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Do pension funds beat inflation? Assessing trend-dependent risk and dominance techniques

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Abstract
In this paper, we address whether pension fund investments are losing value over time. We propose a new methodology for pension fund risk and performance evaluation based on the trend-risk measurement concept. We analyze the two-sided and downside deviations from a given trend. In the long term, inflation and consumer price changes significantly affect an investor's wealth. For this reason, we consider these macroeconomic indicators to represent a time-dependent trend, which pension funds should outperform. Furthermore, we propose the concept of Time-Cumulative Dominance. This methodology serves as a valuable tool for both portfolio managers and regulators. In the empirical part, we study this new methodology across various pension funds in Lithuania while reflecting on various market conditions and regimes detected by Hidden Markov Models. The results highlight the impact of portfolio composition on the ability to outperform inflation and consumer price changes in the long-term period. We also observe a negative effect during market anomalies.

Keywords Dominance · Inflation · Pension funds · Portfolio performance · Trend-risk analysis

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1 Introduction

The ageing of the population and the growing scepticism over the sustainability of public pension systems in the West have led to a sharp growth in pension fund investment in recent decades. These funds serve as significant contributors to the overall economy by channelling substantial investments into various sectors, fostering economic growth, and supporting capital market development (Zeng, 2016). In particular, pension funds help mitigate social welfare burdens on governments by reducing dependency on public pension systems, thereby promoting fiscal sustainability and long-term economic stability. As stated in Thomas and Spataro (2016), due to being able to foster national and individual savings or increase the amount of long-term capital, pension funds are essential to the development of capital markets and the acceleration of economic growth.

Additionally, there is a vast amount of literature demonstrating that pension funds boost financial development and allocate investor savings through efficient investment, see Kumara and Pfau (2013); Grujić (2019); Babalos and Stavroyiannis (2020); Bayar et al. (2022). Likewise, another stream of literature is concerned with evaluating the performance of these funds Thomas and Tonks (2001); Hinz and Yermo (2010); Adami et al. (2014); Gonzalez et al. (2020). For example, Gonzalez et al. (2020) studied the impact of activity and patience in pension fund investments. In addition, most of them target emerging or developed markets. Recently, Martí-Ballester (2020) dealt with the financial performance of classical pension funds compared to those aligned with the goals of sustainable development. We aim to contribute to the stream of financial risk and performance measurement with this work.

In this research, we analyze the necessity of evaluating portfolios of pension funds based on their ability to outperform inflation and consumer price changes. The overarching objective is to propose a new methodology based on trend-risk[1] analysis applied to pension funds' risk and performance evaluation. In addition, we aim to present valuable insights that can enrich individual investors, guide policy formulation to enhance pension fund systems, and aid market regulators in promoting financial stability and resilience within broader economic frameworks, such as those published by, for example, Gnabo and Soudant (2022); Kopa et al. (2022) and the literature therein. Generally, we use inflation because this indicator is scrutinized by regulators (Federal Reserve, European Central Bank, Bank of Lithuania, etc.) and is typically monitored by investors over the long term. As stated by the European Insurance and Occupational Pensions Authority (EIOPA), inflation impacts investors' financial situation and reduces their purchasing power from a long-term perspective (EIOPA, 2023). The impact of inflation on portfolio valuation and protection has already been analysed by many researchers, such as Brière and Signori (2012); Bampinas and Panagiotidis (2016); Vukovic et al. (2022).

Furthermore, we present a new concept of Trend-Cumulative Dominance and its weaker version, which allows the ordering of pension fund portfolios. This type of dominance stems from stochastic dominance (Rothschild & Stiglitz, 1970) applied to cumulative returns. A similarly directed approach motivated by the cumulative prospect theory was applied by Baucells and Heukamp (2006). While Baucells and Heukamp (2006) applied stochastic dominance to cumulative distributions within a behavioral framework, we apply Trend-

[1] At this point, we would like to stress that due to their properties, the trend-risk measures also imply time-dependent or time-reflecting risk measures, as discussed by Ruttiens (2013) or Neděla et al. (2024).

Cumulative Dominance directly on cumulative return series. Thus, it explicitly respects the time dimension by evaluating dominance over the entire trajectory of cumulative returns. The weaker form of dominance is motivated by the concept of almost stochastic dominance (Leshno & Levy, 2002; Tzeng et al., 2013).

According to economic theory, inflation is the rate at which the general level of prices of goods and services rises and, consequently, the purchasing power of the population falls. Whereas annual average inflation, defined as the consumer price changes, indicates the average variation in prices over the preceding 12 months in comparison to the corresponding period of the previous 12 months.

The imperative for evaluating pension funds according to their capacity to outperform inflation stems from the fundamental concern regarding the preservation of investors' real wealth and financial security over time. The literature has also provided such studies Zhang and Ewald (2010); Yao et al. (2013); Baltas et al. (2013). Inflation poses a significant threat to the purchasing power of retirement savings, thereby necessitating a thorough assessment of pension fund performance relative to inflationary trends. This analysis is essential for gauging the efficacy of fund management strategies in mitigating the erosion of investors' purchasing power and ensuring the preservation and appreciation of capital in real terms. By employing a scientific approach to evaluating pension funds' ability to outpace inflation, investors can gain valuable insights into the effectiveness of investment strategies and risk management practices in navigating inflationary pressures. Such assessments serve as crucial benchmarks for evaluating the long-term viability and sustainability of pension fund investments, ultimately contributing to informed decision-making processes and the attainment of investors' financial objectives within the context of retirement planning and wealth management.

Furthermore, recognizing the utility of evaluating pension funds in terms of their ability to outperform inflation extends beyond individual investors to encompass broader implications for policymakers and market regulators, including, for example, central banks. Our concept serves as a valuable tool for policymakers in assessing the overall health and resilience of pension fund systems within an economy. By understanding the extent to which pension funds can effectively combat inflationary pressures, policymakers can formulate targeted strategies to enhance the stability and sustainability of retirement savings vehicles, see Zhang and Ewald (2010); Baltas et al. (2022); Madukwe and Okeke (2022). In addition, market regulators, such as central banks, can leverage the insights derived from this analysis to inform regulatory frameworks and monetary policy decisions aimed at fostering a conducive environment for pension fund investments. Incorporating the evaluation of pension funds vis-à-vis inflationary benchmarks into regulatory frameworks can promote greater transparency, accountability, and resilience within financial markets, ultimately contributing to the broader objectives of economic stability and inclusive growth. We should emphasize that the same assumptions apply to consumer price changes.

In the empirical part, we provide an analysis of our proposed trend-inflation and trend-price changes risk frameworks using a dataset sourced from pension funds offered in Lithuania's pension system. The primary motivation is to advance the analysis of pension fund systems in the Baltic region, with particular emphasis on Lithuania. We especially show an in-depth study of how pension funds respond to fluctuating market conditions caused by the COVID-19 pandemic or the following energy crisis. Specifically, we use both the full period (January 2019–September 2022) and various shorter intervals reflecting particular market anomalies. In addition, we study the risk and performance of pension funds within four market regimes detected by using the Hidden Markov Model (HMM) as applied in Kabašinskas et al. (2024) and Kabašinskas (2024). We elucidate the dynamics between the evolution of inflation and pension fund performance.

The remainder of this paper is structured in the following way. In Sect. 2, we describe the methodology for the trend-risk measures and their modification incorporating macroeconomic indicators, dominance constraints, and the methodology to detect various market regimes. In Sect. 3, we characterize the dataset and show the results obtained using the proposed methodology with an explanation of the results. Section 4 discusses the findings as well as further work. The conclusions are summarized in Sect. 5.

2 Methodology

The intention of this section is to explain a methodology describing the concept of trend-risk measurement and its modification using macroeconomic variables. In addition, we define Time-Cumulative Dominance (TCD) and its less strict type, and finally, the approach for identifying different market regimes.

2.1 Trend-including risk measures

Firstly, we generally present the methodology of trend-risk measurement. Specifically, we are motivated by the idea of risk measures originally presented by Ruttiens (2013). In this recent work, A. Ruttiens presented the risk-measuring perspective that incorporates the factor of time. However, in general, this factor shows the trend of the original variable. Since this concept allowed space for modifications and improvements in the calculations, Neděla et al. (2024) proposed modified risk measures that are generally more accurate for optimization purposes. The specificity of this modification is to ignore the mean component in a formula. Finally, downward trend-risk measures are introduced, which more specifically reflect the preferential attitudes of risk-averse individuals.

However, let us start with the asset return formulation. The well-known equation of the classical log return of asset *i* is conceded as follows:

$$r_{i,t} = \ln \frac{P_{i,t}}{P_{i,t-1}},$$

where $P_{i,t}$ means the price of asset *i* at time *t*, for $t = 1, \dots, T$. To capture the trend of individual assets, we have to orient our perspective on the cumulative returns instead of the classical return series, such as proposed by Ruttiens (2013). Note that cumulative returns allow us to approximate the random wealth of the asset. Hence, we denote $c_{i,t}$ as the cumulative return of asset *i* with the *t*-th observation calculated as $c_{i,t} = c_{i,t-1}(1 + r_{i,t})$. From the cumulative perspective, the trend is simplistically represented by the linear equally accrued return $e_{i,t}$ with its *t*-th observation formulated as $e_{i,t} = c_{i,0} + \frac{t}{T}(c_{i,T} - c_{i,0})$.[2] In

[2] For the simplification, we assume that $c_{i,0} = 1$ for each asset, which is commonly taken in the vast literature on portfolio management, see, among others, Rachev et al. (2008), Ortobelli and Tichý (2015), or Neděla et al. (2024).

other words, we can define e_i as a linear function leading from $c_{i,0}$ to $c_{i,T}$. Then, according to the Ruttiens (2013), the "accrued returns variability" (ARV) is defined as:

$$ARV(r_i) \cong \sqrt{\frac{1}{T}\sum_{t=1}^{T}[(c_{i,t} - e_{i,t}) - m_i]^2}. \tag{1}$$

Similarly to theRuttiens concept of ARV,[3] we can obtain just the vector of differences $d_{i,t}$ (or may also be called deviation, spread, etc.) for the *i*-th asset at time $t = 1, \ldots, T$, based on the cumulative returns and their equally accrued version, which is formulated as:

$$d_{i,t} = c_{i,t} - e_{i,t}. \tag{2}$$

Following this idea, differences for market benchmark (index) series $d_{b,t}$ may also be computed, given by $d_{b,t} = c_{b,t} - e_{b,t}$, where $c_{b,t}$ and $e_{b,t}$ is cumulative, respectively, equally accrued return of the given benchmark at time *t*. Essentially, ARV can be further referred to as a modified version of the tracking error indicator, where the benchmark is replaced by a linear trend.

For clearer comprehension, a graphical representation of the concept of trend-risk measures is shown in Fig. 1. In this case, we show the two different cumulative return paths of two assets which begin and end at the same points. Emphasize that the same applies to the portfolio of assets, funds, or indexes.

Figure 1 provides a visual illustration demonstrating how two asset return series, despite exhibiting identical trends, can differ significantly in their risk profiles. Recall that under such circumstances, we assume that *asset1* is more risky than *asset2* because of the higher volatility around the trend. In addition, according to this concept, we are able to analyze periods during which the value of an investment in a given asset outperforms its trend.

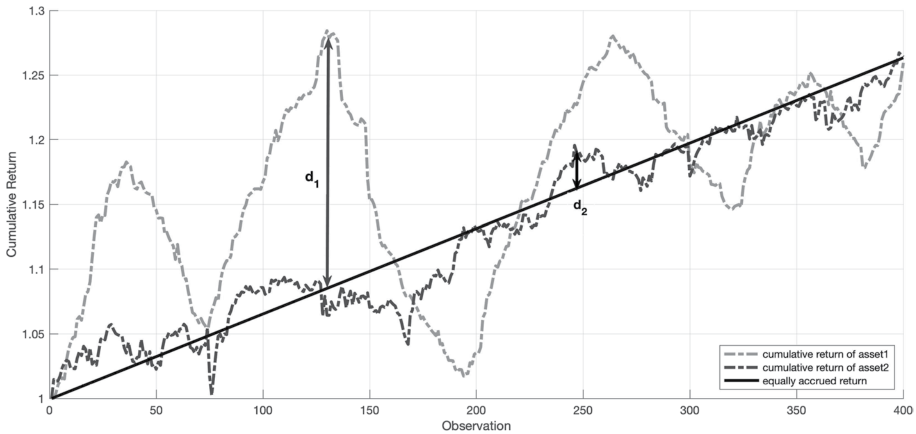


Fig. 1 Cumulative returns of two assets compared to their linear trend line

[3] For a more detailed explanation of ARV and its properties we refer to Ruttiens (2013).

Emphasize that the trend may also be substituted by another factor that will influence the efficiency of the asset (investment).

In this study, we introduce a modification to the conventional measurement of trend risk within portfolios of funds by replacing the equally accrued return with macroeconomic indicators. Essentially, we consider inflation and consumer price changes as key metrics that affect all portfolios over time. Hence, if we denote the *t*-th observation of the inflation rate by π_t,[4] then the modification of the original concept of deviations is formulated as:

$$d_{i,t}^{\pi} = c_{i,t} - \psi_t, \tag{3}$$

where particular observations of cumulative inflation ψ_t at time $t = 1, \ldots, T$ is calculated as $\psi_t = \psi_{t-1}(1 + \pi_t)$. The deviations identified in this manner are, in themselves, useful tools for analyzing the behavior of the portfolio value in relation to the trend. However, for a simpler comparison of various portfolios, we present a risk indicator called Trend-Inflation-Risk (TIR), which is based on the concept proposed by Neděla et al. (2024). Thus, *TIR* is mathematically defined by the following formula:

$$TIR = \frac{1}{T}\sum_{t=1}^{T}(d_{i,t}^{\pi})^2, \tag{4}$$

which expresses the average squared deviation from the trend of inflation.

Now, consider the fact that positive deviations from inflation do not bother investors but rather prefer them. From a financial point of view, relying solely on a two-sided measurement of risk is inadequate because it may misrepresent the potential for significant losses. For this reason, it is more appropriate to use a one-sided risk indicator that includes only negative deviations. Thus, we propose the measure called downTIR, formulated as follows:

$$downTIR = \frac{1}{T}\sum_{t=1}^{T}[d_{i,t}^{\pi}]_{-}^{2}, \tag{5}$$

where $[\cdot]_{-}$ means using only negative values. Examining only negative deviations is a more appropriate approach in financial management and modeling.

Next, we extend the concept of trend inflation risk (TIR) by introducing a complementary metric known as Trend-price change risk (TPR). While TIR primarily focuses on the impact of inflationary trends on investment portfolios, TPR incorporates changes in consumer price levels ω. Again, the cumulative principle of consumer price change τ is considered, which we define such as $\tau_t = \tau_{t-1}(1 + \omega_t)$. By integrating TPR alongside TIR, we aim to provide a more nuanced assessment of portfolio risk, taking into account broader macroeconomic factors that influence asset performance. Hence, TPR using $d_{i,t}^{\tau} = c_{i,t} - \omega_t$, where ω_t represents consumer prices changes at time *t*, is defined as follows:

[4] Here, we assume that the inflation rate is expressed over the same interval as the portfolio returns.

$$TPR = \frac{1}{T}\sum_{t=1}^{T}(d_{i,t}^{\tau})^2. \tag{6}$$

Similarly to *downTIR* proposed above, we define the *downTPR* in the following way:

$$downTPR = \frac{1}{T}\sum_{t=1}^{T}[d_{i,t}^{\tau}]_{-}^{2}. \tag{7}$$

where $[\cdot]_{-}$ again takes into account only the negative values of the given vector.

While a single value obtained from risk metrics can provide a useful snapshot of investment riskiness, analyzing deviations from the selected trend offers a more nuanced understanding of investment behavior. The former provides a static measure, and the latter captures the dynamic fluctuations and deviations from price changes, offering valuable insights into the underlying drivers of risk. For these reasons, we also put emphasis on the analysis of d^{π} and d^{τ}. By scrutinizing these deviations, investors and regulators can better assess the resilience and adaptability of their investment portfolios to changing market conditions, enabling more informed decision-making and risk management strategies. We also emphasize that the proposed methodology and its conceptualization do not rely on Gaussian assumptions of normality in time series, as many concepts in finance do.

2.2 Time-cumulative dominance

In this subsection, we discuss the methodology for comparing assets and their trend (benchmark) as well for comparison with one another. Since asset returns are modeled as random variables, the resulting deviations also inherit stochastic properties (non-stationarity and non-normality). This naturally leads to the application of stochastic dominance theory (Hadar & Russell, 1969; Levy, 2006). In particular, we use its cumulative form. By evaluating the cumulative return paths over time, we define a new concept called Time-Cumulative Dominance, which extends traditional stochastic dominance by incorporating the temporal dimension explicitly.

First of all, therefore, it is essential to delineate and analyze certain properties of the deviation time series *d*. Due to the original time series $r \in \mathbb{R}$ and trend $e \in \mathbb{R}$, it is evident that also $d_t \in \mathbb{R}$, which indicates that the deviation values are from set of real numbers. Furthermore, under the assumption that the returns of assets *r* are random variables, it follows that *d* is also a random variable. Importantly, *d* exhibits characteristics of non-stationarity and deviates from a normal distribution.

When examining the localization of the variables based on their cumulative returns, we are able to study new types of stochastic dominance.[5] Due to using cumulative returns, we incorporate the time dependency into this tool. Thus, we call it Time-Cumulative Dominance. In particular, as we already mentioned above, we study deviations of assets between the given trend or between each other. Starting with the analysis of deviations for a particu-

[5] Detailed characterization of different types of stochastic dominance and their application in finance is also proposed, for example, by Bampinas and Panagiotidis (1992); Ogryczak and Ruszczyński (1999); Bampinas and Panagiotidis (2003); Kopa and Chovanec (2008).

lar asset *i*, we can answer the question of whether the asset time-cumulatively dominates another asset. Thus, we present the following definition.

Definition 1 An asset *i* time-cumulatively dominates asset *j* denoted as $i \succ j$, for $i \neq j$, if all differences between cumulative returns of asset *i* and asset *j* are non-negative and at least one is higher than 0, that is:

$$i \succ j \Leftrightarrow c_{i,t} - c_{j,t} \geq 0, \forall t = 1, \ldots, T \text{ with at least one strict inequality.} \tag{8}$$

One can substitute asset *j* with any benchmark such as macroeconomic variables (for example inflation, consumer price change, index, or interest rates). In other words, we say that the examined time series dominates a particular series, for example, trend, inflation, or price changes, while taking into account the aspect of time. Considering the vector of return series r_i and the vector of inflation rates π, we can denote it as $i \succ \pi$.

Note that this type of TCD is very strict because it does not allow any negative deviation. Thus, we can extend this concept by the Time-Cumulative-Almost Dominance (TCAD). For this purpose, we assume the critical value $\epsilon \in [0, 1]$, which proportionally defines the violation subset. If $\epsilon = 0$, i.e. there is no violation subset, then the TCD is fulfilled.

Definition 2 For a given ϵ, an asset *i* time-cumulatively-almost dominates asset *j*, for $i \neq j$, denoted as $i \succ^{\epsilon} j$ if there exists $S \subset \{1, 2, \ldots, T\}$ such that $card(S) \leq \epsilon T$ and

$$c_{i,t} - c_{j,t} \geq 0, \text{ for all } t \in \{1, \ldots, T\} \setminus S \text{ with at least one strict inequality.} \tag{9}$$

Based on the proposed dominance condition, we can not only test return series, but we are also able to rank them accordingly. Basically, we can investigate dominance among assets as well by examining the position of the deviation vectors of two assets. Beyond merely identifying whether an asset's returns time-cumulatively dominate the benchmark, this methodology enables us to sort the assets based on the extent and consistency of their dominance. Assets with more frequent and higher positive deviations are ranked higher, providing a clear hierarchical structure. This dual capability of testing and sorting based on dominance offers a comprehensive tool for analyzing and comparing asset performance relative to macroeconomic benchmarks.

2.3 Market regime identification

Identifying market regimes (conditions) and incorporating them into risk analysis is crucial for accurately assessing and managing investment risk. Market regimes, characterized by distinct economic and financial conditions, significantly influence asset performance and risk profiles. By recognizing and defining these regimes, such as periods of stability, stock market shocks, bond market shocks, and simultaneous shocks, analysts and investors can gain a deeper understanding of how different market environments affect their portfolios. Consequently, integrating market regimes into risk analysis enhances the robustness of risk assessment.

In this paper, we analyze the performance of pension funds within the entire period as well as four specific market regimes: no shock detected, shocks observed in stock markets, shocks detected in bond markets, and shocks observed in stock and bond markets simultaneously. The methodology of regime detection is motivated by the proposed framework of Kabašinskas et al. (2024). In particular, we assume that the Hidden Markov Model determines the fluctuation of pension fund returns (Lindgren, 1978). This approach assumes that there exist only two market situations (also referred to as states) $S = \{"no - stress", "stress"\}$, which are observed indirectly. These situations are called Hidden Markov ones. In Kabašinskas (2024), it is mentioned that only return series of pension funds for each day are available, while unobservable market states are hidden from users.

Assume that a sequence of observations $X = (x_1, \ldots, x_t)$ for $x_t \in \mathcal{R}^d$ is generated by a finite-state Markov chain with hidden states $S = (s_1, \ldots, s_t)$ for $s_t \in \{1, \ldots, M\}$, where M is the number of states consistent during the period. The HMM is then specified by three segments: the initial probability vector $\pi_i = \Pr(S_1 = i), \; i = (1, \ldots, M)$, a transition probability matrix $P_t = (p_{ij})_t = \Pr(s_{t+1} = j \mid s_t = i)$, as well as the emission probabilities B, which can be any distribution conditioned on the current hidden state. Then, we can define the joint likelihood of observations X and hidden states S with model parameters ϕ as:

$$f(X, S \mid \phi) = \pi \cdot b_{s_1}(x_1) \prod_{t=1}^{T-1} P_t b_{s_t}(x_{t+1}), \tag{10}$$

where b_{s_t} is a vector of observation densities $b^j_{s_t}(x_t) = \Pr(x_t \mid s_t = j)$ that provide the conditional densities of observations x_t associated with the hidden state j, $j = 1, \ldots, M$ at time $t = 1, \ldots, T$. The set of parameters (π, P, B) is estimated from the observed sequence X.

First, hidden market states are detected separately using the historical data of stock funds and bond funds. In particular, using the method designed by Lavielle and Lebarbier (2001), we label no-crisis and crisis periods. To detect these situations, the stock index MSCI World index (MSCI) and the bond index Bloomberg Barclays Euro 1–5 year Bond index (BB EURO) are incorporated. To estimate transition probabilities of hidden states, the methodology proposed by Visser and Speekenbrink (2010) was applied. In this work, we may distinguish between two techniques based on the information used. For the first one, only the historical time series of given data serves for the identification of hidden states. On the contrary, the second technique may be employed for the detection of regimes using historical data as well as other indicators, such as time series, as regressors, see Kabašinskas et al. (2024). Emphasize that in this paper, the first approach is applied.

Considering the first technique, let L^S_t and L^B_t denote the hidden states of the stock and bond markets (MSCI and BB EURO), respectively, at time $t \in \{1, \ldots, T\}$. Following Kabašinskas et al. (2024), these series are combined into a single data set following such an algorithm:

- if no shock is observed in any market, i.e., $P(L^S_t = \text{"no-crisis"}) 0.5$ and $P(L^B_t = \text{"no-crisis"}) 0.5$, then Regime 1 is assumed;
- if shock is observed in stock indices and no shock is observed in bonds, i.e., $P(L^S_t = \text{"crisis"}) > 0.5$ and $P(L^B_t = \text{"no-crisis"}) > 0.5$, then we assume Regime 2;
- if no shock is observed in stock indices and shock is observed in bonds, i.e.,

$P(L_t^S = \text{"no-crisis"}) > 0.5$ and $P(L_t^B = \text{"crisis"}) > 0.5$, then we assume Regime 3;
- if shocks are observed in stock indices and bonds, i.e., $P(L_t^S = \text{"crisis"}) > 0.5$ and $P(L_t^B = \text{"crisis"}) > 0.5$, then we assume Regime 4.

In such a way, one can obtain a historical series of market regimes. Emphasize that the threshold probability of 0.5 may be increased to a higher value $\alpha \in (0.5, 1)$ if stronger statistical evidence is required to confirm the presence of a hidden state.

The second approach, combining various states L_t^n from several indices, is based on the application of the following aggregation:

$$L_t^{\mathrm{S}} = \sum_{n=1}^{N_s} w_n^s L_t^n, \text{ and } L_t^{\mathrm{B}} = \sum_{n=1}^{N_b} w_n^b L_t^n, \tag{11}$$

where N_s and N_b denote the number of stock and bond indices considered, respectively, see Kabašinskas (2024). The weights w_n^s and w_n^b represent the relative importance of each index within the stock or bond category. To reflect the global market situation, the weights are uniformly distributed as $w^s = \frac{1}{N_s}$ and $w^b = \frac{1}{N_b}$. Emphasize that $L_t^{\mathrm{S}} \in [1, 2]$ and $L_t^{\mathrm{B}} \in [1, 2]$. Thus, implementing the methodology in Kabašinskas (2024), if $L_t^{\mathrm{S}} \geq 1.3$ and $L_t^{\mathrm{B}} \geq 1.5$, we assume shock in the particular market. For a more regionally focused analysis, these weights and threshold parameters could be adjusted.

3 Empirical analysis

This section begins with a description of the pension funds data used for further empirical analysis. Then, we present the results of the proposed risk metrics on the selected dataset and discuss the findings.

In our empirical analysis, we segment the dataset based on prevailing market conditions. Initially, we conducted a comprehensive examination using the entire dataset to capture the overarching trends and patterns in pension fund performance. Subsequently, we delve deeper into specific market scenarios, notably focusing on the COVID-19 crisis period, to assess the impact of exceptional circumstances on pension fund behavior. Furthermore, to evaluate the efficacy of our proposed measures, we conduct a monthly analysis and compute averages, providing a granular understanding of pension fund performance dynamics over time and under varying market conditions. Finally, we examine pension fund performance across different market regimes as defined by the framework established by Kabašinskas et al. (2024).

3.1 Data description

For the empirical analysis, we primarily utilize a dataset comprising daily returns of selected Lithuanian pension funds that participate in the IInd pillar pension system.[6] The period

[6] The Baltic region, including Lithuania, remains underrepresented in the academic literature compared to Western European economies such as the UK, Germany, or France. By focusing on Lithuania, this study contributes to a broader understanding of pension fund performance in less-explored markets.

analyzed is from January 2019 (the introduction of life-cycle pension funds) to September 2022. The selected period encompasses a highly volatile market environment, characterized by significant macroeconomic and geopolitical disruptions, including the COVID-19 pandemic, the global energy crisis, and the economic repercussions of regional military conflicts. Although the examined time span is not wide, these circumstances allow us to conduct a more precise analysis in an unstable market environment. For a comprehensive overview of the Lithuanian pension system and descriptive statistics related to the performance of second pillar pension funds, see, for example, the studies by Kabašinskas et al. (2024); Kabašinskas (2024).

Summarizing the data set, it must be emphasized that there are 40 time series with observations of returns of Lithuanian IInd pillar pension funds. Each fund is assigned to one of 8 predefined age groups (depending on the year of birth of clients): 54-60, 61-67, 68-74, 75-81, 92-88, 89-95, 96-02 and T (preservation type). Moreover, these funds are managed by five companies (SEB, Swedbank, Lumino, INVL and Allianz). Therefore, in this paper, we use the notation "manager yy-yy" or "manager T" to identify a particular pension fund.

In addition to pension fund return data, the dataset is supplemented with inflation and consumer price changes indicators for Lithuania, both sourced directly from the Bank of Lithuania. Inflation data are reported monthly on a year-over-year basis, meaning each value reflects the percentage change relative to the same month of the previous year—a standard methodology across many countries. For analytical consistency, we interpolate these monthly values to a daily frequency to match the granularity of the pension fund return data. Although this approach is common in empirical finance (Jarrow & Yildirim, 2003; Breitung & Roling, 2015), it may introduce minor distortions in the short-term dynamics. The consumer price data are constructed analogously to the inflation series, with each monthly observation representing the year-over-year percentage change relative to the same month in the preceding year. This approach is widely adopted across countries as a standard practice. However, since all data were obtained directly from the central bank, the dataset is regarded as highly credible and reliable.[7] Importantly, the temporal span of the inflation and price change data aligns with that of the pension fund returns, ensuring consistency and comparability in the empirical analysis.

Following the description of the returns data of the pension funds, we computed selected key statistical indicators to analyze the whole time series of returns, see (Table 1). These statistics include the mean return, standard deviation, Value at Risk (VaR) with the significance level 5%, minimum, and maximum. Additionally, the skewness and kurtosis were calculated to assess the asymmetry and the tailedness of the return distribution, respectively. Furthermore, we performed the Jarque-Bera and Kolmogorov–Smirnov tests to check the normality of returns at the 5% significance level. For all return series, the observed p-value for both tests is less than 0.01, suggesting that the time series do not significantly follow a normal distribution at the given significance level. In other words, we observed that for all time series of returns, the normality assumption is rejected according to these tests. (Table 1).

The statistics observed confirm the impact of the portfolio composition on its performance and risk exposure. In general, pension funds designed for the elderly population (year of birth 60 and earlier) are more conservative, characterized by a lower rate of expected

[7] Such a dataset structure is common internationally. Nevertheless, the direct provision by the central bank enhances its credibility and reliability.

Table 1 Selected statistics of pension funds returns

Fund	Mean (%)	Std (%)	Skew	Kurt	$VaR_{5\%}$(%)	Min	Max
Allianz 54-60	0.0061	0.2073	−1.1770	9.5038	0.3046	−0.0127	0.0095
Allianz 61-67	0.0151	0.4112	−0.7638	8.7335	0.6834	−0.0229	0.0220
Allianz 68-74	0.0327	0.7935	−0.5660	11.4819	1.2240	−0.0453	0.0517
Allianz 75-81	0.0398	0.8697	−0.6671	9.6711	1.4172	−0.0498	0.0507
Allianz 82-88	0.0411	0.8696	−0.6562	9.9583	1.4321	−0.0495	0.0527
Allianz 89-95	0.0410	0.8747	−0.6667	10.0497	1.4562	−0.0496	0.0536
Allianz 96-02	0.0436	0.8863	−0.5841	10.3568	1.4017	−0.0509	0.0575
Allianz T	0.0007	0.1590	−0.9331	8.3629	0.2576	−0.0092	0.0067
INVL 54-60	0.0014	0.2112	−2.7229	26.5489	0.2955	−0.0222	0.0089
INVL 61-67	0.0189	0.5367	−2.0718	21.9974	0.7189	−0.0542	0.0323
INVL 68-74	0.0365	0.8358	−1.4606	15.9313	1.2417	−0.0744	0.0513
INVL 75-81	0.0436	0.9708	−1.3007	14.1747	1.4639	−0.0828	0.0587
INVL 82-88	0.0430	0.9672	−1.3296	14.4760	1.4494	−0.0831	0.0589
INVL 89-95	0.0431	0.9773	−1.3197	14.5059	1.4664	−0.0843	0.0597
INVL 96-02	0.0428	0.9879	−1.2346	13.7408	1.4733	−0.0840	0.0600
INVL T	0.0065	0.2311	−2.0145	21.6300	0.3152	−0.0233	0.0132
Luminor 54-60	0.0045	0.2963	−2.3513	24.9577	0.4276	−0.0301	0.0159
Luminor 61-67	0.0245	0.4769	−1.4690	15.1649	0.7185	−0.0396	0.0273
Luminor 68-74	0.0406	0.6755	−1.1694	11.8634	1.0054	−0.0489	0.0373
Luminor 75-81	0.0411	0.6957	−1.0472	10.8648	1.0455	−0.0485	0.0375
Luminor 82-88	0.0410	0.6957	−1.0395	10.8787	1.0378	−0.0485	0.0379
Luminor 89-95	0.0415	0.7010	−0.9545	10.4639	1.0528	−0.0463	0.0395
Luminor 96-02	0.0416	0.7071	−0.8707	10.1187	1.0891	−0.0438	0.0414
Luminor T	0.0027	0.2036	−2.1720	21.9239	0.2854	−0.0193	0.0114
SEB 54-60	0.0046	0.2497	−1.9735	18.5633	0.4016	−0.0253	0.0110
SEB 61-67	0.0222	0.5104	−1.5900	16.0650	0.8118	−0.0480	0.0257
SEB 68-74	0.0399	0.8029	−1.3873	14.0342	1.2824	−0.0707	0.0362
SEB 75-81	0.0462	0.9062	−1.3239	13.4958	1.4978	−0.0792	0.0417
SEB 82-88	0.0439	0.9035	−1.3325	13.6211	1.4848	−0.0790	0.0421
SEB 89-95	0.0435	0.9098	−1.3357	13.6801	1.4806	−0.0797	0.0431
SEB 96-02	0.0436	0.9279	−1.3358	13.7424	1.5125	−0.0809	0.0472
SEB T	0.0029	0.2161	−2.6016	24.1539	0.3297	−0.0235	0.0083
Swedbank 54-60	0.0057	0.2860	−0.8745	32.1269	0.3683	−0.0276	0.0301
Swedbank 61-67	0.0265	0.6084	−1.3385	15.2639	0.8809	−0.0551	0.0371
Swedbank 68-74	0.0435	0.9611	−1.2651	16.0653	1.4185	−0.0867	0.0661
Swedbank 75-81	0.0466	1.0033	−1.1290	14.1051	1.5382	−0.0868	0.0662
Swedbank 82-88	0.0465	1.0058	−1.1114	13.8059	1.5308	−0.0864	0.0659
Swedbank 89-95	0.0453	1.0090	−1.1076	13.8236	1.5327	−0.0866	0.0663
Swedbank 96-02	0.0427	1.0674	−0.9728	15.1208	1.5939	−0.0902	0.0695
Swedbank T	0.0028	0.2075	−1.5369	15.0186	0.3112	−0.0182	0.0128

return associated with a lower level of risk. In contrast, funds designed for younger investors are characterized by a more aggressive portfolio generating higher expected returns. Consequently, the risk level is logically higher. Surprisingly, for some companies, the best-performing fund is intended for individuals born between 1975 and 1981. We are aware that the period monitored was affected by several market anomalies that may have an impact on the observed values.

3.2 Results

This part of the analysis aims to present the results obtained using the measures proposed in Sect. 2 on the selected dataset of pension funds. As mentioned above, the empirical analysis is divided into several parts reflecting different market situations. In particular, split the whole period into corresponding time windows.

3.2.1 Analysis of the overall data sample

First, we address the entire data sample. In order to compare whether pension funds are able to outperform inflation or price changes and thus not lose investors' money entrusted to them, it is useful first to monitor the deviations d^{π} and d^{τ}. If the deviations are positive (negative), the fund outperforms (does not outperform) the reference indicator, and vice versa. Through this analysis, we can study the behavior of pension fund portfolios during different economic situations or economic cycles. The evolution of the deviations of individual pension funds according to the use of the chosen reference rate is shown in Figs. 2

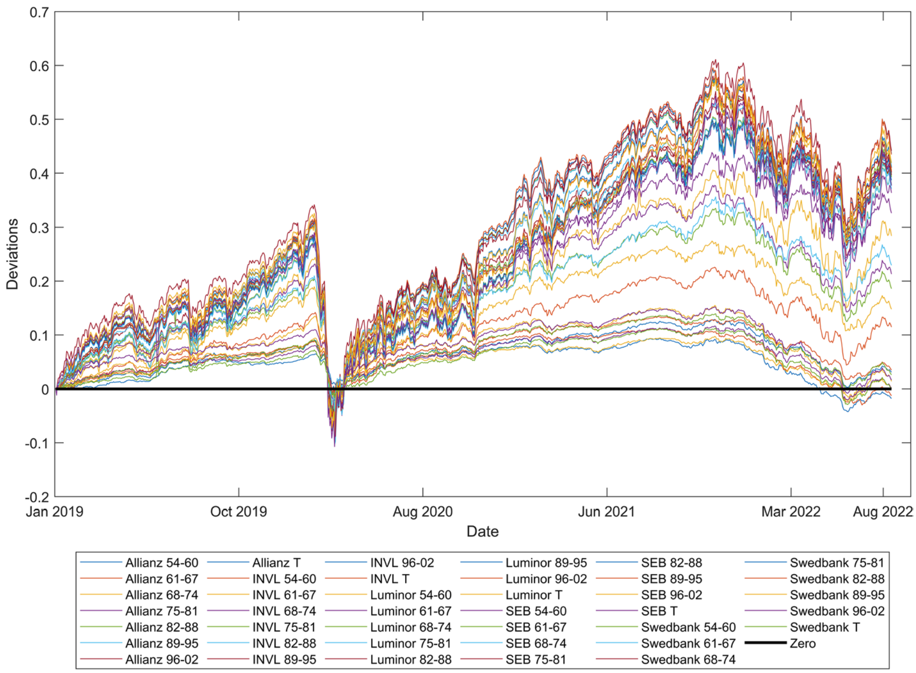


Fig. 2 Deviations of the cumulative funds returns from cumulative inflation d^{π} during the whole period

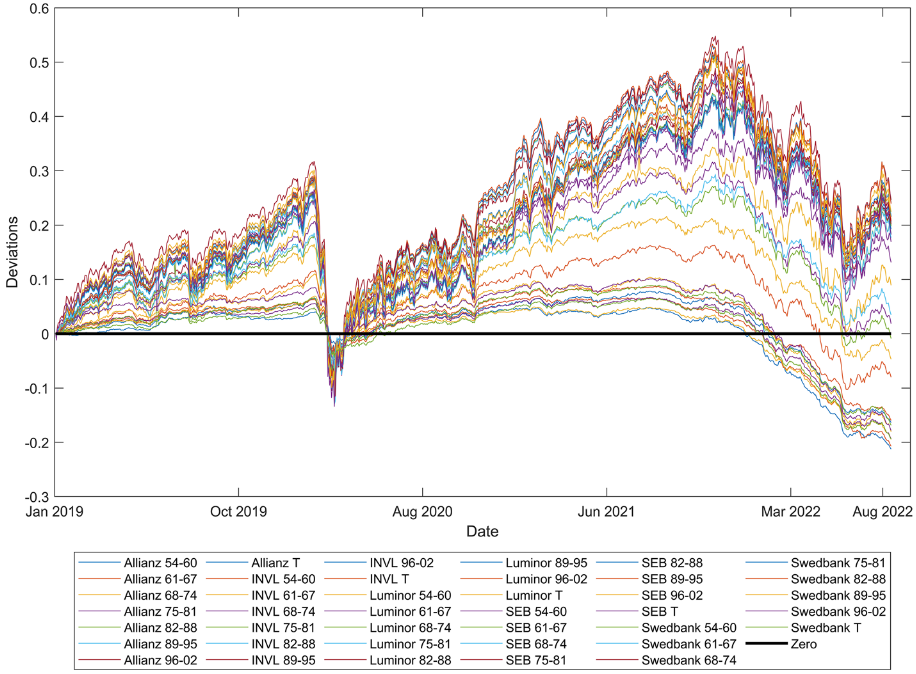


Fig. 3 Deviations of the cumulative funds' returns from cumulative consumer price changes d^{τ} during the whole period

and 3. It is worth noting that the level of price index change is higher than inflation for almost the entire period considered.

The deviations in Fig. 2 indicate that the funds are rather clustered, which depends on the type of pension fund and its portfolio composition. For aggressive funds tailored to relatively young investors (year of birth 89-95 or 96-02), which contain a significant portion of the portfolio invested in stocks, the deviations are larger than for conservative funds labeled 54-60, 61-67, or T. For this reason, funds with more conservative strategies oscillate closer to the zero line.

In addition, all funds faced problems with outperforming inflation during the COVID-19 crisis, even though the inflation rates in Lithuania were lower than 1 %. However, this problem has caused all types of investments globally; therefore, it is not surprising in our context. The only funds that were essentially unable to protect investors' capital against inflation over the entire period under consideration were Allianz T and INVL 54-60. This is due to their negative final deviation value. In addition, for two funds (Luminor T and Swedbank T) the value was very close to zero. In general, we see a relatively solid performance of pension funds in Lithuania compared to inflation, even for different market situations.

Very similar findings can be seen in Fig. 3 when considering changes in the consumer's prices. However, as its values are generally higher than inflation, the curves are shifted downward, and at the end of the period considered, even less conservative funds do not cover this macroeconomic situation. Basically, this consists of fund portfolios for investors with a year of birth of 54-60, preservation types of funds, and some designed for the

Table 2 Analysis of proposed risk measures of all pension funds applied on all historical dataset

Fund	TIR	TPR	downTIR	downTPR
Allianz 54-60	0.0057	0.0034	0.0000	0.0078
Allianz 61-67	0.0163	0.0083	0.0002	0.0035
Allianz 68-74	0.0504	0.0314	0.0019	0.0036
Allianz 75-81	0.0727	0.0486	0.0021	0.0040
Allianz 82-88	0.0763	0.0513	0.0019	0.0037
Allianz 89-95	0.0750	0.0503	0.0021	0.0040
Allianz 96-02	0.0846	0.0579	0.0015	0.0031
Allianz T	0.0032	0.0039	0.0003	0.0121
INVL 54-60	0.0049	0.0044	0.0003	0.0123
INVL 61-67	0.0268	0.0151	0.0005	0.0011
INVL 68-74	0.0651	0.0424	0.0009	0.0022
INVL 75-81	0.0801	0.0538	0.0016	0.0033
INVL 82-88	0.0774	0.0517	0.0018	0.0036
INVL 89-95	0.0787	0.0527	0.0018	0.0035
INVL 96-02	0.0782	0.0525	0.0017	0.0033
INVL T	0.0067	0.0037	0.0000	0.0072
Luminor 54-60	0.0086	0.0057	0.0002	0.0108
Luminor 61-67	0.0462	0.0293	0.0005	0.0009
Luminor 68-74	0.1008	0.0722	0.0008	0.0017
Luminor 75-81	0.0997	0.0713	0.0009	0.0020
Luminor 82-88	0.1010	0.0725	0.0008	0.0018
Luminor 89-95	0.1058	0.0766	0.0008	0.0018
Luminor 96-02	0.1072	0.0778	0.0009	0.0018
Luminor T	0.0037	0.0035	0.0002	0.0110
SEB 54-60	0.0087	0.0055	0.0000	0.0107
SEB 61-67	0.0367	0.0219	0.0006	0.0009
SEB 68-74	0.0890	0.0619	0.0010	0.0024
SEB 75-81	0.1172	0.0853	0.0011	0.0024
SEB 82-88	0.1007	0.0714	0.0017	0.0035
SEB 89-95	0.0989	0.0699	0.0018	0.0035
SEB 96-02	0.0992	0.0703	0.0015	0.0033
SEB T	0.0047	0.0037	0.0001	0.0107
Swedbank 54-60	0.0067	0.0040	0.0000	0.0078
Swedbank 61-67	0.0401	0.0236	0.0004	0.0010
Swedbank 68-74	0.0870	0.0596	0.0016	0.0032
Swedbank 75-81	0.0950	0.0659	0.0016	0.0032
Swedbank 82-88	0.0947	0.0657	0.0016	0.0032
Swedbank 89-95	0.0902	0.0622	0.0018	0.0034
Swedbank 96-02	0.0764	0.0512	0.0026	0.0048
Swedbank T	0.0041	0.0035	0.0002	0.0110

class of 61-67. Such a signal is not very convenient for small investors who are saving for retirement.

Next, we show the results of the trend-risk measures proposed for each pension fund. They allow us to compare the riskiness of these funds based on one value (measure) more precisely. The results are shown in Table 2.

The results of proposed risk measures for various pension funds indicate a rising trend while decreasing investor age. In most cases, less risky portfolios are related to preservation funds, which confirms the findings of the statistics in Table 1. For downTIR and downTPR measures, the values obtained are logically low across all funds, indicating limited downside risks.

Looking at individual pension companies in more depth, the least risky one, according to TIR and TPR, can essentially be considered Allianz. In contrast, the portfolios of the Luminor pension fund management company usually reach the highest values.

However, if we assess only the downside risk (downTIR, down TPR), it is rather surprising that Luminor performs more effectively than other companies. Essentially, it is caused by a small number of negative deviations. Similarly, the opposite tendency is evident with Allianz, whose portfolios tend to be relatively more risky now. Generally speaking, the most risky portfolios now are conservative ones (labeled T and 54-65), which suffer more negative deviations, and the impact of the crisis is more intense for them.

3.2.2 Analysis under various market regimes

In this subsection, we proceed with the analysis of pension funds' performance within different market regimes. A characterization of how market regimes are set up and what methodologies are used is proposed in subsection 2.3. Since the values of pension funds are on a daily basis, we also assign a particular regime to each day.

First, we use the given methodology to detect hidden states for selected financial market indices (MSCI and BB EURO). Motivated by Kabašinskas et al. (2024), we use these indices to represent the financial markets with the highest liquidity, and the pension funds of the Lithuanian IInd pillar usually allocate their funds there. Thus, they are considered appropriate for detecting market shocks. In addition, some fund managers follow indices as a benchmark. Applying the methodology, we detect states using both historical data of the given index. Then, we aggregate particular regimes for the stock and bond indices to receive final evidence of market regimes following the methodology presented in Sect. 2.3.

Here, we present Fig. 4 that illustrates the detection of different market regimes over the given period. This figure identifies and delineates distinct phases within the market, characterized by varying shocks in the stock and bond markets.

When examining the occurrence of the analyzed market regimes during the period under investigation, Regime 1, with no market shocks, occurred most frequently. In contrast, the least frequent regime was the third (only within 5 days), which detected only shocks in the

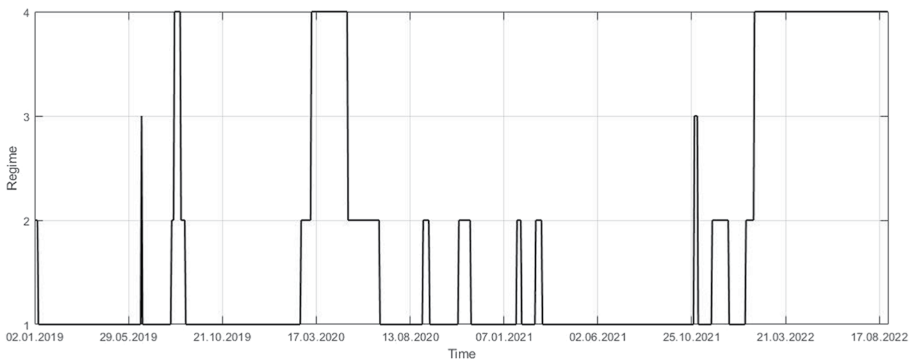
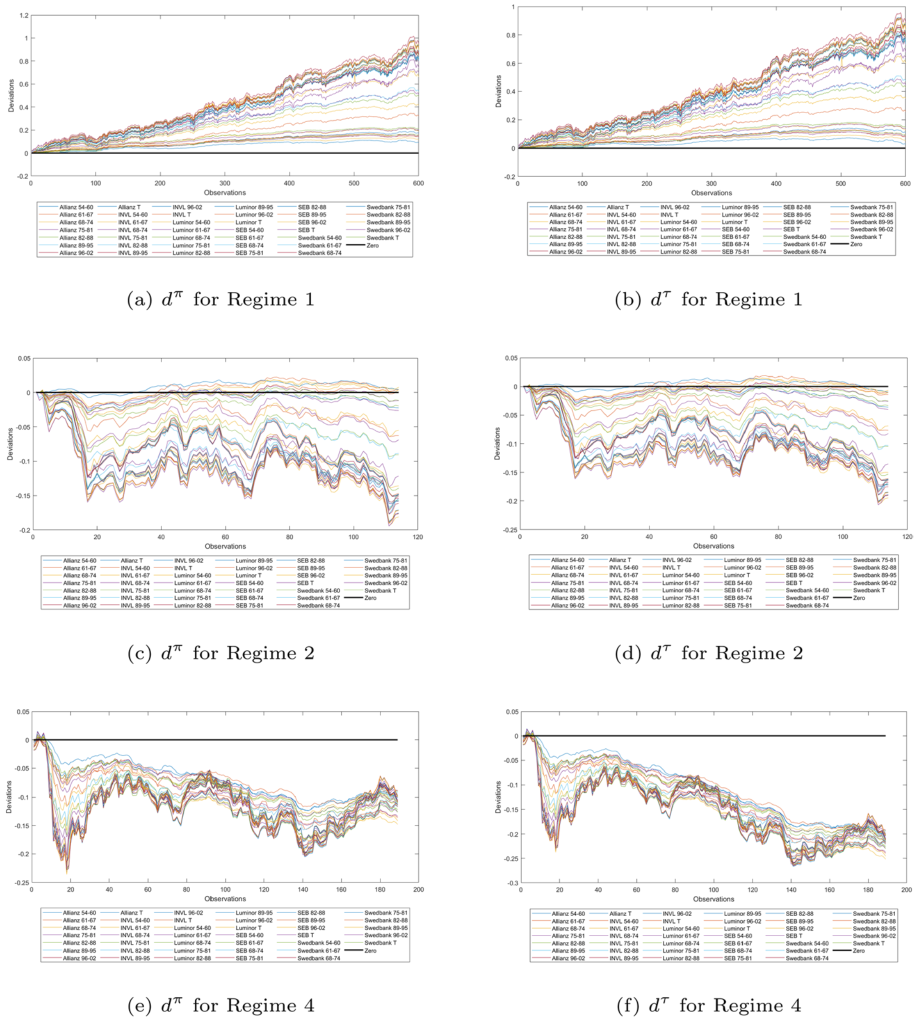


Fig. 4 Identified market regimes within the selected period using aggregated method

bond market. Hence, given this circumstance, we do not consider it relevant to include this type of regime in a further detailed analysis. More meaningful is to compare the performance of funds in the period without shocks with the period of shocks in the stock market (Regime 2) or in both stock and bond markets (Regime 4). An interesting point is that Regime 4 is always bounded by Regime 2. Illustrations of the deviations for individual pension funds from their respective indicators that capture different regimes are presented in Fig. 5. Basically, we aggregate all the data for the days with the respective regime and create a new subset of the data.

Concerning the findings of the regime analysis, they are consistent with financial theories. In particular, during a stable market period, pension fund managers are able to preserve the real value of their clients' investments, mitigating the effects of depreciation due to fac-



(a) d^{π} for Regime 1

(b) d^{τ} for Regime 1

(c) d^{π} for Regime 2

(d) d^{τ} for Regime 2

(e) d^{π} for Regime 4

(f) d^{τ} for Regime 4

Fig. 5 Deviations of the cumulative funds returns from cumulative inflation and consumer price changes for different regimes tors like inflation and market volatility over time. The distribution and clustering of deviations are also apparent depending on the composition of the portfolios.

In addition, we calculate the proposed risk measures for different market regimes. All results obtained are presented in the Appendix, see Table 3.

3.2.3 COVID-19 crisis period

Finally, we turn our attention to the recent crisis associated with the COVID-19 pandemic. The COVID-19 pandemic triggered one of the most volatile periods in recent financial history, deeply affecting global markets and, by extension, the performance of pension funds. Essentially, this crisis could be characterized by a rapid decline in financial markets and a relatively fast recovery. To define the period of the COVID-19 pandemic, we select a time horizon between December 2019 and December 2021.

The behavior of the deviations of individual pension funds from inflation and price changes within the COVID-19 pandemic is shown in Figs. 6 and 7.

Analyzing the performance of Lithuanian IInd pillar pension funds during the COVID-19 crisis reveals significant insights into their resilience and adaptability. From Fig. 6 it is evident that there is a significant short-term fall in investment portfolios during this period. Thereafter, it takes about 1 year for portfolio managers to recover portfolio values from the impact of inflation. However, we can observe that the performance of the conservative portfolios drops below inflation again in early 2021, which was also due to the gradually escalating inflation rate.

A different situation can be observed in the change in consumer prices presented in Fig. 7, where prices increased rapidly during this pandemic. The market downturn at the beginning

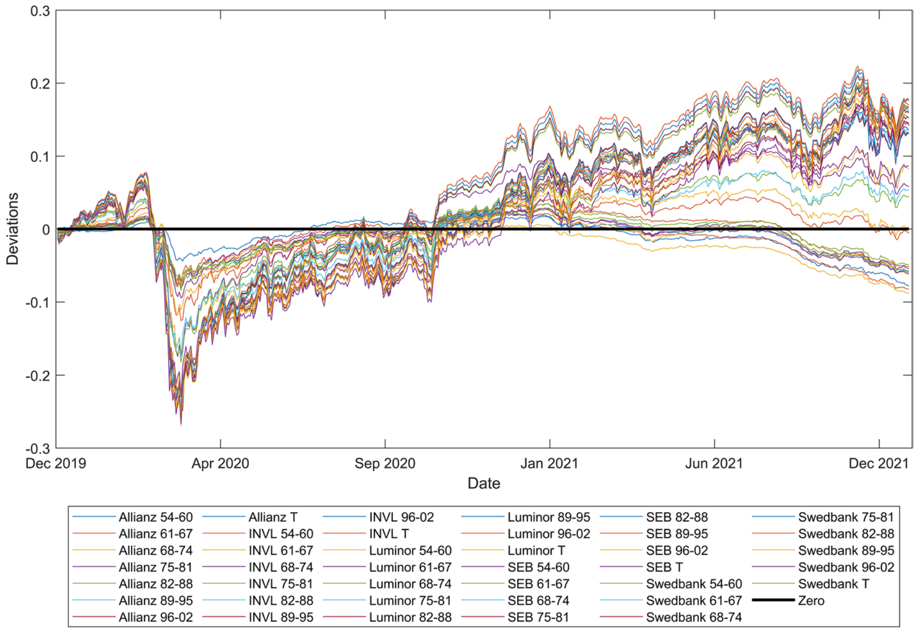


Fig. 6 Deviations of the cumulative funds' values from cumulative inflation during the COVID-19 crisis period

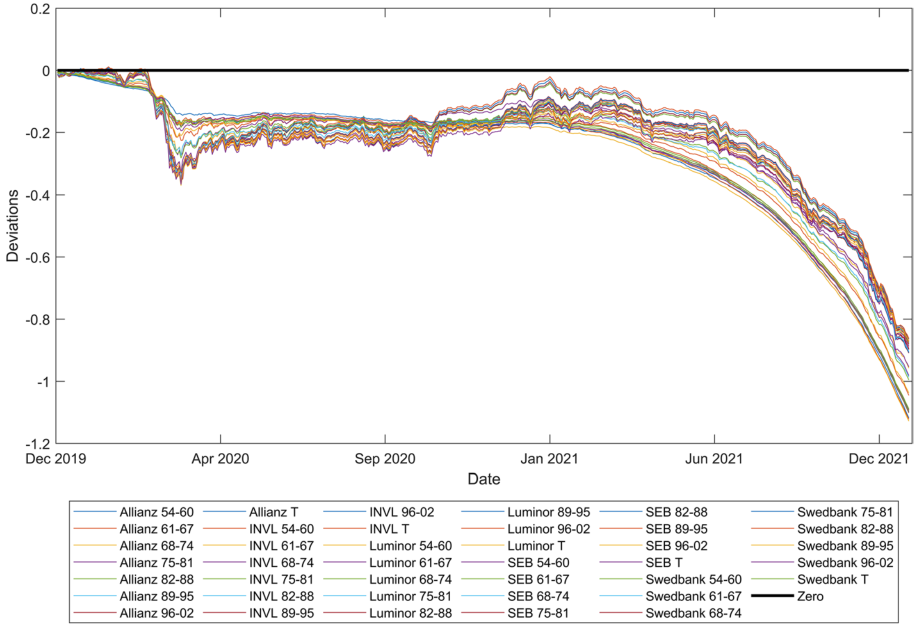


Fig. 7 Deviations of the cumulative funds' values from cumulative consumer price changes during the COVID-19 crisis period

of this period affected the performance of the portfolios and the subsequent increase in consumer prices prevented positive deviations at the end of the pandemic. Even aggressive portfolios of pension funds have not been able to overcome the enormous increase in consumer prices. In this perspective, there is a gradual devaluation of pension investments for all investors caused by such a systemic crisis.

3.3 Results of time-cumulative dominance test

In this subsection, we show the results of whether pension funds' returns Time-Cumulatively Dominate the given trend (π and τ). Mainly, we present results using π as the trend, but we also do the same analysis for τ. Furthermore, we also present the results of the analysis regarding whether pension fund portfolios dominate each other. Subsequently, we relax the strict dominance condition and present the insights for the TCAD concept. To do so, we apply the methodology proposed in Subsection 2.2. As we conduct risk analysis at various time intervals and regimes, we also investigate dominance within those horizons.

Starting with the issue of the strict dominance defined in Definition 1, we can observe that in most periods there is no evidence of this type of dominance. The only exception occurs in the presence of Regime 1, where several pension funds coincidentally dominate both macroeconomic indicators (π and τ). Interestingly, the set of dominating funds consists of a few funds from particular companies (INVL 54-60, INVL 68-74, SEB 54-60, SEB 75-81), but all funds from Swedbank. The non-dominance in other funds for the periods with a particular type of regime is mostly due to negative values among the observations (location behind the zero line).

However, using the Time Cumulative Almost Dominance defined in (9), we are able to relax the strict condition gradually. Thus, we find that pension funds are starting to dominate inflation, mainly for corresponding $\epsilon \in (0, 0.1]$. The results of this analysis for the whole period are shown in Fig. 8. It is evident that allowing the relaxation of $\epsilon \in (0.02, 0.03)$, (i.e., from the interval 2% to 3%) causes the funds to almost dominate inflation. Such findings correspond to the evidence from Fig. 2, where during the COVID-19 crisis, the deviations d^{π} of all funds were negative. In addition, the higher value of ϵ for predominantly conservative funds is also due to negative deviations d^{π} at the end of the period.

Checking this type of dominance for selected regimes with market instability (Regime 2 or Regime 4), we are still no able to observe Time-Cumulative Almost Dominance for the relevant parameter of $\epsilon < 0.5$. This finding means that we should allow for a breach of condition by more than half, which is logically out of the scope of the mathematical as well as economic theory.

Then, we move on to studying the dominance between the portfolios of pension funds with each other. The dominance results for selected periods are shown in Figs. 9, 11, and 12. The dark blue color in the given block indicates that the particular fund on the y-axis dominates the given fund on the x-axis.

The results in Fig. 9 show that only a few funds dominate other funds throughout the entire period. In that respect, the SEB 75-81 Allianz 96-02 are the most successful. Focusing further on the interval with the COVID-19 period, TCD is not observed for any fund.[8]

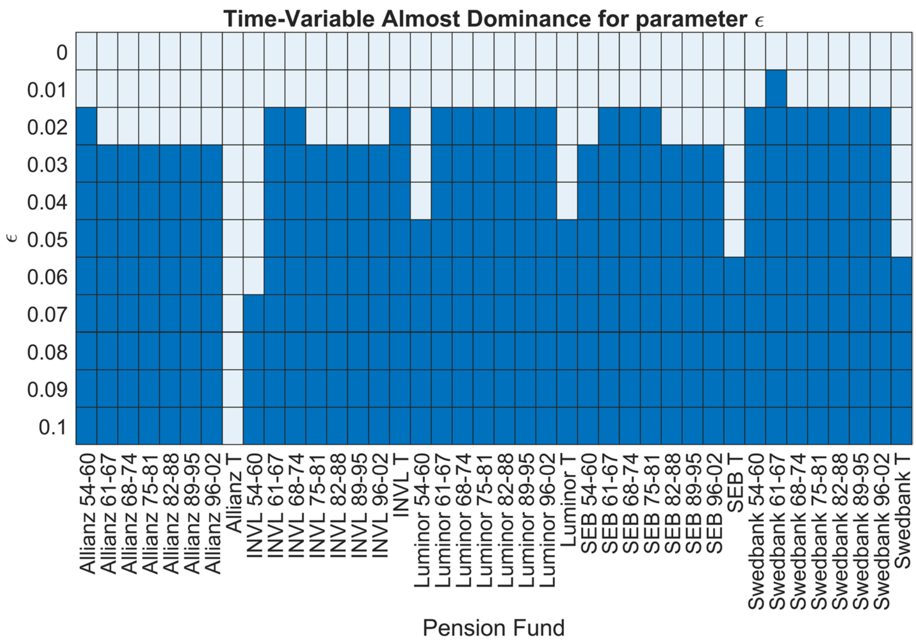


Fig. 8 Behavior of ϵ while computing time-cumulative almost dominance between pension funds and inflation π considering the whole period

[8] For this reason, the table with results is not attached in the Appendix.

Of more interest are the results for individual regimes, where general patterns can be identified. Starting with Regime 1 in Fig. 11, in most cases preservation-type funds are dominated by other more aggressive funds, except Allianz funds. Furthermore, in general, Allianz funds are dominated by other companies' funds, which indicates the poorest performance. For other institutions, there are sporadic cases where aggressive funds for younger participants dominate conservative types of funds.

Moving to Regime 2, the situation is the reverse of Regime 1. Here, conservative funds dominate the dynamic ones, as their return volatility is not as high and during stressful periods they do not lose as much for all their investors. These findings are in the scope of the financial theory. The only funds (even dynamic ones) of the pension company that are not dominated by some exceptions are from Swedbank, which confirms their solid performance throughout the selected period.

Finally, we check again the properties using the less strict dominance proposed by Definition 2. The detailed results obtained for the pairwise analysis using data from the whole period are shown in Fig. 13. To do so, we can observe the possibility of detecting TCAD for the parameter $\epsilon \in (0, 0.3)$. In other words, we admit a violation of the condition for at most 30% of observations. Note that 0 means that the fund does not dominate another fund even with this relaxing condition. It is apparent from the results that aggressive funds dominate conservative ones, even for small values of ϵ.

Again, we can generally observe that, allowing for a slight relaxation of the strict dominance condition by the parameter $\epsilon \in (0.01, 0.05)$, the aggressive funds time-cumulatively almost dominate conservative funds. Focusing on the individual institutions, in most cases, Allianz and INVL funds' portfolios are almost dominated by portfolios of Luminor, SEB and Swedbank. Hence, the resulting outcomes imply that investors should prefer funds provided by these institutions.

We would like to emphasize that we have performed this analysis for each sub-period and regime, with similar conclusions as discussed above. However, we decided to show only selected results in order to reduce the space.

4 Discussion

Here, it is appropriate to study the results obtained for the proposed TCD in relation to stochastic dominance. Generally, since TCD considers using the cumulative principle, second-order stochastic dominance (SSD) can be regarded as a corresponding type of stochastic dominance. The rationale is that the SSD is also based on the cumulative principle (Dupačová & Kopa, 2012). The study of different orders of stochastic dominance, including SSD between pension funds, has been published by, for example, Kopa et al. (2021). To conduct a comparison, we performed the SSD test on our data and reported the results in Fig. 10. The results indicate that the temporal ordering in our proposed approach imposes a stricter dominance criterion compared to the SSD. In particular, it is more probable that the pension funds of one company will SSD dominate similar funds from another company for a given group of investors. Over the entire data sample, joint dominance (TCD and SSD) is rather rare, as it is observed in less than a dozen cases.

In the previous empirical analysis, we analyze the general (usually used) risk measures with the proposed trend-dependent ones. Due to the targeting of funds on different groups of investors, the funds' portfolios should adjust to investors' preferences through the composition of the portfolio. However, some fund managers may address other criteria, such as ESG, and index tracking, which have varying effects on the performance and risk of the constructed portfolio. This could also be the reason for the higher values of trend-risk measures related to middle-aged investors for particular institutions.

The study opens several avenues for further research. The use of our approach can also be beneficial for optimization purposes. For example, the condition of outperforming inflation or the price changes index could be interesting to incorporate into even classical portfolio selection frameworks. In addition, the proposed measures can also be applied as a criterion for the preselection of assets. Similarly, since we also propose new types of dominance, we could set a model condition following this criterion, such as the incorporation of classical stochastic dominance presented, for example, Dupačová and Kopa (2014).

All of the notes above can serve as a space for future research focused mainly on the application in a portfolio and risk management issues. Future studies could also expand the analysis to include a wider range of countries and economic environments, providing a more global perspective on pension fund performance relative to inflation.

Finally, we outline potential applications for regulators and policymakers. To strengthen the long-term sustainability of pension systems, policymakers should consider integrating the proposed risk measures TIR and TPR into supervisory benchmarking frameworks. These tools offer a more accurate assessment of whether pension funds are preserving real value for investors over time. Furthermore, the deviation metrics introduced in this study can serve as early warning indicators, helping to identify funds that consistently underperform relative to inflation or consumer price trends. We also recommend the introduction of standardized reporting requirements for these trend-risk metrics. Such measures would enhance transparency, support more consistent oversight, and enable meaningful comparisons across fund managers. Collectively, these steps could contribute to more resilient, inflation-protected pension systems that better serve the public interest.

5 Conclusion

The evaluation of pension fund portfolios with respect to the change in price level is of paramount importance for clients. Inflation and consumer price changes erode the purchasing power of money, making it crucial for pension funds to not only generate returns but also to outperform these indicators to ensure the financial security of retirees. The necessity to protect the value of these funds against inflationary pressures cannot be overstated, as it directly impacts the sustainability of retirement incomes and the broader economic stability.

In this study, we explore the critical importance of evaluating pension fund portfolios in the context of inflation and consumer price changes. Our proposed methodology, based on trend-inflation and trend-price changes analysis, offered a novel methodology for assessing the performance and resilience of these portfolios. By addressing both the theoretical and practical aspects of inflation's impact on portfolio valuation, we provided a comprehensive framework that can enhance the decision-making processes of individual investors, policymakers, and market regulators. Generally, the integration of such a trend-risk analysis into pension fund evaluations represents a significant advancement over traditional (static) evaluation methods, which often fail to adequately account for the long-term effects of inflation.

In addition, we studied new types of dominance based on the deviations of the cumulative returns of funds to a given trend. Thus, we are able to check whether a particular portfolio of funds dominates the trend or whether the funds' portfolios dominate each other.

To check the efficiency of pension funds, we used data from 40 IInd-pillar life-cycle pension funds (5 pension companies) operating in the Lithuanian market. Concerning the empirical analysis, we segmented the dataset according to prevailing market conditions to capture a comprehensive picture of the performance of the pension funds. Initially, we conducted a broad examination using the entire dataset to identify general trends and patterns. We then focused on the evaluation of pension fund performance across different market regimes detected by the framework using Hidden Markov regime-switching models. Finally, we evaluated portfolios of funds during the COVID-19 crisis period to understand how pension funds behaved in exceptional circumstances. We performed this to ensure the robustness of our methodology in different economic environments.

Our results clearly demonstrate that the most risky funds according to our methodology are those for investors mainly born in the '80 s and '90 s. In contrast, the less risky funds are the conservative ones targeting to early-born investors. Furthermore, we observed that aggressive funds designed for young investors consistently outperformed inflation, while more conservative ones did not. The results obtained showed the variability and risk inherent in pension fund investments during the turbulent market period of COVID-19. It was also apparent that the funds were overwhelmingly unable to dominate the trend, but we could observe almost dominance for relevant time intervals and regimes.

Appendix A

See Table 3 and Figs. 9, 10, 11, 12 and 13.

Table 3 Selected trend-risk measures for different market regimes

	Regime 1				Regime 2				Regime 4			
	TIR	TPR	downTIR	downTPR	TIR	TPR	downTIR	downTPR	TIR	TPR	downTIR	downTPR
Allianz 54-60	0,0117	0,0067	0,0000	0,0000	0,0003	0,0005	0,0003	0,0005	0,0044	0,0127	0,0046	0,0132
Allianz 61-67	0,0350	0,0257	0,0000	0,0000	0,0024	0,0028	0,0024	0,0028	0,0048	0,0130	0,0050	0,0135
Allianz 68-74	0,1277	0,1092	0,0000	0,0000	0,0119	0,0129	0,0119	0,0129	0,0067	0,0149	0,0069	0,0155
Allianz 75-81	0,1688	0,1473	0,0000	0,0000	0,0149	0,0160	0,0149	0,0160	0,0060	0,0134	0,0063	0,0139
Allianz 82-88	0,1720	0,1503	0,0000	0,0000	0,0148	0,0159	0,0148	0,0159	0,0056	0,0126	0,0059	0,0131
Allianz 89-95	0,1716	0,1499	0,0000	0,0000	0,0148	0,0159	0,0148	0,0159	0,0059	0,0132	0,0063	0,0137
Allianz 96-02	0,1832	0,1607	0,0000	0,0000	0,0144	0,0155	0,0144	0,0155	0,0056	0,0125	0,0059	0,0130
Allianz T	0,0044	0,0016	0,0000	0,0000	0,0001	0,0000	0,0001	0,0001	0,0049	0,0136	0,0052	0,0142
INVL 54-60	0,0115	0,0065	NaN	NaN	0,0001	0,0002	0,0002	0,0002	0,0090	0,0197	0,0093	0,0205
INVL 61-67	0,0721	0,0583	0,0000	0,0000	0,0056	0,0062	0,0056	0,0062	0,0093	0,0197	0,0097	0,0205
INVL 68-74	0,1774	0,1553	NaN	NaN	0,0193	0,0205	0,0193	0,0205	0,0059	0,0140	0,0062	0,0145
INVL 75-81	0,2225	0,1978	0,0000	0,0000	0,0256	0,0271	0,0256	0,0271	0,0050	0,0119	0,0051	0,0123
INVL 82-88	0,2176	0,1931	0,0000	0,0000	0,0258	0,0272	0,0258	0,0272	0,0050	0,0119	0,0051	0,0123
INVL 89-95	0,2217	0,1970	0,0000	0,0000	0,0262	0,0277	0,0262	0,0277	0,0050	0,0119	0,0052	0,0123
INVL 96-02	0,2225	0,1978	0,0000	0,0000	0,0267	0,0282	0,0267	0,0282	0,0048	0,0118	0,0050	0,0121
INVL T	0,0125	0,0073	0,0000	0,0000	0,0004	0,0005	0,0004	0,0005	0,0042	0,0123	0,0043	0,0128
Luminor 54-60	0,0202	0,0133	0,0000	0,0000	0,0007	0,0009	0,0007	0,0009	0,0104	0,0216	0,0108	0,0225
Luminor 61-67	0,0956	0,0796	0,0000	0,0000	0,0051	0,0057	0,0051	0,0057	0,0092	0,0193	0,0095	0,0201
Luminor 68-74	0,2260	0,2011	0,0000	0,0000	0,0139	0,0150	0,0139	0,0150	0,0088	0,0181	0,0091	0,0188
Luminor 75-81	0,2244	0,1995	0,0000	0,0000	0,0141	0,0151	0,0141	0,0151	0,0082	0,0171	0,0085	0,0178
Luminor 82-88	0,2270	0,2020	0,0000	0,0000	0,0143	0,0154	0,0143	0,0154	0,0082	0,0172	0,0085	0,0178
Luminor 89-95	0,2334	0,2080	0,0000	0,0000	0,0140	0,0151	0,0140	0,0151	0,0085	0,0176	0,0088	0,0183
Luminor 96-02	0,2345	0,2091	0,0000	0,0000	0,0139	0,0149	0,0139	0,0149	0,0082	0,0172	0,0085	0,0179
Luminor T	0,0079	0,0039	0,0000	0,0000	0,0002	0,0003	0,0002	0,0003	0,0055	0,0143	0,0057	0,0147
SEB 54-60	0,0187	0,0121	NaN	NaN	0,0007	0,0009	0,0007	0,0009	0,0070	0,0167	0,0072	0,0174
SEB 61-67	0,0860	0,0709	0,0000	0,0000	0,0079	0,0087	0,0079	0,0087	0,0065	0,0154	0,0068	0,0160
SEB 68-74	0,2173	0,1928	0,0000	0,0000	0,0231	0,0245	0,0231	0,0245	0,0049	0,0120	0,0052	0,0125

Table 3 (continued)

	Regime 1				Regime 2				Regime 4			
SEB 75-81	0,2669	0,2396	NaN	NaN	0,0244	0,0259	0,0247	0,0261	0,0050	0,0118	0,0056	0,0124
SEB 82-88	0,2543	0,2277	0,0000	0,0000	0,0273	0,0288	0,0273	0,0288	0,0051	0,0119	0,0056	0,0125
SEB 89-95	0,2542	0,2276	0,0000	0,0000	0,0275	0,0290	0,0275	0,0290	0,0053	0,0122	0,0057	0,0128
SEB 96-02	0,2643	0,2372	0,0000	0,0000	0,0288	0,0304	0,0288	0,0304	0,0054	0,0125	0,0058	0,0130
SEB T	0,0100	0,0054	0,0000	0,0000	0,0002	0,0004	0,0002	0,0004	0,0055	0,0144	0,0057	0,0150
Swedbank 54-60	0,0150	0,0092	NaN	NaN	0,0009	0,0012	0,0009	0,0012	0,0052	0,0138	0,0054	0,0144
Swedbank 61-67	0,0832	0,0683	NaN	NaN	0,0076	0,0084	0,0078	0,0086	0,0047	0,0126	0,0049	0,0130
Swedbank 68-74	0,2047	0,1808	NaN	NaN	0,0222	0,0236	0,0226	0,0240	0,0051	0,0122	0,0052	0,0126
Swedbank 75-81	0,2174	0,1928	NaN	NaN	0,0228	0,0242	0,0232	0,0246	0,0048	0,0115	0,0050	0,0119
Swedbank 82-88	0,2168	0,1922	NaN	NaN	0,0228	0,0241	0,0232	0,0246	0,0047	0,0114	0,0050	0,0118
Swedbank 89-95	0,2111	0,1868	NaN	NaN	0,0228	0,0242	0,0232	0,0246	0,0049	0,0117	0,0051	0,0121
Swedbank 96-02	0,1946	0,1714	NaN	NaN	0,0235	0,0249	0,0240	0,0254	0,0054	0,0125	0,0056	0,0130
Swedbank T	0,0094	0,0050	NaN	NaN	0,0003	0,0005	0,0003	0,0005	0,0061	0,0153	0,0063	0,0159

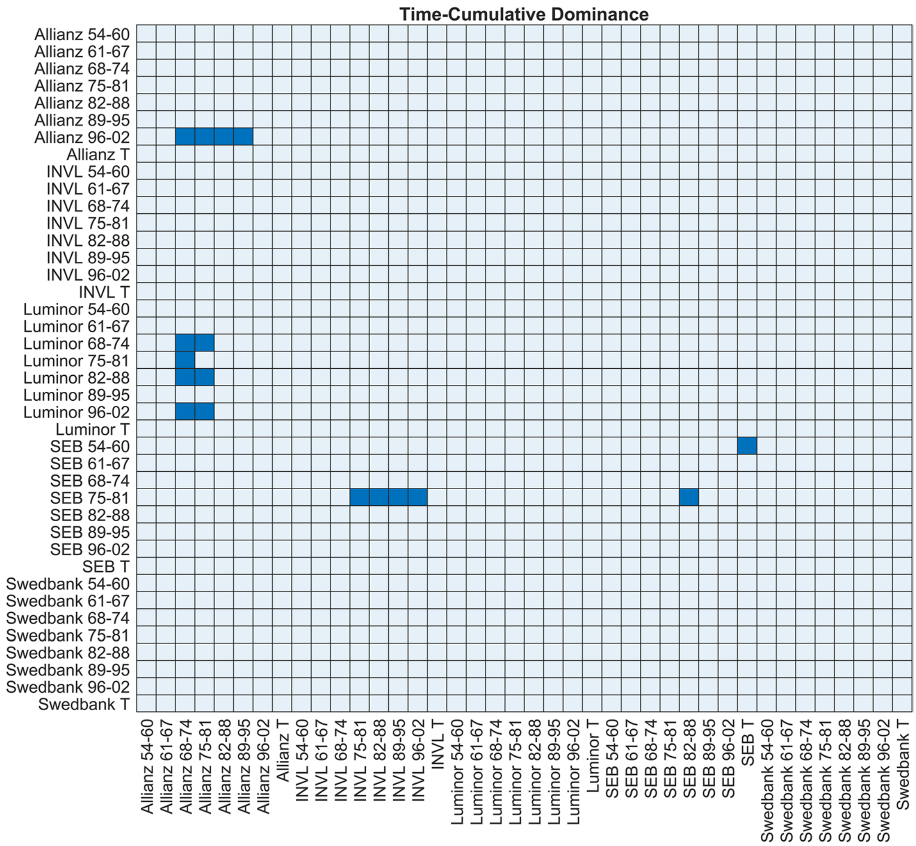


Fig. 9 Results of time-cumulative dominance between funds for whole period. Note: A dark blue box indicates compliance with the time-cumulative dominance condition

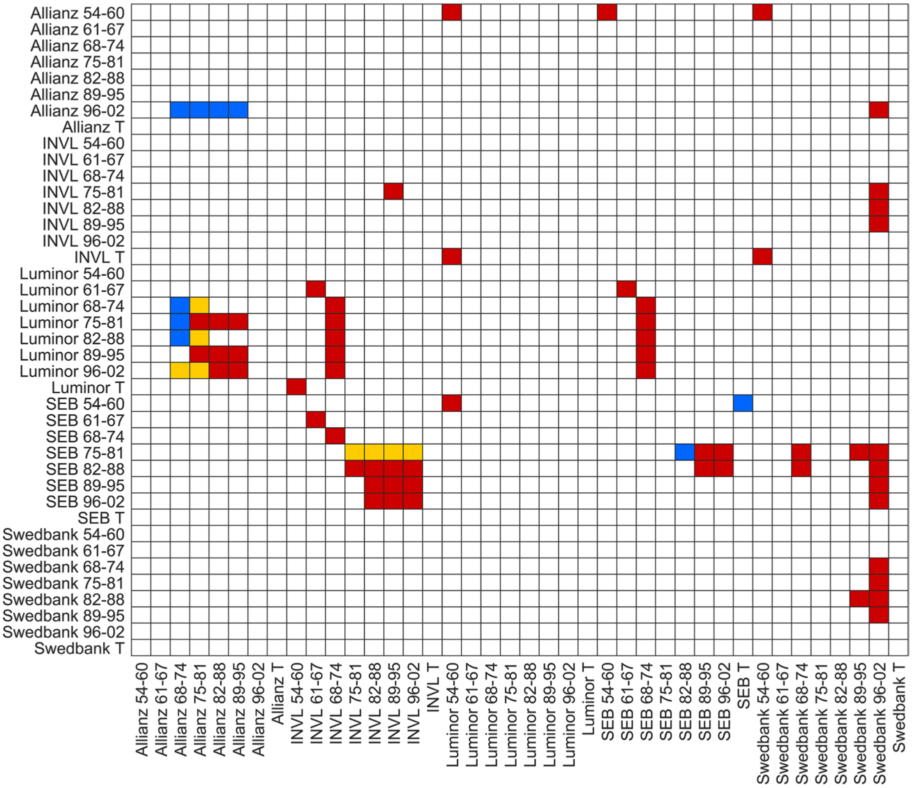


Fig. 10 Overlap of time-cumulative dominance and second order stochastic dominance using data of the whole period. Note: The colour of the box means the following dominance between funds: white—no dominance type observed; blue—only TCD observed; red—only SSD observed; orange—both types of dominance observed

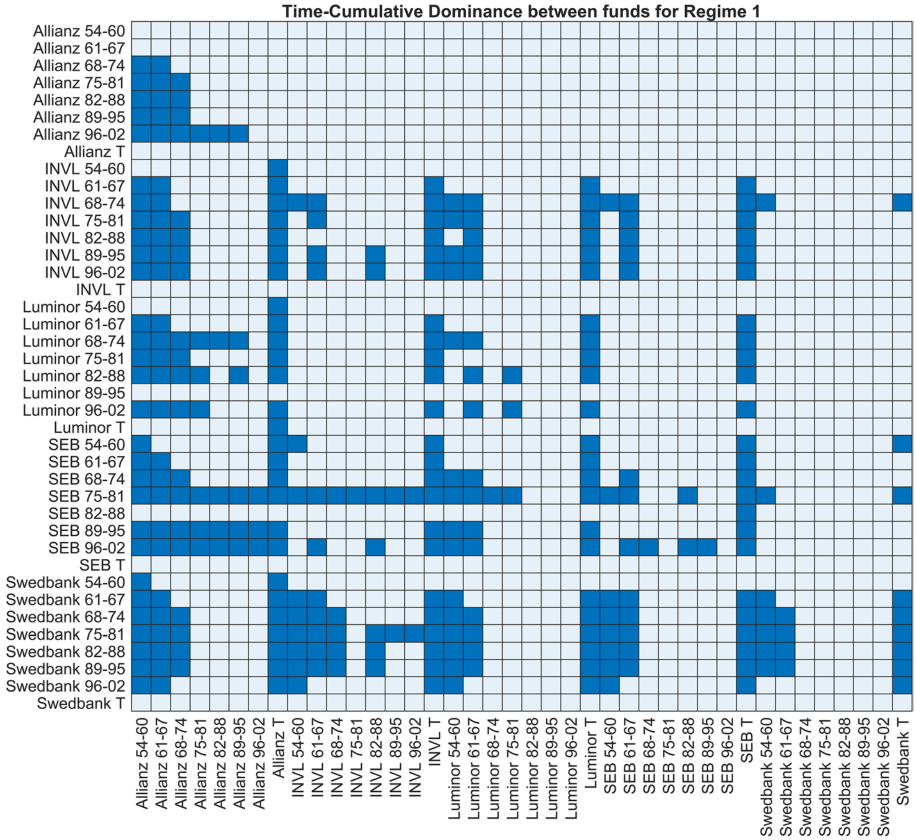


Fig. 11 Results of time-cumulative dominance between funds for Regime 1 period

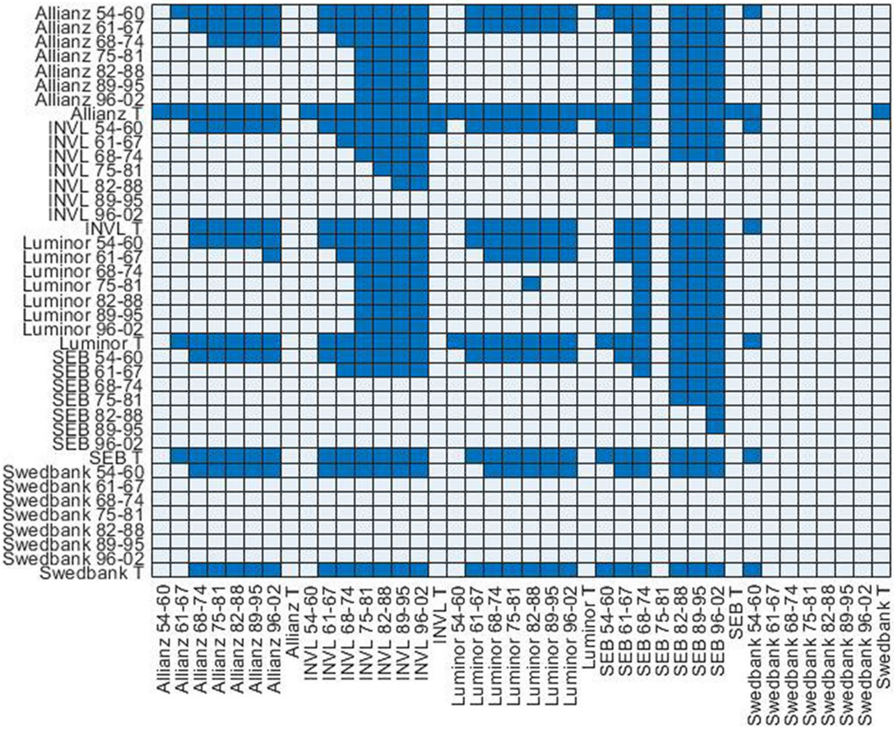


Fig. 12 Results of time-cumulative dominance between funds for Regime 2 period. Note: A dark blue box indicates compliance with the time-cumulative dominance condition

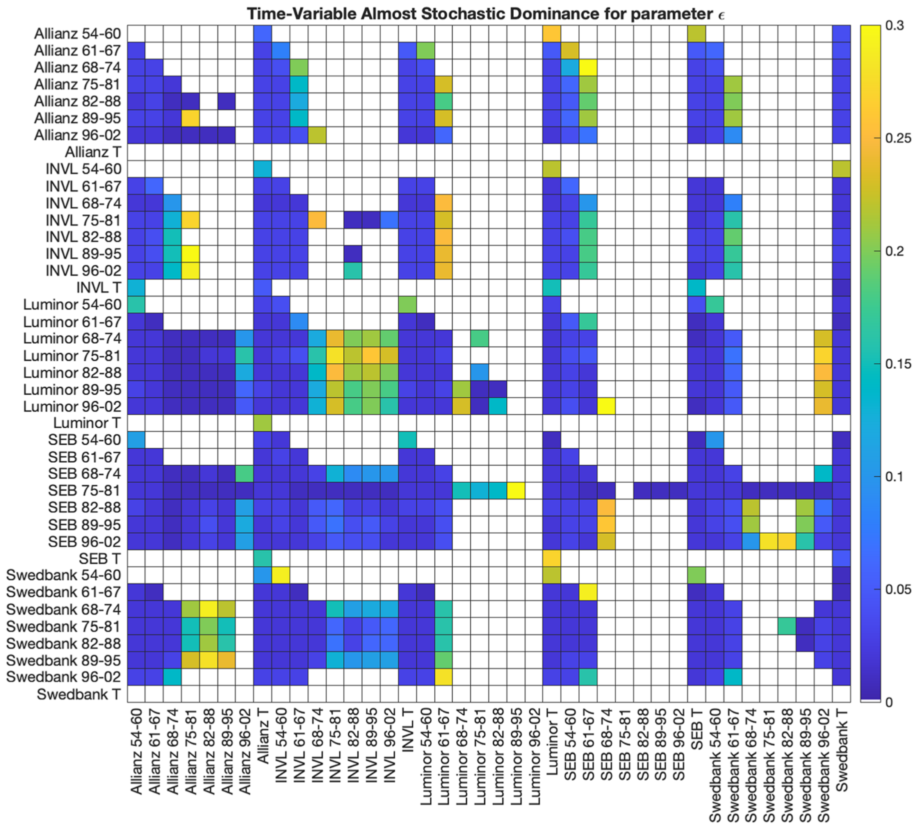


Fig. 13 Results of time-cumulative almost dominance between funds for the whole period. Note: values closer to yellow ($\epsilon \to 0.3$) suggest weaker dominance (up to 30% allowable violation of the dominance condition), while values closer to dark blue ($\epsilon \to 0$) indicates stronger dominance even under this relaxed criterion. White color indicates a failure to meet even the Time-Cumulative Almost Dominance

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Author contributions The authors affirm that this work is an original work that is not published or under consideration elsewhere and is a genuine collaboration.

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Data availibility Data will be made available on request.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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