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Dynamics of wave vibrational motors

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1. Introduction

Wave type vibrational motors are widely used in precise techniques and they change high frequency oscillations into purposeful movement of the rotor. These mechanisms are of simple and technological design. Vibrational motors allow changing the speed of the rotor in a wide range and to realize small changes even up to one thousandth part of a micrometer [1]. The principle of the oscillation transformation of ultrasonic travelling waves in to directional movement was described in [1-3]. Many types of the vibrational motors (Ultrasonic Motors) [3-5] were developed. The dynamics of non shock regimes of the wave type vibrational motors is analyzed in this article. Presented research is aimed to develop universal models of vibrational motors of different design. Wave type vibrational motors are created at Kaunas University of Technology [1, 2].

2. Constructions

The wave type vibrational motors are simple mechanisms with the principle of action based on the transformation of high frequency wave type oscillations into directional movement of a rotor. The schemes of the vibrational motors are presented in Figs. 1, 2.

The piezoceramic ring *1* is interacting with the rotor *2* in a friction manner. The contact of the piezoceramic ring (stator) and the rotor can be concentrated

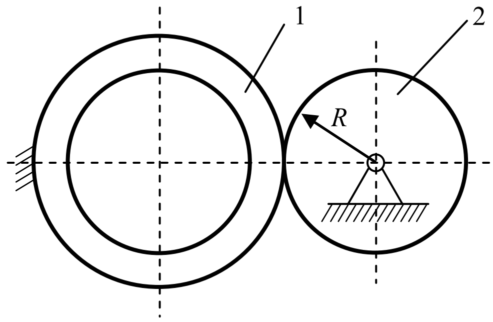

Fig. 1 Vibrational motor with concentrated stators and rotors contact: *1* - piezoceramic ring; *2* - rotor

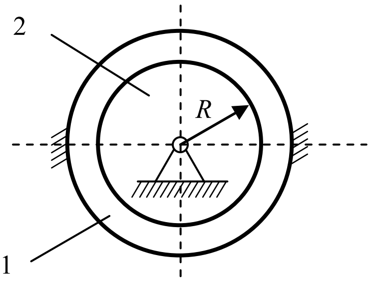

Fig. 2 Vibrational motor with distributed stators and rotors contact: *1* - piezoceramic ring; *2* - rotor

(Fig. 1), or distributed (Fig. 2). A thin silver layer is printed on the both sides of the piezoceramic ring in order to apply an electric field to it. These silver electrodes are divided into segments. A high frequency voltage drives each segment of the electrodes with a phase difference. Travelling wave oscillations of the piezoceramic ring *1* are exited in this way and movement of rotor *2* is realized.

3. Models

Movement of the rotor *2* of vibrational motor has been realized by exiting travelling wave-type high frequency oscillations of piezoceramic ring *1*

$$y_l = A_l \sin\left(n\varphi + \omega t\right) \tag{1}$$

$$y_t = A_t \sin\left(n\varphi + \omega t + \varphi_0\right) \tag{2}$$

where y_l, y_t are longitudinal and transversal deformations of the ring *1*, m; A_l, A_t are amplitudes of longitudinal and transversal oscillations, m; n is the wave number, φ is angular coordinate, rad; ω is angular frequency of oscillations, rads^{-1}; φ_0 is phase difference of transversal oscillations with respect to longitudinal, rad; t is time, s.

Oscillations (1) and (2) cause the reaction N and friction F forces

$$N = N_0 + A_t Z \sin\left(n\varphi + \omega t + \varphi_0 + \phi\right) \tag{3}$$

$$F = f N sign\left(R\frac{d\psi}{dt} - \frac{dy_l}{dt}\right) \tag{4}$$

where N_0 is the reaction of initial stretching of piezoceramic ring and rotor, N; Z and ϕ are respectively module and argument of dynamical stiffness of the rotor, Nm^{-1}, rad; f is coefficient of friction; R is radius of the rotor, m; ψ is angular coordinate of the rotor, rad.

The non shock regime of interaction of piezoceramic ring *1* and rotor *2* is realized when

$$N_0 \ge A_t Z \tag{5}$$

Movement of the rotor *2* is described by the differential equation

$$I\frac{d^2\psi}{dt^2} + fR\left[N_0 + A_t Z \sin\left(n\varphi + \omega\, t + \varphi_0 + \phi\right)\right]\times$$
$$\times sign\left[R\frac{d\psi}{dt} - \omega A_l \cos\left(n\varphi + \omega\, t\right)\right] + M = 0 \tag{6}$$

where I is inertia moment of the rotor, kgm^2; M is the moment of resistance to the movement, Nm.

Eq. (5) describes slow directional movement of the rotor *2* and high frequency oscillations, caused by the exited waves. Slow movement can be described after definition of the average friction force in the period of oscillation $T = 2\pi / \omega$

$$I\frac{d^2\psi}{dt^2}+\frac{fR}{T}\int_{t_0}^{t_0+T}\left[N_0+A_tZ\,sin\left(n\varphi+\omega\,t+\varphi_0+\phi\right)\right]\times$$
$$\times sign\left[R\frac{d\psi}{dt}-A_l\omega\,cos\left(n\varphi+\omega\,t\right)\right]dt+M=0 \qquad (7)$$

After the definition the moments of time t_1, t_2, and t_3, when the friction force changes its direction

$$t_1=\frac{1}{\omega}arc\,cos\frac{R}{A_l\omega}\frac{d\psi}{dt}-\frac{n\varphi}{\omega}$$
$$t_2=\frac{2\pi}{\omega}-\frac{1}{\omega}arc\,cos\frac{R}{A_l\omega}\frac{d\psi}{dt}-\frac{n\varphi}{\omega}$$
$$t_3=\frac{2\pi}{\omega}+\frac{1}{\omega}arc\,cos\frac{R}{A_l\omega}\frac{d\psi}{dt}-\frac{n\varphi}{\omega} \qquad (8)$$

Eq. (7) can be transformed

$$I\frac{d^2\psi}{dt^2}-\frac{fR}{T}\int_{t_1}^{t_2}\left[N_0+A_tZ\,sin\left(n\varphi+\omega\,t+\varphi_0+\phi\right)\right]\times$$
$$\times dt+\frac{fR}{T}\int_{t_2}^{t_3}\left[N_0+A_tZ\,sin\left(n\varphi+\omega\,t+\varphi_0+\phi\right)\right]dt+$$
$$+M=0 \qquad (9)$$

After integration of the Eq. (9), the slow rotors movement is described as

$$\xi''+f\left[n_0arc\,sin\,\xi'-c_1y_1\sqrt{1-(\xi')^2}\,sin\,\varphi_N\right]+$$
$$+m=0 \qquad (10)$$

where

$$\left.\begin{aligned}&\tau=\omega\ t;\quad \xi=\frac{R\psi}{A_l};\quad '=\frac{d}{d\tau};\ n_0=\frac{4R^2N_0}{I\,A_l\omega^2};\\ &c_1=\frac{4R^2Z}{I\omega^2};\quad m=\frac{MR}{I\,A_l\omega};\quad \varphi_N=\varphi_0+\phi\end{aligned}\right\} \qquad (11)$$

If the contact of the piezoceramic ring *1* and rotor *2* is continuous, oscillations (1) and (2) cause distributed reaction q and distributed friction force f_d

$$q=q_0+A_t\,z\,sin\left(\omega t+\varphi_0+\phi\right) \qquad (12)$$

$$f_d=f\,q\,sign\left(R\frac{d\psi}{dt}-\frac{dy_l}{dt}\right) \qquad (13)$$

where q_0 is distributed reaction force, Nm^{-1}, caused by initial stretching of piezoceramic ring *1* and rotor *2*; z is the coefficient, proportional to the module of dynamical stiffness of the rotor, Nm^{-2}.

Non shock regime is realized if

$$q_0\geq A_t z \qquad (14)$$

Movement of the rotor is described by the equation

$$I\frac{d^2\psi}{dt^2}+f\,R\int_0^{2\pi}\left[q_0+A_tz\,sin\left(n\varphi+\omega\,t+\varphi_0+\varphi\right)\right]\times$$
$$\times sign\left[R\frac{d\psi}{dt}-A_l\omega\,cos\left(n\varphi+\omega\,t\right)\right]d\varphi+M=0 \qquad (15)$$

After definition of the coordinates φ_1, φ_2, and φ_3

$$\varphi_1=\frac{1}{n}arc\,cos\frac{R}{A_l\omega}\frac{d\psi}{dt}-\frac{\omega t}{n} \qquad (16)$$

$$\varphi_2=\frac{2\pi}{n}-\frac{1}{n}arc\,cos\frac{R}{A_l\omega}\frac{d\psi}{dt}-\frac{\omega t}{n} \qquad (17)$$

$$\varphi_3=\frac{2\pi}{n}+\frac{1}{n}arc\,cos\frac{R}{A_l\,\omega}\frac{d\psi}{dt}-\frac{\omega t}{n} \qquad (18)$$

where the friction force changes its direction, Eq. (14) can be transformed

$$I\frac{d^2\psi}{dt^2}-n\,f\,R\int_{\varphi_1}^{\varphi_2}\left[q_0+\right.$$
$$\left.+A_tz\,sin\left(n\varphi+\omega\ t+\varphi_0+\phi\right)\right]d\varphi+n\,f\,R\int_{\varphi_2}^{\varphi_3}\left[q_0+\right.$$
$$\left.+A_tz\,sin\left(n\varphi+\omega\ t+\varphi_0+\phi\right)\right]d\varphi+M=0 \qquad (19)$$

Movement of the rotor is described

$$\xi''+f\left[n_0arc\,sin\,\xi'-c_1y_1\sqrt{1-(\xi')^2}\,sin\,\varphi_N\right]+$$
$$+m=0 \qquad (20)$$

where

$$\left.\begin{aligned}&\tau=\omega t;\quad \xi=\frac{R\psi}{A_l};\quad '=\frac{d}{d\tau};\\ &n_0=\frac{4R^2q_0}{I\,A_l\omega^2};\quad c_1=\frac{4R^2z}{I\omega^2};\quad m=\frac{MR}{I\,A_l\omega};\\ &\varphi_N=\varphi_0+\phi.\end{aligned}\right\} \qquad (21)$$

4. Results

The Eq. (20) is identical to the Eq. (10). Slow movement of the rotor of the vibrational motor of concentrated contact is identical to the movement of the rotor of the vibrational motor with distributed contact. Characteristics of the movement velocity of the rotor are presented in Fig. 3.

Taking into account that at stationary regime of the movement, acceleration $\xi_s''=0$, velocity of the move-

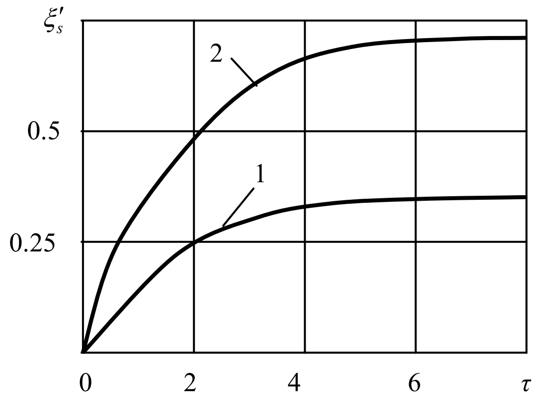

Fig. 3 Characteristics of movements velocity, $f = 0.5$, $c_1 y_1 / n_0 = 1$: *1* - $m = 0$; *2* - $m/n_0 = 0.5$

ment ξ_s' can be expressed by the equation

$$\xi_s' - sin\left[\frac{c_1 y_1}{n_0}\sqrt{1-(\xi_s')^2}\, sin\,\varphi_N - \frac{m}{f\, n_0}\right] = 0 \qquad (22)$$

The amplitude of longitudinal oscillations A_l, the resistance moment to movement M (Fig. 4), and the relation $A_t Z / N_0$ (Fig. 5) are the major parameters on which the velocity of rotor movement depends.

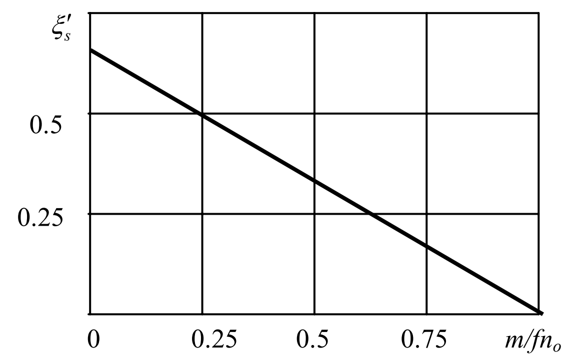

Fig. 4 Dependence of movement's stationary velocity on resistance force, $c_1 y_1 / n_0 = 1$

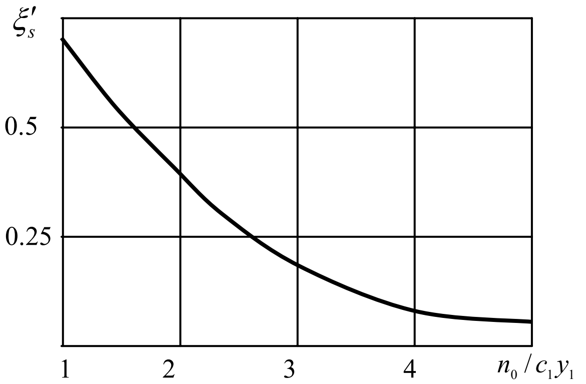

Fig. 5 Dependence of movements stationary velocity on the force of initial stretching, $m = 0$

At initial stage of the movement, initial velocity $\xi_i' = 0$ and initial (maximal) acceleration can be described

$$\xi_i'' = f\, c_1 y_1\, sin\,\varphi_N - m \qquad (23)$$

The amplitude of transversal oscillations A_t, the resistance moment to movement M, and phase difference of reaction φ_N with respect to the longitudinal oscillations are the major parameters on which the acceleration of rotor movements depends.

5. Conclusions

1. Slow movement of the rotor of vibrational motor of concentrated contact is identical to the movement of the rotor of vibrational motor with distributed contact. The mathematical models of wave type vibrational motors are developed.

2. The amplitude of longitudinal vibration and the resistance moment to movement are the major parameters on which the velocity of rotor movement depends.

3. The amplitude of transversal vibration A_t, the resistance moment to movement M, and phase difference of reaction with respect to the longitudinal vibration are the major parameters on which the acceleration of rotor movement depends.

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BANGINIŲ VIBRACINIŲ VARIKLIŲ DINAMIKA

R e z i u m ė

Straipsnyje analizuojama banginių vibracinių variklių dinamika. Vibraciniai varikliai yra nesudėtingos konstrukcijos mechanizmai, transformuojantys aukštojo dažnio virpesius į kryptingą judesį. Aprašytos banginių vibracinių variklių konstrukcijos, sudaryti šių mechanizmų dinaminiai modeliai. Analitiniu būdu nustatytos vibracinių variklių pagrindinių dinaminių charakteristikų priklausomybės nuo konstrukcijos parametrų. Gautos formulės judėjimo greičiui ir pagreičiui apskaičiuoti. Banginiai vibraciniai varikliai sukurti Kauno technologijos universitete.

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DYNAMICS OF WAVE VIBRATIONAL MOTORS

S u m m a r y

Dynamics of wave type vibrational motors is analyzed in the article. The vibrational motors are mechanisms transforming high frequency oscillations into directional movement. The constructions of the vibrational motors are described and mathematical models are developed. The formulas to describe the velocity of rotor movement and the acceleration are presented. The major dynamical parameters of vibrational motors are analytically defined. Wave type vibrational motors are developed at Kaunas University of Technology.

Р. Курила

ДИНАМИКА ВОЛНОВЫХ ВИБРОДВИГАТЕЛЕЙ

Р е з ю м е

В настоящей статье анализируется динамика волновых вибродвигателей. Вибродвигатели это механизмы простой конструкции, преобразующие высокочастотные вибрации в направленное движение. Представлены конструкции волновых вибродвигателей и составлены их динамические модели. Аналитически определены зависимости основных динамических параметров вибродвигателей от их конструктивных параметров. Получены формулы для определения скорости и ускорения выходного движения. Волновые вибродвигатели разработаны в Каунасском технологическом университете.

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