Multiple conjugate smoothing for calculation of dynamic excitation

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Introduction

Electrostatic excitation is used for excitation of wave motion in a number of vibrating systems [1, 2]. The determination of the dielectrophoretic force and of the travelling wave force acting on a spherical particle in an electric field requires calculation of higher derivatives of the electrostatic potential [1]. Thus, their determination from a conventional finite element formulation would require dense meshing for producing a sufficiently smooth field.

Here application of the technique of multiple conjugate approximation with smoothing for calculation of the second derivatives of the electrostatic potential is developed. This smoothing technique is similar to conjugate approximation used for the calculation of nodal values of stresses in [3, 4] and enables to obtain the second derivatives of the electrostatic potential of acceptable quality on a rather coarse mesh by using the conventional finite element formulation for the calculation of the electrostatic potential.

Calculation of electrostatic excitation

The electrostatic field is described by the equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 ,\qquad(1)$$

where ϕ is the electrostatic potential, x and y are the orthogonal Cartesian coordinates. This problem is solved by applying the conventional finite element formulation [3].

The application of the procedure of multiple conjugate smoothing for calculation of the second derivatives consists of two parts:

1. Calculation of the first derivatives of the potential;
2. Calculation of the second derivatives of the potential on the basis of the first derivatives obtained previously.

So the first problem is to obtain the gradients of the potential of acceptable quality.

The gradients of the potential at the points of numerical integration of the finite element are calculated in a usual way:

$$\begin{Bmatrix} \dfrac{\partial \phi}{\partial x} \\[2mm] \dfrac{\partial \phi}{\partial y} \end{Bmatrix} = [B]\{\delta\} ,\qquad(2)$$

where $\{\delta\}$ is the vector of nodal potentials obtained as a result of solving of the system of linear algebraic equations of the conventional finite element formulation of Eq. 1 as described in [3]; $[B]$ is the matrix relating the gradients of the potential with the nodal potentials. The potential is continuous at inter-element boundaries, but the calculated gradients of the potential due to the operation of differentiation are discontinuous.

The appropriate gradients of the potential are obtained by minimising the following errors:

$$\frac{1}{2}\iint \left(\begin{array}{l} ([N]\{\delta_x\} - \phi_x)^2 + \\ + \lambda \left(\left(\dfrac{\partial \phi_x}{\partial x}\right)^2 + \left(\dfrac{\partial \phi_x}{\partial y}\right)^2 \right) \end{array} \right) dxdy =$$

$$= \frac{1}{2}\iint \left(\begin{array}{l} ([N]\{\delta_x\} - \phi_x)^2 + \\ + \lambda \{\delta_x\}^T [B*]^T [B*]\{\delta_x\} \end{array} \right) dxdy ,$$

$$\frac{1}{2}\iint \left(\begin{array}{l} ([N]\{\delta_y\} - \phi_y)^2 + \\ + \lambda \{\delta_y\}^T [B*]^T [B*]\{\delta_y\} \end{array} \right) dxdy ,\qquad(3)$$

where λ is the smoothing parameter; $\{\delta_x\}$ is the vector of nodal values of $\phi_x = \dfrac{\partial \phi}{\partial x}$; $\{\delta_y\}$ is the vector of nodal values of $\phi_y = \dfrac{\partial \phi}{\partial y}$; $[N]$ is the row of the shape functions of the finite element; $[B*]$ is the matrix of the derivatives of the shape functions (the first row with respect to x; the second – with respect to y).

This leads to the following systems of linear algebraic equations for the determination of each of the components of the gradient of the potential:

$$\iint \left([N]^T [N] + [B*]^T \lambda [B*] \right) dxdy \cdot \{\delta_x\} =$$
$$= \iint [N]^T \phi_x dxdy ,$$

$$\iint \left([N]^T [N] + [B*]^T \lambda [B*] \right) dxdy \cdot \{\delta_y\} =$$
$$= \iint [N]^T \phi_y dxdy .\qquad(4)$$

The second problem is to obtain the second derivatives of the potential of acceptable quality. For this purpose the vector $\{\bar{\delta}\}$ is formed as the first component of the vector $\{\delta_x\}$, the first component of the vector $\{\delta_y\}$, the second component of the vector $\{\delta_x\}$.

The second derivatives of the potential at the points of numerical integration of the finite element are calculated as:

$$\left\{ \begin{array}{c} \dfrac{\partial^2 \phi}{\partial x^2} \\ \dfrac{\partial^2 \phi}{\partial y^2} \\ \dfrac{\partial^2 \phi}{\partial x \partial y} \end{array} \right\} = [\overline{B}]\{\overline{\delta}\}, \qquad (5)$$

where $[\overline{B}]$ is the matrix relating the second derivatives of the potential with the vector $\{\overline{\delta}\}$:

$$[\overline{B}] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & 0 & \dots \\ 0 & \dfrac{\partial N_1}{\partial y} & \dots \\ \dfrac{1}{2}\dfrac{\partial N_1}{\partial y} & \dfrac{1}{2}\dfrac{\partial N_1}{\partial x} & \dots \end{bmatrix}, \qquad (6)$$

where N_I, are the shape functions of the finite element. The previously obtained gradients of the potential are continuous at inter-element boundaries, but the calculated second derivatives of the potential due to the operation of differentiation are discontinuous.

The appropriate second derivatives of the potential are obtained by minimising the following errors:

$$\frac{1}{2}\iint \left(\begin{array}{c} ([N]\{\delta_{xx}\} - \phi_{xx})^2 + \\ + \lambda \left(\left(\dfrac{\partial \phi_{xx}}{\partial x}\right)^2 + \left(\dfrac{\partial \phi_{xx}}{\partial y}\right)^2 \right) \end{array} \right) dxdy =$$

$$= \frac{1}{2}\iint \left(\begin{array}{c} ([N]\{\delta_{xx}\} - \phi_{xx})^2 + \\ + \lambda \{\delta_{xx}\}^T [B*]^T [B*]\{\delta_{xx}\} \end{array} \right) dxdy,$$

$$\frac{1}{2}\iint \left(\begin{array}{c} ([N]\{\delta_{yy}\} - \phi_{yy})^2 + \\ + \lambda \{\delta_{yy}\}^T [B*]^T [B*]\{\delta_{yy}\} \end{array} \right) dxdy,$$

$$\frac{1}{2}\iint \left(\begin{array}{c} ([N]\{\delta_{xy}\} - \phi_{xy})^2 + \\ + \lambda \{\delta_{xy}\}^T [B*]^T [B*]\{\delta_{xy}\} \end{array} \right) dxdy, \qquad (7)$$

where $\{\delta_{xx}\}$ is the vector of nodal values of $\phi_{xx} = \dfrac{\partial \phi_x}{\partial x}$;

$\{\delta_{yy}\}$ is the vector of nodal values of $\phi_{yy} = \dfrac{\partial \phi_y}{\partial y}$; $\{\delta_{xy}\}$ is

the vector of nodal values of $\phi_{xy} = \dfrac{1}{2}\left(\dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \right)$.

This leads to the following systems of linear algebraic equations for the determination of each of the components of the second derivatives of the potential:

$$\iint \left([N]^T [N] + [B*]^T \lambda [B*]\right) dxdy \cdot \{\delta_{xx}\} =$$

$$= \iint [N]^T \phi_{xx} dxdy,$$

$$\iint \left([N]^T [N] + [B*]^T \lambda [B*]\right) dxdy \cdot \{\delta_{yy}\} =$$

$$= \iint [N]^T \phi_{yy} dxdy,$$

$$\iint \left([N]^T [N] + [B*]^T \lambda [B*]\right) dxdy \cdot \{\delta_{xy}\} =$$

$$= \iint [N]^T \phi_{xy} dxdy. \qquad (8)$$

The choice of the smoothing parameter is performed interactively from a qualitative view of the isolines of the second derivatives of the potential. When the smoothing parameter is too small, the images are of unacceptable quality because of the unphysical behaviour of the isolines as a result of their calculation from the conventional finite element formulation. When the smoothing parameter is too big, over-smoothed images of isolines are obtained which may look acceptable but be incorrect. So the best value of the parameter might be considered when most of the images of isolines are of acceptable quality without the unphysical behaviour produced by the approximation.

The choice of the smoothing parameter is based on the numerical effectiveness of the formulation: when the same value of the smoothing parameter is applied in both stages of the analysis the matrixes of the systems of linear algebraic Eq. 4 and 8 are the same. Numerical experiments show that this choice produces acceptable results and thus is to be recommended.

Numerical results

The rectangular domain is analysed. On the left and on the right boundaries periodic boundary conditions are assumed: that is the values of the electrostatic potential for the same y coordinates on those boundaries are mutually equal. On the upper boundary the electrostatic potential is assumed to be equal to zero. On the lower boundary the electrostatic potential is assumed to change as a period of a sin function.

The unsmoothed isolines of the second derivatives of the potential are shown in the Fig 1, Fig. 2 and Fig.3. The figures show the unphysical oscillations of the results produced by the approximation.

The smoothed isolines of the second derivatives of the potential are shown in the Fig.4, Fig. 5 and Fig. 6. It is evident that the smoothing procedure suppresses the unphysical oscillations produced by the approximation.
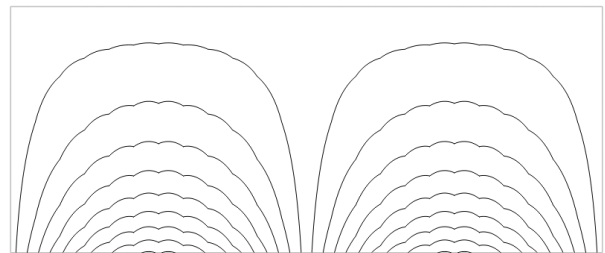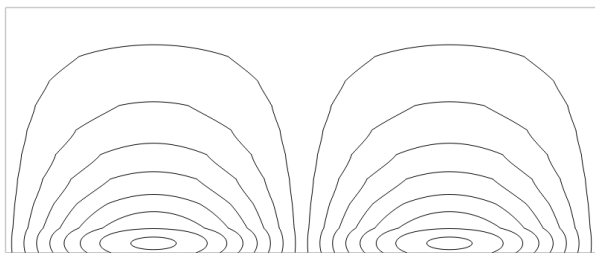

Fig. 1. **Unsmoothed isolines of the values of** ϕ_{xx}

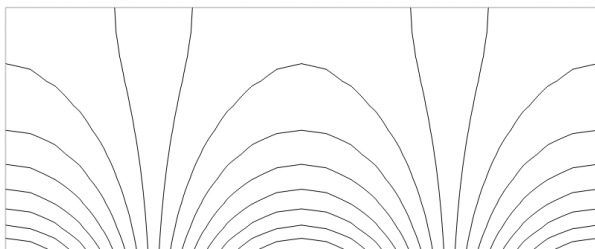Fig. 2. **Unsmoothed isolines of the values of** ϕ_{yy}

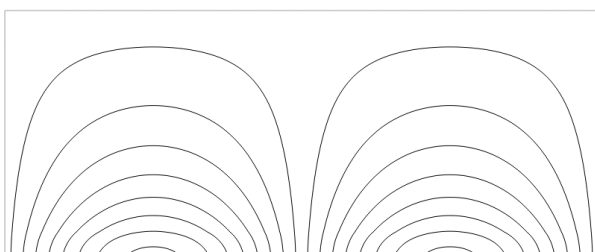
Fig. 3. **Unsmoothed isolines of the values of** ϕ_{xy}
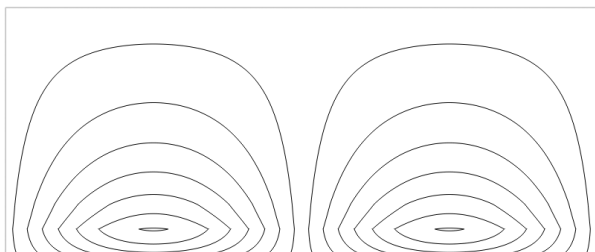

Fig. 4. **Smoothed isolines of the values of** ϕ_{xx}

Fig. 5. **Smoothed isolines of the values of** ϕ_{yy}
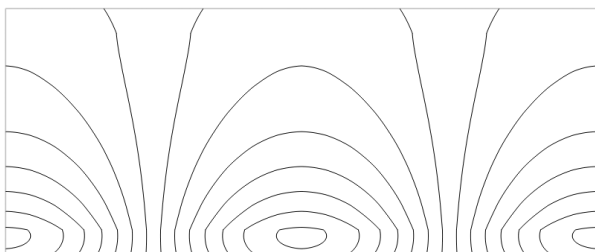

Fig. 6. **Smoothed isolines of the values of** ϕ_{xy} .

Conclusions

For precise investigations of the second derivatives of the potential the representation of the results by the digital image of isolines is to be used.

The application of the procedure of multiple conjugate approximation with smoothing is proposed for the elimination of the unphysical oscillations produced by the approximation. This smoothing procedure enables the calculation of the second derivatives of the potential on rather coarse conventional finite element meshes. Those fields of the higher derivatives of the potential are necessary for calculation of some types of forces acting in vibrating mechanical systems.

The choice of the smoothing parameter is based on the numerical effectiveness of the formulation: when the same value of the smoothing parameter is applied in both stages of the analysis the matrixes of all of the systems of linear algebraic equations used in the procedure of multiple conjugate smoothing are the same. Numerical experiments show that this choice produces acceptable results and thus is to be recommended.

References

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Kartotinis jungtinis glotninimas dinaminiam sužadinimui apskaičiuoti

Reziumė

Elektrostatinis sužadinimas naudojamas virpamosiose mechaninėse sistemose. Jėgos, veikiančios sferinę dalelę, išreiškiamos per aukštesnes elektrostatinio potencialo išvestines. Tiriamas kartotinės jungtinės aproksimacijos glotninimu taikymas antrosioms elektrostatinio potencialo išvestinėms apskaičiuoti.