Motion of a part on a horizontally vibrating plane

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1. Introduction

Main stages of automated assembly are matching of connective surfaces and joining of the parts to be assembled. For automated assembly of the parts mutually dependent orienting of the parts and matching of their connective surfaces is necessary. Under competitive conditions the need arises to seek for reliable and uncostly technological means for assembly operations. One of the advanced assembly methods is vibratory assembly which allows simplifying the design of assembly devices and reducing expenses for assembly operations.

Orienting and matching of the connective surfaces by search technique is an advanced method which allows increasing the reliability and efficiency of the process [1]. The application field of devices operating by search technique is not limited to assembly of common pegs and bushings. Nowadays in the world various search techniques are used: as a body is elastically based, blind search, as position of one of the parts is not exactly defined, vibratory and etc. During the search one of the parts to be assembled is subjected to the motion in perpendicular to joining direction by a certain trajectory, while the other part is elastically based. Choosing assembly method it is necessary to take into account the shape and complexity of the parts to be assembled, required accuracy of the assembly and parameters of the connective surfaces matching.

Parts feeding and orienting on a horizontal plane has been investigated numerous times. D.S. Reznik and J.F. Canny [2,3] proposed an universal planar manipulator. which by means of vibrating plane moves the parts from one location on the plane to the other. The universal planar manipulator consists of 4 motors and flat plate. Depending on the motion law of the motors, excitation frequency and layout schema the character, direction and velocity of the part motion on the plane are dependent. The motors are used to vibrate the plane and due to friction forces between the surfaces of the part and the plane translation and rotation of the part is possible. Changing the motors' connection schemas and excitation frequencies various manipulations of the parts are possible. In papers [2,3] feeding, orienting and picking of the parts on vibrating plane are analysed, though this is not related to assembly operations.

Winney Y. Du proposed the other scheme of vibrating plane [4]. The device consists of a flat vibrating plate, the base, permanent magnet and the coils. When current goes through the coils the generated electromagnetic force moves the coils which collide with the plate and push the latter. So the plate is moved side to side (x direction) or back and forth (y direction), depending both on the current's direction and the coil layout. By adjusting the magnitude and direction of the input current it is possible to control vibrations of the plane, simultaneously controlling motion of the part. Also a vision system is used to track part's motion and perform real-time estimation of the part location. This information is transferred to controller which controls input signal. So the plane and also the part are pushed to the desirable position. Such manipulator is able to cope with rather precise feeding and positioning of various parts, has simple design and is inexpensive. However, the mentioned manipulator is not applicable to orienting and assembly of the parts.

This paper investigates the motion of a part on a horizontally vibrating plane, when excitation by two perpendicular directions is provided. Changing excitation amplitude, frequency and direction, and the coefficient of friction it is possible to perform easy and fast orienting of the part towards the predetermined position, to provide the part with the motion by different trajectories and so perform the positioning of the part in respect of the connecting part.

2. Dynamic model and equations of motion

Proposed here the assembly method by vibratory search is based on a body motion on a vibrating plane. Vibratory motion in two perpendicular directions is provided to the plane. Changing amplitudes of perpendicular components of vibrations, frequencies and phase angles between the components of vibrations it is possible to obtain different character laws of plane motion. Depending on the plane motion law a character of the body motion on the plane changes.

Investigated here is the motion of a body on a vibrating base. Let us assume that $\zeta O\eta$ is an immovable coordinate system located in a horizontal plane. In the same plane a coordinate system xO_1y related to the vibrating plane is located. The coordinate axes x and y are parallel to the axes ζ and η respectively (Fig. 1, a). Let us suppose that the plane motion in $\zeta O\eta$ coordinate system is uniform and runs in such a way that every point traces a circle of radius R_e . Thus motion of any point of the plane is determined by the equations:

$$\begin{aligned} \xi &= \xi_0 + R_e \cos \omega t \\ \eta &= \eta_0 + R_e \sin \omega t \end{aligned}$$
 (1)

where ω is the frequency of harmonic motion, t is time.

For every point of the plane acceleration projections onto immovable axes ξ and η are expressed by the dependencies

$$\ddot{\xi} = -R_e \omega^2 \cos \omega t$$

$$\ddot{\eta} = -R_e \omega^2 \sin \omega t$$
(2)

A body of mass m having coordinates of the center of gravity in the movable system denoted by x and y,

rests on the plane. When the body slides with respect to the plane x=x(t), y=y(t) and acceleration projections of the center of gravity are expressed by two components

$$\begin{array}{l} a_{\xi} = \ddot{x} + \ddot{\xi} = \ddot{x} - R_{e}\omega^{2}\cos\omega t \\ a_{\eta} = \ddot{y} + \ddot{\eta} = \ddot{y} - R_{e}\omega^{2}\sin\omega t \end{array}$$
(3)

The first component determines relative acceleration of the body, and the second component – lifting acceleration.



Fig. 1 Dynamic scheme of vibratory search: a – location of the coordinate axes; b – direction of the force acting upon the body

The body moving on the plane is influenced by resistant friction force *F*, which has the opposite direction to the relative velocity direction. Projections of the relative velocity of the body are denoted by \dot{x} and \dot{y} . Then the projections of friction force onto ξ and η axes (also onto the *x*, *y* axes) are

$$F_{\xi} = -\mu mg \cos(v, x) = \frac{v_x}{v} = \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}}$$

$$F_{\eta} = -\mu mg \sin(v, y) = \frac{v_y}{v} = \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}$$
(4)

where μ is coefficient of dry friction between the surfaces of the body and the plane.

Differential equations of the body motion on a plane are $ma_{\xi}=F_{\xi}, ma_{\eta}=F_{\eta}$

Substituting the expressions (3) and (4) into the body motion equations the following is obtained

$$\ddot{x} + \mu g \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = R_e \omega^2 \cos \omega t$$

$$\ddot{y} + \mu g \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = R_e \omega^2 \sin \omega t$$
(5)

These equations are valid as the body slides on the plane, i.e. as $\dot{x}^2 + \dot{y}^2 \neq 0$.

It is necessary to determine such a trajectory of the body motion, which results the connective surfaces search time to be the smallest.

3. Simulation of the part motion on the circularly vibrating plane

To solve the equation (5) a calculation program was written applying MATLAB software. The examples of part's center motion trajectories along x and y directions are shown in Fig. 2. It was determined by the mentioned trajectories that the part moves on xy plane towards the positioning point vibrating near the position of equilibrium. Summarized trajectory of transitional motion regime of the part, before circular motion of the part gets steady, may be of different character (Fig. 3). Both the character of the part motion trajectory and the direction of the motion are influenced by initial velocity. Due to initially provided velocity, at the beginning the part slips towards the positioning point by higher velocity, but later slip velocity decreases and the part rotation about the positioning point starts.



Fig. 2 Characteristic vibrations of the part, as $\mu=0.1$; $R_e=0.01$ m; $\omega=20$ s⁻¹: a – along x axis direction; b – along y axis direction



Fig. 3 Trajectory of transitional and steady motion regimes of the part on vibrating plane

It was determined, that direction of the part motion may be controlled by changing the initial conditions of excitation, i.e. the initial impulse of excitation providing a certain direction relatively to x and y axes. For example, if the excitation has positive x and y directions, then the part moves by trajectory 1 (Fig.4). If x axis related excitation has negative direction while along y – positive, then the body moves by trajectory 2. When the plane along both axes is excited in negative direction, then the part moves by trajectory 3, and if x axis related excitation has negative – along the trajectory 4. By controlling direction of the excitation it is possible to orient a part on the plane in any direction targeting towards the immovable fixed body.



Fig. 4 Dependence of the part motion direction on excitation direction as μ =0.1; R_e =0.005m; ω =25s⁻¹

For accurate and fast matching of the parts' connective surfaces it is necessary to determine the parameters that are influencing the radius of the body's slip motion near positioning point, because the mentioned radius predetermines if the parts would be joined. It was noticed, that changing the excitation frequency, amplitude and coefficient of friction, the character of the part motion trajectory changes also. When the excitation amplitude and frequency are increasing, the radius of the part rotation near the positioning point increases also (Fig. 5). As the excitation frequency reaches 50 s^{-1} value radius of the part rotation is no longer increasing. As the coefficient of friction is increased, the radius of rotation decreases (Fig. 6).



Fig. 5 Dependence of rotation radius *R* near the positioning point on excitation frequency ω for different amplitudes of excitation R_e , as $\mu=0.1$



Fig. 6 Dependence of rotation radius R near the positioning point on coefficient of friction μ for different excitation frequencies, as R_e =0.01 m

The trajectory of the part motion may be varied by changing the coefficient of friction μ and excitation frequency (Fig. 7). By increasing the coefficient of friction μ , displacement of the part from the initial position to the positioning point gets shorter (Fig. 7, a, b). When μ =0.08, the part moves by a helical twisting trajectory (Fig. 7, c), and further increasing the coefficient of friction, the part is no longer sliding, but only rotates circularly (Fig. 7, d). Character of the trajectory is influenced not only by the coefficient of friction μ , but also by the excitation amplitude R_e and frequency ω . When choosing the higher values of the excitation frequency and/or amplitude the body resumes moving by translational circular trajectory.

Considering character of the motion trajectories it is possible to identify 5 motion regimes of the body motion, taking into account the sets of parameters R_e , and ω , and also μ and ω . Zones of occurrence, where such regimes exist, were determined (Figs. 8 and 9).

With given parameters from the first zone the part initially moves by a looping trajectory, which later changes



Fig. 7 Character of trajectories as $R_e=0.01$ m, $\omega=10$ s⁻¹: a - $\mu=0.01$; b - $\mu=0.05$; c - $\mu=0.1$; d - $\mu=0.15$

into circular motion (Fig. 7, a). In the second zone characteristic for the part is the motion, when it rotates circularly and slides, and the distance from the initial position up to the steady center of rotation is larger than the rotation radius near the positioning point (Fig. 7, b). In the third zone the part has characteristic helical unwinding motion (Fig. 7, c). The fourth zone characterizes circular motion of the body (Fig. 7, d). When parameters R_e and ω are within the fifth zone, the part moves chaotically, because both the excitation frequency and amplitude are too small to make circular motion of the part possible.



Fig. 8 Zones of motion regimes in coordinates R_e and ω , as μ =0.05



Fig. 9 Zones of motion regimes in coordinates μ and ω , as $R_e=0.005$ m

Due to frictional resistance forces between the plane and the body, resting on the later, motion of the part is delayed in time with respect to the motion of the plane. Possible is the characterization of such a lag by a phase difference, which is determined as the body motion trajectory reaches the steady state (Fig. 10). It was determined, that phase difference δ depends on coefficient of friction, excitation frequency and amplitude. Fig. 11 presents the dependence of phase difference on friction coefficient μ . As friction between the contacting surfaces increases the

phase difference gradually decreases. An increase of the excitation frequency ω also results in phase difference decrease. Therefore, as excitation amplitude R_e is increased, the phase difference increases (Fig. 12).



Fig. 10 Phase difference determination scheme, where l – vibrations of the plane near equilibrium position along η direction; 2 – vibrations of the part near equilibrium position in y direction; δ – phase difference



Fig. 11 Dependence of phase difference on friction coefficient, as R_e =0.005 m



Fig. 12 Dependence of phase difference on excitation amplitude R_e , as $\omega=20 \text{ s}^{-1}$

The main criterion that characterizes the effectiveness of automated assembly of the parts is the amount of time needed to assemble the parts. Often high influence on assembly time has positioning time. It was determined, that increasing the excitation amplitude R_e , and frequency ω , positioning time t_1 gets longer (Fig. 13). At increasing the friction coefficient μ , positioning time gets shorter (Fig. 14).



Fig. 13 Dependencies of the time t_1 , within which motion trajectory of the part reaches steady state near positioning point, on excitation frequency ω , as μ =0.1





4. Numerical simulation of a part motion on a plane vibrating along elliptical trajectory

As excitation of the plane in one direction has higher amplitude than in the other direction, in the system of coordinates' $\xi O\eta$ every point of the plane traces an ellipse. Then the motion of any point of the plane is determined by the equations

$$\xi = \xi_0 + A_e \cos \omega t$$

$$\eta = \eta_0 + B_e \sin \omega t$$
(6)

where A_e is length of major axis of the ellipse, B_e is the length of minor axis of the ellipse.

Motion of the part on the plane vibrating along elliptical trajectory is expressed by the equations

$$\ddot{x} + \mu g \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_e \omega^2 \cos \omega t$$

$$\ddot{y} + \mu g \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = B_e \omega^2 \sin \omega t$$
(7)

As the plane is subjected to elliptical vibrations, the trajectory of the steady state motion regime of the part is also elliptical (Fig. 15). The character of the transitional motion regime, which describes movement of the part, while the plane moves along elliptical or circular trajectory, practically does not differ.



Fig. 15 Graph showing motion of the part along elliptic trajectory

It is possible to obtain different motion trajectories of the plane providing the excitation by sine law with a phase shift. The motion law of the plane is written as follows

$$\xi = \xi_0 + A_e \sin(\omega t + \alpha_1))$$

$$\eta = \eta_0 + A_e \sin(\omega t + \alpha_2))$$
(8)

where α_1 is phase shift along *x* axis, α_2 is phase shift along *y* axis.

Then the equations of part motion on the plane are

$$\ddot{x} + \mu g \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_e \omega^2 \sin(\omega t + \alpha_1)$$

$$\ddot{y} + \mu g \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_e \omega^2 \sin(\omega t + \alpha_2)$$
(9)

In this case the shape of motion trajectory and sliding direction of the body depends not on the values of phases α_1 and α_2 , but depends on phase difference $\theta = \alpha_1 - \alpha_2$. By changing the phase difference linear, circular and elliptic motion trajectories of the body are obtained (Fig. 16).



Fig. 16 Dependence of the part motion trajectories on phase difference, as $\mu = 0.2$; $A_e = 0.01$ m; $\omega = 20$ s⁻¹

When the plane vibrates along the elliptical trajectory, similarly to the case of circular excitation, the character of part motion trajectory depends on excitation frequency, amplitude and coefficient of friction. It was determined, that the dependencies for major and minor axes of the ellipse on mentioned parameters are the same, and therefore only the dependency graphs for the major axis are presented. When friction coefficient is increased, the major axis of the ellipse gets shorter (Fig. 17, a), and increasing the excitation amplitude and frequency the major axis of the ellipse gets longer (Fig. 17, b and c). But, as the excitation frequency reaches 45 s⁻¹ value, the major axis of the part motion trajectory does not increase anymore.

It was determined that tilt angle φ of the ellipse depends on friction coefficient, excitation frequency and amplitudes (Fig. 18). As the coefficient of friction is increased the tilt angle rapidly decreases (Fig. 19). Within the range of frequencies 20-30 s⁻¹ an increase of the tilt angle of the ellipse is more significant than that of higher frequencies range (Fig. 20).



Fig. 17 Dependencies of the major axis A of the ellipse during motion near the positioning point, as $\alpha_1 = \pi$, $\alpha_2 = \pi/4$: a – on the coefficient of friction μ , as the excitation amplitude $A_e = 0.01$ m; b – on excitation amplitude A_e , along x direction as the excitation frequency $\omega = 30$ s⁻¹; c – on excitation frequency ω , as the coefficient of friction $\mu = 0.2$



Fig. 18 Tilt angle of the ellipse as $\alpha_1 = \pi$; $\alpha_2 = \pi/4$; $A_e = -0.01 \text{ m}; \omega = 20 \text{ s}^{-1}; \mu = 0.2$



Fig. 19 Dependence of tilt angle of the ellipse on friction coefficient μ , as $\alpha_1 = \pi$; $\alpha_2 = \pi/4$; $A_e = 0.01$ m



Fig. 20 Dependence of tilt angle of the ellipse on excitation frequency ω , as $\alpha_1 = \pi$; $\alpha_2 = \pi/4$; $\mu = 0.1$

5. Motion trajectories when vibratory excitation of different frequencies along the perpendicular directions is provided to the plane

When excitation along ξ and η axes has different frequencies the trajectories of plane motion obtain a complex character. Let us express the excitation by equations

$$\xi = \xi_0 + A_e \sin(\omega_1 t - \alpha)$$

$$\eta = \eta_0 + A_e \sin(\omega_2 t - \alpha)$$
(10)

Then the equations of the part motion are

$$\ddot{x} + \mu g \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_e \omega_1^2 \sin(\omega_1 t - \alpha)$$

$$\ddot{y} + \mu g \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = A_e \omega_2^2 \sin(\omega_2 t - \alpha)$$
(11)

Practically most common is the case as frequency along one direction is twice that of the other. Then the trajectories of part motion become complex and highly dependent on phase angle α (Fig. 21). As $\alpha = \pi/2$ the part moves by similar to parabola segment trajectories. Those curves are so-called Lissajous patterns. When frequencies are commensurable, steady state periodic motion of the part by closed curves is obtained, because the part center is repeating traces of the same curve again.



Fig. 21 Motion trajectories of the part as $\mu=0.15$; $\omega_2=20s^{-1}$; $\omega_1=2\omega_2$; $A_e=0.01$ m



Fig. 22 Complex trajectories of the part motion as $\mu=0.1$; $\omega_2=15 \text{ s}^{-1}$; $\omega_1=k\omega_2$; $A_e=0.01 \text{ m}$; $\alpha=\pi$; k is coefficient, that determines excitation frequency ratio

If frequencies are not commensurable, then center point of the part never reaches the same place, i.e. it would be making new loops again and again and therefore the trajectory never becomes closed (Fig. 22).

Having an aim to match connective surfaces of the parts and make it possible joining of cylindrical parts, the center of the part must fall into the zone of allowable error, which is characterized by the clearance between the parts to be assembled [1]. When a bushing moves along circular or elliptic trajectory, depending on the position of the circle's center relatively to the allowable error zone, the part rotation radius and the clearance between the parts to be assembled, an axis of the bushing either fall into the mentioned zone or not. However, with the given elliptic trajectory, a higher probability exists for the axis of the bushing to fall into the assembly zone, as if minor axis of the ellipse is smaller than the diameter of allowable error zone the part center will cross the mentioned zone twice during one rotation cycle. Providing vibrations to the part in mutually perpendicular directions and of different frequencies assembly possibilities get better, as when the part performs loops in some other place, higher probability exists, that the bushing axis will fall into the zone of allowable error.

6. Conclusions

1. The character of part motion on a vibrating plane, when the plane moves along a circular, elliptical or complex trajectory was analysed. It was determined that a part on a plane can move along a certain steady trajectory and can slip by transitional motion regime towards the positioning point.

2. It was determined that changing initial conditions of excitation along x and y axes it is possible to control sliding direction of the part.

3. When both the amplitude and frequency of plane excitation in perpendicular directions increase radius of the part rotation about the positioning point increases. However, as excitation frequency obtains 50 s⁻¹ value radius of the part rotation remains constant. As the coefficient of friction increases the rotation radius decreases.

4. By taking into account combination of parameters R_e , and ω , and also μ and ω parameters combinations 5 regimes of the body motion and zones, where such regimes exist, were defined.

5. Numerical simulation shows that motion of the part falls behind the motion of the plane. Phase difference δ between the mentioned motions depends on friction coefficient μ , excitation frequency ω and amplitude R_{e} . As the friction and the excitation frequency are increased the phase difference decreases gradually. Though as excitation amplitude R_{e} , is increased the phase difference increases.

6. Performed analysis shows that circular and elliptic excitation of the plane results in steady state motion of the part near the positioning point. The increase in excitation amplitude R_e , A_e , and frequency ω , results in longer time t_1 , within which steady state motion near the positioning point is reached, while increasing the coefficient of friction μ - mentioned time gets shorter.

7. It was noticed that exciting the plane by harmonic law with existing phase difference, the inclination angle φ of the ellipse depends on excitation frequency ω , amplitude R_e and on friction coefficient μ . 8. Trajectories of the body motion were obtained as the plane along ξ and η directions was excited with different frequencies. It was determined that the best and fastest parts matching occurs as the part repeatedly performing loops in some other place passes $2A_e$ size square.

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DETALĖS JUDĖJIMAS HORIZONTALIAI VIRPANČIA PLOKŠTUMA

Reziumė

Straipsnyje nagrinėjamas detalės judėjimas statmenomis kryptimis žadinama horizontalia plokštuma. Sudaryti vibracinės paieškos matematinis bei dinaminis modeliai, kai plokštuma juda apskritimu, elipse ir sudėtingomis uždaromis trajektorijomis. MatLab paketu parašytos lygčių sprendimo programos, nustatyta, kaip kūno, judančio apskritimu žadinama plokštuma, judesio charakteristikos priklauso nuo trinties koeficiento, žadinimo dažnio ir amplitudės. Detalės judėjimo režimai suskirstyti į 5 grupes, nustatytos šių judėjimo režimų egzistavimo zonos. Kūnui judant elipsine trajektorija, nustatytos elipsės ilgosios ašies ilgio bei jos posvyrio kampo priklausomybės nuo trinties koeficiento ir žadinimo parametrų. Nustatytos judėjimo trajektorijos, žadinant plokštumą skirtingų dažnių virpesiais statmenomis kryptimis. Pastebėta, kad trajektorijų pobūdis priklauso nuo fazės kampo tarp žadinimo dedamųjų bei koeficiento k, nustatančio žadinimo dažnių santykį.

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MOTION OF A PART ON A HORIZONTALLY VIBRATING PLANE

Summary

Motion of a part on an excited in two perpendicular directions horizontal plane is analyzed in the paper.

Both the mathematical and dynamic models for vibratory search, when the plane moves along circular, elliptic or complex closed trajectories, were formed. Using MatLab software the programs for equations solving were written and motion characteristics of the body moving on circularly excited plane, depending on friction coefficient and both on excitation frequency and amplitude, were determined. The regimes of part motion were distributed into 5 groups and the zones were determined, where those motion regimes exist. For moving the body along elliptic trajectory, were determined dependencies of the length of major axis of the ellipse and tilt angle of the ellipse on coefficient of friction and excitation parameters. Motion trajectories were determined for the plane excited in perpendicular directions by the vibrations of different frequencies. It was noticed, that character of the trajectories depends both on the phase angle between the components of excitation and on coefficient k, which determines the relationship between the excitation frequencies.

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ДВИЖЕНИЕ ДЕТАЛИ НА ГОРИЗОНТАЛЬНОЙ ВИБРИРУЮЩЕЙ ПЛОСКОСТИ

Резюме

В статье рассматривается движение детали на горизонтальной плоскости, возбуждаемой в двух перпендикулярных направлениях. Составлены математическая и динамическая модели вибрационного поиска при движении плоскости по круговой, эллиптической и сложными закрытыми траекториями. С помощью пакета MatLab составлены программы для решения уравнений движения, определены характеристики движения тела на плоскости, совершающей круговое перемещение, в зависимости от частоты и амплитуды возбуждения, коэффициента трения. Режимы движения тела распределены на пять групп, определены зоны их существования. Для случая движения тела по эллиптической траектории определены зависимости длины большой оси эллипса и угла наклона от коэффициента трения и параметров возбуждения. Определены траектории движения при возбуждении плоскости в перпендикулярных направлениях вибрациями различной частоты. Выявлено, что характер движения тела зависит от фазового угла между составляющими возбуждения и коэффициента k, выражающего отношение частот возбуждения.

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