the model's performance. The threefold cross validation test is employed to ascertain the optimal parameters of the algorithm. A grid search approach was employed to automate this operation and efficiently explore a wide range of hyperparameters. Search spaces for optimal hyperparameters and the selected hyperparameters is provided in Appendix 1. The hardware configuration consists of an Intel(R) Core (TM) i7-1165G7 CPU@ 2.80 GHz processor and 16 G memory, and the experimental algorithm is primarily implemented using Python 3.11 and the scikit-learn toolkit.

Annual data MSE of 0,064 and R^2 Score of 0,742 suggest that the model produced precise predictions on the validation set, as demonstrated by the minimal error. The model's predictive potential is reasonably strong, as shown by the R^2 score of approximately 0,74, which suggests that it explains 74% of the variance in the target variable.

XGBoost output was interpreted using the SHAP method, which provided a direct visual comprehension of feature contributions (Fig. 8). According to the findings, the Expected Shortfall is the most significant. There is a strong indication that the predictions made by

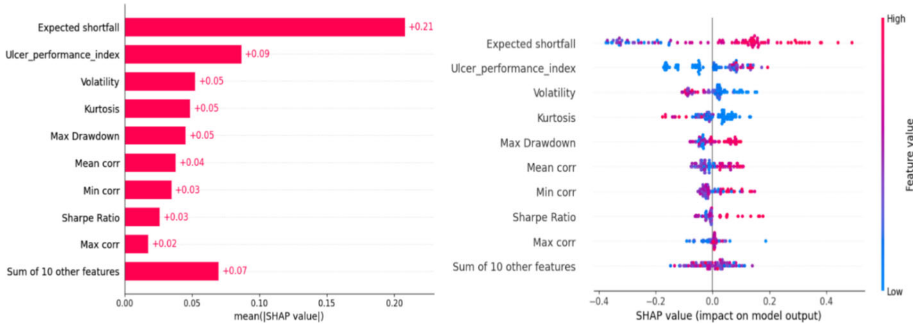

Fig. 8 Plot of the SHAP summary for the annual data XGBoost regression model **a** A standard bar chart depicting the importance of features based on the mean absolute SHAP value of each feature; **b** A SHAP beeswarm plot that illustrated the distribution of SHAP values for each feature. Higher feature values are represented by red, while lower feature values are represented by blue. (Color figure online)

the model are highly sensitive to this variable. The Ulcer performance index is the attribute that has the second most significant impact. Figure 8b displays the colour gradients, which demonstrate the extent to which the high and low values of every feature are correlated with changes in prediction. As the Expected Shortfall increases in value, it contributes to the upward movement of predictions (red dots appear on the right). In contrast, predictions are diminished when low values (blue points) are present. There is a correlation between high values (red) and increased forecasts (positive SHAP values) for volatility and kurtosis, which all show similar patterns.

Semi-annual data The XGBoost regression model was assessed using a semi-annual dataset, using the same analytical methods as the prior model; however, the outcomes varied. The performance of the model diminished when trained and validated on six months data, evidenced by an increase in MSE and a fall in the R^2 score. The MSE of 0.078 signifies a marginal increase in error relative to the dataset organised by year (MSE 0.0637). The R^2 score of 0.6257 shows that the model explains for about 62.6% of the variation in the target variable. This represents a decline from 74.2% seen with the annual dataset. The semi-annual dataset may contain increased noise or irrelevant characteristics, and the NetSD ratio derived from this data, despite its larger size, adversely affects model performance.

SHAP analysis identified six factors that significantly influenced the predictions: maximum drawdown, semi-variance, mean correlation, minimum correlation, kurtosis, and projected shortfall. Figure 9 illustrates that these factors emerged as significant predictors. Among these, Expected Shortfall continually appeared as a critical element, retaining its importance throughout the study.

5 Discussions

Using the NetSD ratio, a clear dominance among asset categories is demonstrated based on the three stochastic dominance principles. Generally, stocks all display positive NetSD ratios, which means they more often dominate other asset classes than are dominated by them. This dominance indicates that stocks could be an effective option for long-term portfolio building, as they attract a wide range of investors with differing risk tolerances . However, cryptocur-

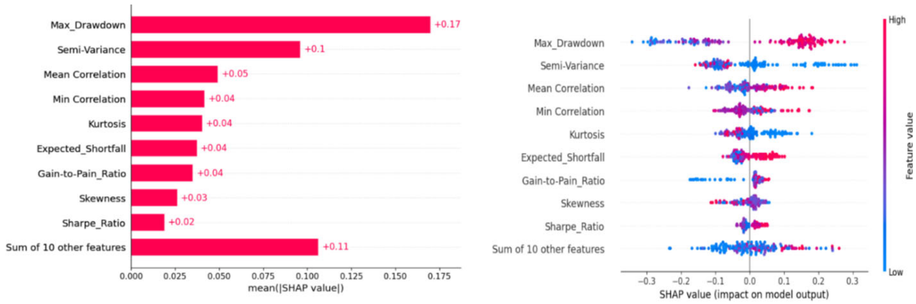

Fig. 9 Plot of the SHAP summary for the semi-annual data XGBoost regression model **a** A standard bar chart depicting the importance of features based on the mean absolute SHAP value of each feature; **b** A SHAP beeswarm plot that illustrated the distribution of SHAP values for each feature. Higher feature values are represented by red, while lower feature values are represented by blue. (Color figure online)

rencies are more often dominated than dominates the other assets, which underscores their extreme volatility and high-risk factor. This finding emphasises the risky characteristics of cryptocurrencies, making them less attractive to risk-averse investors and highlighting the necessity for thorough risk management in portfolio inclusion. Commodities emerge as a non-dominated asset class, especially during economic instability, highlighting their importance as a diversification tool in portfolio construction. This argument is consistent with the idea that commodities have historically functioned as buffers against economic instability. The investigated dominant patterns of asset classes may influence to make investors better educated decisions about asset allocation and risk management, therefore improving portfolio performance and stability.

A fundamental expectation in analysing the NetSD ratio is that the average return serves as the principal predictor, as stochastic dominance relies on utility theory, in which returns are an essential factor in evaluating the choice between investments. However, our research indicates that various performance and risk variables exert more influence on the NetSDR. This discovery contests traditional beliefs and highlights the intricacy of the linkages influencing the NetSDR. It shows that elements besides returns, such as the significance of distributional characteristics (Kurtosis) and risk metrics (Max Drawdown and Expected Shortfall), may be more crucial in elucidating fluctuations in the NetSD ratio.

In addition to enhancing our comprehension of the NetSD ratio, this unexpected result underscores the significance of utilising sophisticated analytical methods to challenge and improve standard financial theories. By investigating the more extensive implications of these discoveries for portfolio optimisation and risk management strategies, future research could expand upon these insights.

To further enhance this research, additional asset classes could be included to provide a more comprehensive understanding of dominance profiles. Furthermore, employing a wider range of machine learning methods could uncover new patterns.

6 Conclusions

The fact that stocks are preferred to cryptocurrencies based on NetSDR, highlights the different risk profiles and market behaviours of both asset classes, providing useful insights for making investment decisions and managing portfolios. Despite the scope of the analysed data time window, commodities can be regarded as a rarely dominated asset class.

The XGBoost model accounts for about 74.2% of the variation in the NetSDR within the yearly data window, signifying a robust fit to the data. The model's efficiency was somewhat reduced for the semi-annual data window, suggesting that it explains roughly 62.57% of the variation in the semi-annual NetSD ratio. The asymmetry in prediction results between the semi-annual and annual datasets suggests that the model may be more proficient in recognising patterns and correlations in the annual data. This shows the benefits of utilising annual data, as it displays more consistent tendencies. In comparison to quantile regression, XGBoost exhibits superior prediction accuracy as a result of its advanced internal algorithmic architecture, which is specifically designed to model complex, non-linear connections in data.

SHAP analysis indicates that Expected Shortfall is the principal predictor, underscoring its impact on annual NetSD ratio forecasts, contributing 0.21 units to the model's predictions for the NetSD ratio, and reflecting the average positive magnitude of this feature's influence. Additional significant features for annual data were the Ulcer Performance Index and metrics of volatility, skewness, and kurtosis. In semi-annual data, Maximum Drawdown, Semi-Variance, and correlation metrics become significant, indicating variations in feature importance based on the dataset's time frame.

Quantile regression analysis further supported these findings. For yearly data, Average Return, Ulcer Performance Index, and Kurtosis emerged as the most significant predictors, while semi-annual data highlighted the importance of Expected Shortfall, Maximum Drawdown, Recovery Factor, and Gain-to-Pain Ratio.

In summary, the adaptability of our model to different time windows reinforces its advantage in capturing time-specific market dynamics. By emphasising the differentiation of asset class dominance based on NetSDR, we show how our approach provides tailored insights for portfolio management. The XAI approach highlights the impact of performance and risk indicators, thus providing insights into the inner workings of the XGBoost model and effectively addressing the often-cited "black box" problem of machine learning.

In accordance with the preceding conclusions, we propose several relevant suggestions. Future research may concentrate on expanding the data window and include a wider array of asset classes, so enhancing the comprehension of the causation of the network stochastic dominance ratio. These results highlight the importance for investors of harmonising the depth of data and feature selection to enhance portfolio management and risk evaluation. Further research could explore the possibility of constructing a portfolio that incorporates both the true NetSDR and the derived from the XGBoost model. It would enable a direct comparison of the impact of these methodologies on essential portfolio performance metrics and would be beneficial to examine the differences in asset allocation, total portfolio returns, and volatility.

Appendix 1: Hyperparameters that are employed for tuning an XGBoost model

Hyperparameters that are employed for tuning an XGBoost model			
Parameters	Values that are searchable	Selected values for annual data	Selected values for semi-annual data
learning_rate	[0.01, 0.1]	0.1	0.1
max_depth	[3, 5, 7, 10]	10	5
'min_child_weight	[1, 3, 5]	5	5
Subsample	[0.5, 0.8, 1.0]	0.8	0.5
colsample_bytree	[0.5, 0.8, 1.0]	0.8	0.5
n_estimators	[100, 200, 500]	100	100
Objective	'reg:squarederror'	reg:squarederror	reg:squarederror

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Declarations

Conflict of interest Jurgita Černevičienė declares that she has no conflict of interest. Audrius Kabašinskas declares that he has no conflict of interest. Miloš Kopa declares that he has no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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