ELECTRONICS

T170 ————————————————

ELEKTRONIKA

Defining Random Search Termination Conditions

E. Bareiša, V. Jusas, K. Motiejūnas, R. Šeinauskas

Software Engineering Department, Kaunas University of Technology
Studentų St. 50-406, Kaunas, Lithuani;, e-mail: eduardas.bareisa@ktu.lt, vacius.jusas@ktu.lt, kestutis.motiejunas@ktu.lt,
rimantas.seinauskas@ktu.lt

1. Introduction

Test generation is a complex problem with many interacting aspects e.g. the cost of test generation, test length and the quality of generated test. Test generation can be accomplished at different levels: micro-level, gate-level and functional level [2]. Functional test generation is usually based on simulation during which output values are computed for given input stimuli. The problems arising in this context can be solved by random and deterministic search methods for discrete optimization.

Pure random search consists of sampling a stream of independent and identically distributed random vectors and then selecting the best one as a solution. Pure random search is very easy to implement. Unfortunately, the convergence is extremely slow in most cases of interest. Much attention has been devoted to modifying pure random search to improve its convergence rate. There are approaches involving adaptive construction of distribution, which assign more mass to promising regions of the search space or shrink the domain by some factor. The rigorous mathematical comparisons of different approaches are reported in [2].

Most adaptive random search methods differ in several respects. In particular, they differ in the choice of the neighborhood structure, in the mode of selecting a candidate for solution, in the way the next point is determined and in the way the estimate of the optimal solution is defined [3]. The genetic algorithms, which are based on the biological process of natural selection, are modern heuristic approaches for stochastic optimization. Note that typically a stopping criterion is not included in the algorithm for solving deterministic and stochastic optimization problems. A key reason for this is that the convergence rate for these algorithms is asymptotic as the number of iterations goes to infinity. However, in practice, it is obviously necessary to select an appropriate criterion to stop search.

Random search methods that require only a small number of attempts per iteration can move more rapidly towards the optimal solution than random search methods for which each iteration involves a substantial amount of computer effort. Most (adaptive) random search methods for discrete optimization choose the current estimate as the optimal solution after *m* iterations have been completed with the same solution [4].

A test generation task formulated as the optimization problem can be solved using various stochastic optimization methods [5]. However the most convenient strategy and criteria have to be chosen for every problem being solved. The attempts to apply methods based on search area restrictions, which strive to perform the search only in the most promising areas [6], were not shown to be successful for the test generation problem.

The aim of this paper is to formulate the test generation problem as an optimization problem and to define random search termination conditions. The main attention will be paid to criteria for search termination.

2. Formulating test generation problem as an optimization problem

When dealing with the development of test generation methods one usually faces various optimization problems. In the general case, the functional test generation problem can be formulated in the following way.

The input stimulus to the functional module M having n input and m output variables is described by the vector X $= <x_1,x_2,x_3, \ldots,x_i,\ldots,x_n>$, and the output response is described by the vector $Z= <z_1,z_2,z_3,\ldots,z_j,\ldots,z_m>$, where Z values directly depend on the X values, $x_i \in \{0,1\}$ and $z_j \in \{0,1\}$. In general, 2^n input stimuli may occur. The collection of all possible sets of input stimuli is denoted by X^D. A set of input stimuli is denoted by $X^\square$, where $X^\square \in X^D$, and its cardinality (the number of stimuli) - by $|X^\square|$. Suppose there is given a set S of conditions that have to be fulfilled by input stimuli of the set $X^\square$. An input stimulus $X \in X^\square$ may fulfill several conditions $s \in S$. A condition s may be fulfilled by many input stimuli $X \in X^\square$. In order to assess the fulfillment of the conditions $s \in S$ by the set of input stimuli $X^\square$, the estimate function F^s is defined. If at least one input stimulus $X \in X^\square$ fulfills the condition s, then the estimate function F^s has the value 1, i.e. $F^s (X^\square) = 1$,

otherwise $F^s (X^\square) = 0$. The number of conditions fulfilled by an input stimuli set $X^\square$ is equal to the sum of values $F^s (X^\square)$ taken over all conditions $s \in S$. On the base of the estimate function F^s, the objective function Ψ is defined as follows:

$$\Psi = \alpha \sum F^s (X^\square) - \beta \mid X^\square \mid,$$
$$s \in S,$$

where α, β are positive coefficients.

The test generation problem asks for a set of input stimuli at which the function Ψ is maximized:

$$\text{Max} (\alpha \sum F^s (X^\square) - \beta \mid X^\square \mid),$$
$$X^\square \in X^D \quad s \in S.$$

Specific test generation problems may be obtained and solved by changing conditions that have to be fulfilled. When the number of fulfilled conditions is more important factor than the number of input stimuli, we can take the coefficient $\beta=0$. An important aspect of functional test generation is that the fulfillment of the conditions cannot be evaluated analytically, and instead it has to be estimated using simulation techniques only.

3. Random Search Termination Conditions

As it is well known, random search applied to an optimization problem requires some termination condition to be defined. The simplest termination condition is the number of randomly generated input stimuli; the best solution is chosen from these stimuli. The number of randomly generated input stimuli for finding the best solution depends in large part on an instance of the problem being solved.

Having a solution obtained after performing a fixed number of random search iterations nothing can be said about its quality. In practice, frequently it is not possible to obtain an optimal solution; often one has no such purpose. First of all one faces limited time and computer resources. However, in practice it is always worth to evaluate how much the solution could be improved, and how much time and computer resources it would take. Usually such an evaluation is considerably more expensive than finding the solution. When looking for random search termination conditions one has to evaluate both aspects – cost of solution finding and its quality estimation.

More information about the solution quality is gained when a sequence of solutions is constructed. Comparing the distribution of objective function values of these solutions one can decide whether the time allotted for random search is enough. Wide scattering of objective function values shows that termination of random search is premature. Theoretically, when the time allotted for random search is long enough, optimal or close to optimal solutions are to be found.

Suppose we have N solutions with objective function values Ψ_1, Ψ_2, Ψ_3, ..., Ψ_i,... Ψ_N. Let Ψ_{max} denote the maximum of these values. The distribution of objective function values is characterized by the following quantity expressed in percents:

$$D= (((\textstyle\sum_{i=1}^{N} (\Psi_{max} - \Psi_i))/N)/ \Psi_{max})*100$$

For the experiment we will use the benchmarks of logical circuits ISCAS'85. Optimal solutions of the objective function Ψ [7] for these circuits were obtained analytically for the case, when the coefficients are $\alpha=1$ and $\beta=0$. For each of these circuits million input stimuli were generated ten times and input stimuli that fulfill the conditions $s \in S$ were selected. During every run of the search procedure we recorded the number „$|X^\square|$" of the selected input stimuli and current number of the last selected stimulus „Last". The analytically obtained optimal values of the objective function Ψ (for $\alpha=1$ and $\beta=0$) are listed in column „Best" of Table 1. The sum of entries of relationship matrix [8] was used as the value of the objective function. In order to unify the values of the objective function with [7], the sums were doubled. These sums, which correspond to the values of the objective function when the coefficients are $\alpha=1$ and $\beta=0$, are listed in the penultimate column. As we see, the best solutions were obtained generating a million of random input stimuli for all the circuits except c2670 and c7552. There was a success of obtaining the best solution for the circuit c2670 when generating 20 million input stimuli; whereas no best solution was obtained for the circuit c7552 even generating 90 million random stimuli. The results of these generations are presented in Table 1 in an additional second line under the circuit's results. Table 1 shows the largest and the smallest amount of the selected stimuli (Columns 5 and 6), the largest and the smallest current number of the last selected stimulus (Columns 2 and 3) from all 10 random generations of every circuit. As we see, the striving for the best solution required generating a very different amount of input stimuli (from several thousands to nearly a million) for different circuits. The value that defines the scattering of the results is presented in the column that is the third to the end. The magnitude 0 indicates that the same objective function value was obtained for all 10 random searches. This result testifies that a million input stimuli randomly generated for these circuits is an amount sufficient for obtaining the best result. Ten random searches in turn achieving the solution with the same objective function value signify that there is a high possibility for this solution to be the best.

In total, 100 million input stimuli were generated for all the circuits in Table 1. The current number of the last selected stimulus is presented in the third column of the table. It indicates how many random stimuli had to be generated for obtaining the same solution ten times. Based on the numbers of this column, we can conclude that there was no need in generating a million input stimuli randomly for some circuits. Therefore, the random search may be worth of performing several times enlarging the space of random stimuli for the circuits when the obtained solutions were not uniform during ten runs. In order to explore the last prediction, the random search was performed for each circuit ten times, generating 100 000 random stimuli for each one. The results are presented in Table 2, which has the same format as Table 1. In this case, the best solutions were obtained only for three circuits. In general, ten million random stimuli had to be generated for all the circuits. Therefore generating a million input stimuli for every circuit where the best solution was not achieved, the same result will be obtained after generating 80 million

input stimuli in total. The results would be better if the space of the first random search was enlarged to 300 000 or if more than two search levels of different spaces were used. It is not easy to predict what generation steps are to be included in order to obtain the same result under the minimum amount of generated input stimuli.

Table 1. 1000000 random stimuli

Circuits	Last (Min)	Last (Max)	Last (Max)/ Last (Min)	$|X^{\square}|$ (Min)	$|X^{\square}|$ (Max)	D%	Ψ	Best
C432	2862	4899	1,7	55	67	0	540	540
C499	82877	102442	1,2	485	522	0	5184	5184
C880	80123	192373	2,4	171	218	0	1326	1326
C1355	70645	88129	1,2	470	511	0	5184	5184
C1908	96861	124203	1,3	282	329	0	3004	3004
C2670	965000	984508	1,0	184	194	3,73	3016	3320
20 million	16550550	16550550	1,0	257	257	?	3320	3320
C3540	90093	438846	4,9	227	263	0	2588	2588
C5315	83650	205820	2,5	552	605	0	10540	10540
C6288	81668	649648	7,9	115	131	0	3068	3068
C7552	958935	981376	1,0	633	686	2,65	9334	12188
90 million	82999170	82999170	1,0	851	851	?	10564	12188

Table 2. 100000 random stimuli

Circuits	Last (Min)	Last (Max)	$|X^{\square}|$ (Min)	$|X^{\square}|$ (Max)	D%	Ψ	Best
C432	2708	5251	56	76	0	540	540
C499	79255	89611	478	519	0	5184	5184
C880	84411	99638	173	200	>0	1326	1326
C1355	71877	82246	452	521	0	5184	5184
C1908	92973	99951	284	340	>0	3002	3004
C2670	63253	98654	130	144	>0	2788	3320
C3540	79747	99995	233	275	>0	2588	2588
C5315	84272	99651	548	593	>0	10540	10540
C6288	56950	96385	105	134	>0	3068	3068
C7552	98259	99468	494	560	>0	10564	12188

Table 3. Generation based on the number of selected stimuli

Circuits	K=1000				K=5000				Best
	Min, gen.	Max. gen.	D%	Ψ	Min. gen.	Max. gen.	D%	Ψ	
C432	58000	65000	0	540					540
C499	458000	503000	0	5184					5184
C880	146000	206000	0,15	1326	181084	1060000	0	1326	1326
C1355	492000	514000	0	5184					5184
C1908	278000	338000	0	3004					3004
C2670	108000	147000	6,13	2676	988263	1015000	5,77	3072	3320
C3540	240000	263000	0,15	2588	486098	1375000	0	2588	2588
C5315	559000	597000	0	10540					10540
C6288	109000	130000	0,65	3068	181084	1060000	0	3068	3068
C7552	623000	651000	3,43	8970	2523409	2530000	2,69	9872	12188
Total		3414000				4359938			

It is worth to bind the condition of the termination of random search to the number of the selected input stimuli. The generation can be terminated when the total number of the generated input stimuli exceeds the number of the selected input stimuli multiplied by the coefficient K. The results of the experiment are presented in Table 3. Ten independent random searches were accomplished for every circuit. The left part of Table 3 contains the minimum and the maximum numbers of the generated input stimuli, size D, and the values of objective function Ψ, when K=1000. The random search was repeated for the circuits where there was no success in obtaining the same solution ten times, when K=5000 (the right part of Table 3). Note that to achieve the same best solutions or even better ones (as in Tables 1 and 2), less than 78 million of input stimuli had to be generated in total. Termination of random search generation based on the number of the selected input stimuli allows adjusting to resource requirements necessary for the generation more flexibly and effectively.

When the same value of the objective function is achieved during several random searches, it is evident that the best solution is obtained. The more times the same value of the objective function is achieved during several independent searches, the higher the possibility that the obtained solution is the best. However, the space of the random search is increased as well. If the best solution was not obtained in all the cases, then it remains unclear what the next size of the space of random search has to be used.

The dependence of the size D on the enlargement of the space of random search was analyzed. The possibilities to predict the size of the space of random search when the value of D will become 0 were analyzed, too. It was noticed that the convergence is very slow; the law of the decrease of size D is asymptotical. Therefore, any prediction showed to be very imprecise.

The completeness of the search can be defined by the ratio of how many the last input stimuli selections are rarer than in the beginning of the search. Let's assume R_i is the number of selected stimuli when i random stimuli were generated. The percent $P = ((R_i - R_{i/c})/R_i)*100$, where C>1, has a tendency to decrease during the random search. $R_{i/c}$ is the number of selected stimuli when i/C (i divided by C) input stimuli were generated. The difference $R_i - R_{i/c}$ shows how many input stimuli were selected after generating C times more random stimuli. As the space of the random search increases, P decreases to zero. The rate of decrease depends on the coefficient C. The bigger coefficient means the slower approximation to zero. The value P can be calculated for every random input stimulus which has an index larger than C. If we assume that the termination condition of generation is P=0, this termination condition will be stricter when the value of the coefficient C is larger. The dependence of P value on the increase of the space of random stimuli can be determined and on the basis of this dependence one can evaluate how many

random stimuli are required till P obtains a zero value. If a required amount of random stimuli cannot be generated due to the limitation of calculation resources, there is a possibility to evaluate how far the obtained solution can differ from the best one according to the P value. The values of P obtained during one instance of random search are plotted in Fig. 1. The number of selected stimuli is shown on X axis, and the coefficient P is shown on Y axis.
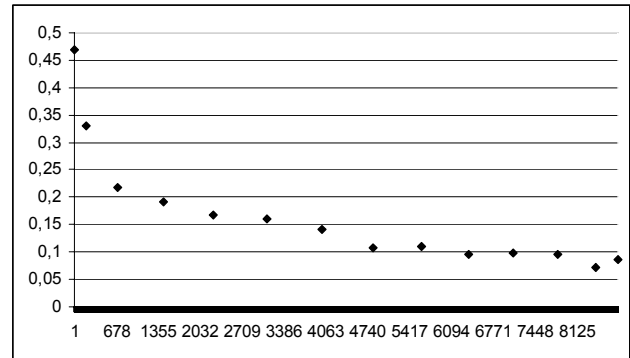

Fig. 1. P values and the number of selected stimuli

However, the analysis of P values indicated in Fig. 1 shows that the estimate of P value becoming zero is quite problematic.

Table 4. Generation according to the number of selected stimuli

Circuit.	C=2				C=3			Best
	Min.	Max.	K	Worst Ψ	Min.	Max.	K	
C432	5744	8930	10	540	8622	11757	10	540
C499	163880	181774	10	5184	252240	291060	10	5184
C880	193948	355604	9	1324	302022	551604	10	1326
C1355	143778	164968	10	5184	222822	256842	10	5184
C1908	194186	242276	10	3004	282540	371805	10	3004
C2670	22760	29965284	4	2488	28043244	42046896	10	3320
C3540	177504	526998	9	2586	275688	1040529	10	2588
C5315	171400	466498	10	10540	294960	573948	10	10540
C6288	183186	710416	7	3060	375942	1679799	10	3068
C7552	*	*	*	*	*	*	*	12188
Total	1256386	32622748		34754	30058080	30092834		34754

Table 4 reports the results of ten random generations when termination condition of generation was based on the number of selected stimuli. In the case of coefficients C=2 and C=3, random generation reached zero value of P for all the circuits, except circuit c7552, for which random generation had come short of calculation resources. Nevertheless, the generation did not achieve the best solution for the circuits c880, c2670, c3540 and c6288 when coefficient C was 2, though the difference from the best value was tiny (except the circuit c2670). The worst obtained solution is shown in Column „Worst Ψ". The number of experiments (out of ten) we succeeded to obtain the best solution is shown in columns „K". The smallest and the largest amounts of input stimuli till the termination condition was fulfilled (P=0) are presented in columns „Min." and "Max.", respectively. The increase of coefficient C from 2 to 3 allowed obtaining the best solutions for all the circuits in all ten runs. On this basis, we recommend to use only the value 3 of coefficient C while solving such problems in practice, and to perform

the random search only one time thereby reducing the need for computer resources. The total number of the analyzed random stimuli when the coefficient C=3 was in use evaluating the maximum number of generated random stimuli for every circuit would be approximately 30 millions only. The generation which fails in fulfilling the termination condition of the generation has to be terminated when it runs out of resources, and the approximation to the best solution is to be evaluated according to the value P.

4. Conclusions

The random search may last very long. The quality of the solution depends on the tackled task. The random search that spans too short may produce not qualitative solution, however, long random search may be inefficient and waste computer resources. It is especially relevant when the task is solved for the first time. Therefore, the defining of random search termination conditions is

essential and integral part of the optimization problem. In many cases the termination condition determines quality of the solution. It is demonstrated that during functional test generation there may be used various random search termination conditions and experimentally evaluated the quality of the obtained solution. The proposed random search termination conditions may be used in solving of other optimization problems. The presented research results enable reasonably to choose the appropriate termination conditions for proper search scope and precision of the solution.

References

1. **Bareisa E., Motiejunas K., Seinauskas R.** Identifying Legal and Illegal States in Synchronous Sequential Circuits Using Test Generation INFORMATICA 14 (2): 135-154, 2003.

2. **Appel M. J., Radulovic D.** Accelerated Random Search, Proceedings of 16th IMACS World Congress 2000 on Scientific Computing, Lausane, Switzerland, August, 2000, P. 329-343.

3. **Andradottir S.** Accelerating the Convergence of Random Search Methods for Discrete Stochastic Optimization. ACM Transactions on Modeling and Computer Simulation, Vol. 9, No. 4, October 1999, P. 349-380.

4. **Bareisa E., Jusas V., Motiejunas K., Seinauskas R.** The Realization-Independent Testing Based on the Black Box Fault Models. INFORMATICA 16 (1): 19-36, 2005.

5. **Spall J. C.** Introduction to Stochastic Search and Optimization: Estimation, Simulation, and Control, Wiley, Hoboken, NJ, 2003

6. **Kushner H. J. and Yin G. G.** Stochastic Approximation and Recursive Algorithms and Applications (2nd ed.), Springer-Verlag, New York, 2003

7. **Michael M., Tragoudas S.** ATPG Tools for Delay Faults at the Functional Level. ACM Transactions on Design Automation of Electronics Systems, Vol. 7, No. 1, January 2002, P. 33-57.

8. **Bareisa E., Jusas V., Motiejunas K., Seinauskas R.** Functional Test Generation Tool. Proceedings of the 8th euromicro conference on digital system design (DSD'2005), 30August - 3 September, 2005, Porto, Portugal, P. 192-195.

E. Bareiša, V. Jusas, K. Motiejūnas, R. Šeinauskas. Defining Random Search Termination Conditions // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 2(66). – P. 26-31

Test generation can be accomplished at different levels: micro-level, gate-level and functional level. Functional test generation is usually based on simulation during which output values are computed for given input stimuli. The problems arising in this context can be solved by random and deterministic search methods for discrete optimization. The aim of this paper is to formulate the test generation problem as an optimization problem and to define random search termination conditions. In many cases the termination condition determines quality of the solution. It is demonstrated that during functional test generation there may be used various random search termination conditions and experimentally evaluated the quality of the obtained solution. The proposed random search termination conditions may be used in solving of other optimization problems. The presented research results enable reasonably to choose the appropriate termination conditions for proper search scope and precision of the solution. Ill. 1, tabl. 4, ref. 8 (in English, Abstracts in English, Russian and Lithuanian).

Э. Барейша, В. Юсас, К. Мотеюнас, Р. Шейнаускас. Определение условий окончания случайного поиска // Электроника и электротехника. – Каунас: Технология, 2006. – №. 2(66). – С. 26–31

Генерация тестов может быть проведена на разных уровнях абстракции: микро, вентильном и функциональном уровне. Функциональная генерация чаще всего опирается на моделирование. Проблемы, возникающие в этом контексте, могут быть решены опираясь на регулярные и случайные методы дискретной оптимизации. Основная цель данной статьи – сформулировать проблему генерации тестов как проблему оптимизации и определить условия окончания случайного поиска. Во многих случаях условие окончания поиска определяет качество решения. Показано, что при решении задачи генерации функционального теста могут быть использованы разные условия окончания поиска, экспериментально оценено качество решения. Предлагаемые условия окончания случайного поиска могут быть использованы и при решении других задач оптимизации. Представленные результаты позволяют обоснованно подобрать подходящий вариант условий окончания случайного поиска, принимая во внимание качество решения и объем поиска. Ил. 1, табл. 4, библ. 8 (на английском языке; рефераты на английском, русском и литовском яз.).

E. Bareiša, V. Jusas, K. Motiejūnas, R. Šeinauskas. Atsitiktinės paieškos pabaigos sąlygų nustatymas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 2(66). – P. 26–31

Testų gali būti generuojami įvairiais abstrakcijos lygiais: mikrolygiu, ventiliniu ir funkciniu lygiais. Funkcinis testų generavimas dažniausiai remiasi modeliavimu. Problemos, iškylančios šiame kontekste, gali būti sprendžiamos taikant diskretinės optimizavimo deterministinius ir atsitiktinius paieškos metodus. Šio straipsnio pagrindinis tikslas – suformuluoti testų generavimo problemą kaip optimizavimo problemą ir apibrėžti atsitiktinės paieškos pabaigos sąlygas. Daugeliu atvejų paieškos pabaigos sąlyga lemia sprendinio kokybę. Parodyta, kad, sprendžiant funkcinio testo generavimo uždavinį, gali būti naudojamos įvairios paieškos pabaigos sąlygos, eksperimentiškai įvertinta gaunamo sprendinio kokybė. Siūlomos atsitiktinės paieškos pabaigos sąlygos gali būti naudojamos ir sprendžiant kitus optimizavimo uždavinius. Pateikti eksperimentinio tyrimo rezultatai leidžia pagrįstai pasirinkti tinkamą atsitiktinės paieškos pabaigos sąlygos variantą, vertinant sprendinio tikslumą ir paieškos apimtį. Il. 1, lent. 4, bibl. 8 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).